Name : ____________

Number : ____________

HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12 2009

MATHEMATICS

TRIAL HIGHER SCHOOL CERTIFICATE

Examiners : P. Biczo, S. Hackett, D. Crancher, S. Faulds, J. Dillon

General Instructions

- Reading time : 5 minutes
- **Working time : 3 hours**
- Attempt **all** questions
- **Start a new answer booklet for each question**
- All necessary working should be shown
- This paper contains 10 questions worth 12 marks each. Total Marks: **120 marks**
- Marks may not be awarded for careless or badly arranged work
- Board approved calculators and mathematical templates may be used
- This examination paper must **not** be removed from the examination room

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2009 HSC Mathematics Examination.

Marks

QUESTION 1. *Start a new answer booklet.*

a) Factorise $3x^3 + 24$ **2**

b) Rationalise the denominator and simplify: $\dfrac{2}{5-\sqrt{3}}$ **2**

c) Express $\dfrac{3x+2}{2} - \dfrac{x-1}{5}$ as a single fraction, in simplest form **2**

d) Solve the inequality, graphing your solution on a number line: $|2x-3| < 7$ **2**

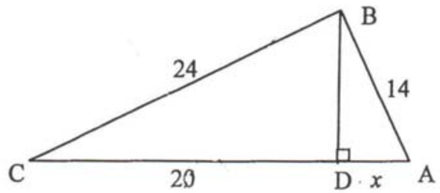
e) Write, in scientific notation, correct to 2 significant figures: $\dfrac{e^{-3.5}}{4}$ **2**

f) Sketch the graph of $y = \cos x$ for $0° \le x \le 360°$ **2**

QUESTION 2. *Start a new answer booklet.*

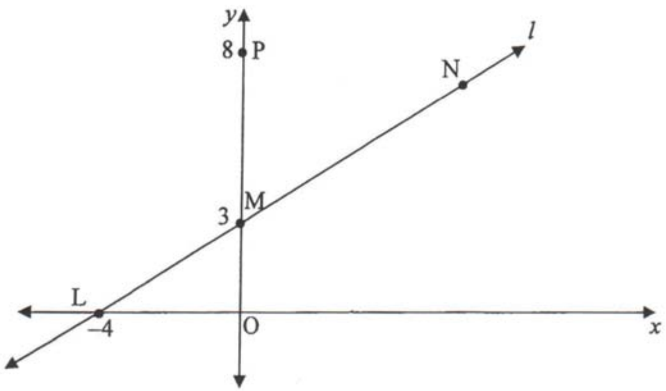
Marks

a)



Find x, the length of AD, as an exact value. Justify your answer. **2**

b)



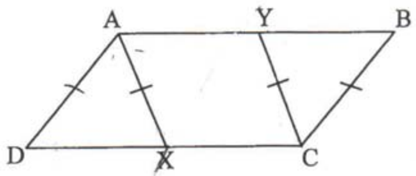
For the diagram shown, M is the midpoint of the interval LN

(i) Find the coordinates of the point N. **2**

(ii) Show that $\angle NPL$ is a right angle. **2**

(iii) Find the equation of the circle that passes through the points N, P and L. **2**

c) ABCD is a parallelogram. The point X lies on CD, the point Y lies on AB, and AX = CY = BC, as shown in the diagram.



(i) Explain why $\angle ADX = \angle CBY$. **1**

(ii) Show that AD = AX. **1**

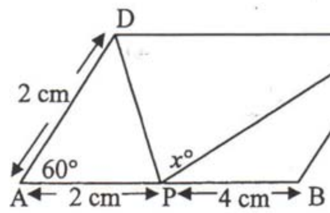
(iii) Show that triangles ADX and CBY are congruent. **2**

Marks

QUESTION 3. *Start a new answer booklet.*

a) Use the formula $a^2 = b^2 + c^2 - 2bc\cos A$ to find A, correct to the nearest degree, when $a = 22$, $b = 12$, $c = 13$, and A lies between $0°$ and $180°$. **2**

b)



NOT TO SCALE

In the figure (not to scale), ABCD is a parallelogram in which $AB = 6\text{cm}$, $AD = 2\text{cm}$, and $\angle DAB = 60°$. The point P on AB is such that $AP = 2\text{cm}$, and $\angle DPC = x°$.

(i) Find the length of DP, giving reasons. **2**

(ii) Use the cosine rule for each of the triangles PBC, PCD to show that $\cos x° = \dfrac{-\sqrt{7}}{14}$. **3**

c) Find all the values of θ, for $0° \le \theta \le 180°$, such that $\cos 2\theta = \dfrac{1}{2}$. **2**

(d) Sketch the graph of: **3**

$$f(x) = \begin{cases} 5 & \text{if } x > 2 \\ x^2 & \text{if } 0 \le x \le 2 \\ 2x - 1 & \text{if } x < 0 \end{cases}$$

QUESTION 4. *Start a new answer booklet.*

Marks

a) Given that $\log_k 5 = 0\cdot 627$ and $\log_k 2 = 0\cdot 270$, find the value of:

(i) $\log_k 10$ **1**

(ii) $\log_k 25$ **1**

b) Differentiate the following with respect to x:

(i) $\left(e^{2x}+1\right)^3$ **2**

(ii) $\dfrac{\log_e x}{x+1}$ **2**

c) Find $\displaystyle\int \frac{x}{x^2+2}\,dx$ **2**

d) Evaluate $\displaystyle\int_0^1 \left(e^{2x}-x\right)dx$ **2**

e) Consider the equation $\log_e y = x\log_e\left(\dfrac{1}{2}\right)$. **2**

Write an expression for $y = f(x)$.

Marks

QUESTION 5. *Start a new answer booklet.*

a) The first three terms of a geometric series are:

$$4a^2b^2,\ x\ ,\ a^2+2ab+b^2$$

Find x in terms of a and b. **2**

b) A hole in a water reservoir wall will allow through it 50L more for each hour that it remains undetected. At the moment that this hole was detected, water was leaking through it at the rate of 1200L/h.

(i) Write down the first three terms of a series which represents the water lost through the hole for each of the first three hours. **1**

(ii) For how long had the water been leaking when the hole was detected? **2**

(iii) What was the total volume of water lost through the hole, up to the time when it was detected? **1**

c) For the series:

$$1+\sin A+\sin^2 A+\sin^3 A+\ldots$$

(i) Explain why the series has a limiting sum. **1**

(ii) Find the exact value of this limiting sum when $A = \dfrac{4\pi}{3}$. **2**

d) An investment fund intends to pay interest at the rate of 6% p.a. every six months.

(i) If an investment of $250 is made today, what amount (ie. principal plus interest) will be available for withdrawal in 10 years time? **1**

(ii) If nineteen further investments of $250 are made every six months, show that the amount available for withdrawal in ten years time will be $6919 to the nearest dollar. (Assume that no withdrawals are made from the fund during this time.) **2**

QUESTION 6. *Start a new answer booklet.*

a) Consider the following statement:

"In 2008, Fiji had a population of approximately 906 000. The number of fatalities on Fijian roads was 78. If similar figures were to be maintained into the future, the probability of a Fijian, chosen at random, being killed in a road accident in any particular year is approximately 1 in 12 000."

Comment upon the validity of this statement. **2**

b) A General Practitioner has compiled statistics on patients visiting his practice. He has determined that during the winter months, there is a 45% chance that a patient will require treatment for influenza and a 15% chance that a patient will require treatment for food poisoning.

(i) What is the probability that one of the doctor's patients will require treatment for influenza ***and*** food poisoning during the same season. **1**

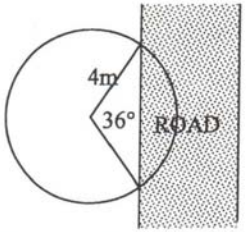
(ii) What is the probability that one of the doctor's patients will require treatment for either influenza ***or*** food poisoning during the same season. **2**

c) A coin is specially weighted so that the probability of tossing a "head" on any single toss is twice that of tossing a "tail".

(i) What is the probability of tossing a "head" with this coin. **1**

(ii) Calculate the probability of tossing at least one "tail" if the coin is tossed five times. **1**

d)



A sprinkler spraying water in a circular pattern of radius 3m is watering a lawn adjacent to a straight section of road as shown in the diagram. The angle subtended by the road at the sprinkler head is 36°.

(i) Convert 36° to radians. Give your answer in terms of π. **1**

(ii) Find the area of road being watered in square metres correct to 2 decimal places. **2**

(iii) Calculate the volume of water being wasted each hour if the sprinkler delivers 3.5kL per hour. Give your answer to the nearest litre. (Assume the sprinkler disperses water evenly over its spray area.) **2**

Marks

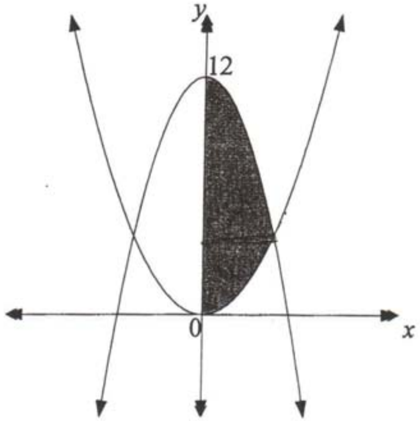
QUESTION 7. *Start a new answer booklet.*

a) $\int (2x+3)^3 \, dx$ **1**

b) Evaluate $\int_1^4 \frac{x^2+8}{x^2} \, dx$ **3**

c) Given $f'(x) = 3x^2 + x$, find $f(x)$ given $f(-2) = 4$ **2**

d) The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram.



(i) Find the points of intersection of the curves **1**

(ii) Calculate the area between the two curves **2**

(iii) The shaded region between the curves and the y axis is rotated about the y axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed. **3**

Marks

QUESTION 8. *Start a new answer booklet.*

a) If $A^m = 3$, find the value of $A^{4m} - 5$ — 2

b) Use Simpsons Rule, with three function values, to approximate $\int_3^5 x\log_e x \, dx$ — 2

c) Consider the function $y = xe^{-x}$.

(i) Find the coordinates of any stationary point(s) and determine their nature — 3

(ii) Describe the behaviour of the function as x:

- increases to negative infinity
- increases to positive infinity — 2

(iii) Does the graph pass through the origin? Justify your answer. — 1

(iv) Sketch the graph of this function — 2

QUESTION 9. *Start a new answer booklet.*

a) Find the equation of the tangent to the curve $y = \sqrt{x-1}$ at the point (2,1). — 2

b) Solve for x: $x^2 - 8x + 12 > 0$ — 2

c) Point $P(x, y)$ is a point on the parabola $y = x^2$.

(i) Show that the distance, S, from the line $y = 2x - 5$ is given by $S = \left|\frac{2x - x^2 - 5}{\sqrt{5}}\right|$. — 2

(ii) Show that $f(x) = 2x - x^2 - 5$ is negative definite. — 2

(iii) Hence show that $S = \frac{x^2 - 2x + 5}{\sqrt{5}}$. — 1

(iv) Hence find the shortest distance possible between the point $P(x, y)$ and the line $y = 2x - 5$. — 3

QUESTION 10. *Start a new answer booklet.*

a) For the parabola $(x-2)^2 = 8(y+1)$, find:

(i) the focal length — 1

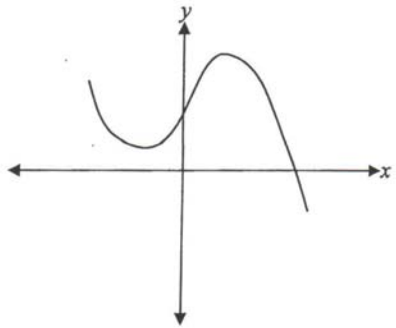
(ii) the co-ordinates of the vertex — 1

(iii) the co-ordinates of the focus — 1

b) The parabola $y^2 = 4x$ is reflected in the y axis. What is the equation of the resultant parabola formed? — 1

c) Show that the locus of a point, $P(x, y)$, which is always 5 units from the point $A(3,6)$ is $x^2 - 6x + y^2 - 12y + 20 = 0$. — 3

d) The diagram show a graph of a certain function $y = f(x)$.



(i) Copy this graph into your writing booklet.

(ii) On the same set of axes, draw a sketch of the derivative $f'(x)$ of the function. — 2

e) The quadratic equation $x^2 + mx + 10 = 0$ has one root twice the other.

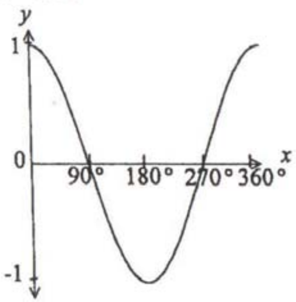
(i) If one of the roots is α, write expressions for the sum and product of the roots. — 1

(ii) Hence, or otherwise, find the value of m. — 2

END OF EXAMINATION

Outcomes Addressed in this Question

P3 Performs routine arithmetic & algebraic manipulation involving surds & simple rational expressions
P4 Chooses and applies appropriate arithmetic, algebraic & trigonometric techniques
P5 Understands the relationship between a function and its graph
H3 Manipulates algebraic expressions involving logarithmic & exponential functions

Outcome	Solutions	Marking Guidelines
P3	a) $3x^3+24=3(x^3+8)$ $=3(x+2)(x^2-2x+4)$	2 marks : factorises correctly twice 1 mark: factorises correctly once
P3	b) $\frac{2}{5-\sqrt{3}}=\frac{2}{5-\sqrt{3}}\times\frac{5+\sqrt{3}}{5+\sqrt{3}}$ $=\frac{2(5+\sqrt{3})}{22-3}$ $=\frac{5+\sqrt{3}}{11}$	2 mark: finds common denominator and correctly simplifies 1 mark : significant progress towards correct answer
P4	c) $\frac{3x+2}{2}-\frac{x-1}{5}=\frac{5(3x+2)-2(x-1)}{10}$ $=\frac{15x+10-2x+2}{10}$ $=\frac{13x+12}{10}$	2 marks : correct answer 1 mark : significant progress towards correct answer
P4	d) $\lvert 2x-3\rvert<7$ $\therefore -7<2x-3<7$ $\therefore -4<2x<10$ $\therefore -2<x<5$  	2 marks: correct answer 1 mark : partially correct answer
H3	e) $\frac{e^{-3.5}}{4}=0.007549\ldots$ $=7.5\times10^{-3}$ to 2 significant figures	2 mark : correct answer 1 mark: correct calculation, incorrect rounding
P5	f) $y=\cos x$  	2 marks: correct answer 1 mark : partially correct answer

Outcomes Addressed in this Question H2

Outcome	Sample Solution	Marking Guidelines
H2	a) $BD^2+20^2=24^2$ *(Pythagoras' Theorem)* $BD=\sqrt{176}$ $x^2+(\sqrt{176})^2=14^2$ *(Pythagoras' Theorem)* $\therefore x=\sqrt{20}$ $=2\sqrt{5}$	**2 mark** ~ Correct answer with reasons **1 mark** ~ Correct answer without reasons
H2	b) i) $\frac{-4+x}{2}=0$ $\quad$ $\frac{0+y}{2}=3$ $x=4$ $\quad$ $y=6$ $\therefore N(4,6)$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H2	ii) $m_{NP}=\frac{8-6}{0-4}=-\frac{1}{2}$ $\quad$ $m_{PL}=\frac{8-0}{0+4}=2$ $m_{NP}\times m_{PL}=-\frac{1}{2}\times2$ $=-1$ $\therefore NP\perp PL$ $\therefore \angle NPL=90°$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H2	iii) *Circle centre* $M(0,3)$, *radius* 5 $x^2+(y-3)^2=25$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H2	c) i) $\angle ADX=\angle CBY$ *(opposite ∠'s of parallelogram)* ii) $AD=BC$ *(opposite sides of parallelogram)* $AX=BC$ *(given)* $\therefore AD=AX$	**1 mark** ~ correct reasons given **1 mark** ~ correct reasons given
H2	iii) $AD=AX$ $\therefore \Delta ADX$ *is isosceles* $BC=YC$ $\therefore \Delta CBY$ *is isosceles* $\therefore \angle ADX=\angle AXD$ *and* $\angle CBY=\angle CYB$ *(equal angles in isosceles Δ's)* $\angle ADX=\angle CBY$ *(shown in (i))* $AX=BC$ *(given)* $\angle AXD=\angle CYB$ *(equals of* $\angle ADX$ *and* $\angle CBY$*)* $\therefore \Delta ADX\equiv\Delta CBY$ *(AAS)*	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.

Year 12 Mathematics Extension 1 Task 4 2009

Question No. 3 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

P5 understands the concept of a function and the relationship between a function and its graph

P6 relates the derivative of a function to the slope of its graph

Outcome	Solutions	Marking Guidelines
P4	**3. a)**	1 mark awarded for partial correct solution 2 marks awarded for complete correct solution

$$a^2 = b^2 + c^2 - 2ab\cos A$$
$$22^2 = 12^2 + 13^2 - 2(12)(13)\cos A$$
$$2(12)(13)\cos A = 12^2 + 13^2 - 22^2$$
$$\cos A = \frac{12^2 + 13^2 - 22^2}{2(12)(13)}$$
$$\therefore A = 123.2351...^\circ \text{(by calculator)}$$
$$\therefore A = 123^\circ \text{ to nearest degree}$$

P4 **b)**

(i) $DP = 2cm$ (ΔDAP is equilateral)

Or

$DP = 2cm$ (using cosine rule)

Marking guidelines: 1 mark for partial correct solution. 2 marks for complete correct solution

(ii)

Now $\angle PBC = 180^\circ - 60^\circ = 120^\circ$ (co-interior to $\angle DAP$; $DA \parallel CB$)

and $BC = AD = 2cm$ (opposite sides in parallelogram)

In ΔPBC:

$$PC^2 = PB^2 + BC^2 - 2(PB)(BC)\cos\angle PBC$$
$$PC^2 = 4^2 + 2^2 - 2(4)(2)\cos 120^\circ$$
$$PC^2 = 16 + 4 - 16\left(-\frac{1}{2}\right)$$
$$PC^2 = 28$$
$$PC = \sqrt{28}$$
$$PC = 2\sqrt{7}$$

Marking guidelines: 1 mark awarded for partial correct solution leading to the correct value of PC.

Now $DC = AB = 6cm$ (opposite sides of parallelogram)

In ΔPCD

$$\cos x^\circ = \frac{DP^2 + PC^2 - DC^2}{2 \times DP \times PC}$$
$$\cos x^\circ = \frac{2^2 + \left(2\sqrt{7}\right)^2 - \left(6^2\right)}{2 \times 2 \times 2\sqrt{7}}$$
$$\cos x^\circ = \frac{4 + 28 - 36}{8\sqrt{7}}$$
$$\cos x^\circ = \frac{-4}{8\sqrt{7}}$$
$$\cos x^\circ = \frac{-1}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$
$$\cos x^\circ = \frac{-\sqrt{7}}{14}$$

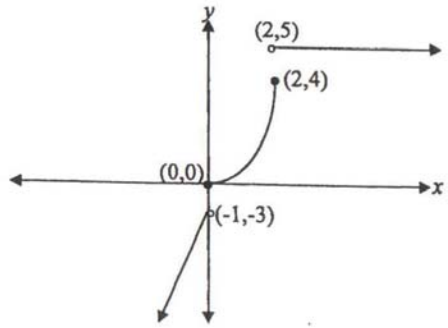
Marking guidelines: 2 marks awarded for a further correct partial solution using the cosine rule and giving any correct value for $\cos x^\circ$. 3 marks awarded for complete correct solution

P3 **(c)**

$$\cos 2\theta = \frac{1}{2} \text{ for } 0^\circ \le \theta \le 180^\circ$$
$$\therefore 2\theta = 60^\circ \text{ or } (360^\circ - 60^\circ) \text{ for } 0^\circ \le 2\theta \le 360^\circ$$
$$\therefore 2\theta = 60^\circ \text{ or } 300^\circ$$
$$\therefore \theta = 30^\circ \text{ or } 150^\circ \text{ for } 0^\circ \le \theta \le 180^\circ$$

Marking guidelines: 1 mark awarded for partial correct solution. 2 marks awarded for complete correct solution

P5 **(d)**



Marking guidelines: 1 marked awarded for sketching $f(x) = 5$ for $x > 2$. 1 marked awarded for sketching $f(x) = x^2$ for $0 \le x \le 2$

Year 12 **Mathematics** **Trial HSC 2009**

Question No. 4 **Solutions and Marking Guidelines**

Outcomes Addressed in this Question

H3 **manipulates algebraic expressions involving logarithmic and exponential functions**

H5 **applies appropriate techniques from the study of calculus**, geometry, probability, trigonometry and series **to solve problems**

Outcome	Solutions	Marking Guidelines
(a) (i) H3	$\log_k 10 = \log_k(2\times 5) = \log_k 2 + \log_k 5 = 0.270 + 0.627$ $= 0.897$	**Award 1** for correct answer.
(ii) H3	$\log_k 25 = \log_k 5^2 = 2\log_k 5 = 2\times 0.627$ $= 1.254$	**Award 1** for correct answer.
(b) (i) H3, H5	$\frac{d}{dx}\left(\left(e^{2x}+1\right)^3\right) = 3\left(e^{2x}+1\right)^2 .2e^{2x}$ $= 6e^{2x}\left(e^{2x}+1\right)^2$	**Award 2** for correct solution. **Award 1** for attempting to use an appropriate process.
(ii) H3, H5	$\frac{d}{dx}\left(\frac{\log_e x}{x+1}\right) = \frac{(x+1).\frac{1}{x} - \log_e x.1}{(x+1)^2}$ $= \frac{x+1-x\log_e x}{x(x+1)^2}$	**Award 2** for correct solution. **Award 1** for attempting to use an appropriate process.
(c) H3, H5	$\int \frac{x}{x^2+2}\,dx = \frac{1}{2}\int \frac{2x}{x^2+2}\,dx$ $= \frac{1}{2}\log_e\left(x^2+2\right)+c$	**Award 2** for correct solution. **Award 1** for attempting to use an appropriate process.
(d) H3, H5	$\int_0^1 \left(e^{2x}-x\right)dx = \left[\frac{1}{2}e^{2x} - \frac{x^2}{2}\right]_0^1$ $= \frac{1}{2}e^2 - \frac{1}{2} - \left(\frac{1}{2}e^0 - 0\right)$ $= \frac{1}{2}e^2 - \frac{1}{2} - \left(\frac{1}{2}\right)$ $= \frac{1}{2}e^2 - 1$	**Award 2** for correct solution. **Award 1** for finding a primitive and correctly performing the substitution.
(e) H3	$\log_e y = x\log_e\left(\frac{1}{2}\right)$ $\log_e y = \log_e\left(\frac{1}{2}\right)^x$ $\therefore y = \left(\frac{1}{2}\right)^x = 2^{-x}$	**Award 2** for correct solution. **Award 1** for attempting to use an appropriate process.

ear 12 Mathematics Trial HSC Examination 2009

uestion No. 5 Solutions and Marking Guidelines

Outcomes Addressed in this Question

5 applies appropriate techniques from the study of series to solve problems

utcome	Solutions	Marking Guidelines
H5	**(a)** Since the series is geometric: $\frac{T_3}{T_2} = \frac{T_2}{T_1}$ *ie.* $\frac{a^2+2ab+b^2}{x} = \frac{x}{4a^2b^2}$ $x^2 = 4a^2b^2(a^2+2ab+b^2)$ $= 4a^2b^2(a+b)^2$ $x = \pm 2ab(a+b)$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution
H5	**(b)** **(i)** $T_1 = 50$ $T_2 = 100$ $T_3 = 150$	**1 mark** Correct terms shown
H5	**(ii)** Since the above series is arithmetic: $T_n = 1200$ $a = 50$ $d = 50$ $T_n = a+(n-1)d$ $1200 = 50+(n-1).50$ $= 50+50n-50$ $= 50n$ $n = \frac{1200}{50}$ $= 24$ $\therefore$ The water had been leaking for 24 hours when the leak was detected.	**2 marks** Correct solution **1 mark** Correctly identifies series as arithmetic, giving first term, common difference and demonstrating substantial knowledge or required formula.
H5	**(iii)** Total volume of water lost = S_{24} $S_n = \frac{n}{2}(a+l)$ where $l = T_{24}$ $S_{24} = \frac{24}{2}(50+1200)$ $= 12.1250$ $= 15000$ $\therefore$ 15000L of water had been lost when the leak was detected.	**1 mark** Correct solution
H5	**(c) (i)** $1+\sin A+\sin^2 A+\ldots$ is a geometric series where $r = \sin A$ Since $-1\le \sin A \le 1$ and provided $\sin A \ne \pm 1$ series will have a limiting sum as limiting sum exists where $-1 < r < 1$	**1 mark** Correct justification

H5	**(ii)** $r=\sin\frac{4\pi}{3}$ $=-\frac{\sqrt{3}}{2}$ $S_\infty=\frac{a}{1-r}$ $=\frac{1}{1+\frac{\sqrt{3}}{2}}$ $=\frac{2}{2+\sqrt{3}}$ $=4-2\sqrt{3}$	**2 marks** Correct solution **1 mark** Gives correct value for sine ratio **OR** uses incorrect value in correct formula for limiting sum.
H5	**(d) (i)** $A_1=P(1+r)^n$ $=250(1.03)^{20}$ $=451.53$ **OR** $A_1=P(1+r)^n$ $=250(1.06)^{10}$ $=447.71$ \$447.71 will be available for withdrawal in 10 years.	**1 mark** Correct answer. Accept both interest compounded 6 monthly and compounded yearly.
H5	**(ii)** At the end of 6 months $A_1=250(1.03)$ At the end of 12 months (ie. 1 year) $A_2=250(1.03)(1.03)+250(1.03)$ $=250(1.03)^2+250(1.03)$ $=250(1.03)(1+1.03)$ Similarly, after 3 time periods (18 months) $A_3=250(1.03)^3+250(1.03)^2+250(1.03)$ $=250(1.03)(1+1.03+1.03^2)$ After 20 time periods ie. 10 years $A_{20}=250(1.03)^{20}+250(1.03)^{19}+250(1.03)^{18}+...+250(1.03)^2+250(1.03)$ $=250(1.03)(1+1.03+1.03^2+...+1.03^{19})$ Now, $1+1.03+1.03^2+...+1.03^{19}$ is a geometric series with $a=1, r=1.03$ and $n=20$ $S_{20}=\frac{a(r^n-1)}{r-1}$ $=\frac{1.03^{20}-1}{0.03}$ $=26.8704$ $\therefore A_{20}=250(1.03)(26.8704)$ $=6919$ ie. \$6919 is available for withdrawal after 10 years, as required.	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of probability and trigonometry to solve problems

Outcome	Solutions	Marking Guidelines
H5	**(a)** The statement is not valid. It assumes that all Fijians have access to motor vehicles with similar safety levels and the roads being driven on are in a similar state of repair. In fact, many rural dwelling Fijians rarely use motor vehicles, so the probability of them being killed in a road accident is almost zero, whereas a city dwelling Fijian would have a comparably higher probability of being a road fatality. ie. the event of being killed on the road is not equally likely for each Fijian.	**2 marks** Correct assessment of validity of statement **1 mark** Some aspects of assessment are valid.
	(b) Probability of requiring treatment for food poisoning $=P(F)$ $=0.15$ Probability of requiring treatment for influenza $=P(I)$ $=0.45$	
H5	(i) $P(F \text{ and } I)$ $=P(F)\times P(I)$ $=0.15\times0.45$ $=0.0675$	**1 mark** Correct answer.
H5	(ii) $P(F \text{ or } I)$ $=P(F)+P(I)-P(F \text{ and } I)$ $=0.15+0.45-0.0675$ $=0.5325$ Note: These events are not mutually exclusive	**2 marks** Correct solution. **1 mark** Solution substantially correct.
H5	**(c)** (i) $P(H)=\frac{2}{3}$	**1 mark** Correct answer.
H5	(ii) $P(\text{at least 1 tail})=1-P(5\text{ heads})$ $=1-\left(\frac{2}{3}\right)^5$ $=1-\frac{32}{243}$ $=\frac{211}{243}$	**1 mark** Correct solution.
H5	**(d)** (i) $36^\circ=\frac{36\pi}{180}$ radians $=\frac{\pi}{5}$ radians	**1 mark** Correct answer.
H5	(ii) $A=\frac{1}{2}r^2(\theta-\sin\theta)$ $=\frac{1}{2}\times4^2\left(\frac{\pi}{5}-\sin\frac{\pi}{5}\right)$ $=0.32$ $\therefore 0.32\text{m}^2$ of the road is being watered.	**2 marks** Correct solution **1 mark** Correct formula and substitution.

H5	(iii) Water wasted $=\dfrac{\text{area of road watered}}{\text{area of circle}}$ $=\dfrac{0.32}{\pi r^2}\times 3.5\text{kL}$ $=\dfrac{0.32}{\pi\times 4^2}\times 3.5\text{kL}$ =22L per hour		**2 marks** Correct solution **1 mark** Substantial progress towards correct solution.

Outcomes Addressed in this Question

H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
H8	a) $\int(2x+3)^3dx=\dfrac{(2x+3)^4}{4\times 2}+c$ $=\dfrac{(2x+3)^4}{8}+c$	1 mark : correct integral
H8	b) $\int_1^4\dfrac{x^2+8}{x^2}\,dx=\int_1^4\left(\dfrac{x^2}{x^2}+\dfrac{8}{x^2}\right)dx=\int_1^4\left(1+8x^{-2}\right)dx$ $=\left[x-8x^{-1}\right]_1^4$ $=\left[x-\dfrac{8}{x}\right]_1^4$ $=4-2-(1-8)=9$	3 marks: correct solution 2 marks: significant progress towards correct solution 1 mark: progress towards correct solution
H8	c) $f'(x)=3x^2+x$ $\therefore f(x)=\dfrac{3x^3}{3}+\dfrac{x^2}{2}+c$ $\therefore f(x)=x^3+\dfrac{x^2}{2}+c$ $f(-2)=4, \therefore -8+2+c=4.\ \therefore c=10$ $\therefore f(x)=x^3+\dfrac{x^2}{2}+10$	2 marks : correct answer with justification 1 mark: one of above
H8	d) (i) $y=x^2$ and $y=12-2x^2$ meet when $x^2=12-2x^2$. $\therefore 3x^2=12$ $\therefore x^2=4 \quad x=\pm 2$ $\therefore$meet at $(-2,4)$ and $(2,4)$	1 mark : correct answer
H8	(ii) Area = Area under top curve − area under bottom curve $=\int_{-2}^{2}\left(12-2x^2-x^2\right)dx=2\int_0^2\left(12-3x^2\right)dx$ $=2\left[12x-x^3\right]_0^2$ $=2(24-8)=32\text{ units}^2$	2 marks : correct answer with justification 1 mark : significant progress towards correct answer
H8	(iii) Volume,in relation to y axis $=\pi\int_a^b x^2dy$ V = sum of the volume when the area between $y=x^2$ and the y axis between $y=0$ and $y=4$ is rotated about the y axis, and the volume when the area between $y=12-2x^2$ and the y axis between $y=4$ and $y=12$ rotated about the y axis	3 marks: correct solution 2 marks: significant progress towards correct solution 1 mark: progress towards correct solution

From $y=x^2$, ie. $x^2=y$

From $y=12-2x^2$, $2x^2=12-y$, $\therefore x^2=6-\dfrac{y}{2}$.

$$\therefore V=\pi\int_0^4 y\,dy+\pi\int_4^{12}\left(6-\frac{y}{2}\right)dy$$

$$\therefore V=\pi\left[\frac{y^2}{2}\right]_0^4+\pi\left[6y-\frac{y^2}{4}\right]_4^{12}$$

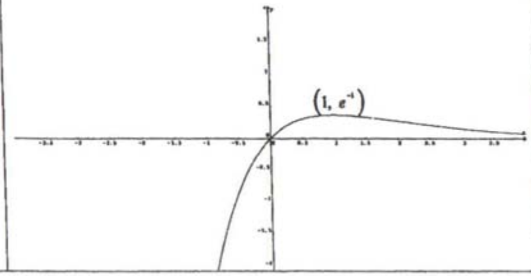
$$\therefore V=8\pi+\pi\left(72-36-(24-4)\right)$$

$$\therefore V=24\pi \text{ units}^3$$

Outcomes Addressed in this Question

H3 **manipulates algebraic expressions involving logarithmic and exponential functions**

H5 **applies appropriate techniques from the study of calculus,** geometry, probability, trigonometry and series **to solve problems**

Outcome	Solutions	Marking Guidelines
(a) H3	$A^{4m}-5=\left(A^m\right)^4-5$ $=3^4-5$ $=76$	**Award 2** for correct solution. **Award 1** for attempting to use an appropriate process.
(b) H3, H5	$\int_3^5 x\log_e x\,dx\approx\frac{1}{3}(3\log_e 3+5\log_e 5+4\times4\log_e 4)$ $=11.17457874$	**Award 2** for correct solution. **Award 1** for attempting to use Simpson's rule.
(c) (i) H3, H5	$y=xe^{-x}$ $\frac{dy}{dx}=x.-e^{-x}+e^{-x}.1=e^{-x}(1-x)$ $\frac{d^2y}{dx^2}=e^{-x}(-1)+(1-x).-e^{-x}=e^{-x}(x-2)$ Stationary point(s) occur @ $\frac{dy}{dx}=0$ $e^{-x}(1-x)=0$ $\therefore 1-x=0\quad(\because e^{-x}\neq0)$ $\therefore x=1$ Test $x=1$ $\frac{d^2y}{dx^2}=e^{-1}(1-2)=-e^{-1}<0$ $\therefore$ Relative maximum turning point @ $\left(1,\ e^{-1}\right)$	**Award 3** for correct stationary point, with full justification. **Award 2** for correct stationary point, without full justification. **Award 1** for attempting to find the stationary point.
(ii) H3	$\lim_{x\to\infty}\left(xe^{-x}\right)=0$ or function approaches $y=0$. $\lim_{x\to-\infty}\left(xe^{-x}\right)=-\infty$ or function gets very big and negative.	**Award 2** for correct solutions. **Award 1** for only one correct solution.
(iii) H3, H5	When $x=0$, $y=0e^{-0}=0$ $\therefore$ Graph passes through the origin.	**Award 1** for correct solution.
(iv) H3, H5		**Award 2** for correct graph, showing relevant details from (i), (ii) and (iii). **Award 1** for correct graph, but lacking sufficient detail.

Question No. 9 Solutions and Marking Guidelines

Outcomes Addressed in this Question H5

Outcome	Sample Solution	Marking Guidelines
H5	a) $y=\sqrt{x-1}$ $y=(x-1)^{\frac{1}{2}}$ $y'=\frac{1}{2}(x-1)^{-\frac{1}{2}}.1$ $y'=\frac{1}{2\sqrt{x-1}}$ $when\ x=2\ \ y'=\frac{1}{2}$ $eqn\ of\ tangent:\ y-1=\frac{1}{2}(x-2)$ $y=\frac{1}{2}x\ \ or\ \ x-2y=0$	**2 mark** ~ Correct equation **1 mark** ~ Substantial progress towards correct solution
H5	b) $x^2-8x+12>0$ $(x-6)(x-2)>0$ $\therefore x>6,\ x<2$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H5	c) i) $S=\left\|\frac{2x-x^2-5}{\sqrt{2^2+(-1)^2}}\right\|$ $=\left\|\frac{2x-x^2-5}{\sqrt{5}}\right\|$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H5	ii) $f(x)=2x-x^2-5$ $=-x^2+2x-5$ $For\ negative\ definite, a<0\ and\ \Delta<0$ $a=-1 \qquad \Delta=4-4\times(-1)\times(-5)$ $=4-20$ $=-16$ $\therefore f(x)\ is\ negative\ definite.$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H5	iii) since $f(x)$ is negative definite, $\|f(x)\|=-f(x)$ $\therefore \|2x-x^2-5\| =-(2x-x^2-5)$ $=x^2-2x+5$ $\therefore S=\frac{x^2-2x+5}{\sqrt{5}}$	**1 mark** ~ correct reasons given
H5	iv) $S=\frac{x^2-2x+5}{\sqrt{5}}$ $S'=\frac{2x-2}{\sqrt{5}}$ $S''=\frac{2}{\sqrt{5}}$ $stat.pt\ \ S'=0 \quad \frac{2x-2}{\sqrt{5}}=0$ $x=1$ $when\ x=1\ S''=\frac{2}{\sqrt{5}}>0 \therefore Minimum\ at\ x=1$ $when\ \ x=1\ \ S=\frac{1^2-2+5}{\sqrt{5}}=\frac{4}{\sqrt{5}}$ $\therefore$ Shortest possible distance is $\frac{4}{\sqrt{5}}$ units.	**3 marks** ~ Correct solution **2 marks** ~ Substantial progress towards correct solution. **1 mark** ~ Some progress towards correct solution.

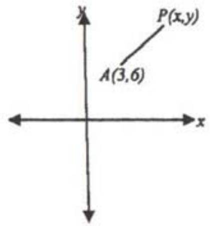
estion No. 10 Solutions and Marking Guidelines

Outcomes Addressed in this Question

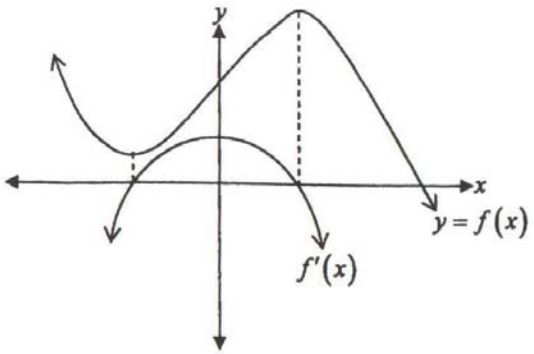
P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities

P5 understands the concept of a function and the relationship between a function and its graph

P6 relates the derivative of a function to the slope of its graph

Outcome	Solutions	Marking Guidelines
P5	**10.** **(a)** **(i)** $4a=8$ $\therefore a=2$ $\therefore$ focal length is 2	1 mark awarded for correct answer
	(ii) **Vertex (2,-1)**	1 mark awarded for correct answer
	(iii) **Focus (2,1)**	1 mark awarded for correct answer
P5	**(b)** $y=-4x$	1 mark awarded for correct answer
P5	**(c)**  $PA=\sqrt{(x-3)^2+(y-6)^2}$ comdition is $PA=5$	1 mark awarded for partial correct solution
	$5=\sqrt{(x-3)^2+(y-6)^2}$ $\therefore (x-3)^2+(y-6)^2=25$	2 marks awarded for a further correct partial solution
	$\therefore x^2-6x+9+y^2-12y+36=25$ $\therefore x^2-6x+y^2-12y+20=0$ The locus is $x^2-6x+y^2-12y+20=0$	3 marks awarded for complete correct solution

P6 (d)



1mark awarded for partial correct graph

2 marks awarded for complete correct graph

P3 (e)

(i)

Roots are $\alpha, 2\alpha$

Sum of roots: $3\alpha = -m$

Product of roots: $2\alpha^2 = 10$

1 mark awarded for correct solution

(ii)

$3\alpha = -m \ldots\ldots\ldots\ldots (A)$

$2\alpha^2 = 10 \ldots\ldots\ldots\ldots (B)$

From (A):

$\alpha = \frac{-m}{3} \ldots\ldots\ldots\ldots (C)$

substitute (C) into (B):

$2\left(\frac{-m}{3}\right)^2 = 10$

$\frac{2m^2}{9} = 10$

$m^2 = 45$

$m = \pm\sqrt{45}$

$m = \pm 3\sqrt{5}$

1 mark for partial correc solution.

2 marks for complete correct solution