HORNSBY GIRLS HIGH SCHOOL

Mathematics

Year 12 Higher School Certificate
Trial Examination Term 3 2017

STUDENT NUMBER: ______________________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators and drawing templates may be used
- A reference sheet is provided separately
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination room

Total marks – 100

Section I Pages 3 – 6

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 7 – 17

90 marks

Attempt Questions 11 – 16

Start each question in a new writing booklet

Write your student number on every writing booklet

Question	***1-10***	***11***	***12***	***13***	***14***	***15***	***16***	***Total***
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/15*	*/15*	*/100*

This assessment task constitutes 45% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 Evaluate $\dfrac{3}{\sqrt{1+\log_{10} 11}}$, correct to 3 significant figures.

(A) 1.63

(B) 1.627

(C) 2.10

(D) 2.100

2 The solutions to the equation $\sin x(2\cos x - 1) = 0$ for $0 \le x \le 2\pi$ are:

(A) $x = 0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}, 2\pi$

(B) $x = 0, \dfrac{\pi}{6}, \pi, \dfrac{11\pi}{6}, 2\pi$

(C) $x = 0, \dfrac{\pi}{3}, \dfrac{2\pi}{3}\pi, 2\pi$

(D) $x = 0, \dfrac{\pi}{3}, \pi, \dfrac{4\pi}{3}, 2\pi$

3 The value(s) of k for which $x^2 - 2kx + 4 = 0$ has equal roots are:

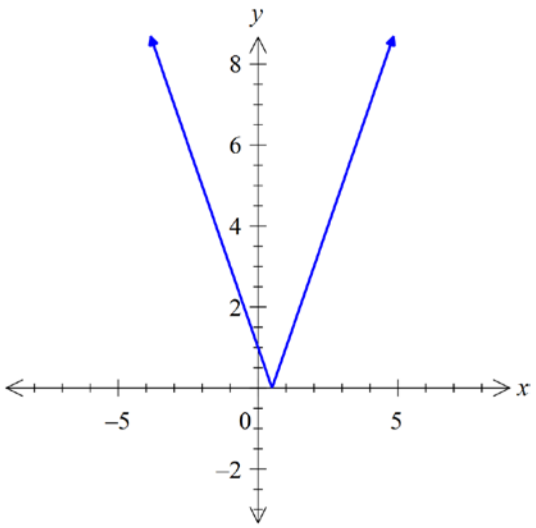
(A) $k = \pm 1$

(B) $k = \pm 2$

(C) $k = 1$

(D) $k = 2$

4 The graph of $y = |2x - 1|$ is drawn below to scale.



The solution to $|2x - 1| < 4 - x$ is:

(A) $x < -3$ or $x > \dfrac{5}{3}$

(B) $x < -3$ or $x > 2$

(C) $-3 < x < \dfrac{5}{3}$

(D) $-3 < x < 2$

5 The primitive function of $\dfrac{x+2}{x^2 + 4x + 3}$ is:

(A) $\log_e\left(x^2 + 4x + 3\right) + C$

(B) $\dfrac{1}{2}\log_e\left(x^2 + 4x + 3\right) + C$

(C) $\left(x + 4x + 3\right)^{-2} + C$

(D) $\dfrac{x+2}{\sqrt{x^2 + 4x + 3}}$

6 The sum of the first 10 terms of an arithmetic sequence with first term 7 and common difference 4 is:

(A) 47

(B) 250

(C) 355

(D) 500

7 The period of $y = \tan\frac{2x}{3}$ is:

(A) π

(B) 2π

(C) $\frac{3\pi}{2}$

(D) $\frac{2}{3}$

8 The Cartesian Equation of the locus of a point $P(x, y)$ that moves so that it is equidistant from $(0,2)$ and $(2,0)$ is:

(A) $y = x$

(B) $y = 2x + 2$

(C) $(x-2)^2 + y^2 = 4$

(D) $x^2 + (y-2)^2 = 4$

9 The quadratic equation $2x^2 + 3x - 7 = 0$ has roots α and β.
The value of $\alpha^2 + \beta^2$ is:

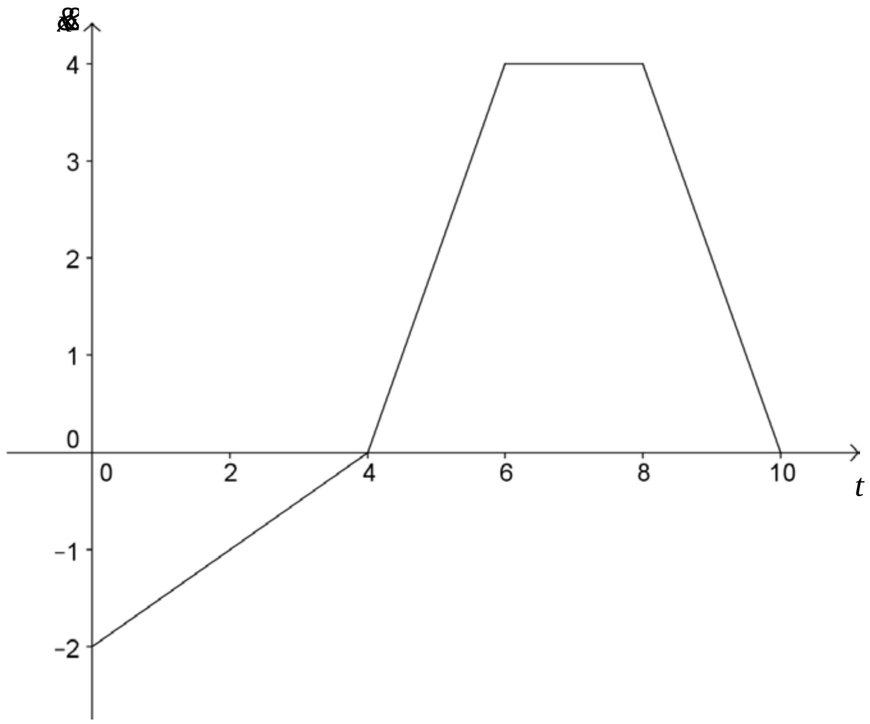
(A) $-\frac{3}{2}$

(B) $-\frac{7}{2}$

(C) $\frac{49}{4}$

(D) $\frac{37}{4}$

10 The acceleration of a particle at time *t* seconds is shown in the diagram below.
The particle begins at rest at the origin, with initial acceleration $2\ \text{ms}^{-2}$ in the negative direction.



The first time, in seconds, the particle will be stationary again after $t = 0$ seconds is:

(A) 2

(B) 4

(C) 6

(D) 8

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet

(a) Solve $\frac{x}{3} - \frac{x-1}{4} = 10$. **2**

(b) Differentiate $\frac{x+1}{x}$, giving your answer in simplest form. **2**

(c) Evaluate $\int_{-1}^{2} (2x+1)\,dx$. **2**

(d) Find the centre and radius of the circle with equation $x^2 + 2x + y^2 - 6y = 2$. **3**

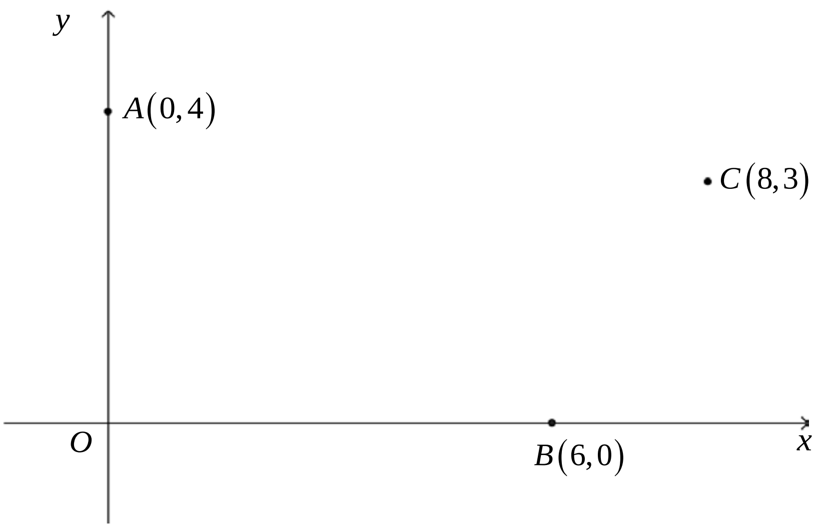
(e) Sketch the parabola $(x-1)^2 = 8y$, showing the vertex, focus and directrix. **2**

(f) Solve $x^2 - 5x + 4 \le 0$. **2**

(g) If $f(x) = \begin{cases} x+1, & \text{for } x > 0 \\ -x^2, & \text{for } x \le 0 \end{cases}$, evaluate $f(3) + f(-2)$. **2**

Question 12 (15 marks) Start a new writing booklet

(a) Three points $A(0,4)$, $B(6,0)$ and $C(8,3)$ lie in the coordinate plane.
The equation of AB is $2x+3y-12=0$.



NOT TO SCALE

Copy or trace the diagram to your writing booklet

(i) Find the shortest distance between AB and the origin. Leave your answer in exact form. **2**

(ii) Show that M, the midpoint of AB, has coordinates $(3,2)$. **1**

(iii) Prove that ΔMCB is a right-angled isosceles. **2**

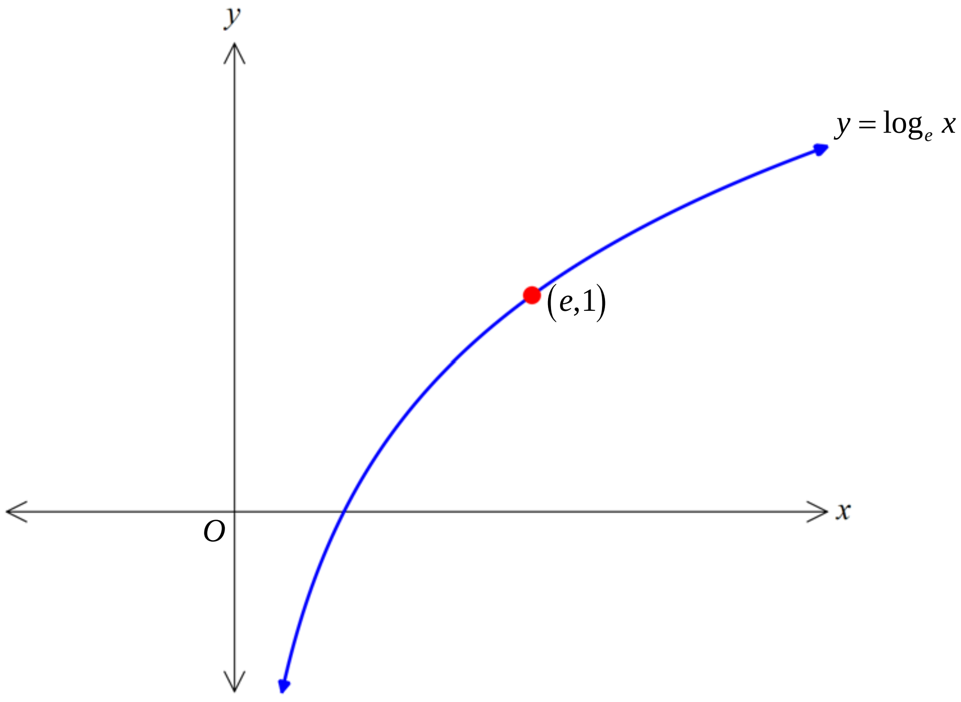
(b) A circular metal plate is being heated. The rate of increase of the area, $A\,\text{cm}^2$, of the plate is given by $\frac{dA}{dt}=\frac{\pi}{32}t$, where t is time in hours. Find the exact area of the plate after 8 hours, if initially the plate had a radius of *7* cm. **3**

Question 12 continues on page 9

Question 12 (continued)

(c) Show that $\dfrac{d}{dx}\left(x\log_e x - x\right) = \log_e x$. **2**

(d) The diagram below shows the curve $y = \log_e x$ passing through the point $(e,1)$.



NOT TO SCALE

(i) Find the equation the tangent at $(e,1)$, and show that it passes through the origin. **3**

(ii) Using the result in part (c), show that the area of the region bounded by the curve $y = \log_e x$, $y = 0$ and the tangent is $\left(\dfrac{1}{2}e - 1\right)$ square units. **2**

End of Question 12

Question 13 (15 marks) Start a new writing booklet

(a) The curve $y = x^3 - x^2 - x + 3$ has a local maximum at A and a point of inflexion at B.

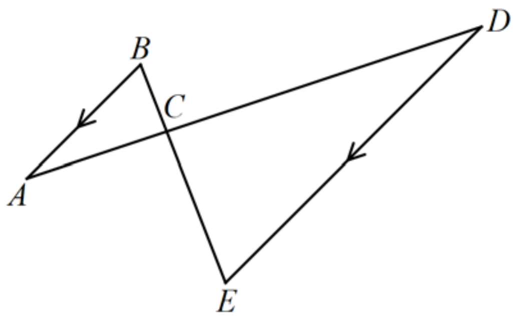
(i) Show that $\dfrac{d^2y}{dx^2} = 6x - 2$. **1**

(ii) Show that there are two stationary points, and determine the coordinates of A. **3**

(iii) Show that B has coordinates $\left(\dfrac{1}{3}, \dfrac{70}{27}\right)$. **1**

(iv) State the maximum value of the function in the domain $-3 \le x \le 2$. **1**

(b) In the diagram below, $DE = 2AB$, $AB \parallel DE$, BE and AD intersect at C.



NOT TO SCALE

(i) Prove that triangle ABC is similar to triangle DEC. **2**

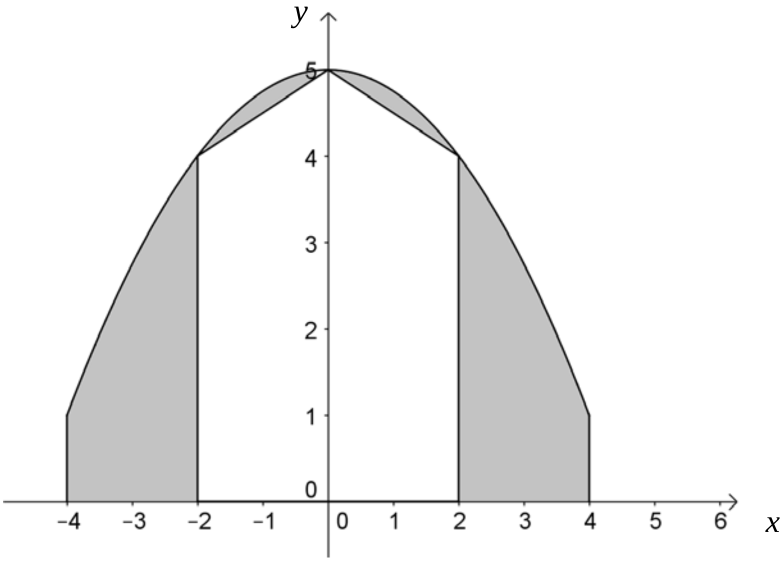
(ii) If the area of triangle DEC is 10 square units, find the area of triangle ABC. **2**

Question 13 continues on page 11

Question 13 (continued)

(c) An architect is designing an entry way to the school's new multi-purpose room.

The doorway will be in the shape of two trapeziums as shown in the diagram below.



Around the door will be a decorative window, as shaded in the diagram. The window will be bounded by the top and sides of the two trapeziums, the parabola $y = 5 - \frac{x^2}{4}$, and the lines $x = \pm 4$, where x and y are in metres .

(i) Use two applications of the trapezoidal rule to show that the area of the doorway is 18 square metres. **1**

(ii) Use Simpson's Rule with 3 function values to show that the area bounded by the parabola $y = 5 - \frac{x^2}{4}$, the x-axis and the lines $x = \pm 4$ is approximately $\frac{88}{3}$ square metres. **2**

(iii) Hence, find the approximate cost of the glass for the decorative window if it costs $\$133$ per square metre. **1**

(iv) Would finding the area in part (ii) be different using Simpson's Rule with 5 function values? Give reasons for your answer. **1**

End of Question 13

Question 14 (15 marks) Start a new writing booklet

(a) The common ratio of a geometric series is 2^{x-1}.

(i) Sketch $y = 2^{x-1}$, showing any intercepts and asymptotes. **2**

(ii) Hence, or otherwise, state the set of values of x for which the sum to infinity of the series exists. **1**

(iii) If the first term of the series is 35, find the value of x for which the sum to infinity is 40. **2**

(b) A rumour spreads exponentially through Hornsby Girls High School, so that the number of people who know it (N) can be modelled by the equation $N = Ae^{kt}$, where t is the time, in minutes, after the beginning of Period 1 at 8:48 am.

When Period 1 begins, 18 people know the rumour. By the end of Period 1, at 10:06, 42 people know the rumour.

(i) Write down the value of A. **1**

(ii) Show that $k = 0.01086$, correct to five decimal places. **2**

(iii) How many people know the rumour by the beginning of lunch at 1:07 pm? **2**

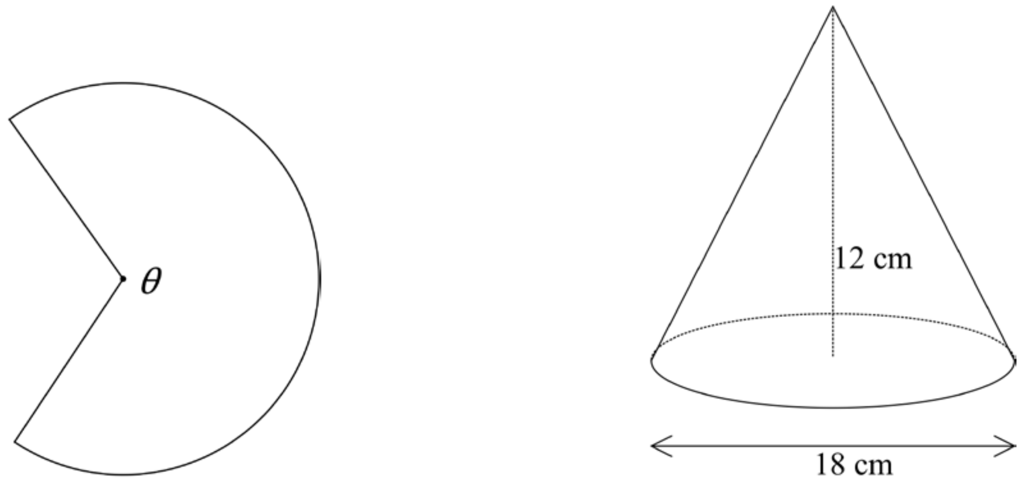
Give your answer correct to the nearest whole number.

(iv) There are 780 people in the school. According to the exponential model, at what time will everyone know the rumour? **2**

Question 14 continues on page 13

Question 14 (continued)

(c) An open cone is made by rolling a piece of paper as shown in the diagram below. **3**



NOT TO SCALE

The cone is to have height 12 cm and a base diameter 18 cm.
Find the size of the angle θ in degrees.

End of Question 14

Question 15 (15 marks) Start a new writing booklet

(a) Justin does a bungee jump from a platform 50 m above a river. Let h be his height above the river, in metres, at time t seconds after stepping off the platform.
His velocity is given by $v = 2t^2 - 10t$.

(i) After stepping off the platform, what time is Justin's velocity zero? **1**

(ii) How close to the river does Justin get? **2**

(iii) What distance does Justin travel in the first seven seconds? **2**

(b) Jack and Jill take out a home loan of \$1 000 000 with interest fixed at 6% per annum, compounded monthly. Jack and Jill want to pay it off over 25 years by equal monthly instalments.

Interest is added at the end of each month, just before the repayment is made. Let \$ M be the monthly repayment amount and let $\$A_n$ be the amount owing after n months.

(i) Show that $A_2 = 1000000 \times 1.005^2 - M\left(1 + 1.005\right)$ **1**

(ii) Show that $A_n = 1000000 \times 1.005^n - M\left(\dfrac{1.005^n - 1}{0.005}\right)$ **2**

(iii) Find M, correct to the nearest cent. **2**

After 12 months of making repayments, Jack and Jill owe \$982 200.
Do NOT prove this amount.

The Federal Government then passes the Bank Levy. The bank passes this fee onto consumers, meaning that Jack and Jill's monthly interest rate increases by 0.1% for the remainder of the loan.

Jack and Jill decide they can afford to increase their repayment to \$7000 per month.

(iv) How many more months will it take for Jack and Jill to now pay off the loan? **2**

Question 15 continues on page 15

Question 15 (continued)

(c) The region bounded by the curve $y = \dfrac{3}{\sqrt{x}}$, the x-axis, $x = 1$ and $x = a$, where $a > 1$, **3**

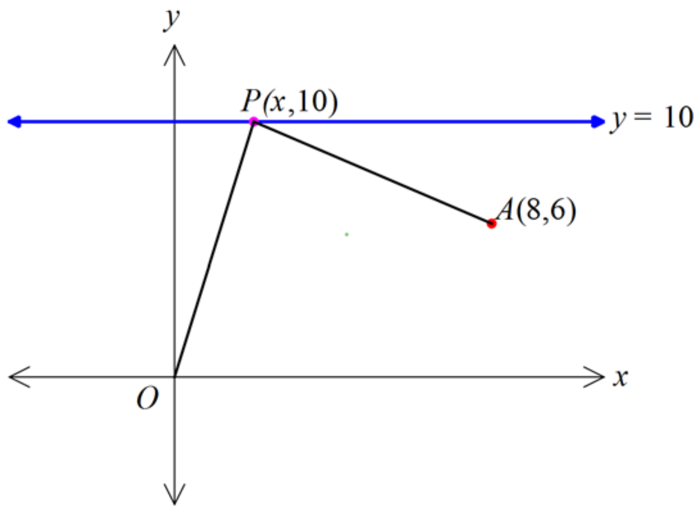
is rotated about the x-axis. The volume of the resulting solid is $\pi \log_e \left(\dfrac{64}{27} \right)$ cubic units.

Find the exact value of a.

End of Question 15

Question 16 (15 marks) Start a new writing booklet

(a) In the diagram below, the points $O(0,0)$ and $A(8,6)$ are fixed. $\angle OPA$ varies as the point $P(x,10)$ moves along the horizontal line $y=10$. The distance OP is $\sqrt{x^2+100}$ units and the distance AP is $\sqrt{x^2-16x+80}$ units.



NOT TO SCALE

(i) Show that $\cos\angle OPA = \dfrac{x^2-8x+40}{\sqrt{(x^2-16x+80)(x^2+100)}}$. **2**

(ii) Consider the case when $x=8$.

(α) Show that $\angle OPA = \cos^{-1}\left(\dfrac{5}{\sqrt{41}}\right)$. **2**

(β) Hence, find the exact value of $\sin\angle OPA$ and the exact area of ΔOPA. **2**

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(iii) Let $f(x) = \cos\angle OPA = \dfrac{x^2-8x+40}{\sqrt{(x^2-16x+80)(x^2+100)}}$, $0 \le x \le 15$.

Consider $f(x)=1$.

(α) Explain, in terms of the position of *O*, *A* and *P*, why this equation has a solution. **1**

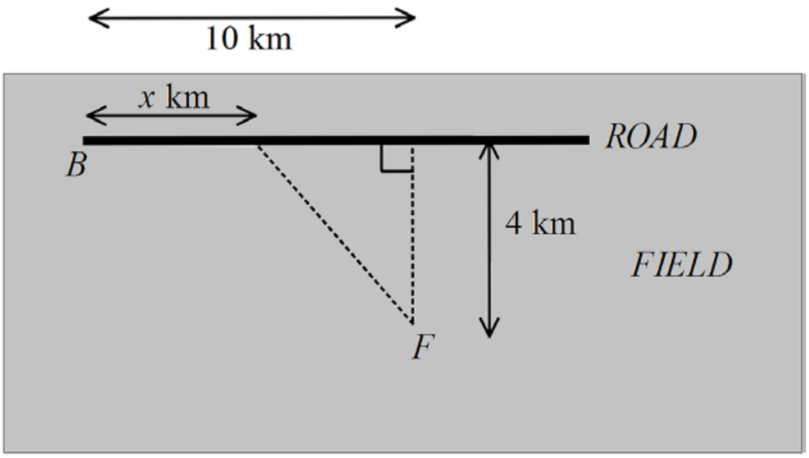
(β) Find the exact solution to the equation. **2**

Question 16 continues on page 17

Question 16 (continued)

(b) Bill's house, B, is on a main road. Frank's house, F, is in a field. Bill knows that from his house he can ride 10 km along the road, then walk 4 km through the field to get to Frank's house.

Bill can cycle only on paved surfaces, at a speed of 10 km/h. Bill can walk through the field at a speed of 5km/h. He decides to cycle for x km along the road, then walk in a straight line through the field to Frank's house, as shown in the diagram below.



NOT TO SCALE

(i) Show that the time it takes for Bill to get to Frank's house is given by: **2**

$$T = \frac{x}{10} + \frac{1}{5}\sqrt{16 + (10 - x)^2}.$$

Bill wishes to get to Frank's house in the shortest possible time.

(ii) Show that the distance, x, he should cycle satisfies $3(10 - x)^2 = 16$. **3**

(iii) Hence, find exactly how far Bill should cycle. **1**

End of Paper

Year 12 Mathematics (2 unit) HSC Trial Examination 2017

Multiple Choice

1.

$$\frac{3}{\sqrt{1+\log_{10}11}} = 2.09970...$$

$$= 2.10\ (3sf)$$

Answer:(C)

2.

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

Answer:(A)

3.

$\Delta = (-2k)^2 - 4\times4\times1$

$= 4k^2 - 16$

Let $\Delta = 0$

$4k^2 - 16 = 0$

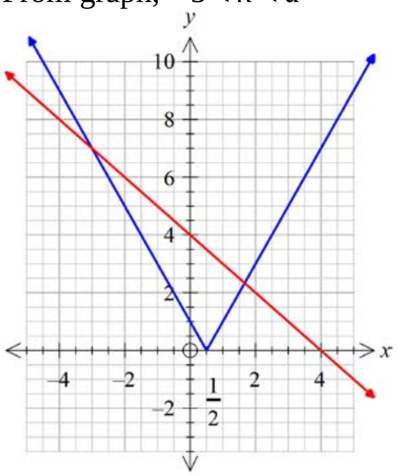
$k^2 = 4$

$k = \pm2$

Answer:(B)

4.

From graph, $-3 < x < a$



Finding a:

Let $|2x-1| = 4 - x$

$2x - 1 = 4 - x$

$3x = 5$

$x = \frac{5}{3}$

$2x - 1 = x - 4$

$x = -3$

$\therefore -3 < x < \frac{5}{3}$

Answer:(C)

5.

$$\int \frac{x+2}{x^2+4x+3}dx = \frac{1}{2}\int \frac{2x+4}{x^2+4x+3}dx$$

$$=\frac{1}{2}\log_e\left(x^2+4x+3\right)+C$$

Answer:(B)

6.

$a=7, d=4$

$$S_{10}=\frac{10}{2}(2\times7+9\times4)$$
$$=5(14+36)$$
$$=5\times50$$
$$=250$$

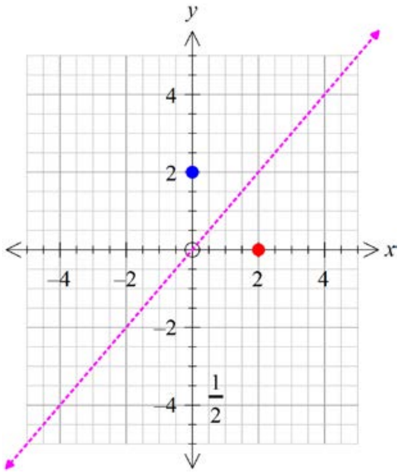
Answer:(B)

7.

$$period=\frac{\frac{\pi}{2}}{3}$$
$$=\frac{3\pi}{2}$$

Answer:(C)

8.



By observation, $y=x$

Answer:(A)

9.

$2x^2+3x-7=0$

$$\alpha+\beta=\frac{-3}{2}, \alpha\beta=\frac{-7}{2}$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2(\alpha\beta)$$
$$=\left(\frac{-3}{2}\right)^2-2\left(\frac{-7}{2}\right)$$
$$=\frac{37}{4}$$

Answer:(D)

10. Area under *t*-axis:

$$A = \frac{1}{2} \times 2 \times 4$$
$$= 4\, units^2$$

Want first t after 4 seconds that makes area above *t*-axis equal.

Try $t = 6$

$$A = \frac{1}{2} \times 2 \times 4$$
$$= 4\, units^2$$

Answer: (C)

Question 11	
(a) $\frac{x}{3}-\frac{x-1}{4}=10$ $\frac{4x-3(x-1)}{12}=10$ $4x-3x+3=120$ $x=117$	
(b) $\frac{d}{dx}\left(\frac{x+1}{x}\right)=\frac{x\times 1-1\times(x+1)}{x^2}$ $=\frac{x-x-1}{x^2}$ $=-\frac{1}{x^2}$ OR $\frac{d}{dx}\left(\frac{x+1}{x}\right)=\frac{d}{dx}\left(1+x^{-1}\right)$ $=-1x^{-2}$ $=-\frac{1}{x^2}$	
(c) $\int_{-1}^{2}(2x+1)\,dx=\left[x^2+x\right]_{-1}^{2}$ $=\left(2^2+2\right)-\left((-1)^2+(-1)\right)$ $=6$	
(d) $x^2+2x+y^2-6y=2$ $x^2+2x+1+y^2-6y+9=2+1+9$ $(x+1)^2+(y-3)^2=12$ Centre $(-1,3)$ and radius $\sqrt{12}=2\sqrt{3}\ units$	

(e)

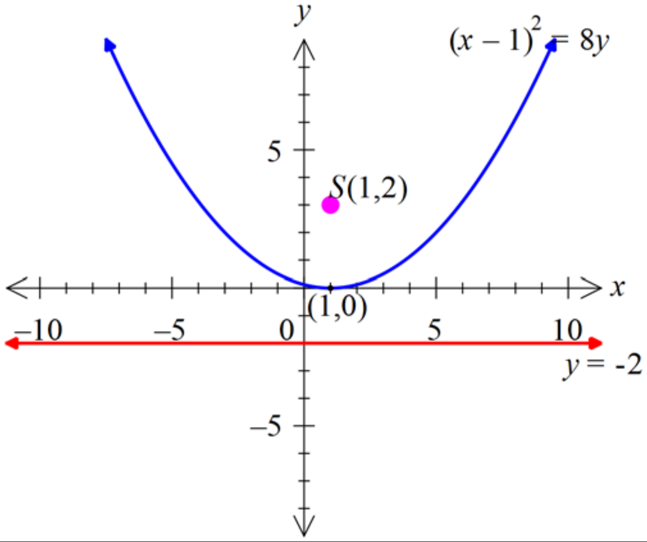
$(x-1)^2 = 8y$

Vertex $(1,0)$

Focal Length 2

Focus $(1,2)$

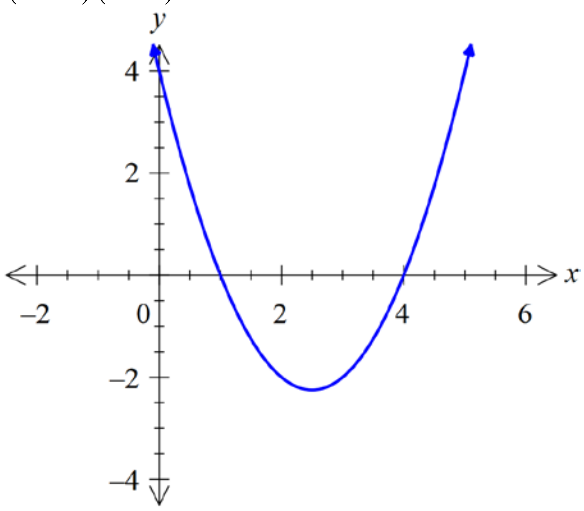
Directrix $y=-2$



(f)

$x^2-5x+4\le 0$

$(x-4)(x-1)\le 0$



$\therefore 1\le x\le 4$

(g)

$$\begin{aligned} f(3)+f(-2) &= (3)+1-(-2)^2 \\ &= 4-4 \\ &= 0 \end{aligned}$$

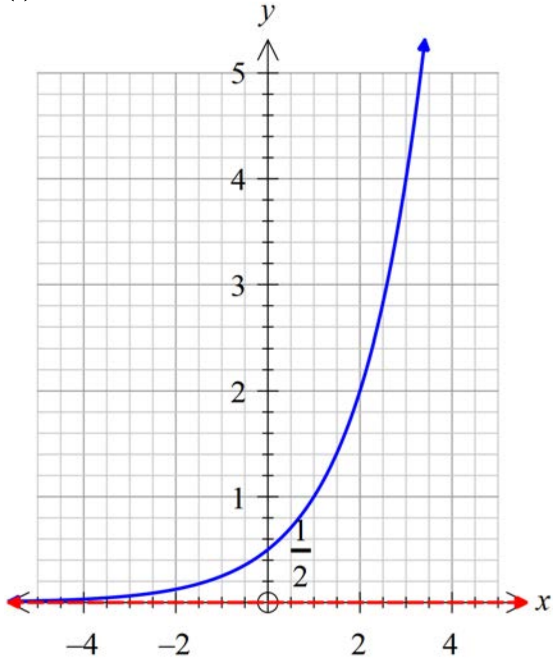
Question 12	
(a) (i) $a=2, b=3, c=-12$ $x_1=0, y_1=0$ $d=\frac{\lvert 2\times 0+3\times 0-12\rvert}{\sqrt{2^2+3^2}}$ $=\frac{12}{\sqrt{4+9}}$ $=\frac{12\sqrt{13}}{13}\ units$	
(ii) $A(0,4) \qquad B(6,0)$ $M=\left(\frac{0+6}{2},\frac{4+0}{2}\right)$ $=\left(\frac{6}{2},\frac{4}{2}\right)$ $=(3,2)$	
(iii) $m_{MB}=\frac{2-0}{3-6}$ $=-\frac{2}{3}$ $m_{BC}=\frac{3-0}{8-6}$ $=\frac{3}{2}$ $m_{MB}\times m_{BC}=-\frac{2}{3}\times\frac{3}{2}$ $=-1$ $\therefore MB\perp BC$ $BC=\sqrt{(8-6)^2+(3-0)^2}$ $=\sqrt{2^2+3^2}$ $=\sqrt{13}$ $BM=\sqrt{(6-3)^2+(0-2)^2}$ $=\sqrt{3^2+2^2}$ $=\sqrt{13}$ $=BC$ Since $BC=BM$ and $\angle MBC=90°$, ΔMBC is right-angled isosceles.	

(b) $\frac{dA}{dt}=\frac{\pi}{32}t$ $A=\frac{\pi t^2}{64}+C$ When $t=0, r=7, A=49\pi$ $49\pi=0+C$ $\therefore A=\frac{\pi t^2}{64}+49\pi$ When $t=8$, $A=\frac{64\pi}{64}+49\pi$ $=50\pi\ cm^2$	
(c) $\frac{d}{dx}\left(x\log_e x-x\right)=1\times\log_e x+x\times\frac{1}{x}-1$ $=\log_e x+1-1$ $=\log_e x$	
(d) (i) $y=\log_e x$ $\frac{dy}{dx}=\frac{1}{x}$ At $(e,1)$, $\frac{dy}{dx}=\frac{1}{e}$ Equation of tangent: $y-1=\frac{1}{e}(x-e)$ $y-1=\frac{x}{e}-1$ $y=\frac{x}{e}$ Sub $(0,0)$, $LHS=0$ $RHS=0$ The tangent passes through the origin.	
(ii) $A=\frac{1}{2}\times e\times 1-\int_1^e \log_e x dx$ $=\frac{e}{2}-\left[x\log_e x-x\right]_1^e$ $=\frac{e}{2}-\left[(e-e)-(\log_e 1-1)\right]$ $=\frac{e}{2}-1\ square\ units$	

Question 13	
(a) (i) $y = x^3 - x^2 - x + 3$ $\frac{dy}{dx} = 3x^2 - 2x - 1$ $\frac{d^2y}{dx^2} = 6x - 2$	
(ii) Let $\frac{dy}{dx} = 0$ $0 = 3x^2 - 2x - 1$ $0 = 3x^2 - 3x + x - 1$ $0 = 3x(x-1) + 1(x-1)$ $0 = (3x+1)(x-1)$ $\therefore x = \frac{-1}{3}, x = 1$ When $x = \frac{-1}{3}$, $\frac{d^2y}{dx^2} = 6\left(\frac{-1}{3}\right) - 2$ $= -2 - 2$ $= -4 < 0$ $\therefore$ Maximum when $x = \frac{-1}{3}$ When $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 2$ $= 4 > 0$ $\therefore$ Minimum when $x = 1$ $x = \frac{-1}{3}$ When $y = \left(\frac{-1}{3}\right)^3 - \left(\frac{-1}{3}\right)^2 - \left(\frac{-1}{3}\right) + 3$ $= \frac{86}{27}$ $\therefore A\left(\frac{-1}{3}, \frac{86}{27}\right)$	
(iii) Let $\frac{d^2y}{dx^2} = 0$ $6x - 2 = 0$ $6x = 2$ $x = \frac{1}{3}$	

When $x = \frac{1}{3}$ $y = \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) + 3$ $= \frac{70}{27}$ $\therefore$ Point of inflexion at $B\left(\frac{1}{3}, \frac{70}{27}\right)$	
(iv) When $x = 2$, $y = 2^3 - 2^2 - 2 + 3$ $= 5$ When $x = -3$, $y = (-3)^3 - (-3)^2 - (-3) + 3$ $= -30$ Therefore maximum value is 5.	
(b) (i) In ΔABC and ΔDEC $\angle ACB = \angle DCE$ (vertically opposite angles) $\angle BAC = \angle EDC$ (alternate angles, $AB \parallel DC$) $\therefore \Delta ABC \ \vert\vert\vert \ \Delta DEC$ (equiangular)	
(ii) $DE = 2AB$ $\frac{DE}{AB} = 2$ $\frac{Area_{\Delta DCE}}{Area_{\Delta ACB}} = 4$ $\frac{10}{Area_{\Delta ACB}} = 4$ $Area_{\Delta ACB} = 2.5 \text{ square units}$	

(c) (i) $A=\frac{2}{2}(4+5)+\frac{2}{2}(5+4)$ $=4+5+5+4$ $=18\ \textit{square metres}$	
(ii) $A=\int_{-4}^{4}\left(5-\frac{x^2}{4}\right)dx$ $\approx\frac{4}{3}(1+4\times5+1)$ $=\frac{4}{3}(2+20)$ $=\frac{4}{3}\times22$ $=\frac{88}{3}\ \text{m}^2$ (iii) $C=133\left(\frac{88}{3}-18\right)$ $=\$1507.33\ (\textit{nearest cent})$	
(iv) Since $y=5-\frac{x^2}{4}$ is a parabola, Simpson's Rule gives an exact approximation of the area, so it won't matter whether you use one application or two applications, because either will be exact. OR $\int_{-4}^{4}\left(5-\frac{x^2}{4}\right)dx=\frac{2}{3}(1+4\times4+5)+\frac{2}{3}(5+4\times4+1)$ $=\frac{2}{3}\times22+\frac{2}{3}\times22$ $=\frac{4}{3}\times22$ $=\frac{88}{3}$ Which is the same as the approximation using three function values.	

Question 14	
(a) (i) 	
(ii) From graph, $x < 1$	
(iii) $a = 35$, $S_\infty = 40$ $r = 2^{x-1}$ $40 = \frac{35}{1-2^{x-1}}$ $1-2^{x-1} = \frac{35}{40}$ $2^{x-1} = 1-\frac{35}{40}$ $2^{x-1} = \frac{1}{8}$ $2^{x-1} = 2^{-3}$ $x-1 = -3$ $x = -2$	
(b) (i) $N = Ae^{kt}$ When $t = 0$, $N = 18$ $18 = Ae^0$ $A = 18$	

(ii) $N = 18e^{kt}$ When $t = 66 + 12 = 78$ $42 = 18e^{78k}$ $\frac{7}{3} = e^{78k}$ $78k = \ln\left(\frac{7}{3}\right)$ $k = \frac{1}{78}\ln\left(\frac{7}{3}\right)$ $= 0.01086279...$ $= 0.01086\ (5dp)$	
(iii) Let $t = 4 \times 60 + 19 = 259$ $N = 18e^{259k}$ $= 299.7988524...$ $= 299\ people\ (nearest\ whole\ number)$	
(iv) Let $N = 780$ $780 = 18e^{kt}$ $\frac{130}{3} = e^{kt}$ $t = \frac{1}{k}\ln\left(\frac{130}{3}\right)$ $= 377.046...$ $=$ 5 hours 47 minutes (nearest minute) Everyone in the school will know at 2:35 pm	
(c) $r^2 = 12^2 + 9^2$ $r^2 = 15^2$ $r = 15$ $C = 2\pi \times 9$ $= 18\pi$ $l = r\theta$ $18\pi = 15 \times \theta$ $\theta = \frac{18\pi}{15}$ $= 216°$	

Question 15	
(a) (i) $v = 2t^2 - 10t$ Let $v = 0$ $0 = 2t^2 - 10t$ $0 = 2t(t-5)$ $t = 5\,(t > 0)$ $\therefore$ Stationary at 5 seconds.	
(ii) $h = \int (2t^2 - 10t)\,dt$ $h = \frac{2t^3}{3} - 5t^2 + C$ When $t = 0, h = 0\ \ \therefore C = 50$ $h = \frac{2t^3}{3} - 5t^2 + 50$ Let $t = 5$ $h = \frac{2\times 5^3}{3} - 5\times 5^2 + 50$ $= \frac{-125}{3} + 50$ $= \frac{25}{3}$ $\therefore$ He is approximately 8.33 metres from the river	
(iii) When $t = 7$, $h = \frac{2\times 7^3}{3} - 5\times 7^2 + 50$ $= \frac{101}{3}$ $d = 50 - \frac{25}{3} + \frac{101}{3} - \frac{25}{3}$ $= 67\ metres$	

(b) (i) $A_1 = 1000000(1.005) - M$ $A_2 = A_1 \times 1.005 - M$ $= 1000000 \times 1.005^2 - M \times 1.005 - M$ $= 1000000 \times 1.005^2 - M(1 + 1.005)$	
(ii) $A_3 = A_2 \times 1.005 - M$ $= 1000000 \times 1.005^3 - M(1.005 + 1.005^2) - M$ $= 1000000 \times 1.005^3 - M(1 + 1.005 + 1.005^2)$ $\therefore A_n = 1000000 \times 1.005^n - M(1 + 1.005 + 1.005^2 + ... + 1.005^{n-1})$ $= 1000000 \times 1.005^n - M \times \dfrac{(1.005^n - 1)}{1.005 - 1}$ $= 1000000 \times 1.005^n - \dfrac{M(1.005^n - 1)}{0.005}$	
(iii) Let $A_{300} = 0$ $M\dfrac{(1.005^{300} - 1)}{0.005} = 1000000 \times 1.005^{300}$ $= \dfrac{1000000 \times 1.005^{300}}{\frac{(1.005^{300} - 1)}{0.005}}$ $= \$6443.01$ (nearest cent)	
(iv) Let $P = 982200$ $B_n = P \times 1.006^n - \dfrac{7000(1.006^n - 1)}{0.006}$ Let $B_n = 0$ $0 = P \times 1.006^n - \dfrac{7000(1.006^n - 1)}{0.006}$ $0 = 0.006 \times P \times 1.006^n - 7000(1.006^n) + 7000$ $1.006^n(7000 - 0.006 \times P) = 7000$ $1.006^n = \dfrac{7000}{7000 - 0.006P}$ $n = \dfrac{\ln\left(\frac{7000}{7000 - 0.006P}\right)}{\ln 1.006}$ $= 308.3274...$ $= 308\ months(nearest\ month)$	

(c) $y = \dfrac{3}{\sqrt{x}}$ $V = \pi \int_1^a \left(\dfrac{3}{\sqrt{x}}\right)^2 dx$ $= \pi \int_1^a \dfrac{9}{x} dx$ $= 9\pi \left[\ln x\right]_1^a$ $= 9\pi \left(\ln a\right)$ Let $V = \pi \ln\left(\dfrac{64}{27}\right)$ $9\pi \ln a = \pi \ln\left(\dfrac{64}{27}\right)$ $\ln a^9 = \ln\left(\dfrac{4^3}{3^3}\right)$ $a^9 = \dfrac{4^3}{3^3}$ $a^3 = \dfrac{4}{3}$ $a = \sqrt[3]{\dfrac{4}{3}}$	

Question 16	
(a) (i) $\cos C = \frac{a^2+b^2-c^2}{2ab}$ $\cos\angle OPA = \frac{OP^2+AP^2-OA^2}{2\times OP\times AP}$ $= \frac{\left(\sqrt{x^2+100}\right)^2+\left(\sqrt{x^2-16x+80}\right)^2-\left(\sqrt{6^2+8^2}\right)^2}{2\times\sqrt{x^2+100}\sqrt{x^2-16x+80}}$ $= \frac{x^2+100+x^2-16x+80-100}{2\sqrt{x^2+100}\sqrt{x^2-16x+80}}$ $= \frac{2x^2-16x+80}{2\sqrt{x^2+100}\sqrt{x^2-16x+80)}}$ $= \frac{2\left(x^2-8x+40\right)}{2\sqrt{x^2+100}\sqrt{x^2-16x+80)}}$ $= \frac{x^2-8x+40}{\sqrt{x^2+100}\sqrt{x^2-16x+80)}}$	
(ii) (α) $\cos\angle OPA = \frac{8^2-8\times 8+40}{\sqrt{8^2+100}\sqrt{8^2-16\times 8+80}}$ $= \frac{40}{\sqrt{2624}}$ $\angle OPA = \cos^{-1}\left(\frac{40}{\sqrt{2624}}\right)$ $= \cos^{-1}\left(\frac{8\times 5}{\sqrt{64}\sqrt{41}}\right)$ $= \cos^{-1}\left(\frac{5}{\sqrt{41}}\right)$	
(iii) (β) $\sin^2\theta = 1-\cos^2\theta$ $= 1-\frac{25}{41}$ $= \frac{16}{41}$ $\therefore \sin\angle OPA = \frac{4}{\sqrt{41}}\ \left(\angle OPA\ acute\right)$	

$A=\frac{1}{2}\times OP\times AP\times\frac{4}{\sqrt{41}}$ $=\frac{1}{2}\times\sqrt{164}\sqrt{16}\times\frac{4}{\sqrt{41}}$ $=\frac{1}{2}\times 4\times 4\times\sqrt{41}\sqrt{4}\times\frac{1}{\sqrt{41}}$ $=\frac{1}{2}\times 4\times 4\times 2$ $=16\ units^2$	
(iii) (α) $\cos\angle OPA=1$ $\therefore\angle OPA=0°$ The solution to the equation will be the x value of P such that O, P and A are collinear. (β) Equation OA $y=\frac{6}{8}x+0$ $=\frac{3}{4}x$ When $y=10$, $10=\frac{3}{4}\times x$ $x=\frac{40}{3}$	

(b) (i) $\text{time}=\frac{\text{distance}}{\text{speed}}$ $cycling: t=\frac{x}{10}$ Walking: $d=\sqrt{(10-x)^2+4^2}$ $=\sqrt{16+(10-x)^2}$ $t=\frac{\sqrt{16+(10-x)^2}}{5}$ $\therefore T=\frac{x}{10}+\frac{1}{5}\sqrt{16+(10-x)^2}$	
(ii) $\frac{dT}{dx}=\frac{1}{10}+\frac{1}{5}\times\frac{1}{2}\left(16+(10-x)^2\right)^{\frac{-1}{2}}\times 2(10-x)^1\times -1$ $=\frac{1}{10}-\frac{10-x}{5\sqrt{16+(10-x)^2}}$ $Let\ \frac{dT}{dx}=0$ $\frac{1}{10}=\frac{10-x}{5\sqrt{16+(10-x)^2}}$ $\sqrt{16+(10-x)^2}=2(10-x)$ $16+(10-x)^2=4(10-x)^2$ $16=3(10-x)^2$	
(iii) $3(10-x)^2=16$ $(10-x)^2=\frac{16}{3}$ $10-x=\pm\frac{4}{\sqrt{3}}$ $x=10\pm\frac{4}{\sqrt{3}}$ But $x<10$ $\therefore x=10-\frac{4}{\sqrt{3}}\ km$	