HORNSBY GIRLS HIGH SCHOOL

Mathematics

Year 12 Higher School Certificate
Trial Examination Term 3 2013

STUDENT NUMBER: ____________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks – 100

Section I Pages 3 – 5

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 6 – 15

90 marks

Attempt Questions 11 – 16. Start each question in a new writing booklet. Write your student number on every writing booklet

Question	*1-10*	*11*	*12*	*13*	*14*	*15*	*16*	*Total*
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/15*	*/15*	*/100*

This assessment task constitutes 45% of the Higher School Certificate Course School Assessment

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 What is 25.09582 correct to 4 significant figures?

(A) 25.09

(B) 25.10

(C) 25.095

(D) 25.096

2 The solutions of $8x - x^2 < 0$ are:

(A) $0 < x < 8$

(B) $x < 0, x < 8$

(C) $x < 0, x > 8$

(D) $x > 0, x < 8$

3 Factorise completely $a^3b - ab^3 - 4a^2 + 4b^2$

(A) $ab\left(a^2 - b^2\right) - 4\left(a^2 + b^2\right)$

(B) $ab(a^2 - b^2) - 4\left(a^2 - b^2\right)$

(C) $(ab - 4)\left(a^2 - b^2\right)$

(D) $(a - b)(ab - 4)(a + b)$

4 If α and β are the roots of the equation $2x^2 + 5x - 4 = 0$, then $\alpha^2 + \beta^2 =$

(A) $6\frac{1}{4}$

(B) $10\frac{1}{4}$

(C) $2\frac{1}{4}$

(D) 4

5 The period of the function $y = 3\tan\left(\frac{x}{3}\right)$ is:

(A) $\frac{\pi}{3}$

(B) π

(C) 3π

(D) 6π

6 What is the equation of the tangent to the curve $y = x^2 - 5x$ at the point $(1, -4)$?

(A) $y = -3x - 1$

(B) $y = -3x - 7$

(C) $y = 3x + 7$

(D) $y = 3x - 7$

7 Solve $\sin 2\theta = \frac{-\sqrt{3}}{2}$ in the domain $0° \le \theta \le 180°$:

(A) $\theta = 60°$

(B) No solutions

(C) $\theta = 120°$

(D) $\theta = 120°, 150°$

8 What is the solution to the equation $\log_e(x+2) - \log_e x = \log_e 4$?

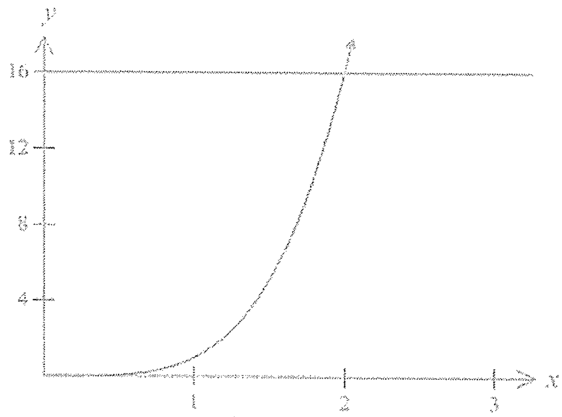
(A) $\frac{2}{5}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

9 A region in the diagram is bounded by the curve $y = x^4$, the y - axis and the line $y = 16$.



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Which of the following expressions is correct for the volume of the solid of revolution when this region is rotated about the y -axis?

(A) $V = \pi \int_0^{16} y^{\frac{1}{2}}\, dy$

(B) $V = \pi \int_0^{16} x^8 dx$

(C) $V = \pi \int_0^{2} y^{\frac{1}{2}} dy$

(D) $V = \pi \int_0^{2} x^8 dx$

10 The value of $\sum_{n=2}^{5} 2n^2$ is:

(A) 50

(B) 108

(C) $205\frac{1}{32}$

(D) $362\frac{21}{32}$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)   Start a new writing booklet

(a) Evaluate $\sqrt[3]{\left(\frac{a}{b}\right)^b}$, correct to three decimal places, where $a = 3$ and $b = 2$. **2**

(b) Solve the equation $4x^2 = x$. **2**

(c) Solve the equation $|4 - x| = 2x$. **2**

(d) Express $\frac{1}{4-\sqrt{13}}$ in the form $a + b\sqrt{13}$, where a and b are rational numbers. **2**

(e) Find $\int e^{3x}dx$. **1**

(f) Sketch the parabola $x^2 = -4y + 8$, showing the vertex, focus and directrix. **3**

(g) Differentiate x^3 from first principles. **3**

Question 12 (15 marks) Start a new writing booklet

(a) Differentiate the following, with respect to x :

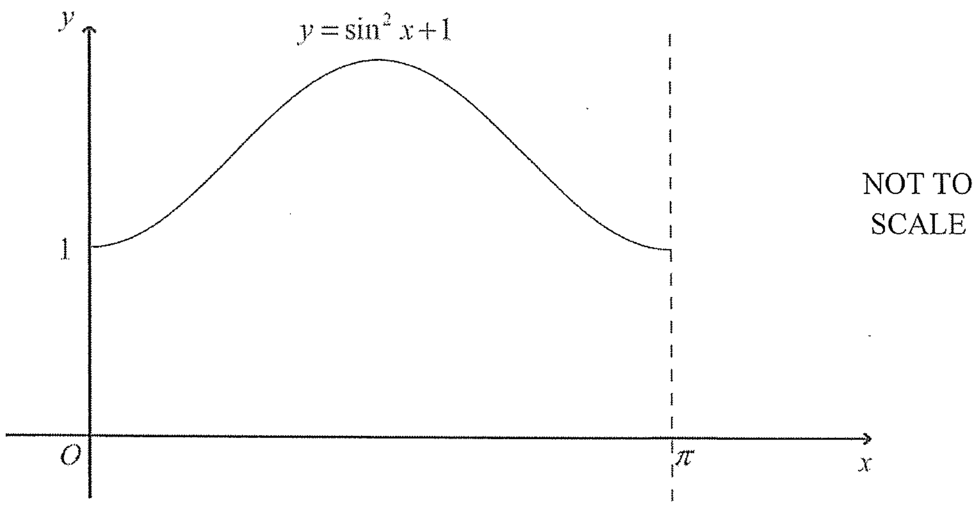
(i) $e^x \sin x$. **2**

(ii) $\dfrac{\log_e x}{x^3}$. **2**

(b) Find the exact value of $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos x}{\sin x + 3}\,dx$. **3**

(c) A stained-glass window can be modelled as the region bounded by the curve $y = \sin^2 x + 1$, the coordinate axes and $x = \pi$.

The graph of $y = \sin^2 x + 1$ for $0 \le x \le \pi$ is shown below.



(i) Copy and complete the table with exact values in your writing booklet. **1**

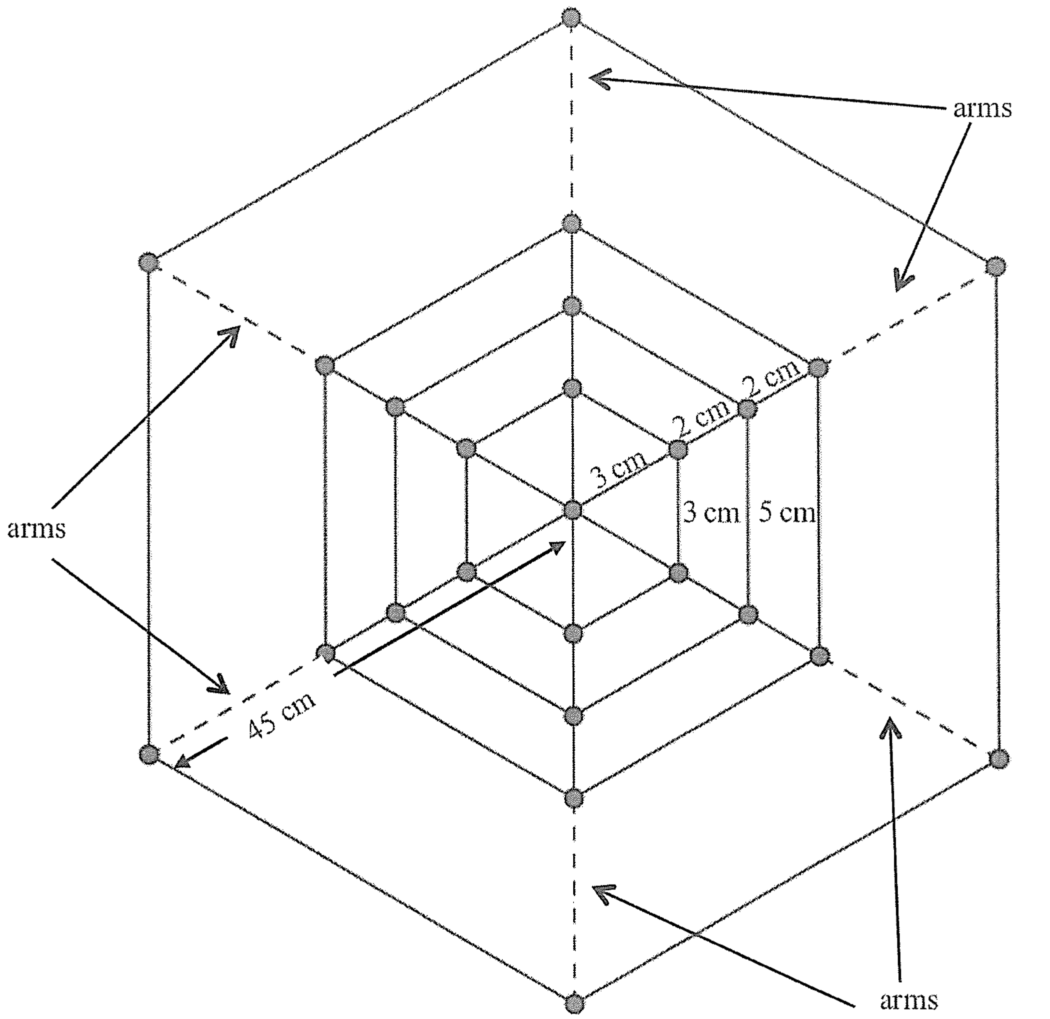
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1				1

(ii) Use Simpson's Rule with 5 function values to calculate the approximate area of glass needed for the window. Answer correct to 2 decimal places. **2**

Question 12 continues on page 8

Question 12 (continued)

(d) Incey Wincey spider makes a web in the shape of concentric regular hexagons. First he makes the 6 arms which are each 45 cm long. He then makes the sides of each regular hexagon. The first is 3 cm from the centre along each arm. Each successive regular hexagon is 2 cm further along the arm. The last hexagon is at the end of the arms.



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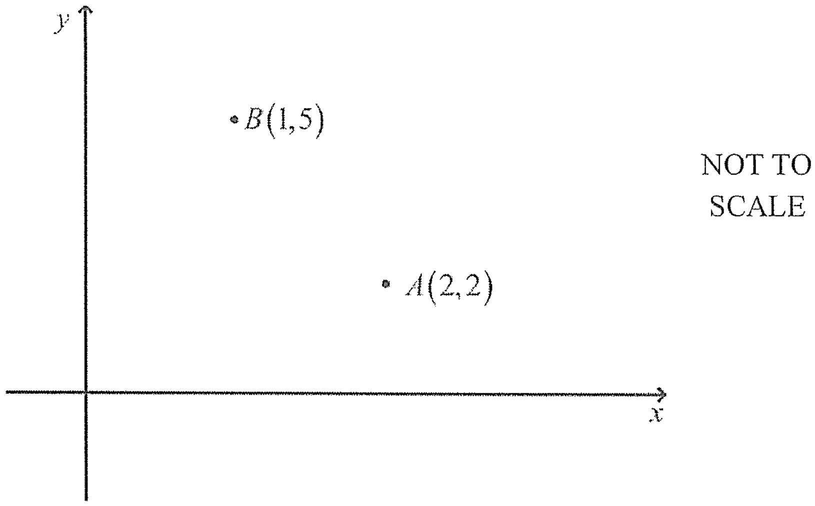
(i) How many regular hexagons does Incey Wincey create? **2**

(ii) Find the total length of the web, including the arms, created by Incey Wincey. **3**

End of Question 12

Question 13 (15 marks) Start a new writing booklet

(a) The diagram below shows two points $A(2,2)$ and $B(1,5)$ on the number plane.



Copy the diagram into your writing book.

(i) Find the coordinates of *M*, the midpoint of *AB*. **1**

(ii) Show that the equation of the perpendicular bisector of *AB* is $x-3y+9=0$. **2**

(iii) Find the coordinates of the point *C* that lies on the $y-$axis and is equidistant from *A* and *B*. **1**

(iv) The point *D* lies on the intersection of the line $y=5$ and the perpendicular bisector $x-3y+9=0$. Find the coordinates of *D*, and mark the position of *D* on your diagram in your writing booklet. **1**

(v) Find the area of triangle *ABD*. **2**

Question 13 continues on page 10

(b) (i) Find the points of intersection of the parabola $y = x^2 + 3x - 5$ and the line $y = 2x + 1$. **2**

(ii) Hence find the area enclosed between the parabola $y = x^2 + 3x - 5$ and the line $y = 2x + 1$. **2**

(c) A school softball team has a probability of 0.2 of winning any match.

(i) Find the probability the team wins exactly one of its first two matches. **2**

(ii) What is the least number of consecutive matches the team must play to be 90% certain that it will win **at least one** match? **2**

End of Question 13

Question 14 (15 marks) Start a new writing booklet

(a) Consider the curve $y = x^3 + 4x^2 - 3x + 2$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **2**

(ii) Find the coordinates of the stationary points and determine their nature. **2**

(iii) Find the coordinates of any points of inflexion. **2**

(iv) Sketch the graph of $y = x^3 + 4x^2 - 3x + 2$, clearly showing any stationary points, points of inflexion and the y-intercept. **2**

(v) Hence find the number of solutions to the equation $x^3 + 4x^2 - 3x + 2 = -1$. **1**

(b) The mass M kg of a radioactive substance present after t years is given by $M = 20e^{-kt}$, where k is a positive constant. After 200 years, the mass has reduced to 10kg.

(i) What is the initial mass? **1**

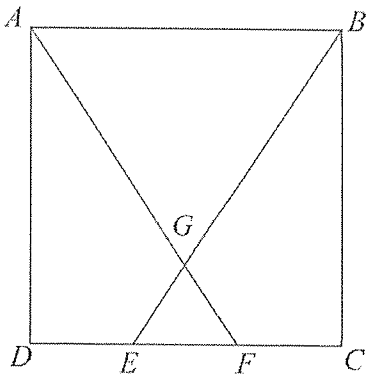
(ii) Find the exact value of k. **2**

(iii) What amount of radioactive substance would remain after a period of 2000 years? **1**

(c) For what values of k does the equation $x^2 - 2x + 3 = k$ have real roots? **2**

Question 15 (15 marks) Start a new writing booklet

(a) $ABCD$ is a square. Points E and F lie on DC such that $DE = CF$.



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Copy or trace the diagram into your writing booklet

(i) Prove that $\Delta AFD \equiv \Delta BEC$. **3**

(ii) Hence or otherwise, prove that $GE = GF$. **1**

(b) The velocity of a particle is given by $\frac{dx}{dt} = t^2 - 5t + 6$, $t \geq 0$, where x is displacement in metres and t is time in seconds. Initially the particle is 3 metres to the left of the origin.

(i) Find when the velocity of the particle is zero. **1**

(ii) Find the minimum velocity of the particle. **1**

(iii) Find the displacement x of the particle in terms of t. **2**

(iv) Find the distance travelled by the particle in the first 3 seconds. **2**

Question 15 continues on page 13

(c) Rita takes out a loan of \$30 000 to buy a new car. The loan is to be repaid over 5 years (60 months) in equal monthly repayments (Q) at the end of each month.

Reducible interest is charged at 9% per annum, calculated monthly.

Let $\$A_n$ be the amount owing after the nth payment.

(i) Write an expression for the amount owing after 1 month, $\$A_1$. **1**

(ii) Show that $\$\,A_n = 30000(1.0075)^n - \dfrac{400Q\left(1.0075^n - 1\right)}{3}$. **2**

(iii) What will the monthly repayment be to the nearest dollar? **2**

End of Question 15

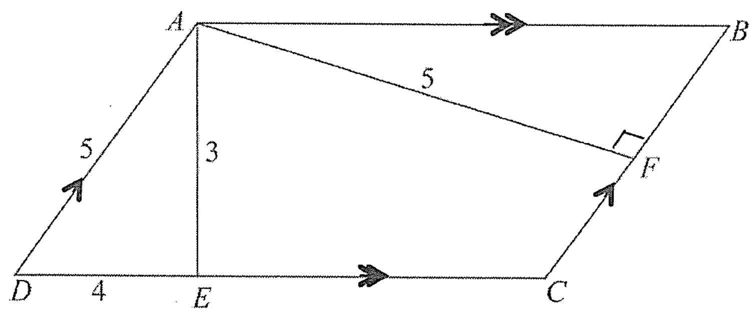
Question 16 (15 marks) Start a new writing booklet

(a) If $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$, find the values of A, B and C. **2**

(b) (i) Given that $a^2 + b^2 = 7ab$, show that $\left(\frac{a+b}{3}\right)^2 = ab$. **1**

(ii) Hence, write $\log\left(\frac{a+b}{3}\right) - \frac{1}{4}(\log a + \log b)$ in simplest form. **2**

(c) In the diagram below, ABCD is a parallelogram. $AD = 5$, $AE = 3$, $DE = 4$, $AF = 5$ and $AF \perp BC$.



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Copy or trace the diagram into to your writing booklet

(i) Prove that $\angle ADE = \angle EAF$. **2**

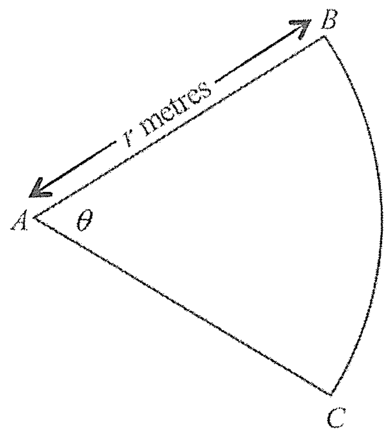
(ii) Find the exact length of EF. **2**

Question 16 continues on page 15

(d) In the figure below, AB and AC are radii of length r metres of a circle with centre A.

The arc BC of the circle subtends an angle of θ at A.

The perimeter of the figure ABC is 12 metres.



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(i) Show that the area Y square metres of the sector ABC is given by $Y = \dfrac{72\theta}{(\theta+2)^2}$. **2**

(ii) Hence, show that the maximum area of the sector is 9 square metres. **4**

End of Paper

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Hornsby Girls High School Mathematics Trial Examination 2013 Solutions and Marking Criteria

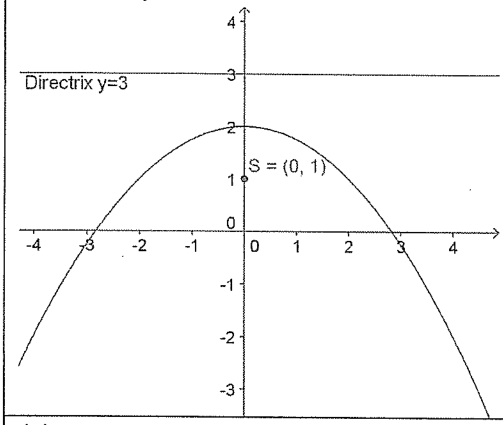
Objective Response Questions 1 – 10

Solutions	**Marking Criteria**
Question 1 $25.0983 \approx 25.10\ (4sf)$ **Option B**	*Students should not be getting this wrong at this point! You must speak to your teacher if you need help with rounding.*
Question 2 $8x - x^2 < 0$ $x(8-x) < 0$ $\therefore x < 0, x > 8$ **Option C**	
Question 3 $a^3b - ab^3 - 4a^2 + 4b^2 = ab(a^2 - b^2) - 4(a^2 - b^2)$ $= (a^2 - b^2)(ab - 4)$ $= (a-b)(a+b)(ab-4)$ **Option D**	*A lot of students failed to recognise the difference of two squares.*
Question 4 $2x^2 + 5x - 4 = 0$ $\alpha + \beta = \frac{-5}{2} \quad \alpha\beta = -2$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= \frac{25}{4} + 4$ $= \frac{41}{4}$ $= 10\frac{1}{4}$ **Option B**	
Question 5 $T = \frac{\pi}{n}$ $= \pi \div \frac{1}{3}$ $= 3\pi$ **Option C**	*Many students just assumed the period was $\frac{2\pi}{n}$ for trig functions.*
Question 6 $y = x^2 - 5x$ $y' = 2x - 5$ When $x = 1$, $y' = -3$ Equation of tangent: $y + 4 = -3(x-1)$ $y = -3x + 3 - 4$ $y = -3x - 1$ **Option A**	

Question 7 $\sin 2\theta = \frac{-\sqrt{3}}{2}$ $0° \le \theta \le 180°$ $0° \le 2\theta \le 360°$ ← $2\theta = 240°, 300°$ $\theta = 120°, 150°$ **Option D**	*Students should adjust the domain to show restrictions on* 2θ.
Question 8 $\ln(x+2) - \ln x = \ln 4$ $\ln\left(\frac{x+2}{x}\right) = \ln 4$ $\frac{x+2}{x} = 4$ $x+2 = 4x$ $2 = 3x$ $x = \frac{2}{3}$ **Option B**	
Question 9 $V = \pi\int_a^b x^2 dy$ $= \pi\int_0^{16} y^{\frac{1}{2}} dy$ **Option A**	
Question 10 $2(2^2 + 3^2 + 4^2 + 5^2) = 108$ **Option B**	

Question 11

Solutions	Marking Criteria
(a) $\sqrt[3]{\left(\frac{3}{2}\right)^2} = 1.31037...$ $= 1.310\ (3dp)$	
(b) $4x^2 = x$ $4x^2 - x = 0$ $x(4x-1) = 0$ $x = 0,\ x = \frac{1}{4}$	
(c) $\lvert 4-x \rvert = 2x$ Case 1 $4 - x = 2x$ $4 = 3x$ $x = \frac{4}{3}$ Case 2: $4 - x = -2x$ $x = -4$ Testing Solutions When: $x = \frac{4}{3}$ $LHS = \left\lvert 4 - \frac{4}{3} \right\rvert$ $= \left\lvert \frac{8}{3} \right\rvert$ $= \frac{8}{3}$ $RHS = 2 \times \frac{4}{3}$ $= \frac{8}{3}$ $= LHS$ $x = -4$ $LHS = \lvert 4 - -4 \rvert$ $= 8$ $RHS = 2 \times -4$ $= -8$ $\neq LHS$ The only solution is $x = \frac{4}{3}$	*Too many students are not testing solutions.*
(d) $\frac{1}{4-\sqrt{13}} \times \frac{4+\sqrt{13}}{4+\sqrt{13}} = \frac{4+\sqrt{13}}{3}$ $= \frac{4}{3} + \frac{\sqrt{13}}{3}$ $a = \frac{4}{3},\ b = \frac{1}{3}$	

(e) $\int e^{3x}dx = \frac{1}{3}\int 3e^{3x}dx$ $= \frac{1}{3}e^{3x} + C$	
(f) $x^2 = -4y + 8$ $x^2 = -4(y-2)$ Focus: $S = (0,1)$ Directrix $y = 3$ 	
(g) Change solutions $f(x) = x^3$ $f(x+h) = (x+h)^3$ $= x^3 + 3hx^2 + 3h^2x + h^3$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\frac{d}{dx}(x^3) = \lim_{h \to 0} \frac{(x^3 + 3hx^2 + 3h^2x + h^3) - x^3}{h}$ $= \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3}{h}$ $= \lim_{h \to 0}(3x^2 + 3hx + h^2)$ $= 3x^2$	*Too many students do not know definition of "from first principles" and use incorrect notation.*

Question 12

<table>
<tr><th>Solutions</th><th>Marking Criteria</th></tr>
<tr><td>

(a)

(i)

$$\frac{d}{dx}\left(e^x \sin x\right) = e^x \cos x + e^x \sin x$$

$$= e^x(\sin x + \cos x)$$

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① Correct product rule method
① Correct answer

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(ii)

$$\frac{d}{dx}\left(\frac{\log_e x}{x^3}\right) = \frac{x^3 \times \frac{1}{x} - 3x^2 \log_e x}{x^6}$$

$$= \frac{x^2 - 3x^2 \log_e x}{x^6}$$

$$= \frac{1 - 3\log_e x}{x^4}$$

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① Correct rule
① Correct answer

Some students had numerator around the wrong way.

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(b)

$$\int_0^{\frac{\pi}{4}} \frac{\cos x}{\sin x + 3}\,dx = \left[\ln(\sin x + 3)\right]_0^{\frac{\pi}{4}}$$

$$= \ln\left(\frac{1}{\sqrt{2}} + 3\right) - \ln 3$$

$$= \ln\frac{1 + 3\sqrt{2}}{\sqrt{2}} - \ln 3$$

$$= \ln\left(\frac{1 + 3\sqrt{2}}{3\sqrt{2}}\right)$$

$$= \ln\left(\frac{\sqrt{2} + 6}{6}\right)$$

</td><td>

A lot of students did not recognise

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left(f(x)\right) + C$$

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(c) (i)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1	$1\frac{1}{2}$	2	$1\frac{1}{2}$	1

$$A \approx \frac{h}{3}\left(y_0 + y_4 + 4 \times (y_1 + y_3) + 2 \times y_2\right)$$

$$= \frac{\frac{\pi}{4}}{3}\left(1 + 1 + 4 \times \left(1\frac{1}{2} + 1\frac{1}{2}\right) + 2 \times 2\right)$$

(ii) $= \frac{\pi}{12}\left(2 + 12 + 4\right)$

$$= \frac{\pi}{12} \times 18$$

$$= \frac{3\pi}{2}$$

$$\approx 4.71 \text{ } units^2$$

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① All correct answers
① Correct formula & substitution
① Correct result

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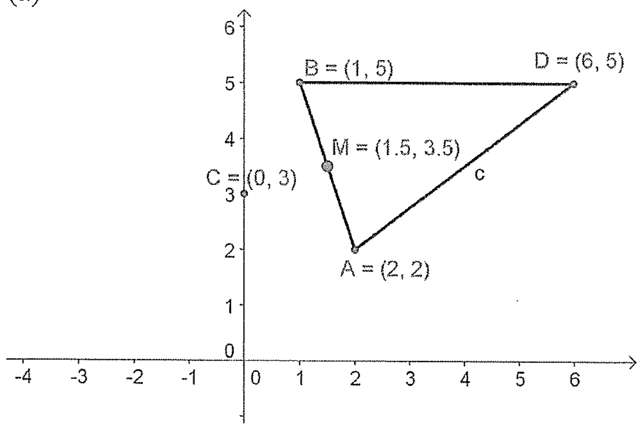
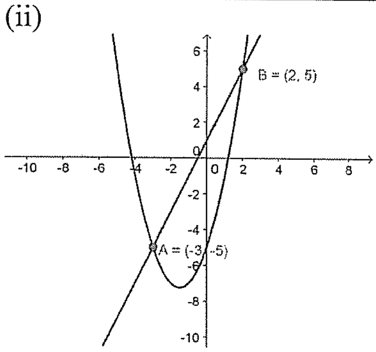
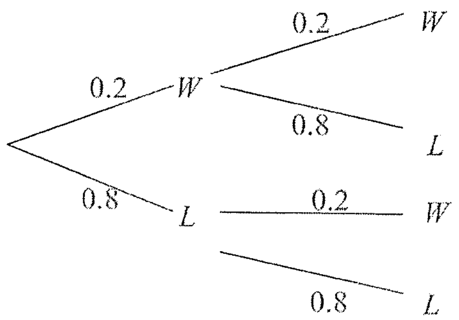
</table>

(d) (i) Change solutions $T_1 = 3$ $T_2 = 5$ $T_3 = 7$ $a = 3, d = 2$ $T_n = 3 + 2(n-1)$ $= 3 + 2n - 2$ $= 2n + 1$ $2n + 1 = 45$ $2n = 44$ $n = 22$ $\therefore$ 22 regular hexagons	① Show pattern as an arithmetic sequence ① Correct answer *Some students did by trial/error OR counting which tended to lead to errors.*
(ii) $Hexagon\ perimter = 6\times3 + 6\times5 + \ldots 6\times45$ $= 6(3 + 5 + \ldots + 45)$ $= 6\times\frac{22}{2}(3 + 45)$ $= 66\times48$ $= 3168$ Total Length $= 3168 + 6\times45$ $= 3438\ cm$	① Show arithmetic series ① Correct formula ① Correct result *Some students forgot arms.* *Some failed to see question as an arithmetic series.* *Some had one sixth of answer for hexagons.*

Question 13

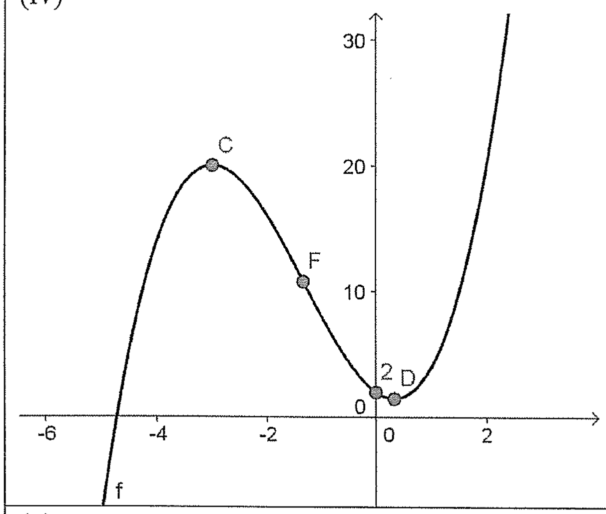
Solutions	Marking Criteria
(a)  (i) $M=\left(\frac{1+2}{2},\frac{5+2}{2}\right)$ $=\left(\frac{3}{2},\frac{7}{2}\right)$ (ii) $m_{AB}=\frac{5-2}{1-2}$ $=\frac{3}{-1}$ $=-3$ Required gradient is $\frac{1}{3}$ Equation of perpendicular bisector $y-\frac{7}{2}=\frac{1}{3}\left(x-\frac{3}{2}\right)$ $3y-\frac{21}{2}=x-\frac{3}{2}$ $6y-21=2x-3$ $0=2x-6y+18$ $x-3y+9=0$	① Required gradient ① Correct equation.
(iii) Let $x=0$ $-3y+9=0$ $y=3$ $C=(0,3)$	*Many students failed to recognise that every point on the perpendicular bisector of AB will be equidistant from A and B – so C is the y intercept of the perpendicular bisector.*
(iv) Sub $y=5$ $x-15+9=0$ $x=6$ $D=(6,5)$	

(v) $h = 5-2$ $= 3$ $b = 5$ $A = \frac{1}{2}\times 5\times 3$ $= 7.5\ units^2$	① for any relevant and correct calculation. ① Correct answer *Much easier to do with the aid of a diagram.*
(b) (i) $y = x^2+3x-5 \ \ldots(1)$ $y = 2x+1 \ \ldots\ (2)$ Equating (1) and (2) $2x+1 = x^3+3x-5$ $0 = x^2+x-6$ $0 = (x+3)(x-2)$ $x=-3,\ x=2$ When $x=2,\ y=5$ and $x=-3,\ y=-5$ The points of intersection are $(2,5)$ *and* $(-3,-5)$	① for quadratic equation ① for points of intersection
(ii)  $A = \int_{-3}^{2}\left((2x+1)-(x^2+3x-5)\right)dx$ $= \int_{-3}^{2}\left(-x^2-x+6\right)dx$ $= \left[\frac{-x^3}{3}-\frac{x^2}{2}+6x\right]_{-3}^{2}$ $= \left(\frac{-8}{3}-2+12\right)-\left(\frac{27}{3}-\frac{9}{2}-18\right)$ $= \frac{125}{6}\ units^2$	① for correct integration (including limits) ① for answer *Some students used the y values for the limits*
(c)  	① for either $P(WL)$ or $P(LW)$ ① Correct answer

$P(one\ win) = 0.2\times0.8 + 0.8\times0.2$ $= 2\times0.2\times0.8$ $= 0.32$	
(ii) Win first 0.2 Lost first win second 0.2×0.8 Lose first, lose second, win third $0.2\times0.8\times0.8$ P(at least one)= $0.2+0.2\times0.8+0.2\times0.8^2+\ldots$ $a=0.2, r=0.8$ Let $S_n=\frac{9}{10}$ $\frac{9}{10}=\frac{0.2(1-0.8^n)}{1-0.8}$ $0.18=0.2\times(1-0.8^n)$ $0.9=1-0.8^n$ $0.8^n=0.1$ $n=\frac{\ln 0.1}{\ln 0.8}$ ≈ 10.31 Therefore need to play 11 matches to have a 90% chance of winning at least one game. ALTERNATIVE METHOD: $P(\text{win at least one})=1-P(\text{lose all})$ $0.9=1-(0.8)^n$ $0.8^n=0.1$ $\ln 0.8^n=\ln 0.1$ $n\ln 0.8=\ln 0.1$ $n=\frac{\ln 0.1}{\ln 0.8}$	① for setting up $0.8^n=0.1$ ① Correct answer *Over-simplified by many students and poorly done.*

Question 14

Solutions	**Marking Criteria**
(a) (i) $y = x^3 + 4x^2 - 3x + 2$ $\frac{dy}{dx} = 3x^2 + 8x - 3$ $\frac{d^2y}{dx^2} = 6x + 8$	① for $\frac{dy}{dx}$ ① for $\frac{d^2y}{dx^2}$
(ii) Let $\frac{dy}{dx} = 0$ $3x^2 + 8x - 3 = 0$ $3x^2 + 9x - x - 3 = 0$ $3x(x+3) - 1(x+3) = 0$ $(3x-1)(x+3) = 0$ $x = -3,\ x = \frac{1}{3}$ When $x = -3$ $y = (-3)^3 + 4(-3)^2 - 3(-3) + 2$ $= 20$ When $x = \frac{1}{3}$ $y = \left(\frac{-1}{3}\right)^3 + 4\left(\frac{-1}{3}\right)^2 - 3\left(\frac{-1}{3}\right) + 2$ $= \frac{40}{27} (\approx 1.4)$ Testing: When $x = -3$ $\frac{d^2y}{dx^2} = 6(-3) + 8$ $= -10 < 0$ Therefore maximum at $(-3, 20)$ When $x = \frac{1}{3}$ $\frac{d^2y}{dx^2} = 6(\frac{1}{3}) + 8$ $= 10 > 0$ Therefore minimum at $\left(\frac{1}{3}, \frac{40}{27}\right)$	① for minimum stat. pt ① for maximum stat. pt *Students are reminded that if they are using a table of values to determine the nature of the stationary point, NOT to just use + and – signs in the table.*

(iii) Let $\frac{d^2y}{dx^2}=0$ $6x+8=0$ $x=\frac{-4}{3}$ When $x=\frac{-4}{3}$ $y=\left(\frac{-4}{3}\right)^3+4\left(\frac{-4}{3}\right)^2-4\left(\frac{-4}{3}\right)+2$ $=\frac{290}{27}(\approx 10.74..)$ Test $x=-2\ ,\ y''=-4$ $x=-1,\ y''=2$ Therefore, change in concavity at $\left(\frac{-4}{3},\frac{290}{27}\right)$	① for point of inflexion ① for testing change of concavity *Some students did not test for a change in concavity.*
(iv)  	① for shape ① for labelling (especially y intercept) *It was disappointing to see so many poorly drawn graphs.* • *Do NOT use a feathered line.* • *Your graph should be a smooth, freehand CURVE.* • *Show that the graph cuts the x axis.* • *Label key points but no need to annotate with "maximum", "minimum", etc.*
(v) From inspection, there is one solution to the equation $x^3+4x^2-3x+2=-1$	
(b) (i) $M=20e^{-kt}$ When $t=0$ $M=20e^0$ $\quad =20$ $\therefore$ Initial mass is 20 kg	

(ii) $M = 20e^{-kt}$ $When\ t = 200, M = 10$ $10 = 20e^{-200k}$ $\frac{1}{2} = e^{-200k}$ $-200k = \ln\frac{1}{2}$ $k = \frac{-1}{200}\ln\frac{1}{2}$ $k = \frac{1}{200}\ln 2$	① for setting up $\frac{1}{2} = e^{-200k}$ ① Correct answer in exact form
(iii) Let $t = 2000$ $M = 20 \times e^{-2000 \times \frac{1}{200}\ln 2}$ $= 0.1953...$ $= 19.53...\ grams$ $= 20\ grams\ (nearest\ gram)$	
(c) $x^2 - 2x + 3 - k = 0$ $\Delta = 4 - 4 \times 1 \times (3-k)$ Let $\Delta \geq 0$ $0 \leq 4 - 4(3-k)$ $0 \leq 4 - 12 + 4k$ $4k \geq 8$ $k \geq 2$	① for discriminant OR stating condition for real roots to exist. *Many students failed to find the discriminant correctly.* *Students who incorrectly stated that for real roots, $\Delta > 0$ lost a mark.*

Question 15

Solutions	Marking Criteria
(a) (i) In ΔADF and ΔBCE ← 1. $AD = BC$ (*sides of a square equal*) 2. $\angle ADF = 90° = \angle BCE$ (*property of a square*) } ① $DF = DE + EF$ $CE = CF + EF$ 3. *But* $DE = CF$ (*given*) $\therefore DF = CF + EF = CE$ $DF = CE$ } ① (with reasonable explanation) $\therefore \Delta ADF \equiv \Delta BCE$ (SAS) ①	*Some students did not write introduction.* *Some students wrote RHS when there was <u>no</u> hypotenuse.*
(ii) $\angle AFD = \angle BEF$ (*corresponding angles*, $\Delta ADF \equiv \Delta BCE$) *Let* $\angle AFD = \beta = \angle BEF$ $\angle GFE = \beta = \angle GEF$(*same angles*) } ① $\therefore FE = GF$ (*equal sides opposite equal angles in an isosceles triangle*)	*Some students tried to use AF = BE $\therefore$ GE = EF (not acceptable). This assumes AG = BG which was not proved.*
(b) (i) *Let* $\frac{dx}{dt} = 0$ $t^2 - 5t + 6 = 0$ $(t-3)(t-2) = 0$ $t = 2, t = 3$ Velocity is zero at 2 seconds and 3 seconds.	*Too many incorrectly factored* $t^2 - 5t + 6$ *to* $(t-6)(t+1)$
(ii) $\frac{d^2x}{dt^2} = 2t - 5$ Let $\frac{d^2x}{dt^2} = 0$ $t = \frac{5}{2}$ When $t = \frac{5}{2}$ $\frac{dx}{dt} = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 5$ $= -0.25$ Minimum velocity is -0.25 metres per second	*Too many did not realise that* $\frac{dx}{dt}$ *was parabolic* $\therefore$ *min was at* $t = 2\frac{1}{2}$ *Too many used* $t = 2$ *and* $t = 3$ *as times when velocity was a min. from previous question.*
(iii) $x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + C$ When $t = 0, x = -3$ $-3 = C$ $x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t - 3$	*A few students used* $t = 0, x = 3$ *(misread question)* ① for constant ① for correct expression for x
(iv)	*Since particle changed directions at*

When $t=0, x=-3$ When $t=2$ $x=\frac{2^3}{3}-\frac{5\times2^2}{2}+6\times2-3$ $=\frac{5}{3}$ When $t=3$ $x=\frac{3^3}{3}-\frac{5\times3^2}{2}+6\times3-3$ $=1.5$ Distance travelled is $3+\frac{5}{3}+\frac{5}{3}-1.5$ $=\frac{29}{6}$ Distance travelled is $\frac{29}{6}$ metres	*$t=2$ and $t=3$ and started at $t=0$, we need positions for each of these times to see where movement was made.* <u>ALTERNATIVELY:</u> Distance $=\left\|\int_0^2 f(t)dt\right\|+\left\|\int_2^3 f(t)dt\right\|$ $=\frac{29}{6}$m
(c) (i) $A_1=30000\times(1+\frac{0.09}{12})-Q$ $=30000\times1.0075-Q$	*Some confused this as as a savings question (i.e. superannuation)*
(ii) $A_2=1.0075\times A_1-Q$ $=1.0075\times(1.0075\times30000-Q)-Q$ $=30000\times1.0075^2-Q(1+1.0075)$ $A_3=1.0075\times A_2$ $=1.0075(30000\times1.0075^2-Q(1+1.0075))-Q$ $=30000\times1.0075^3-Q(1+1.0075+1.0075^2)$ $A_n=1.0075^n\times30000-Q(1+1.0075+1.0075^2+...+1.0075^{n-1})$ $A_n=1.0075^n\times30000-\frac{Q\times1(1.0075^n-1)}{1.0075-1}$ $=1.0075^n\times30000-Q\times\frac{400}{3}(1.0075^n-1)$ $=30000(1.0075)^n-\frac{400Q(1.0075^n-1)}{3}$	*To show a pattern, A_1, A_2 & A_3 should be shown (A_1 above)* ① for A_n in this form *Preferable if explanation was inserted here i.e. a geometric series with a = 1, r = 1.0075, n = n (although no deduction was made for not doing so)* ① for correct expression
(iii) When $n=60$, $A_{60}=0$	① Correct equation ① Correct answer *Mostly well done question.*

$$0 = 30000(1.0075)^{60} - \frac{400Q(1.0075^{60} - 1)}{3}$$ $$\frac{400Q(1.0075^{60} - 1)}{3} = 30000(1.0075)^{60}$$ $$Q = \frac{3 \times 30000(1.0075)^{60}}{400(1.0075^{60} - 1)}$$ $$= \$622.75 \ (\textit{nearest cent})$$	

Question 16

Solutions	Marking Criteria
(a) $6x^2-11\equiv A(x+2)^2+Bx+C$ $=A(x^2+4x+4)+Bx+C$ $=Ax^2+4Ax+4A+Bx+C$ $=Ax^2+(4A+B)x+(4A+C)$ Equating coefficients: $A=6$; $4A+B=0$, $4(6)+B=0$, $B=-24$; $4A+C=-11$, $4(6)+C=-11$, $C=-35$	*Quadratic identity very poorly answered.*
(b) (i) $a^2+b^2=7ab$ $(a+b)^2=a^2+2ab+b^2$ $=7ab+2ab$ $=9ab$ $\dfrac{(a+b)^2}{9}=ab$ $\left(\dfrac{a+b}{3}\right)^2=ab$	*Not well answered – some students find difficult to SHOW things.*
(ii) $\log\left(\dfrac{a+b}{3}\right)-\dfrac{1}{4}(\log a+\log b)=\log\sqrt{ab}-\dfrac{1}{4}\log ab$ $=\dfrac{1}{2}\log ab-\dfrac{1}{4}\log ab$ $=\dfrac{1}{4}\log ab$	
(c) (i) Let $\angle ABE=\alpha$ $AD^2=25$ $AE^2+DE^2=16+9$ $=25$ $\therefore \angle AED=90^\circ$ (ΔAED *right – angled triangle*) Let $\angle FBE=\alpha$ $\angle BAF+\angle AFB+\angle FBA=180^\circ$ ($\angle$ *sum of triangle*) $\angle BAF+90^\circ+\alpha=180^\circ$ $\angle BAF=90^\circ-\alpha$ $\angle DEF=\angle BAE$ (*alternate* $\angle$'s, $DC \parallel AB$) $\therefore \angle BAE=90^\circ$	*Some terribly long solutions.* *Students must give logical reasons and try to be concise.*

$\angle BAE = \angle EAF + \angle FAB$ $90° = \angle EAB + 90° - \alpha$ $\angle EAB = \alpha$	
(ii) In ΔAED $\cos\angle ADE = \frac{4}{5}$ $\angle ADE = \cos^{-1}\left(\frac{4}{5}\right)$ $EF^2 = 9+25-2\times3\times5\cos\left[\cos^{-1}\left(\frac{4}{5}\right)\right]$ $= 9+25-2\times3\times4$ $=10$ $EF = \sqrt{10}$	*Most did not realise to use the cosine rule – some tried Pythagoras (but no right angle!)*
(d) (i) $AB + AC + arcBC = 12$ $r + r + r\theta = 12$ $2r + r\theta = 12$ $r(2+\theta) = 12$ $r = \frac{12}{2+\theta}$ $Y = \frac{1}{2}r^2\theta$ $Y = \frac{1}{2}\times\left(\frac{12}{2+\theta}\right)^2\times\theta$ $= \frac{144}{2(2+\theta)^2}\times\theta$ $= \frac{72\theta}{(2+\theta)^2}$	
(ii) $Y = \frac{72\theta}{(2+\theta)^2}$ $\frac{dY}{d\theta} = \frac{(2+\theta)^2\times72 - 72\theta\times2(2+\theta)}{(2+\theta)^4}$ Let $\frac{dY}{d\theta} = 0$ $\frac{(2+\theta)^2\times72 - 72\theta\times2(2+\theta)}{(2+\theta)^4} = 0$ $72(\theta+2)[\theta+2-2\theta] = 0$ $72(\theta+2)(2-\theta) = 0$ $\theta = 2\ (\theta > 0)$	

Checking if maximum

θ	1	2	3
$\frac{dY}{d\theta}$	$\frac{8}{3}$	0	$\frac{-72}{125}$

Therefore maximum with $\theta = 2$

$$Y = \frac{72 \times 2}{(2+2)^2}$$

$$= \frac{144}{16}$$

$$= 9\ m^2$$

Many students did not attempt to show max, whilst many did not show values in their table.