Hunters Hill High School

Student Number

Hunters Hill High School

Year 12 Mathematics Advanced

Trial Examination, 2020

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and / or calculations

Total Marks: 100

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6–24)

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 If $f(x) = 5x^3 + 3$, what is the value of $f'(-1)$?

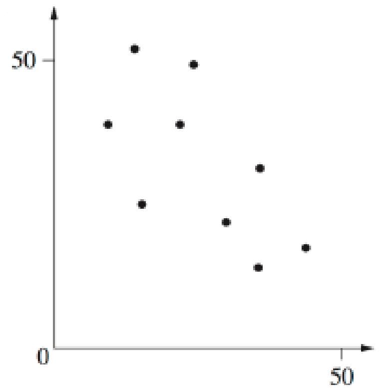
A. -2

B. 18

C. 15

D. -12

2 The graph shows a scatter plot for a set of data.



Which of the following values is the most appropriate Pearson's correlation coefficient of this set of data?

A. -1

B. 0.35

C. -0.35

D. -0.63

3 What is the amplitude, period and phase of $f(x) = -2\cos(5x - \pi)$?

A. Amplitude is -2, period is $\frac{2\pi}{5}$, and phase shift is $\frac{\pi}{5}$ to the left

B. Amplitude is 2, period is 5, and phase shift is π to the left

C. Amplitude is 2, period is $\frac{2\pi}{5}$, and phase shift is $\frac{\pi}{5}$ to the right

D. Amplitude is 2, period is 5, and phase shift is $\frac{\pi}{5}$ to the right

4 A and B are events from a sample space such that $P(A) = p$, where $p > 0$, $P(B|A) = m$ and $P(B|A') = n$. A and B are independent events when

A. $m = n$

B. $m = 1 - p$

C. $m + n = 1$

D. $m = p$

5 Given $f(x) = \ln(2x - 1)$ and $g(x) = x + 2$, the domain of $f\big(g(x)\big)$ is

A. $x \in \left[-\frac{3}{2}, \infty\right)$

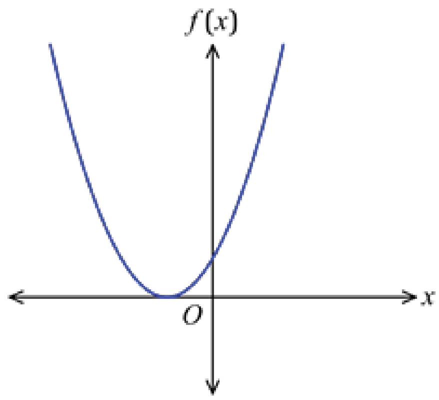
B. $x \in \left[\frac{1}{2}, \infty\right)$

C. $x \in \left(-\frac{3}{2}, \infty\right)$

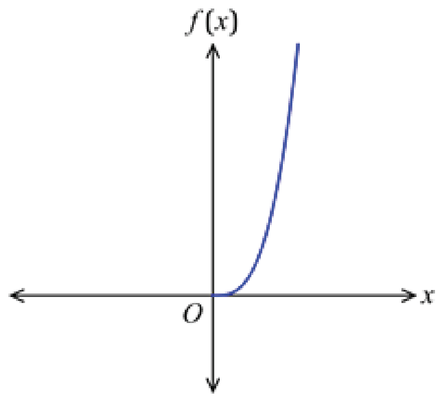
D. $x \in (-2, 0)$

6 Which of the following graphs could NOT represent a probability density function (x) ?

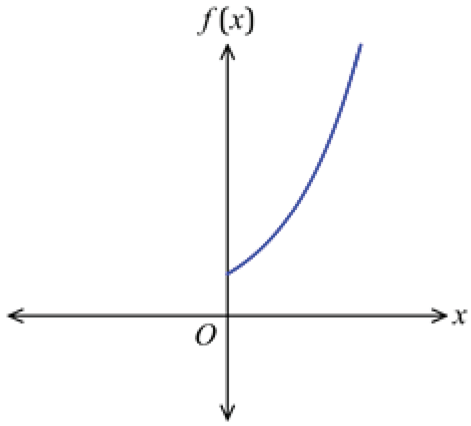
A.



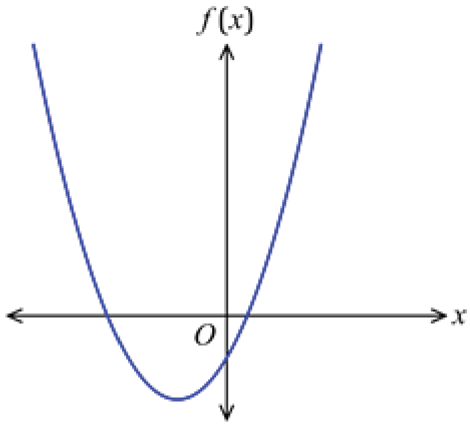
B.



C.



D.



7 The length of a type of ant is approximately normally distributed with a mean of 5.1 mm and a standard deviation of 1.2 mm. A standardised ant length of $z = -0.5$ corresponds to an actual ant length of

A. 13 mm

B. 1 mm

C. 4.5 mm

D. 2.55 mm

8 A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres. At a certain time $\frac{d^2x}{dt^2} = -2 \text{ ms}^{-2}$ and $\frac{dx}{dt} = 1 \text{ ms}^{-1}$.

Which statement describes the motion of the particle at that time?

A. The particle is moving to the right with increasing speed.

B. The particle is moving to the left with increasing speed.

C. The particle is moving to the right with decreasing speed.

D. The particle is moving to the left with decreasing speed.

9 A trigonometric function $f(x)$ satisfies the condition $\int_0^{\frac{\pi}{2}} f(x)dx \neq \int_{\frac{\pi}{2}}^{\pi} f(x)dx$

Which function could be $f(x)$?

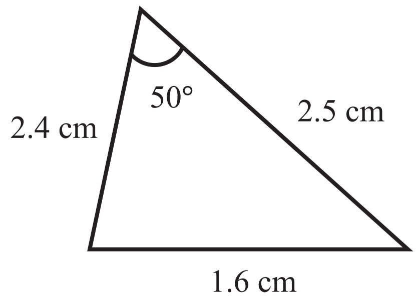
A. $f(x) = \sin x$

B. $f(x) = \cos(2x)$

C. $f(x) = \sin(2x)$

D. $f(x) = \cos(4x)$

10 Which of the following would give the correct value for the area of the triangle?



A. $\frac{1}{2} \times 1.6 \times 2.4 \times \sin 50°$

B. $\frac{1}{2} \times 2.4 \times 2.5 \times \sin 50°$

C. $\frac{1}{2} \times 1.6 \times 2.4 \times 50°$

D. $\frac{1}{2} \times 2.4 \times 2.5 \times 50°$

Question 11 (2 marks)

Evaluate $\sum_{n=1}^{\infty} 2\left(\frac{1}{5}\right)^{n}$ **2**

Question 12 (3 marks)

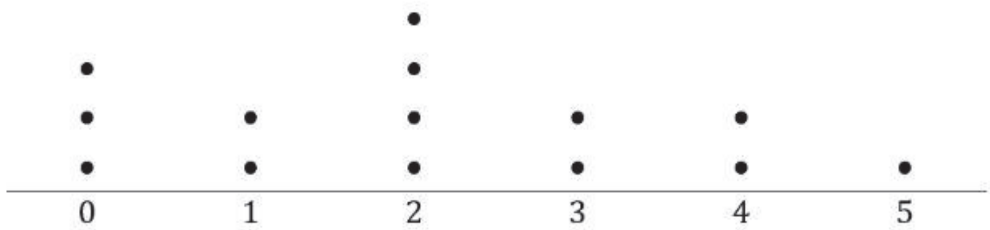
Find the equation of the normal to the curve $f(x) = \ln(x - 1) + 2$ at the point $(2, 0)$. **3**

Question 13 (2 marks)

Describe the transformations applied to $y = x^2$ in order to give the transformed function of $y = (4x + 2)^2$. **2**

Question 14 (3 marks)

A sample of 14 people were asked to indicate the time (in hours) they had spent watching television on the previous night. The results are displayed in the dot plot below: **3**



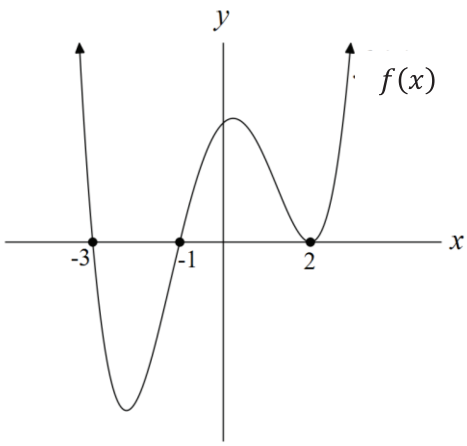
Find the mean and sample standard deviation of these times. Give your answers correct to one decimal place.

Question 15 (2 marks)

Differentiate $e^{x^2 \cos x}$ **2**

Question 16 (2 marks)

The diagram shows the graph of $f(x)$. **2**



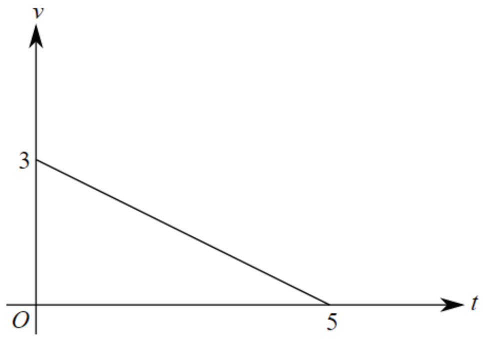
Sketch $f'(x)$ below, showing all critical features.

Question 17 (2 marks)

If $\cos\alpha = -\frac{4}{5}$ and $\sin\alpha < 0$, find the exact value of $\tan\alpha$. **2**

Question 18 (5 marks)

The velocity of a particle moving along the x-axis at v metres per second at t seconds, is shown in the graph below:



Initially, the displacement x is equal to 13 metres.

(a) Write an equation that describes the displacement, x, at time t seconds. **3**

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(b) Draw a graph that shows the displacement of the particle, x metres from the origin, at a time t seconds between $t = 0$ and $t = 6$. Label the coordinates of the endpoints of your graph. **2**

Question 19 (3 marks)

A bag contains 9 blue lollies and 7 green lollies.

Georgia takes out a lolly from the bag without looking and eats it. She then takes out another lolly without looking and eats it.

(a) What is the probability of Georgia choosing a blue lolly in her first selection? **1**

(b) By drawing a tree diagram, or otherwise, find the probability that Georgia eats two lollies of different colours. **2**

Question 20 (2 marks)

If $y = xe^{2x}$, prove that $\dfrac{dy}{dx} - 2y = e^{2x}$ **2**

Question 21 (5 marks)

A continuous random variable X has a function f given by

$$f(x) = \begin{cases} 2x^3 - x + a, & 0 \le x \le 1 \\ 0 \quad\quad\quad\quad , & otherwise \end{cases}$$

(a) Find the value of a which makes $f(x)$ a valid probability density function **2**

(b) Find the expected value and variance of X. **3**

Question 22 (4 marks)

In an arithmetic sequence, the fifth term is 25 and the eighteenth term 181. What is the smallest value of n such that the sum of the first n term in the sequence is at least 27365? **4**

Question 23 (3 marks)

Let $f(x) = \sqrt{x+1}$ for $x \geq 0$

(a) State the domain of $f(x)$ **1**

Question 23 continues on page 14

(b) Let $g(x) = x^2 + 4x + 3$, where $\boldsymbol{x \leq c}$ and $\boldsymbol{c \leq 0}$. Find the largest possible value of c such that the range of $g(x)$ is a subset of the domain of $f(x)$. **2**

Question 24 (10 marks)

Consider the curve $y = \sin x + \cos x$ in the domain $-\pi \leq x \leq \pi$.

(a) Find x-intercepts, express your answer in terms of π. **2**

(b) Find any stationary points, and determine their nature. **3**

Question 24 continues on page 15

(c) Find any points of inflection. **2**

(d) Explain why the points found in (c) are not points of horizontal inflection. **1**

(e) Hence, sketch the graph of $y = \sin x + \cos x$, for $-\pi \leq x \leq \pi$, showing all critical features of the graph. **2**

Question 25 (2 marks)

Prove that $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}=2\sec^2\theta$ **2**

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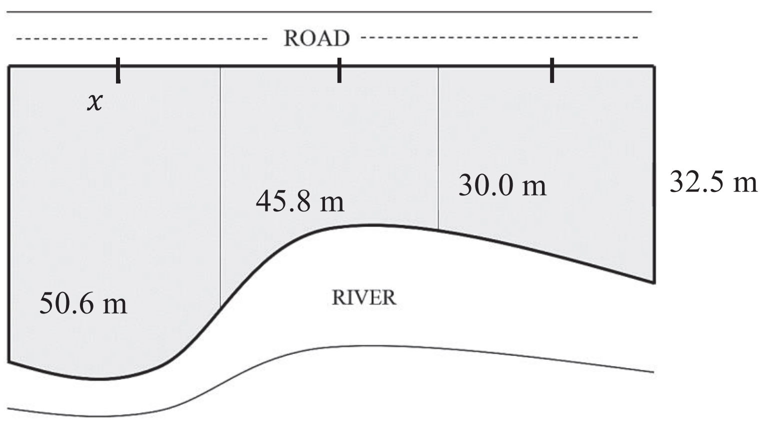
Question 26 (3 marks)

A field (shaded) is bordered on one side by a 120 metre of road and on the other side by a river. Measurements are taken from the road to the river, as shown.



(a) Find the value of x. **1**

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(b) Use the trapezoidal rule to find an approximation to the area of the field. Correct your answer to the nearest square metre. **2**

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Question 27 (3 marks)

The table below shos Bob's scores in PDHPE and Biology examinations, as well as the mean and the standard deviation for each subject. **3**

	PDHPE	**Biology**
μ	65	80
σ	4	3
Score	74	84

Explain which is his strongest subject, show mathematical working.

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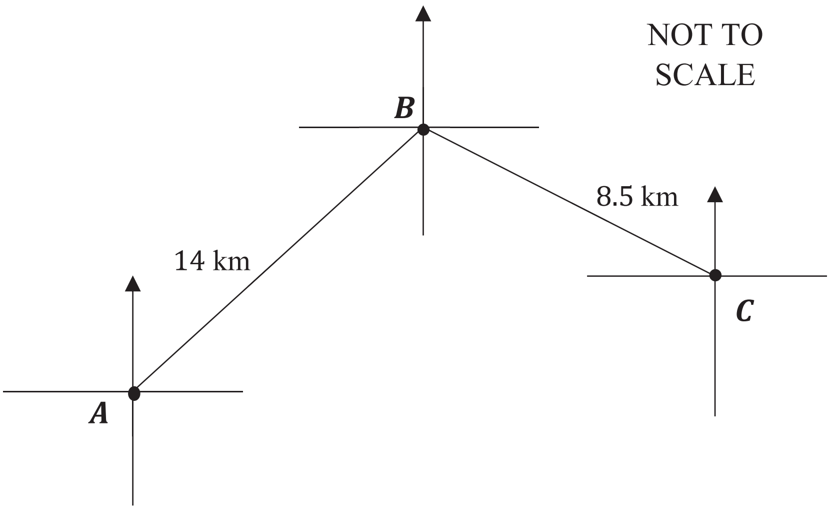
Question 28 (5 marks)

A class is on a hike as part of their sports course. They are given the following directions from starting point A: They are to walk on a bearing of 062° for 14 kilometres to point B. Then, they continue on a bearing of 136° for 8.5 kilometres to point C, before returning to starting point A.



(a) Show that $\angle ABC = 106°$. **2**

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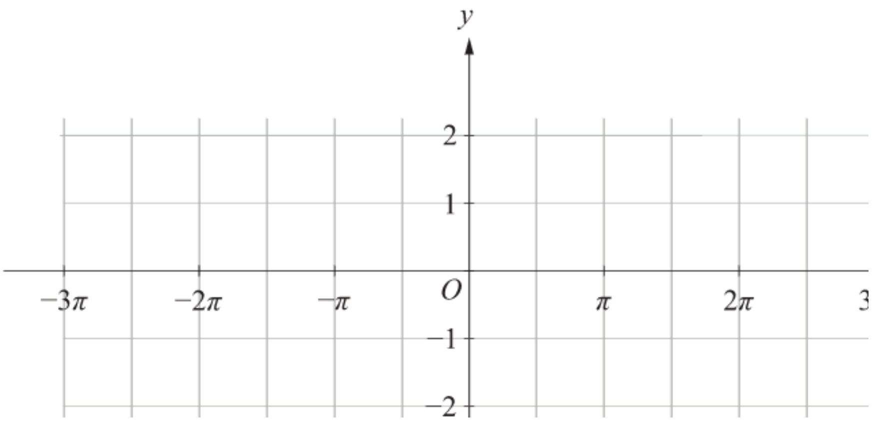
Question 28 continues on page 18

(b) Calculate AC, the distance that the class needs to travel on the final leg of their hike. Give your answer correct to the nearest kilometre. **1**

(c) Find the bearing that the class needs to take from point C to return to starting point A, correct to the nearest degree. **2**

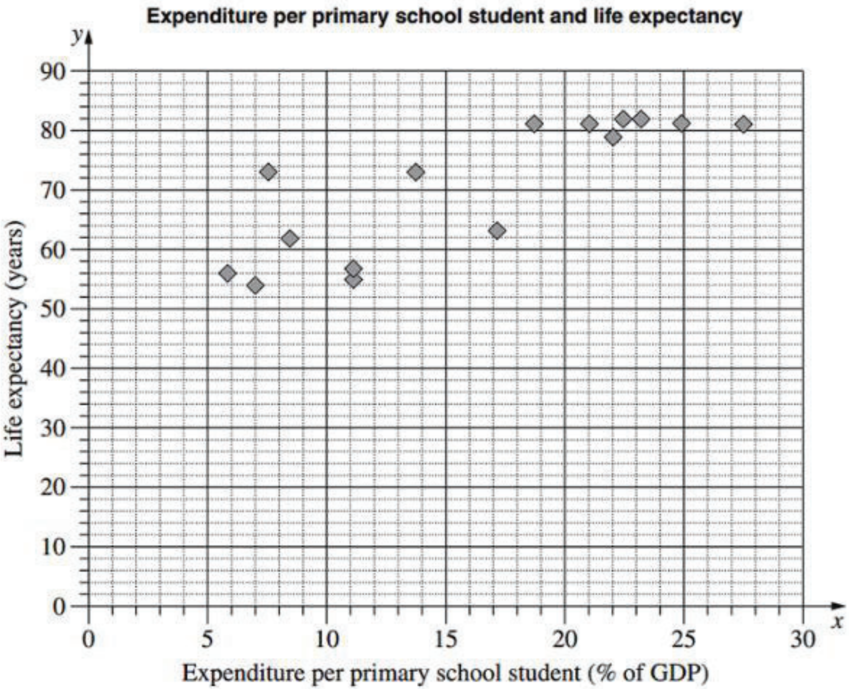
Question 29 (3 marks)

By drawing graphs on the number plane, show how many solution exist for the equation $\cos x = \left|\frac{x-\pi}{4}\right|$ in the domain $(-\infty, \infty)$. **3**



Question 30 (6 marks)

The scatterplot shows the relationship between expenditure per primary school student, as a percentage of a country's Gross Domestic Product (GDP), and the life expectancy in years for 15 countries.



(a) For the data representing expenditure per primary school student, the lower quartile, Q_L is 8.4 and upper quartile, Q_U is 22.5. What is the interquartile range? **1**

(b) Another country has an expenditure per primary school student of 47.6% of its GDP. Would this country be an outlier for this set of data? Justify your answer with calculations. **1**

(c) On the scatterplot, draw the least-squares line of best fit, $y = 1.29x + 49.9$ **2**

Question 30 continues on page 20

(d) Using this line, or otherwise, estimate the life expectancy in a country which has an expenditure per primary school student of 18% of its GDP. **1**

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(e) Why is this line NOT useful for predicting life expectancy in a country which has expenditure per primary school student of 60% of its GDP? **1**

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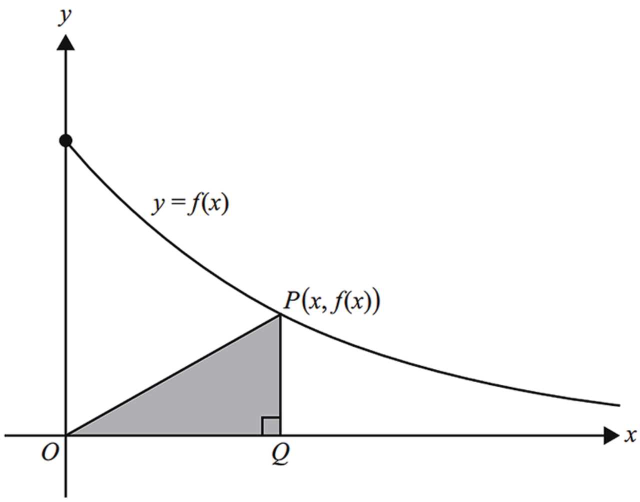
Question 31 (8 marks)

Let $f(x) = 2e^{-\frac{x}{5}}$ for $x \geq 0$. A right-angled triangle OQP has vertex O at the origin, vertex Q on the x-axis and vertex P on the graph of $f(x)$, as shown. The coordinates of P are $(x, f(x))$.



(a) Find the area, A, of the triangle OPQ in terms of x . **1**

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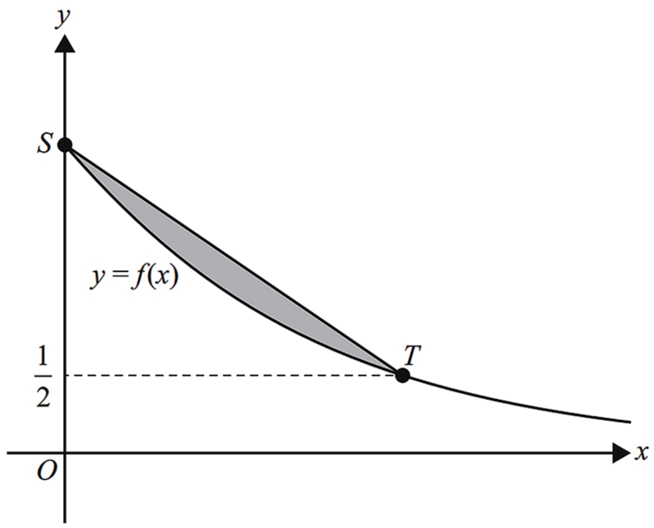
Question 31 continues on page 21

(b) Find the maximum area of triangle OQP and the value of x for which the maximum occurs. **3**

(c) Let S be the point on the graph of $f(x)$ on the y-axis and let T be the point on the graph of $f(x)$ with the y-coordinate $\frac{1}{2}$. Find the area of the region bounded by the graph of $f(x)$ and the line segment ST. **5**



Question 32 (5 marks)

The population of a white-ant colony can be modelled using the equation $P = Ae^{kt}$, where A and k are positive constants and t is time in weeks. Initially, the population is 1000. Two weeks later, the population has increased to become 1500.

(a) Find the value of A and k. Express your answer in exact form. **2**

(b) Find the population after four week, correct to the nearest possible number of white-ants. **1**

Question 32 continues on page 23

(c) White-ants can cause significant structural timber damages. Assume this particular ant colony was found near a timbre structured home, using the model above to explain why the situation is getting worse. Show all mathematical calculations. **2**

Question 33 (6 marks)

The weight, in grams, of beans in a tin is normally distributed with mean μ and standard deviation 7.8. It is known that 10.03% of tins contain less than 200 g.

Table 1: Probability table for calculating the standard normal distribution. The values represent the area to the left of (or less than) the z-score.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952

Question 33 continues on page 23

(a) Use the Probability Table above to find the value of mean value, μ. Correct your answer to 2 decimal places. **2**

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(b) The machine settings are adjusted so that the weight, in grams, of beans in a tin is normally distributed with mean 205 and standard deviation σ. **2**

(i) Given that 98% of tins contain between 200 g and 210 g, draw a normal distribution graph to illustrate this information.

(ii) Use the Probability Table above to find the value of the standard deviation, σ that can be achieved with the new setting. Correct your answer to 2 decimal places. **2**

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End of paper

Section 1

Q1. $f'(-1) = 15x^2$

$= 15(-1)^2$

$= 15$ (C)

Q2. (D) -0.63

Q3. $A = 2$ $f(x) = -2\cos 5(x - \frac{\pi}{5})$

$p = \frac{2\pi}{5}$

shift $= \frac{\pi}{5}$ to the right (C)

Q4. $P(B/A) = P(B) = P(B/A')$

$\therefore m = n$ (A)

Q5. $f(x) = \ln(2x-1)$ $g(x) = x+2$

$f(g(x)) = \ln(2(x+2)) - 1)$

$= \ln 2x+3)$

$2x+3 > 0$ (A)

$2x > -3$

$x > \frac{-3}{2}$

Q6. (D) no -ve pdf.

Q7. $Z = \frac{x - \bar{x}}{\sigma}$

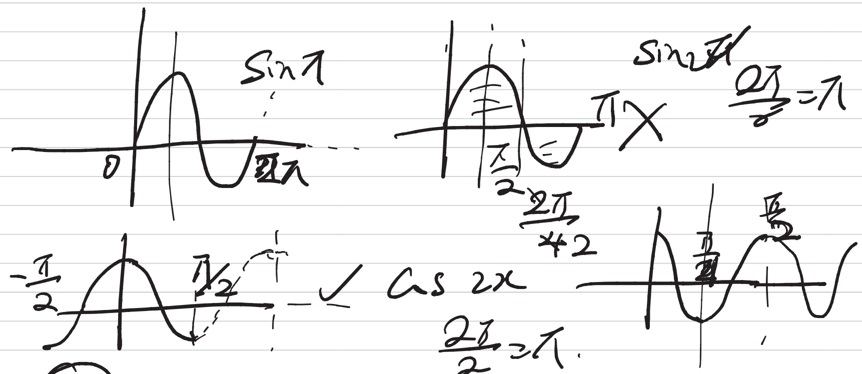
$-0.5 = \frac{x - 5.1}{1.2}$ (C)

$x = 4.5$

Q8 $\frac{d^2x}{dt^2} = -2$, $\frac{dx}{dt} = 1$

$\rightarrow$ (C)

Q9.



(C)

Q10. $A = \frac{1}{2}ab\sin\theta$ (B)

$= \frac{1}{2} \times 2.4 \times 2.5 \times \sin 30^\circ$

section I

Q11. $\sum_{n=1}^{\infty} 2\left(\frac{1}{5}\right)^n$

$= 2 \times \left(\frac{1}{5} + \frac{1}{5}^2 + \frac{1}{5}^3 + \ldots\right)$ (1)

$= 2 \times S_\infty$ where $r = \frac{1}{5}$

$= 2 \times \frac{a}{1-r}$ $a = \frac{1}{5}$

$= 2 \times \frac{\frac{1}{5}}{1-\frac{1}{5}}$ $\frac{1}{5} \times \frac{5}{4}$

$= \frac{1}{2}$ (1)

[2]

Q2 $f(x) = \log_3(x-1) + 2$ at $(2, 0)$

$f'(x) = \frac{1}{x-1}$

at $x = 2$

$f'(2) = \frac{1}{2-1} = 1 = m$ (1)

$m_2 = \frac{-1}{1} = -1$ (1)

[3]

$y - 0 = -1(x-2)$

$y = -x + 2$ (1)

Q13. $y = x^2$ OR $y = (2(2x+1))^2$

$y = (4x+2)^2$ $y = 4(2x+1)^2$

$y = \left(\frac{x+\frac{1}{2}}{\frac{1}{4}}\right)^2$ [2] $\frac{y}{4} = \left(\frac{x+\frac{1}{2}}{\frac{1}{2}}\right)^2$ (1)

dilate horizontally by a factor of $\frac{1}{4}$, translate to the left by $\frac{1}{2}$, (1)

OR $y = \left(\frac{x+2}{\frac{1}{4}}\right)^2$ translate to left by 2, then dilate by $\frac{1}{4}$.

Q14. total point 14.

x	0	1	2	3	4	5
f	3	2	4	2	2	1
p	$\frac{3}{14}$	$\frac{2}{14}$	$\frac{4}{14}$	$\frac{2}{14}$	$\frac{2}{14}$	$\frac{1}{14}$

(1)

$\bar{x} = E(X) = \sum xp$

$= 0 \times \frac{3}{14} + 1 \times \frac{2}{14} + 2 \times \frac{4}{14} + 3 \times \frac{2}{14}$

$+ 4 \times \frac{2}{14} + 5 \times \frac{1}{14}$ (1)

$= 2\frac{3}{14} \simeq 2.1$

$\sigma = \sqrt{Var(X)} = \sqrt{E(X^2) - \mu^2} = \sqrt{\left(0 + \frac{2}{14} + 4 \times \frac{4}{14} + 9 \times \frac{2}{14} + 16 \times \frac{2}{14} + 25 \times \frac{1}{14}\right) - 2.1^2}$

[3]

$= 1.5$ (1)

Q15

let $y = e^{f(x)}$ and $f(x) = x^2 \cos x$

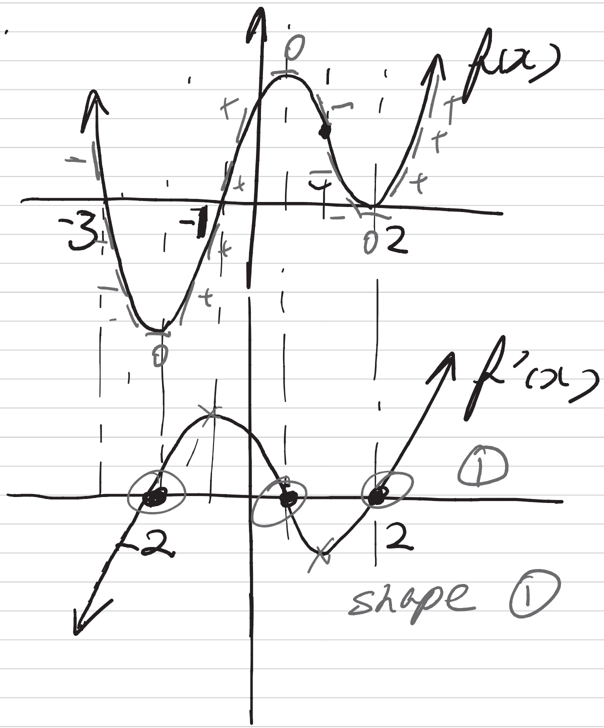
$f'(x) = 2x\cos x - x^2 \sin x$ ①

[2]

$\therefore \frac{dy}{dx} = f'(x) e^{f(x)}$

$= (2x\cos x - x^2 \sin x) e^{x^2 \cos x}$ ①

Q16.



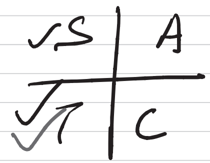
①

shape ①

[2]

Q17

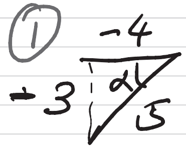
$\cos\alpha = \frac{-4}{5}$ $\sin\alpha < 0$



the angle α is the 3rd quadrant.

[2]

①



$\tan\alpha = \frac{-3}{-4}$

$\tan\alpha = \frac{3}{4}$ ①

Q18. at $t=0$, $v=3$, $x=13$

$t=5$, $v=0$

a) $v - 0 = -\frac{3}{5}(t-5)$

$v = -\frac{3}{5}t + 3$ ①

$x = \int\left(-\frac{3}{5}t + 3\right) dt = -\frac{3}{10}t^2 + 3t + C$ ①

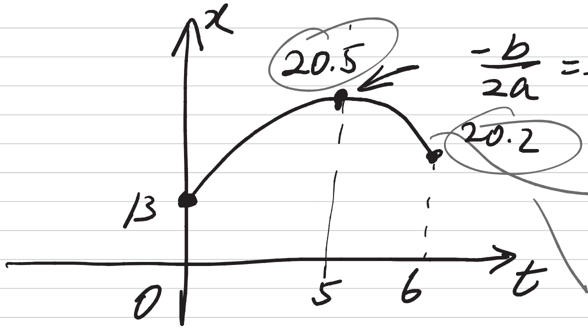
at $t=0$, $x=13$

$\therefore x = 13 = 0 + 0 + C \Rightarrow C = 13$ ①

$\therefore x = \frac{-3}{10}t^2 + 3t + 13$ ①

[3]

b)



$\frac{-b}{2a} = \frac{-3}{\frac{-6}{10}} = 5$

① graph

pts labelled ①

[2]

at $t=6$, $x = 20.2$,
need to check for peak, if not shown 1 mark

Q 19.

a) B : 9 G : 7

total = 16

$P(B) = \frac{9}{16}$ ①

b) $P(BG \text{ or } GB) = \frac{9}{16} \times \frac{7}{15} + \frac{7}{16} \times \frac{9}{15}$ ①

$= \frac{21}{40}$ ① [3]

Q20. $y = xe^{2x}$ show $\frac{dy}{dx} - 2y = e^{2x}$

$\frac{dy}{dx} = e^{2x} + 2xe^{2x}$ ① [2]

LHS: $e^{2x} + 2xe^{2x} - 2xe^{2x}$

$= e^{2x} = RHS$ ①

Q21

a) $F(x) = \int_0^1 2x^3 - x + a = 1$ if pdf.

since $F(x) = 0$ for all other x values

$\left[\frac{x^4}{2} - \frac{1}{2}x^2 + ax\right]_0^1 = 1$ ①

$\frac{1}{2} - \frac{1}{2} + a = 1$ [2]

$a = 1$ ①

b) $E(X) = \mu = \int_0^1 x f(x)\,dx$

$= \int_0^1 x(2x^3 - x + 1)\,dx$

$= \int_0^1 2x^4 - x^2 + x\,dx$ ①

$= \left[\frac{2}{5}x^5 - \frac{1}{3}x^3 + \frac{1}{2}x^2\right]_0^1$

$= \frac{17}{30}$

[3]

$Var(X) = \sigma^2 = \int_0^1 x^2 f(x)\,dx - \mu^2$ ①

$= \int_0^1 (2x^5 - x^3 + x^2)\,dx - \left(\frac{17}{30}\right)^2$

$= \left[\frac{1}{3}x^6 - \frac{1}{4}x^4 + \frac{1}{3}x^3\right]_0^1 - \left(\frac{17}{30}\right)^2$

$= \frac{43}{400}$ ①

Q22 $T_5 = 25$, $T_{18} = 181$

$S_{n} > 27365$ AP.

$T_n = a + (n-1)d$

$25 = a + (5-1)d$

$25 = a + 4d$ — ①

same logic

$181 = a + (18-1)d$

$181 = a + 17d$ — ②

(1)

eqn ② − ①

$156 = 13d$

$d = 12$

sub d into ①

$25 = a + 4 \times 12$

$a = -23$

(1)

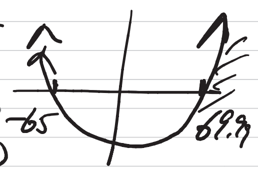
$S_n = \frac{n}{2}(2a + (n-1)d)$

[4]

$S_n = \frac{n}{2}(12n - 58)$

$= 6n^2 - 29n > 27365$

$6n^2 - 29n - 27365 > 0$ (1)

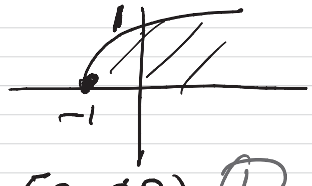


$n = \frac{29 \pm \sqrt{29^2 - 4 \times 6 \times (-27365)}}{2 \times 6}$

$n > 69.99$ ∴ $n = 70$. (1)

Q23

a) $f(x) = \sqrt{x+1}$ for $x \geq 0$



$x + 1 \geq 0$

$x \geq -1$ but

x is also ≥ 0

∴ $x \geq 0$, $x \in [0, \infty)$ (1)

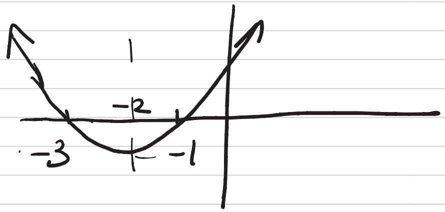
b) $g(x) = x^2 + 4x + 3$ $x \leq c$ & $c \leq 0$

$g(x) \in$ domain of $f(x)$

$g(x) = (x+3)(x+1)$

from (a)

$f(x)$ has a domain of $x \in [0, \infty)$



if $x \in [-1, \infty)$ used, need to show $x \in (-\infty, \infty)$ for part 2. to get a mark.

this also means $0 \leq g(x) < \infty$

∴ $x \geq -1$ or $x \leq -3$ (1)

but also $x \leq 0$

∴ $x \leq c$ and $c = -3$ (1)

[3]

Q24. $y = \sin x + \cos x \qquad -\pi \le x \le \pi$

a) x-intercepts:

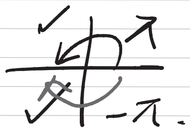
$y = 0 = \sin x + \cos x$

$-\sin x = \cos x$

$-\tan x = 1$

$\tan x = -1$ (1)

$x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$ (1)



[2]

b) stationary points

$y' = 0 = \cos x - \sin x$

$\tan x = 1$

$x = \frac{\pi}{4}$ or $\frac{-3\pi}{4}$

$y'' = -\sin x - \cos x$

when $x = \frac{\pi}{4}$, $y'' = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} < 0$ max (1)

$x = \frac{-3}{4}$, $y'' = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} > 0$ min. (1)

at $x = \frac{\pi}{4}$, $y = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

$x = \frac{-3\pi}{4}$, $y = -\sqrt{2}$

[3]

(1)

$\therefore \left(\frac{\pi}{4}, \sqrt{2}\right)$ is a local maximum turning pt

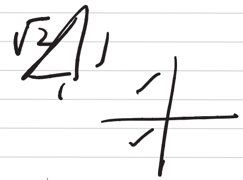
$\left(\frac{-3\pi}{4}, -\sqrt{2}\right)$ is a local minimum turning pt.

c) point of inflection

$y'' = 0 = -\sin x - \cos x$

$\sin x = -\cos x$

$\tan x = -1$



$x = -\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (1)

Test concavity

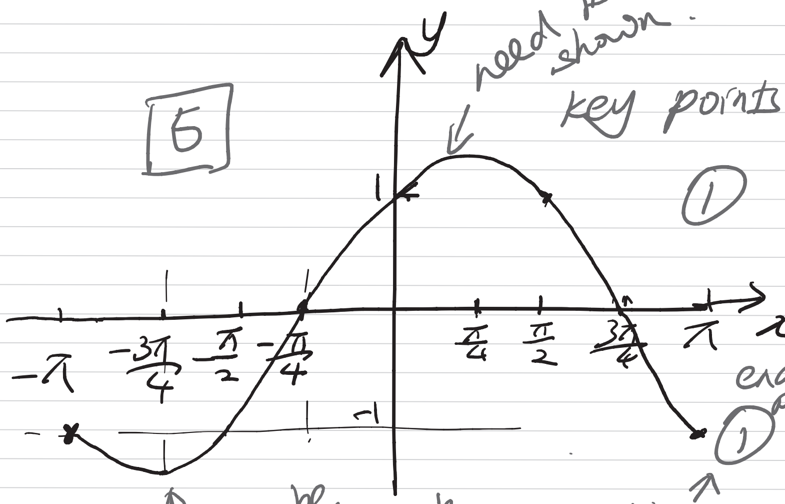
x	$-\frac{2\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	π
y"	1	0	$-\sqrt{2}$	0	1

(1)

$\therefore$ as concavity changes, the points $\left(-\frac{\pi}{4}, 0\right)$ and $\left(\frac{3\pi}{4}, 0\right)$ are points of inflection

d)

[5]



need to be shown key points (1)

end pts (1)

need to be shown to gain a mark

all pts required

Q25

LHS: $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$= \frac{(1-\sin\theta)+(1+\sin\theta)}{1-\sin^2\theta}$ (1)

$= \frac{2}{\cos^2\theta}$ [2]

$= 2\sec^2\theta$ (1)

Q26.

a) $x = 120 \div 3 = 40\text{ m}$ (1)

b) $\int_a^b f(x)dx \approx \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+\ldots f(x_{n-1})]\}$

($\frac{b-a}{2n}$ ↗ $\frac{\delta x}{2}$)

$a=0$, $b=120$, $f(0)=50.6$,
$f(120)=32.5$
$f(x_1)=f(40)=45.8$
$f(x_2)=f(80)=30.0$

$\approx \frac{40}{2}[50.6+32.5+2(45.8+30)]$ (1)

$\approx 4694\ m^2$ (1) [3]

Q27.

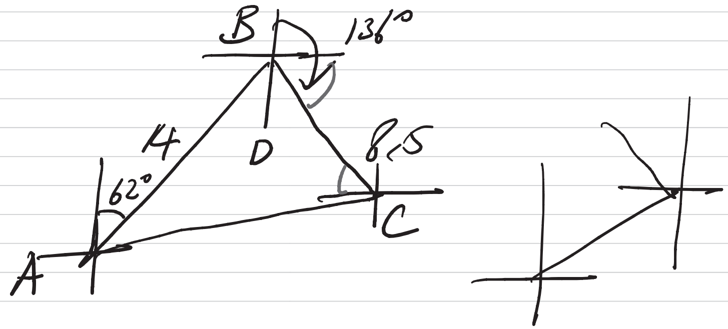
need to standardise results for comparison

PDHPE: $z_P = \frac{74-65}{4} = 2.25$ (1)

Biology: $z_B = \frac{84-80}{3} = 1.33$ (1)

greater the z score, the better his mark is. Therefore, he scored more strongly in PDHPE. (1) [3]

Q28



a) $\angle ABD = 62^\circ$ (alternate angles on parallel lines)

$\angle CDB = 180 - 136^\circ$

$= 44^\circ$ (1)

$\therefore \angle ABC = 62 + 44 = 106^\circ$ (1)

b) use cosine rule

$AC^2 = 14^2 + 8.5^2 - 2 \times 14 \times 8.5 \cos 106$

$AC = 18.27$ (1)

$AC \simeq 18$ km.

c) $\dfrac{\sin \angle BCA}{14} = \dfrac{\sin 106}{18}$

$\angle BCA = 47.437^\circ$ or 132 (not possible)

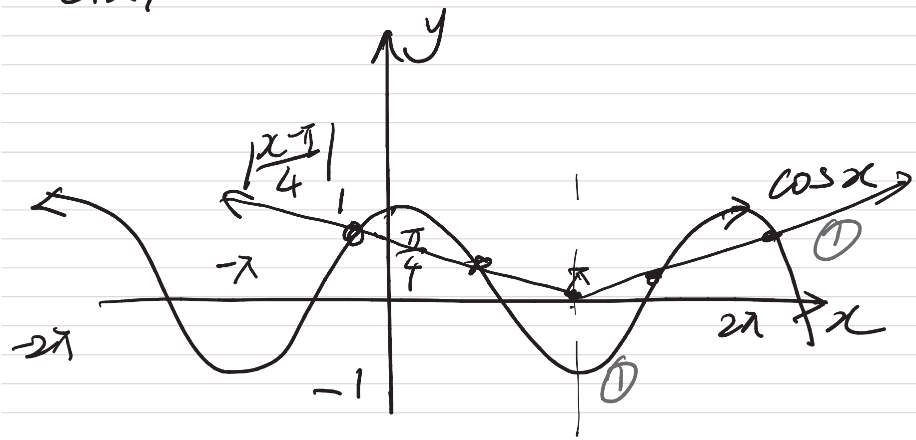
$136 - 90 = 46^\circ$ (1)

bearing $= 270 - (47.437 - 46^\circ)$

$= 268^\circ 33'$ (1)

$\simeq 269^\circ$ T.

Q29



$\cos x$: $A = 1$, $P = 2\pi$, $C = 0$

$\left|\frac{x-\pi}{4}\right| = \left|\frac{x}{4} - \frac{\pi}{4}\right|$ [3]

$m = \frac{1}{4}$, y-intercept $y = \frac{\pi}{4}$

4 solution. (1)

Q30

a) $IQR = Q_U - Q_L$

$= 22.5 - 8.4$

$= 14.1$ (1)

b) $Q_U + 1.5 \times IQR$

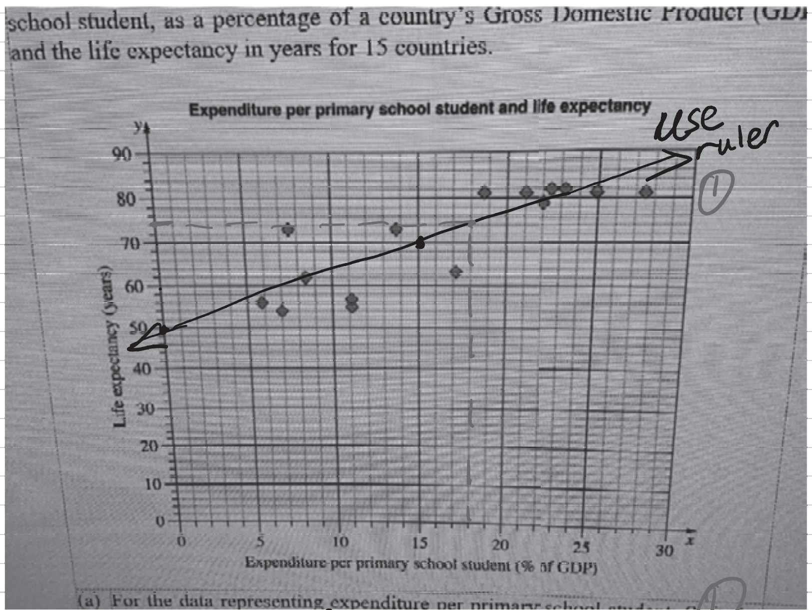
$= 22.5 + 1.5 \times 14.1$

$= 43.65\%$ (1)

Since $47.6\% > 43.65\%$

∴ it is an outlier (1)

c)



(1)

$y = 1.29x + 49.9$ $b = 49.9$, $m = 1.29$

$mx + b$ $x = 15$, $y = 69.25$ $(15, 69.25)$

d) from the diagram, at $x = 18\%$

need to show algebra or diagram to get a mark

$y = 74$ (answ 72–74 all acceptable) (1)

∴ the life expectancy is 74.

e) At 60% GDP, the line predicts a life expectancy over 100 which exceeds the expected life span for most human. So this line of best fit is only predictive in a lower range of GDP expenditure. (1)

Q31

a) $A = \frac{1}{2} \times b \times h$ (1)

$b = x$, $h = f(x) = 2e^{\frac{-x}{5}}$

$A = \frac{1}{2} \times x \times 2e^{\frac{-x}{5}} = xe^{\frac{-x}{5}}$

b) max. area.

$$\frac{dA}{dx} = 0 \quad \& \quad \frac{d^2A}{dx^2} < 0$$

$$x\left(-\frac{1}{5}e^{-\frac{x}{5}}\right) + e^{-\frac{x}{5}} = 0$$

$$e^{-\frac{x}{5}}\left(1 - \frac{x}{5}\right) = 0 \quad (1)$$

$\therefore x = 5$ ($e^{-\frac{x}{2}} > 0$, for all x)

at $x=5$, $A = xe^{-\frac{x}{5}} = 5e^{-1}$ (1)

$$\frac{d^2A}{dx^2} = -\frac{1}{5}e^{-\frac{x}{5}} - \frac{1}{5}e^{-\frac{x}{5}} - \frac{x}{25}e^{-\frac{x}{5}}$$

at $x=5$

$$\frac{d^2A}{dx^2} < 0 \quad (1)$$

$\therefore A_{max} = \frac{5}{e}$ units^2 at $x=5$

c) Find S: $F(0) = 2$

$S(0, 2)$ (1)

Find T: $2e^{-\frac{x}{5}} = \frac{1}{2}$ (1)

$$e^{-\frac{x}{5}} = \frac{1}{4}$$

$$x = -5\ln\frac{1}{4} = 5\ln 4$$

$T(5\ln 4, \frac{1}{2})$

$$A = \text{Area } \triangle - \int_0^{5\ln 4} (2e^{-\frac{x}{5}})\,dx$$

$$= \frac{1}{2}h(a+b) + 10\left[e^{-\frac{x}{5}}\right]_0^{5\ln 4} \quad (1)\ (1)$$

$$= \frac{1}{2} \times 5\ln 4\left(2 + \frac{1}{2}\right) + 10\left[e^{-\ln 4} - 1\right]$$

$$= \frac{25}{4}\ln 4 + 10\left(\frac{1}{4} - 1\right) \qquad [9]$$

$$= \frac{25}{4}\ln 4 - \frac{15}{2} \text{ units}^2 \quad (1)$$

Q32

$$P = Ae^{kt}$$

a) at $t=0$, $P=1000$,
$t=2$, $P=1500$

$1000 = Ae^0$

$A = 1000$ (1) [2]

$1500 = 1000e^{2k}$

$k = \frac{\ln 1.5}{2}$ (1)

b) $P = Ae^{4k}$ $t=4$

$P = 1000e^{4\frac{\ln 1.5}{2}}$

$P = 1000\,(e^{2\ln 1.5})$

$P \approx 2449.49$

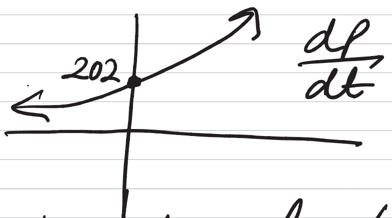
$P = 2450$

no mark for incorrect rounding.

c) $P = Ae^{kt}$

$\frac{dP}{dt} = kAe^{kt}$ ①

$= \frac{\ln 1.5}{2} \times 1000 \times e^{\frac{\ln 1.5}{2}t}$



Since $\frac{dP}{dt} > 0$ for $t > 0$, ① [2]

∴ the rate of change of white-ants population is monotonic increasing as time increases, ~~therefore~~ the population is increasing rapidly and the situation is getting worse.

Q33

a) From the table.

$P(X \le 200) \approx 0.1003 \Rightarrow z = -1.28$ ①

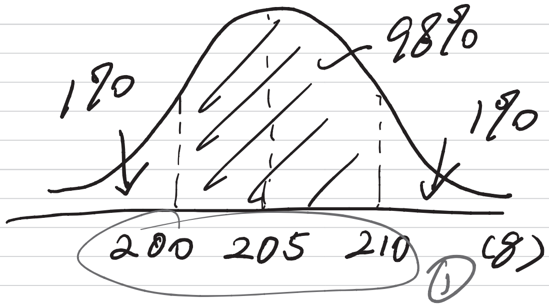
$z = \frac{x-\mu}{\sigma}$

$-1.28 = \frac{200-\mu}{7.8}$ ①

$\mu = 209.984 \approx 210$

b) $\mu = 205,\ \sigma$

i)



①

ii) $z_u = \frac{210-205}{\sigma}$

$z_L = \frac{200-205}{\sigma}$

$$P(X \le z_u) = 99\% \Rightarrow z_u = 2.33$$

①

$$\frac{210-205}{\sigma} = 2.33$$

$$\sigma = \frac{5}{2.33}$$

$$\sigma = 2.145$$ ①

$$\sigma = 2.15 \ (g)$$