Hunters Hill High School

Mathematics

Trial Examination, 2017

Hunters Hill
High School

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided
- The marks for each question are shown on the paper
- Show all necessary working in questions 11-16

Total Marks: 100

Section I Pages 3-5
10 marks

- Attempt Questions 1-10
- Allow about 20 minutes for this section

Section II Pages 6-12
90 marks

- Attempt Questions 11-16
- Allow about 2 hours and 40 minutes for this section

Section I

10 marks Attempt Questions 1–10
Allow about 20 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. What is 2.807956 correct to 3 significant figures?

(A) 2.80
(B) 2.81
(C) 2.807
(D) 2.808

2. A card is drawn at random from a standard deck of playing cards .
What is the probability that the card drawn is a Diamond OR a Queen?

(A) $\frac{13}{52}$
(B) $\frac{4}{52}$
(C) $\frac{17}{52}$
(D) $\frac{16}{52}$

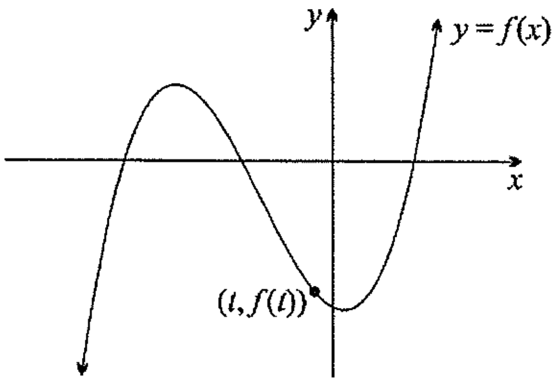
3. The equation $y = 3\cos(\pi x - 3)$ has a period of:

(A) 2π
(B) π
(C) 2
(D) 3

4. When $\frac{3+\sqrt{2}}{3+2\sqrt{2}}$ is expressed in the form $a - \sqrt{b}$, then:

(A) $a = 1, b = \frac{1}{4}$
(B) $a = 9, b = 50$
(C) $a = 5, b = 2$
(D) $a = 5, b = 18$

5. The diagram shows the graph of $y = f(x)$. Which of the following statements is true?



(A) $f'(t) < 0$ and $f''(t) > 0$
(B) $f'(t) < 0$ and $f''(t) < 0$
(C) $f'(t) > 0$ and $f''(t) < 0$
(D) $f'(t) > 0$ and $f''(t) < 0$

6. The maximum value of the expression $-2x^2 - 4x + 7$ is:

(A) -1
(B) 1
(C) 7
(D) 9

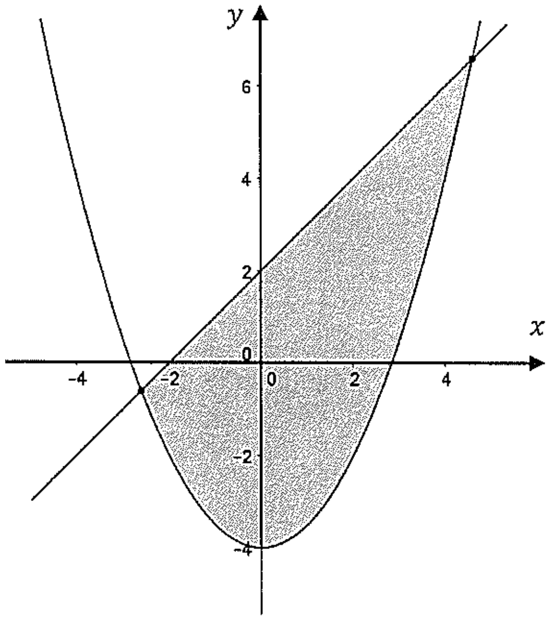
7. For what domain and range is the function $y = \dfrac{1}{\sqrt{x-4}}$ defined?

(A) Domain: $x \geq 4$, Range: $y > 0$
(B) Domain: $x > 4$, Range: $y > 0$
(C) Domain: all real x, Range: all real y
(D) Domain: $x < -2, x > 2$, Range: $y < 0$

8. The values of k for which $x^2 - kx + k + 3 = 0$ has no real roots are:

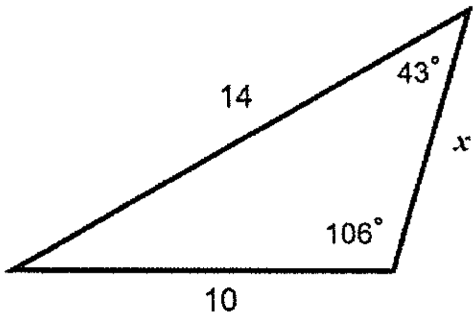
(A) $k < 0$
(B) $-2 < k < 6$
(C) $k < -2, k > 6$
(D) $k = 2, 6$

9. Which statement is consistent with the region shown?



(A) $2y \geq x^2 - 8$ and $x - y + 2 \geq 0$
(B) $2y \leq x^2 - 8$ and $x - y + 2 \geq 0$
(C) $2y \geq x^2 - 8$ and $x - y + 2 \leq 0$
(D) $2y \leq x^2 - 8$ and $x - y + 2 \leq 0$

10. For the triangle below, which of the following statements is true?



(A) $x = \sin 43° . \dfrac{14}{\sin 105°}$

(B) $x = \sin 31° . \dfrac{14}{\sin 43°}$

(C) $x = \sin 31° . \dfrac{10}{\sin 43°}$

(D) $x = \sin 74° . \dfrac{10}{\sin 43°}$

End of Section I

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 40 minutes for this section
Begin each question on a NEW SHEET of paper.

In questions 11 – 16 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 – Use a SEPARATE writing booklet. (15 marks)

a. Solve $|x-7| \geq 3$ **2**

b. Factorise $x^3 - 5x$ completely **2**

c. Simplify $\sqrt{5} + \sqrt{20} - 2\sqrt{45}$ **2**

d. Integrate

i. $2\sin 6x$ **2**

ii. $2e^x + \frac{1}{x}$ **2**

e. Solve simultaneously **2**

$$3x - 2y = 5$$
$$x + 3y = 9$$

f. Find the equation of the tangent to the curve $y = x^2 - 5x + 1$ at the point $(1, -3)$. **2**

g. Express $0.\dot{1}\dot{3}$ as a fraction, showing necessary working. **1**

End of Question 11

Question 12 – Use a SEPARATE writing booklet. (15 marks)

a. Evaluate **2**

$$\lim_{x \to 0} \frac{3x}{\sin 2x}$$

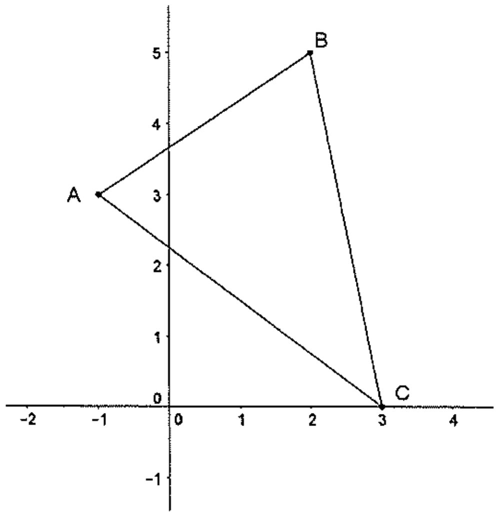
b. Differentiate

i. $e^x \cos x$ **2**

ii. $(1 - \sin 2x)^4$ **2**

c. Find the discriminant of the quadratic equation, $3x^2 + 6x - 1 = 0$, and hence describe the roots of the equation. **2**

d. In the diagram below, the three points $A(-1, 3)$, $B(2, 5)$ and $C(3, 0)$ form the vertices of a triangle.



i. Find the length of the interval BC. **1**

ii. Show that the equation of BC is $5x + y - 15 = 0$. **1**

iii. Find the perpendicular distance from A to the line BC. **1**

iv. Hence, find the area of the triangle ABC. **1**

Question 12 continues on next page

e. An archer fires three arrows at a target. The probability of any single arrow hitting the target is $\frac{4}{5}$.

Find the probability that:

i. The first arrow hits and the next two miss. **1**

ii. The archer hits the target exactly once. **1**

iii. The archer hits the target at least once. **1**

End of Question 12

Question 13 – Use a SEPARATE writing booklet. (15 marks)

a. Find the equation of the normal to the curve $y = 2\cos x$ when $x = \frac{\pi}{4}$. **3**

b. Explain why **1**

$$\int_{-3}^{3} (x^2 + 4)dx = 2\int_{0}^{3} (x^2 + 4)dx$$

c. Given that $\log_5 6 = 1.11$ and $\log_5 3 = 0.68$, find:

i. $\log_5 18$ **1**
ii. $\log_5 2$ **1**
iii. $\log_5 20$ **1**

d. Solve **3**

$$(x+3)^2 + 5(x+3) - 14 = 0$$

e. Given that the quadratic equation $2x^2 - 5x + 6 = 0$ has roots of α and β, find the values of:

i. $\alpha + \beta$ **1**

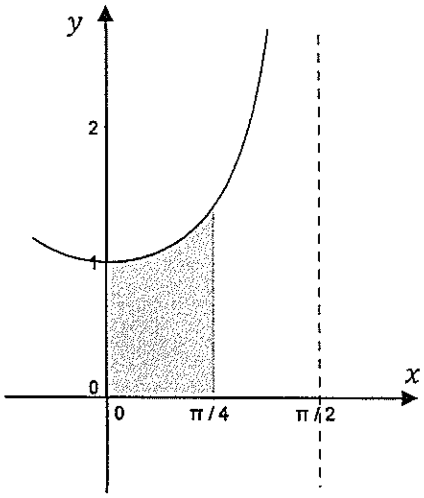
ii. $\frac{1}{\alpha} + \frac{1}{\beta}$ **2**

f. Given that $\tan\theta = -\frac{1}{4}$ and θ is a reflex angle, find the value of $\sin\theta$. **2**

End of Question 13

Question 14 – Use a SEPARATE writing booklet. (15 marks)

a. Find the volume of the solid formed when the area between the curve $y = \sec x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{4}$, is rotated about the x-axis. **3**



b. i. Express $y = \log_3 x$ with a base of e. **1**

ii. Hence, find $\frac{d}{dx}(\log_3 x)$. **1**

c. Given the cubic function, $y = x^3 - 3x^2 + 2x$

i. Factorise completely to find the x-intercepts. **2**

ii. Find the location and nature of the stationary points. **3**

iii. Show that the function has a point of inflexion at $(1, 0)$. **1**

iv. Sketch the cubic function, showing all relevant information. **2**

d. A 2L bucket is being filled with water at a rate, $R = 20 + 2t \text{ cm}^3\text{s}^{-1}$.

i. If it initially contains 500 cm^3 of water, find an expression for the volume of water in the bucket. **1**

ii. Show that the bucket is full after 30 seconds. **1**

End of Question 14

Question 15 – Use a SEPARATE writing booklet. (15 marks)

a. On a small island, an ecologist monitors a colony of rabbits. When first assessed, there are 120 rabbits.
The number of rabbits, $N(t)$, after t years is given by

$$N(t) = 120e^{kt}.$$

i. After 4 years there are 280 rabbits.
Show that $k = 0.2118$, correct to four decimal places. **1**

ii. How many rabbits are in the colony when $t = 6$? **1**

iii. What is the rate of change of the number of rabbits per year when $t = 6$? **1**

iv. How long does it take for the number of rabbits to increase from 120 to 500? **2**

b. Evaluate **3**

$$\int_0^2 \frac{x}{3x^2 + 4}\,dx$$

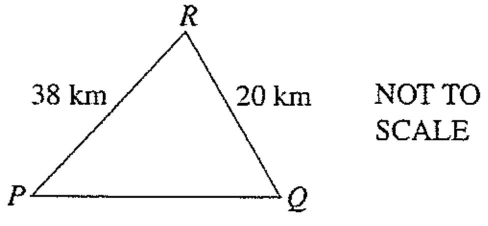
c. Consider the function $y = \ln(x + 1)$ for $x > -1$.

i. Sketch the function, showing its essential features. **1**

ii. Use Simpson's rule with three function values to find an approximation to **2**

$$\int_0^4 \ln(x + 1)\,dx.$$

d. In the diagram below, the point Q is due east of P. The point R is 38 km from P and 20 km from Q. The bearing of R from Q is 315°.



i. What is the size of $\angle PQR$? **1**

ii. What is the bearing of R from P? **3**

End of Question 15

Question 16 – Use a SEPARATE writing booklet. (15 marks)

a. A particle moves along a straight line so that its displacement, x metres, from a fixed point O is given by $x = 1 + 3\cos 2t$, where t is measured in seconds.

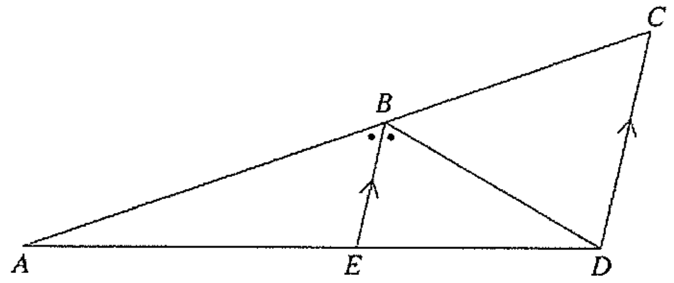
i. What is the initial displacement of the particle? **1**

ii. Sketch the graph of x as a function of t for $0 \leq t \leq \pi$. **2**

iii. Hence, or otherwise, find when the particle first comes to rest after $t = 0$. **1**

iv. Find a time when the particle reaches its maximum speed.
What is this speed? **2**

b. In the diagram below, $BE \parallel CD$ and BE bisects $\angle ABD$.

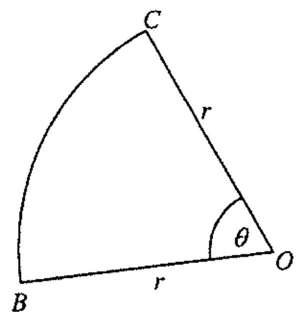


NOT TO SCALE

i. Explain why $\angle EBD = \angle BDC$. **1**

ii. Prove that ΔBCD is isosceles. **2**

c. The diagram below shows a sector OBC of a circle with centre O and radius r cm. The arc BC subtends an angle θ radians at O.



i. Show that the perimeter of the sector is $P = r(2 + \theta)$ **1**

ii. Given that the perimeter of the sector is 36 cm, show that its area is given by **2**

$$A = \frac{648\theta}{(\theta + 2)^2}$$

iii. Hence, show that the maximum area of the sector is 81 cm^2. **3**

End of paper

2017 TRIAL – HAHS – MAT.

SOLUTIONS

SECTION I

1.	B	6.	D
2.	D	7.	~~A~~ B
3.	C	8.	B
4.	D	9.	A.
5.	A	10.	C

SECTION II

a) $|x-7| \geq 3.$ — 1 – resolving abs value

$x-7 \leq 3, \quad x-7 \geq 3$ — 1 – corrections

$x \leq 4 \quad , x \geq 10$

b) $x^3 - 5x = x(x^2-5)$ — 1 – common factor

$= x(x+\sqrt{5})(x-\sqrt{5})$ — 1 – diff 2sq.

c) $\sqrt{5} + \sqrt{20} - 2\sqrt{45} = \sqrt{5} + 2\sqrt{5} - 2.3\sqrt{5}$ — 1 – simplify to $\sqrt{5}$'s

$= -3\sqrt{5}$ — 1 – sum

d) i. $\int 2\sin 6x = 2.\dfrac{(-\cos 6x)}{6} + c$ — 1 – integral, 1 – +c

$= -\dfrac{1}{3}\cos 6x + c$

ii. $\int \left(2e^x + \dfrac{1}{x}\right)dx = 2e^x + \ln x + c$ — 1 – $\int e^x$, 1 – $\int \frac{1}{x}$

e) $3x - 2y = 5$ — i — 1 – progress

$x + 3y = 9$ — ii — 1 – answers

ii × 3

$3x + 9y = 27$ — iii

iii – i $\quad 11y = 22$

$y = 2$

Sub into ii

$x + 3(2) = 9$

$x = 3$

test in i

$LHS = 3(3) - 2(2)$

$= 9 - 4$

$= 5 = RHS$

$\therefore x = 3, y = 2$ is the solution

f) $y = x^2 - 5x + 1$

$y' = 2x - 5$ — 1 – $\frac{d}{dx} \to m$

at $(1, -3)$

$m = 2(1) - 5$ — 1 – equation

$= -3$

by point-gradient,

$y - (-3) = -3(x-1)$

$\therefore 3x + y = 0$ is the equation of the tangent.

g) let $n = 0.\dot{1}\dot{3}$ — i

$100n = 13.\dot{1}\dot{3}$ — ii — 1 – correct process.

ii – i

$99n = 13$

$\therefore n = \dfrac{13}{99}$

Question 12

a) $\lim_{x\to 0} \frac{3x}{\sin 2x} = \frac{3}{2} \lim_{x \to 0} \frac{2x}{\sin 2x}$ — 1 – numerator of 2x

$= \frac{3}{2} \times 1$ — 1 – ans.

$= \frac{3}{2}.$

b) ~~$(1-\sin 2x$~~

i) $\frac{d}{dx}\left(e^x \cos x\right) = e^x \cos x + e^x(-\sin x)$ — 1 – correct $\frac{d}{dx}$; 1 – product rule

$= e^x(\cos x - \sin x)$

ii) $\frac{d}{dx}(1-\sin 2x)^4 = 4(1-\sin 2x)^3(-2\cos 2x)$ — 1 – chain rule; 1 – ans

c). $3x^2+6x-1=0$ — 1 – Δ; 1 – description

$\Delta = 6^2 - 4(3)(-1)$

$= 36 + 12$

$= 48$

∴ roots are real and distinct.

d) i) B(2,5), C(3,0) — 1-ans

by two-point formula.

$y - 0 = \frac{5-0}{2-3}(x-3)$

$y = -5(x-3)$

∴ $5x+y-15=0$ is the equation of BC.

ii) $d_{BC} = \sqrt{(3-2)^2+(0-5)^2}$

$= \sqrt{1+25}$ — 1-ans

$= \sqrt{26}.$

iii) d. A(-1,3) BC. $5x+y-15=0$

$d = \frac{|5(-1)+1(3)-15|}{\sqrt{5^2+1^2}}$ — 1-ans

$= \frac{|-17|}{\sqrt{26}} = \frac{17}{\sqrt{26}}$

iv) $A = \frac{1}{2}bh$ — 1-ans

$= \frac{1}{2}\sqrt{26} \cdot \frac{17}{\sqrt{26}}$

$= \frac{17}{2}$ units^2

e) i) $P(h.t) = \frac{4}{5}$, $P(miss) = \frac{1}{5}$

$P(H, M, M) = \frac{4}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

$= \frac{4}{125}$

ii) $P(\text{one } H) = \frac{4}{125} \times 3$

$= \frac{12}{125}$

iii) $P(\text{at least one } H) = 1 - P(3M)$

$= 1 - \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

$= \frac{124}{125}$

Question 13

a) $y = 2\cos x$

$y' = -2\sin x$

at $x = \frac{\pi}{4}$

$y = 2\cos\frac{\pi}{4}$

$= 2 \cdot \frac{1}{\sqrt{2}}$

$= \sqrt{2}$

$g\, M_T = -2\sin\frac{\pi}{4}$

$= -2 \cdot \frac{1}{\sqrt{2}}$

$= -\sqrt{2}$

$M_N = \frac{-1}{M_T}$

$= \frac{\sqrt{2}}{2}$

by point gradient

$y - \sqrt{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$8y - 8\sqrt{2} = 4\sqrt{2}x - \pi\sqrt{2}$.

$\therefore 4\sqrt{2}x - 8y + \sqrt{2}(8 - \pi) = 0$ is the equation of normal

b) $f(x) = x^2 + 4$ is an even function and the integral has opposite & equal bounds

c) i) $\log_5 6 = 1.11$ $\log_5 3 = 0.68$

$\log_5 18 = \log_5 3 + \log_5 6$

$= 0.68 + 1.11$

$= 1.79$

ii) $\log_5 2 = \log_5 6 - \log_5 3$

$= 1.11 - 0.68$

$= 0.43$

iii) $\log_5 20 = \log_5 5 + \log_5 4$

$= \log_5 5 + 2\log_5 2$

$= 1 + 2 \times 0.43$

$= 1.86$

d) $(x+3)^2 + 5(x+3) - 14 = 0$

let $u = x+3$

$u^2 + 5u - 14 = 0$ — 1 - reduce to quad

$(u+7)(u-2) = 0$

$\therefore u = -7, 2$ — 1 - factorise

1 - solve

$x + 3 = -7, 2$

$\therefore x = -10, -1$

e) $2x^2 - 5x + 6 = 0$

i) $\alpha + \beta = -\frac{(-5)}{2}$

$= \frac{5}{2}$ — 1 - ans

ii) $\alpha\beta = \frac{6}{2}$

$= 3$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$ — 1 - sum fraction

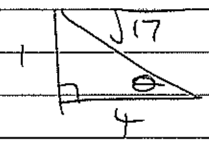
$= \frac{\frac{5}{2}}{3}$ — 1 - ans

$= \frac{5}{6}$

f) $\tan\theta = -\frac{1}{4}$

θ is reflex, so lies in 3^{rd} or 4^{th} quadrant

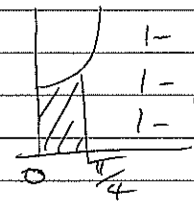
$\tan\theta < 0$ so θ in 3^{rd} quadrant



$\therefore \sin\theta = -\frac{1}{\sqrt{17}}$

Question 14.

a) $y = \sec x$



1 - $\pi \int y^2 dx$

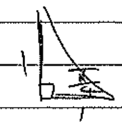
1 - $\int \sec^2 dx$

1 - ans

$V = \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$

$= \pi \left[\tan x \right]_0^{\frac{\pi}{4}}$

$= \pi \left(\tan\frac{\pi}{4} - \tan 0 \right)$



$= \pi (1 - 0)$

$= \pi$ units^3.

b) i) $y = \log_3 x$. — 1 - ans

$= \frac{\ln x}{\ln 3}$

ii) $\frac{d}{dx}(\log_3 x) = \frac{d}{dx}\left(\frac{\ln x}{\ln 3}\right)$ — 1 - ans

$= \frac{1}{\ln 3} \frac{d}{dx}(\ln x)$

$= \frac{1}{\ln 3} \cdot \frac{1}{x} = \frac{1}{x \ln 3}$

iii) $y'' = 6x - 6$ 1 – shown

for inflexion, $y'' = 0$

$\therefore 6x - 6 = 0$

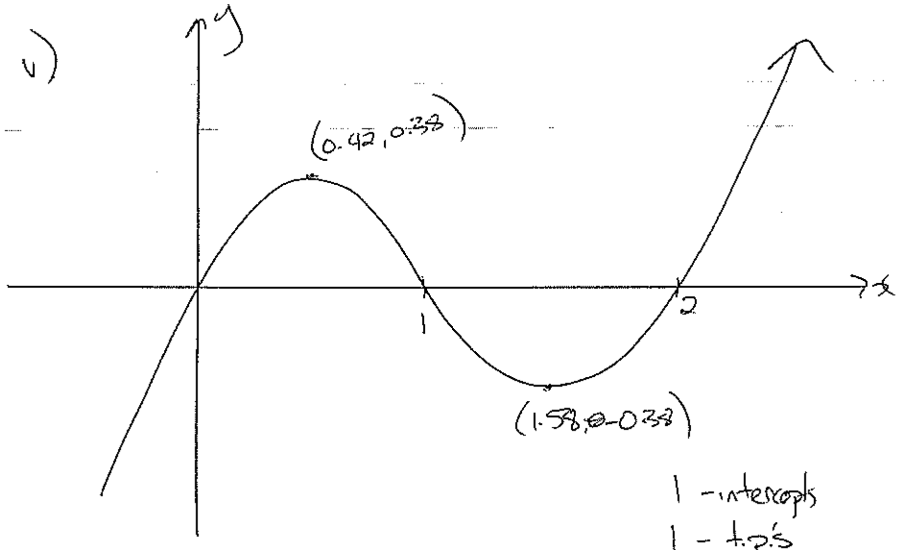
$x = 1$, $y = 1^3 - 3(1)^2 + 2(1)$
$= 0$

test for inflexion

x	0	1	2
y''	−6	0	6
	<0		>0

as concavity changes, inflexion at $(1, 0)$

v)



1 – intercepts
1 – t.p.'s + inflexion at correct location

c) $y = x^3 - 3x^2 + 2x$

i). $y = x(x^2 - 3x + 2)$ 1 – factorised
$= x(x-2)(x-1)$ 1 – x-ints

for x-int, let $y = 0$

$x(x-1)(x-2) = 0$

$\therefore x = 0, 1, 2$. are intercepts

ii) $y' = 3x^2 - 6x + 2$. 1 – x-values

for stationary points, $y' = 0$ 1 – concavities
1 – statement

$\therefore 3x^2 - 6x + 2 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$

$= \frac{6 \pm \sqrt{12}}{6}$

$= \frac{3 \pm \sqrt{3}}{3}$ $(\doteqdot 1.58, 0.42)$

$= 1 \pm \frac{\sqrt{3}}{3}$

$y'' = 6x - 6$

for $x = \frac{3 - \sqrt{3}}{3}$ (0.42)

$y \doteqdot 0.42^3 - 3(0.42)^2 + 2(0.42)$
$= 0.38$ $2\sqrt{3}/9$

$y'' \doteqdot 6(0.42) - 6 < 0$

$\therefore$ concave down

for $x = \frac{3 + \sqrt{3}}{3}$ (1.58)

$y \doteqdot 1.58^3 - 3(1.58)^2 + 2(1.58)$
$= -0.38$

$y'' = 6(1.58) - 6 > 0$

$\therefore$ concave up

$\therefore$ stationary points at $(0.42, 0.38)$ and $(1.58, -0.38)$ are a local maximum and local minimum, respectively.

d) $R = 20 + 2t \text{ cm}^3 s^{-1}$

i) $t=0, V=500$

$V = \int R \, dt$

$= \int (20+2t) \, dt$

$= 20t + t^2 + c.$

when $t=0, V=500$

$500 = 20(0) + 0^2 + c$

$c = 500$ — 1-ans

so $V = 20t + t^2 + 500 \text{ cm}^3$

ii) at $t=30$

$V = 20(30) + 30^2 + 500$

$= 600 + 900 + 500$ — 1-shown

$= 2000 \text{ cm}^3$

$= 2L$

∴ bucket is full after 30sec.

Question 15

a) $N(t) = 120e^{kt}$, $t=0, N(t)=120$

i) at $t=4$, $N(4)=280$

$\therefore 280 = 120e^{4k}$

$e^{4k} = \frac{280}{120}$ — 1-shown

$= \frac{7}{3}$

$4k = \ln(\frac{7}{3})$

$k = \frac{1}{4}\ln(\frac{7}{3})$

$= 0.2118244651$

$= 0.2118$ (4dp)

ii) $N(t) = 120e^{0.2118t}$. — 1-ans

$N(6) = 120e^{0.2118 \times 6}$

$= 427.6442861$ (or 427.7070649)

∴ there are 427 rabbits when $t=6$

iii) $\frac{dN(t)}{dt} = 120e^{0.2118t}(0.2118)$

$\frac{d}{dt}N(6) = 120e^{0.2118(6)}(0.2118)$ — 1-ans

$= 90.59882023$ (or 90.5750598)

at $t=6$, the rate of change ~~of~~ is 90.60 rabbits/year. (2dp)

ii) $N(t) = 120e^{0.2118t}$

for $N(t) = 500$

$120e^{0.2118t} = 500$

$0.2118t = \ln(500/120)$

$t = \dfrac{\ln(500/120)}{0.2118}$

$= \ldots$

$= 6.737259339 \quad (\sim 6.74 \text{ yrs})$

it will take 6·74 years to reach 500 rabbits

[in the '7th year]

1 - progress
1 - ans

b) $\displaystyle\int_0^2 \frac{x}{3x^2+4}\,dx = \frac{1}{6}\int_0^2 \frac{6x}{3x^2+4}\,dx$

$= \frac{1}{6}\left[\ln(3x^2+4)\right]_0^2$

$= \frac{1}{6}\left(\ln(3(2)^2+4) - \ln(3(0)^2+4)\right)$

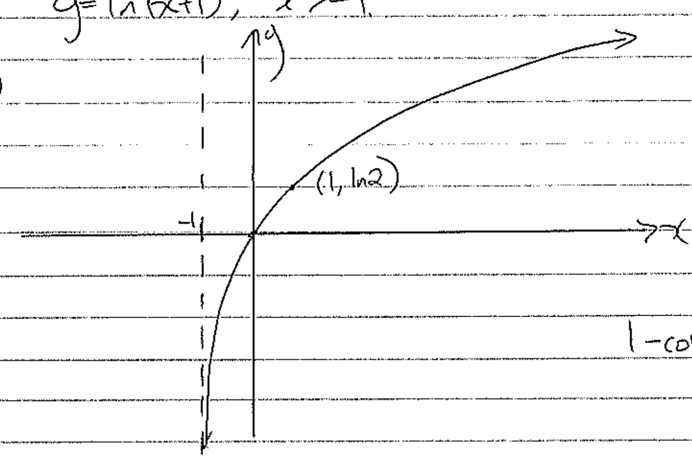
$= \frac{1}{6}(\ln 16 - \ln 4)$ 1 - $\times \frac{6}{6}$

$= \frac{1}{6}\ln 4$ 1 - $\int \rightarrow$ loge.

$= 0.2310490602.$ 1 - value.

c). $y = \ln(x+1),\ x > -1$

i)



1 - correct + labelled

ii)

x	0	2	4
$\ln(x+1)$	$\ln 1$	$\ln 3$	$\ln 4$.

$f(a) + 4f(\frac{a+b}{2}) + f(b)$

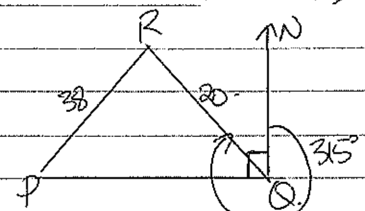
$\displaystyle\int_0^4 \ln(x+1)\,dx \doteq \frac{4-0}{6}\left[\ln 1 + 4\ln 3 + \ln 5\right]$

$= \frac{2}{3}\left[\ln 3^4 + \ln 5\right]$

$= \frac{2}{3}\ln 324$ → 1 - use of simpsons

$= 4.00259\ldots378$ 1 - approx

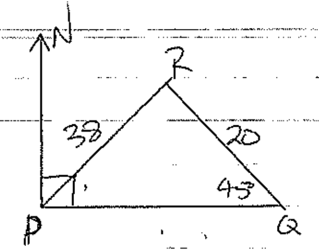
d) i)



$\angle PQR = 315 - 270$
$= 45°.$

1 ans

ii)



by sine rule.

$$\frac{\sin P}{20} = \frac{\sin 45}{38}$$

$$P = \sin^{-1}\left(\frac{20\sin 45}{38}\right)$$

$$= 21.84898225$$

$$= 21°\,50'\,56.34''$$

$$\doteqdot 22° \text{ (nearest degree)}$$

1 – use of sine rule
1 – ∠P
1 – Bearing.

$$\angle NPR = 90 - 22$$
$$= 68°$$

Bearing of R from P is 068°T

Question 16

a) $x = 1 + 3\cos 2t$.

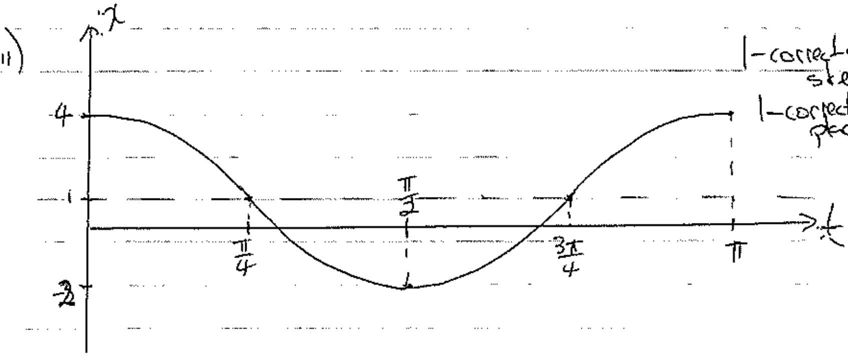
i) when $t = 0$

$$x = 1 + 3\cos 2(0)$$
$$= 1 + 3(1)$$
$$= 4$$

1 – ans

∴ initial displacement is 4 units

ii)



1 – correct period sketch
1 – correct x-axis placement

iii) Particle comes to rest at $\frac{\pi}{2}$ sec. 1 – ans

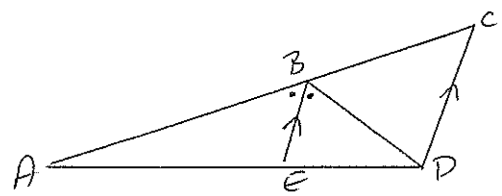
iv) max speed at $t = \frac{\pi}{4}$

$$\dot{x} = -6\sin 2t$$
$$= -6\sin 2\left(\tfrac{\pi}{4}\right)$$
$$= -6\sin\tfrac{\pi}{2}$$
$$= -6$$

1 – time
1 – speed

max speed is $6\,ms^{-1}$

b)



i) alternate angles in parallel lines, EB∥DC, are equal – reason

ii) $\angle BCD = \angle ABE$ (corresponding angles in parallel lines, EB∥DC.)

$= \angle EBD$ (BE bisects ∠ABD)

$= \angle BDC$ (above)

~~BCD is isosceles~~

1 – ∠BCD = ∠ABE + reason.
1 – complete proof

$\therefore \triangle BCD$ is isosceles (pair of equal angles)

c) i) $P = r + r + r\theta$
$= 2r + r\theta$
$= r(2+\theta)$ — shown

ii) $P = 36$
$\therefore r(2+\theta) = 36 \Rightarrow r = \dfrac{36}{2+\theta}$

$A = \frac{1}{2} r^2 \theta$
$= \frac{1}{2} \times \left(\frac{36}{2+\theta}\right)^2 \theta$
$= \dfrac{1296\theta}{2(2+\theta)^2}$
$= \dfrac{648\theta}{(2+\theta)^2}$

1 – expression for r
1 – subst. into A and show

iii) $\dfrac{dA}{d\theta} = \dfrac{(2+\theta)^2 . 648 - 648\theta . 2(2+\theta)}{(2+\theta)^4}$ — quotient rule.
$= \dfrac{648(2+\theta)[(2+\theta) - 2\theta]}{(2+\theta)^4}$
$= \dfrac{648(2+\theta)(2-\theta)}{(2+\theta)^3}$
$= \dfrac{648(2-\theta)}{(2+\theta)^3}$

1 – use of quotient rule
1 – value of θ.
1 – Max shown

for max area, $\dfrac{dA}{d\theta} = 0$

$\therefore 648(2-\theta) = 0$
$\theta = 2.$

$A = \dfrac{1}{2} \cdot \dfrac{648(2)}{(2+2)^2}$
$= 81$

test gradient for max point,

θ	0	2	3
$\frac{dA}{d\theta}$	162	0	−5.184
	/	—	\

Hence, maximum area of sector is $81 cm^2$