HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS

2013

YEAR 12

TRIAL EXAMINATION (TASK 4)

EXAMINERS ~ G RAWSON, P BICZO, J DILLON, S FAULDS, S GUTESA, B MORRISON,

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 3 hours.
- Attempt **all** questions.
- This paper contains ten (10) multiple choice questions in Section I and six (6) free response questions in Section II.
- Each question in Section II is worth 15 marks.
- **All** necessary working should be shown in every question in Section II.
- Board approved calculators and Math aids may be used.
- **Multiple Choice questions to be answered on Multiple Choice answer sheet** which may be removed from the back of this question booklet.
- **Each free response question is to be started in a new answer booklet**. Write the question number and your name and/or student number at the top of each answer booklet.
- You **must** hand in the multiple choice answer sheet as well as an answer booklet for **each question** even if a question has not been attempted.
- This examination must **NOT** be removed from the examination room.

STUDENT NAME: ______________________

TEACHER: ______________________

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided on the back of this booklet for Questions 1-10

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1. Given that *ABCD* is a parallelogram, consider the following statements:

   I: If diagonals *AC* and *BD* are perpendicular, then *ABCD* is a rhombus.

   II: If diagonals *AC* and *BD* are equal, then *ABCD* is a square.

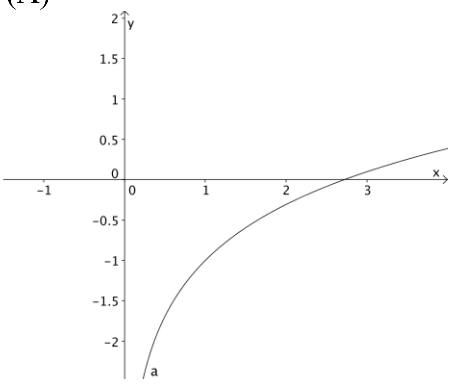
   Which of the statements I and II are correct?

   (A) I only　　　　　　(B) II only

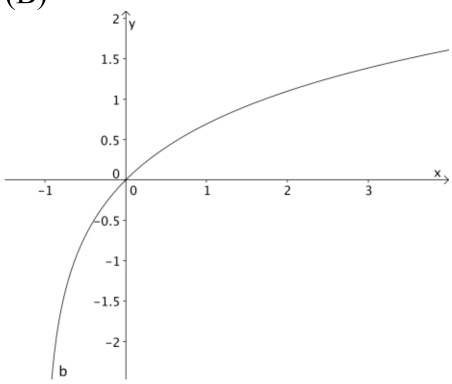
   (C) Both I and II　　　　(D) I and II are both incorrect

2. The graph of the function $y = \ln(x + 1)$ is shown in which of the graphs below:

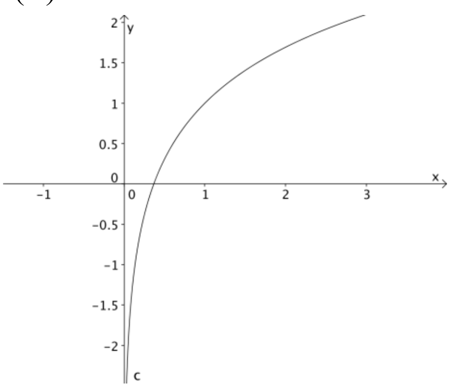
(A)



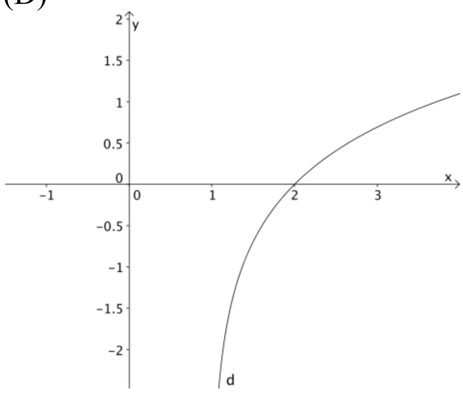
(B)



(C)



(D)



3. The centre of a circle has co-ordinates $(-3,4)$. If the circle passes through the point $(1,2)$, then the **diameter** has length

(A) $2\sqrt{2}$ units (B) $4\sqrt{2}$ units

(C) $2\sqrt{5}$ units (D) $4\sqrt{5}$ units

4. The solution to the inequality $6-x-x^2 \leq 0$ is

(A) $x \leq -3$ or $x \geq 2$ (B) $-3 \leq x \leq 2$

(C) $x \leq -2$ or $x \geq 3$ (D) $-2 \leq x \leq 3$

5. What are the coordinates of the focus of the parabola $x^2 = 2(y-1)$?

(A) $\left(0,\frac{1}{2}\right)$ (B) $\left(0,\frac{3}{2}\right)$

(C) $\left(\frac{1}{2},0\right)$ (D) $\left(\frac{3}{2},0\right)$

6. Which of the following conditions for $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ describe the slowing growth of a variable P?

(A) $\frac{dP}{dt} > 0$ and $\frac{d^2P}{dt^2} > 0$ (B) $\frac{dP}{dt} < 0$ and $\frac{d^2P}{dt^2} < 0$

(C) $\frac{dP}{dt} > 0$ and $\frac{d^2P}{dt^2} < 0$ (D) $\frac{dP}{dt} < 0$ and $\frac{d^2P}{dt^2} > 0$

7. The chance of a fisherman catching a legal length fish is 4 in 5. If three fish are caught at random, what is the probability that exactly one is of legal length?

(A) $\frac{4}{125}$ (B) $\frac{12}{125}$

(C) $\frac{16}{125}$ (D) $\frac{48}{125}$

8. The fourth term of an arithmetic series is 27 and the seventh term is 12. What is the common difference?

(A) –5 (B) 5

(C) 13 (D) 42

9. Solve for $0° \le x \le 360°$, $\sin x = \frac{\sqrt{3}}{2}$,

(A) 60° or 240° (B) 30° or 150°

(C) 30° or 210° (D) 60° or 120°

10. The curve $y = \sin x$ is rotated about the x-axis between $x = 0$ and $x = \frac{\pi}{4}$.

The volume swept out by this rotation is:

(A) $\pi\int_0^{\frac{\pi}{4}} \sin x.dx$ (B) $\pi\int_0^{\frac{\pi}{4}} (\sin x)^2 .dx$

(C) $\pi\int_0^{\frac{\pi}{4}} \sin x.dy$ (D) $\pi\int_{\frac{\pi}{4}}^{0} (1-\sin x)^2.dx$

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) *Commence a new answer booklet*

Marks

(a) Simplify $\dfrac{5}{x-1}-\dfrac{3}{x+1}$ **2**

(b) Solve $|2x-5|>3$ and graph the solution on a number line **3**

(c) Determine the value of a and b if $\dfrac{5}{2+\sqrt{3}}=a+b\sqrt{3}$ **2**

(d) Factorise completely: $3x^2-12y^2$ **2**

(e) Solve simultaneously : $y=x^2$ **3**

$y=2-x$

(f) Simplify $\log_5 125$ **1**

(g) Find the primitive function of $4x^3-7x+5$ **2**

Question 12 (15 marks) *Commence a new answer booklet*

Marks

(a) Find the equation of the line perpendicular to the line $y=-2x+3$ and passing through the point $(3,-4)$. Give your answer in general form. **2**

(b) (i) Find the perpendicular distance of the line *l* with equation $3x+4y+10=0$, to the origin (0, 0). **1**

(ii) **Explain** why the line l is a tangent to the circle $x^2+y^2=4$. **1**

(iii) Find the gradient of the line *l*. **1**

(iv) Calculate to the nearest degree, the angle the line *l* makes with the positive direction of the *x* axis. **1**

(v) Sketch the line *l*, and the circle $x^2+y^2=4$. On your diagram, show the *y* intercept of the line *l*, and call this point *P*. **1**

(vi) Given that the line *l* touches the circle at $Q\left(-\frac{6}{5},-\frac{8}{5}\right)$,

calculate the area of triangle *POQ*. **1**

(c) If the quadratic equation $3x^2-(k+1)x+5=0$ has roots equal in magnitude but opposite in sign, find the value of *k*. **1**

(d) Prove that $3x^2-x+7$ is positive, for all values of *x*.
You must justify your answer with working. **2**

(e) Find values for *a*, *b* and *c* if $x^2-x\equiv a(x+3)^2+bx+c$. **2**

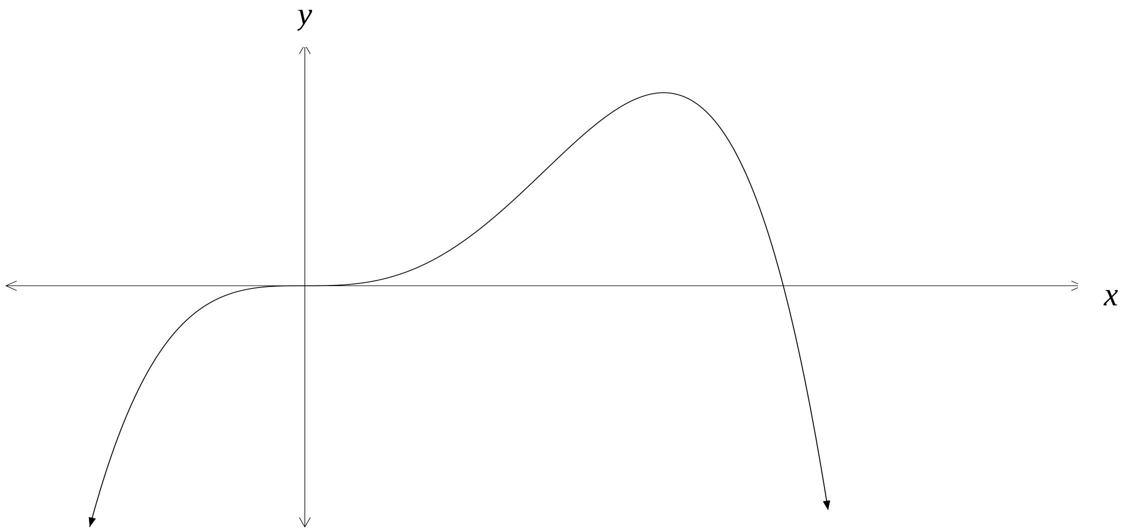
(f) By reducing the following equation to a quadratic, solve for *x*:

$$x^4-x^2=2$$ **2**

Question 13 (15 marks) *Commence a new answer booklet*

Marks

(a) Below is the graph of the function $y = f(x)$. **2**
In your answer booklet copy, or trace the diagram.
On your diagram, draw a possible sketch of $y = f'(x)$.



(b) A function $f(x)$ is defined by $f(x) = x^3 - 3x^2$.

(i) Find all values of x that satisfy $f(x) = 0$. **1**

(ii) Find the coordinates of the stationary points and determine their nature. **3**

(iii) Find the coordinates of any points of inflexion, justifying your answer. **2**

(iv) Sketch the curve $y = f(x)$ showing all important features. **2**

(v) For what values of x is the graph of $y = f(x)$ concave down? **1**

(c) For the parabola $y = 2x^2 - 8x$.

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **1**

(iii) Sketch the curve clearly labeling the vertex and focus. **1**

Question 14 (15 marks) *Commence a new answer booklet*

Marks

(a) Jane's mother puts \$300 into an account at the beginning of each year to pay for Jane's education in 12 years' time. If 6% p.a. interest is paid quarterly, how much money will Jane's mother have at the end of the 12 years? **3**

(b) In a certain city, the probability that the pollution level will be high is $0{\cdot}3$. If the pollution is monitored for 3 successive days, find the probability that the pollution levels will be:

(i) High on each of the 3 days **1**

(ii) High on exactly 2 days **1**

(c) Each week Lucinda and Terry take part in a raffle at their respective workplaces.

The probability that Lucinda wins a prize in her raffle is $\frac{1}{9}$.

The probability that Terry wins a prize in his raffle is $\frac{1}{16}$.

What is the probability that, during the next three weeks, at least one of them wins a prize? **2**

(d) The zoom function in a software package multiplies the dimensions of an image by $1{\cdot}2$. In an image, the height of the building is $50\,\text{mm}$. After the zoom function is applied once, the height of the building in the image is $60\,\text{mm}$. After a second application, its height is $72\,\text{mm}$.

(i) Calculate the height of the building in the image after the zoom function has been applied eight times. Give your answer to the nearest mm. **2**

(ii) The height of the building in the image is required to be more than 400 mm. Starting from the original image, what is the least number of times the zoom function must be applied? **2**

(e) Consider the geometric series $5+10x+20x^2+40x^3+\ldots$

(i) For what values of *x* does this series have a limiting sum? **2**

(ii) The limiting sum of the series is 100. Find the value of *x*. **2**

Question 15 (15 marks) *Commence a new answer booklet*

Marks

(a) By first writing the function $y = 7^x$ in logarithmic form, show that its derivative is $\ln 7 \times 7^x$. **2**

(b) The second derivative of a function is given by $f''(x) = -\frac{3}{x^2}$.

If, $f'\left(\frac{1}{2}\right) = 6$ and $f\left(\frac{1}{2}\right) = 1$, find $f(x)$ in simplest form. **3**

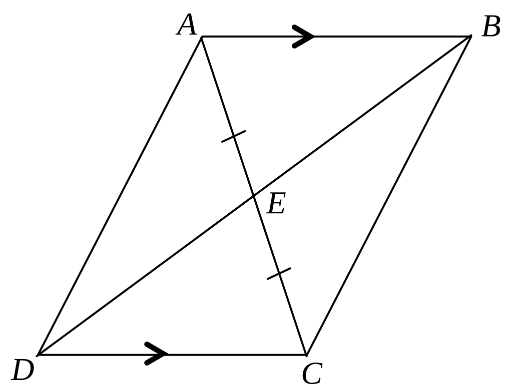
(c) Find the gradient of the curve $y = xe^x$ at the point where $x = \ln 2$. **2**
(Leave your answer in exact form)

(d) (i) Solve the equation $e^{x-1} - 2 = 0$, giving your answer correct to 2 decimal places. **1**

(ii) Evaluate the definite integral $\int_1^2 \left(e^{x-1} - 2\right) dx$. (Answer in exact form) **2**

(iii) Explain why your answer in (ii) does not give the area under the curve $y = e^{x-1} - 2$ between the lines $x = 1$ and $x = 2$.
(Note: it is not necessary to calculate the area). **1**

(e)



(i) In the above diagram, $AB \parallel CD$ and BD bisects AC. Prove that $\Delta ABE \equiv \Delta CED$. **2**

(ii) Hence, or otherwise, prove that $AD \parallel BC$. **2**

Question 16 (15 marks) *Commence a new answer booklet*

Marks

(a) Express 200° in radians, to 3 decimal places. **1**

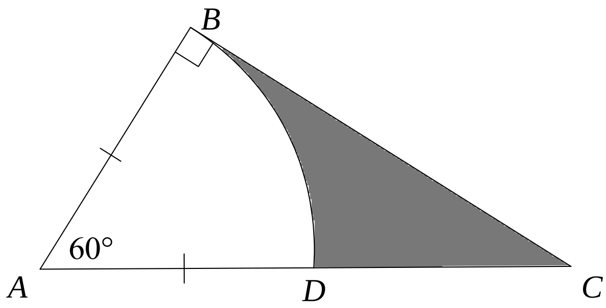
(b) Solve $\tan\theta = \dfrac{1}{\sqrt{2}}$, for $-\pi \le \theta \le \pi$, **2**

(c) Two ships leave the Port of Newcastle at 6 am on the 30th of June.
The first ship travels on a bearing of 060° from the port at a speed of 10 km/hr.
The second ship travels on a bearing of 120° from the port at 15 km/hr

(i) Draw a neat diagram to clearly show this information. **1**

(ii) Determine the distance between the ships after 1 hour of sailing. **2**

(d) In the diagram below, $\angle B = 90°$, $\angle A = 60°$ and AB = AD = 10m.
BD is an arc of the circle with centre A.



Calculate the shaded area in exact form. **3**

(e) Differentiate $x^2\tan^2(x^2)$ with respect to x. **3**

(f) (i) Differentiate ln(sin$x$) with respect to x **1**

(ii) Hence, find the exact value of $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2}{3}\cot x.dx$ **2**

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \qquad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \qquad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \qquad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \qquad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \qquad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \qquad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \qquad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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MATHEMATICS
2013
TRIAL EXAMINATION (TASK 4)

PART A ANSWER SHEET

NAME (or number) :____

Teacher: ____________

- Use this page to mark the answers to the questions in Part A.
- Mark the answer by shading in the circle that matches with the correct answer.
- If you make a mistake, draw a cross through the incorrect answer.

YEAR 12

MATHEMATICS

PART A Answers

1	a	b	c	d
2	a	b	c	d
3	a	b	c	d
4	a	b	c	d
5	a	b	c	d
6	a	b	c	d
7	a	b	c	d
8	a	b	c	d
9	a	b	c	d
10	a	b	c	d

Year 12 Trial Question No.11	Mathematics Solutions and Marking Guidelines	Examination 2013

Outcomes Addressed in this Question

P3 - performs routine arithmetic and algebraic manipulation involving **surds, simple rational expressions** and trigonometric identities

P4 - chooses and applies appropriate **arithmetic, algebraic**, graphical, trigonometric and geometric techniques

Outcome	Solutions	Marking Guidelines
P3	(a) $\frac{5}{x-1}-\frac{3}{x+1}=\frac{5(x+1)-3(x-1)}{(x-1)(x+1)}$ $=\frac{5x+5-3x+1}{x^2-1}$ $=\frac{2x+8}{x^2-1}=\frac{2(x+4)}{(x-1)(x+1)}$	<u>**2 marks**</u> : correct solution <u>**1 mark**</u> : substantial progress towards correct solution
P4	(b) $\lvert 2x-5\rvert>3$ $2x-5<-3$ or $2x-5>3$ $2x<2$ $\quad$ $2x>8$ $x<1$ $\quad$ $x>4$ 	<u>**3 marks**</u> : correct solution <u>**2 marks**</u> : substantially correct solution <u>**1 mark**</u> : partially correct solution (eg, finding one side of the inequality)
P3, P4	(c) $\frac{5}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{10-5\sqrt{3}}{4-3}$ $=10-5\sqrt{3}$ $\therefore a=10,\ b=-5$	<u>**2 marks**</u> : correct solution <u>**1 mark**</u> : substantial progress towards correct solution
	(d) $3x^2-12y^2=3(x^2-4y^2)$ $=3(x-2y)(x+2y)$	<u>**2 marks**</u> : correct solution <u>**1 mark**</u> : substantial progress towards correct solution
P4	(e) $y=x^2 \quad \ldots(1)$ $y=2-x \quad \ldots(2)$ $x^2=2-x \quad \langle \text{sub } (1)\to(2)\rangle$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-1,2$ subbing into (1)... when $x=1,\ y=1$ when $x=-2,\ y=4$ $\therefore$ solutions are $(1,1)$ & $(-2,4)$	<u>**3 marks**</u> : correct solution <u>**2 marks**</u> : substantially correct solution <u>**1 mark**</u> : partially progress towards correct solution *<u>NB</u>: it is important that you explicitly state which x value corresponds to which y value. It is these <u>pairs</u> which are the solutions*

P4	*Question 11 continued...* (f) $\log_5 125 = \log_5 5^3$ $= 3\log_5 5$ $= 3$ (g) $\int \left(4x^3 - 7x + 5\right) dx = x^4 - \frac{7x^2}{2} + 5x + C$	**<u>1 mark</u>** : correct answer **<u>2 marks</u>** : correct solution **<u>1 mark</u>** : substantial progress towards correct solution (leaving out the *C*)

M/C answers

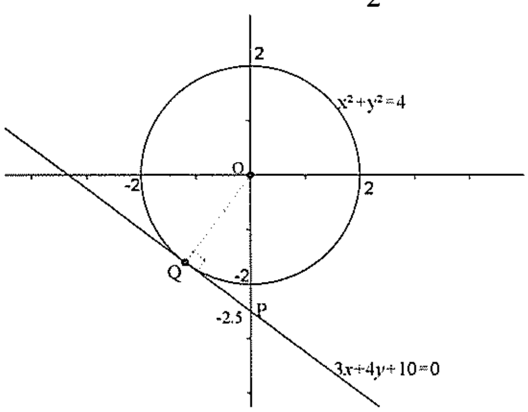
1. A
2. B
3. D
4. A
5. B
6. C
7. B
8. A
9. D
10. B

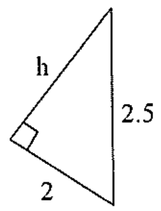
Year 12 Trial Question No.12	Mathematics Solutions and Marking Guidelines	Examination 2013

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, **geometry**, probability, trigonometry and series to solve problems

P4 Chooses and applies appropriate **arithmetic**, algebraic, **graphical**, trigonometric and **geometric techniques**

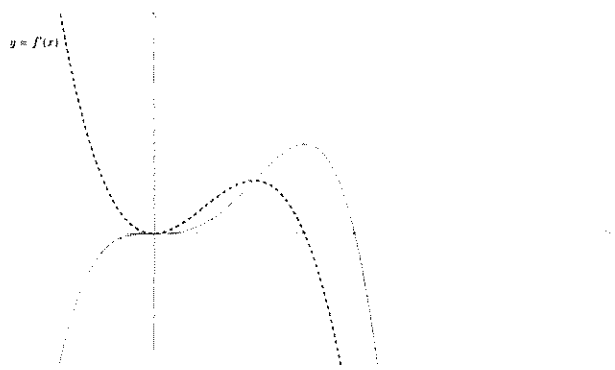
Outcome	Solutions	Marking Guidelines
P4	a) $y=-2x+3$ has gradient -2, $\therefore$ line perpendicular has $m=\frac{1}{2}$. Equation is $y+4=\frac{1}{2}(x-3)$ $2y+8=x-3$ $\therefore x-2y-11=0$	2 marks : correct solution 1 mark : substantial progress towards correct solution
H5	b) (i) $d=\frac{\lvert 0+0+10\rvert}{\sqrt{3^2+4^2}}=\frac{\lvert 10\rvert}{5}=2$ units	1 mark: correct answer
H5	(ii) Since the perpendicular distance **from the centre of the circle** $x^2+y^2=4$ to the line is equal to the radius of the circle, the line is a tangent to the circle.	1 mark: correct explanation
P4	(iii) Rearranging $3x+4y+10=0$, $y=\frac{-3x-10}{4}$ $\therefore m=\frac{-3}{4}$	1 mark: correct answer
P4	(iv) Using $m = \tan\theta$, $\tan\theta=\frac{-3}{4}$ Angle of inclination is $143°$	1 mark: correct answer
H5	(v) l cuts y axis when $x=0$. $4y+10=0$ $y=\frac{-5}{2}$ 	1 mark: correct graph, showing y intercept

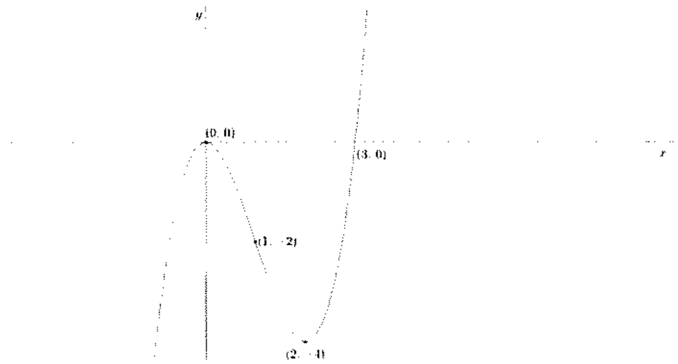
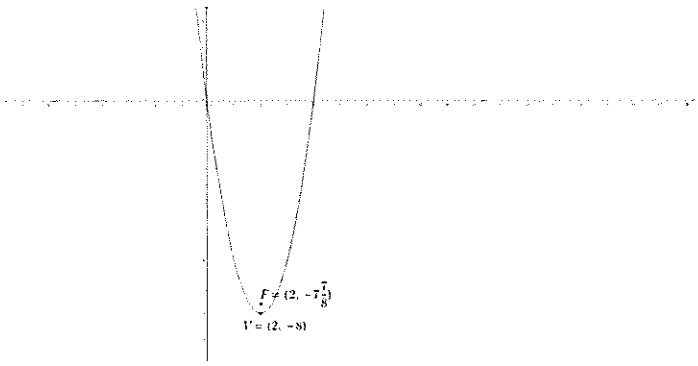
H5	(vi)  $h^2 = 2.5^2 - 2^2$ $h = 1.5$ Area of $\Delta = \frac{1}{2} \times 1\cdot5 \times 2 \ = 1\cdot5\,\text{u}^2$	1 mark : correct answer
H5	(c) Let the roots of $3x^2 - (k+1)x + 5 = 0$ be α and $-\alpha$. Then the sum of the roots = 0. Using sum of roots = $\frac{-b}{a}$, $\frac{k+1}{3} = 0$ $\therefore k = -1$.	1 mark : correct answer
	(d) Using $\Delta = b^2 - 4ac$, discriminant of $3x^2 - x + 7$ is $(-1)^2 - 4 \times 3 \times 7 = -83$ $\therefore$ since the discriminant is negative and the coefficient of x^2 is positive (as = 3), $3x^2 - x + 7$ is positive for all values of x.	2 marks : correct solution 1 mark: significant progress towards correct solution
P4	(e) $x^2 - x \equiv a(x+3)^2 + bx + c$ $x^2 - x \equiv ax^2 + 6ax + 9a + bx + c$ $x^2 - x \equiv ax^2 + (6a+b)x + (9a+c)$ Equating like co-efficients $a = 1$ $6a + b = -1 \quad \therefore 6 + b = -1 \quad \therefore 6 + b = -1$ $9a + c = 0 \quad \therefore 9 + c = 0$ $\therefore a = 1, \quad b = -7, \quad c = -9$	2 marks : correct solution 1 mark : substantial progress towards correct solution
P4	(f) $x^4 - x^2 = 2$ $(x^2)^2 - x^2 - 2 = 0$ Let $y = x^2$, $y^2 - y - 2 = 0$ $\therefore (y-2)(y+1) = 0$ $\therefore y = -1, 2$ $\therefore x^2 = -1, \quad x^2 = 2$ No solution for $x^2 = -1$ $\therefore x = \pm\sqrt{2}$ $\begin{array}{r\|l} & 1 \\ -2 & -2 \\ \hline & -1 \end{array}$	2 marks : correct solution 1 mark : substantial progress towards correct solution

Year 12	Mathematics	Task 4 (Trial HSC) 2013
Question No. 13	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

H6 uses the derivative to determine the features of the graph of a function
H7 uses the features of a graph to deduce information about the derivative
P5 understands the concept of a function and the relationship between a function and its graph

Part	Solutions	Marking Guidelines
(a) H7		**Award 2** for correct graph **Award 1** for graph indicating correct location of stationary point (or similar)
(b) (i) P5	$f(x)=0$ $x^3-3x^2=0$ $x^2(x-3)=0$ $\therefore x=0$ or 3.	**Award 1** for correct solution
(ii) H6	$f'(x)=3x^2-6x$ $f''(x)=6x-6$ Possible stationary points occur @ $f'(x)=0$ $3x^2-6x=0$ $3x(x-2)=0$ $\therefore x=0$ or 2 Test $x=0,\ f''(0)=-6<0$ $\therefore$ Relative maximum @ $(0,0)$ Test $x=2,\ f''(2)=6>0$ $\therefore$ Relative minimum @ $(2,-4)$	**Award 3** for correct solution **Award 2** for substantial progress towards solution **Award 1** for limited progress towards solution
(iii) H6	$f''(x)=0$ $6x-6=0$ $\therefore x=1$ Test $x=0,\ f''(0)=-6$ Test $x=2,\ f''(2)=12$ $\therefore$ Change in concavity either side of $x=1$ $\therefore$ Point of inflexion @ $(1,-2)$	**Award 2** for correct solution **Award 1** for substantial progress towards solution

(iv) H6		**Award 2** for correct graph **Award 1** correct graph, but with insufficient detail
(v) H6	Concave down $f''(x)<0$ $6x-6<0$ $\therefore x<1$	**Award 1** for correct solution
(c) (i) P5	$y=2x^2-8x$ $=2\left(x^2-4x\right)$ $=2\left(x^2-4x+4\right)-8$ $=2\left(x-2\right)^2-8$ $\therefore 2\left(x-2\right)^2-8=y$ $\therefore 2\left(x-2\right)^2=y+8$ $\therefore \left(x-2\right)^2=\frac{1}{2}\left(y+8\right)$ Vertex $=\left(2,-8\right)$	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(ii) P5	$4a=\frac{1}{2}$ $a=\frac{1}{8}$ Focus $=\left(2,-7\frac{7}{8}\right)$	**Award 1** for correct solution
(iii) P5		**Award 1** for correct graph with vertex and focus shown

H5 - applies appropriate techniques from the study of calculus, geometry, *probability*, trigonometry and *series* to solve problems

Outcome	Solutions	Marking Guidelines
H5	**a)** $r = 0.015$ $A_1 = 300(1.015)^4$ $A_2 = (A_1 + 300)(1.015)^4$ $= 300(1.015)^8 + 300(1.015)^4$ $A_3 = (A_2 + 300)(1.015)^4$ $= 300(1.015)^{12} + 300(1.015)^8 + 300(1.015)^4$ ... $A_{12} = 300(1.015)^{48} + 300(1.015)^{44} + 300(1.015)^{40} + ... + 300(1.015)^4$ $= 300(1.015)^4 \underbrace{(1 + 1.015^4 + 1.015^8 + ... + 1.015^{44})}_{\text{G.P. with } a=1,\ r=1.015^4,\ n=12 \quad S_n = \frac{a(r^n-1)}{r-1}}$ $= 300(1.015)^4 \left(\frac{1(1.015^{48} - 1)}{1.015^4 - 1} \right)$ $= \$5414.50$	**(3 marks)** correct solution **(2 marks)** substantial progress towards correct solution **(1 mark)** some progress towards correct solution
	(b) $P(high) = 0.3$ **(i)** $P(H,H,H) = 0.3^3$ $= 0.027$	**(1 mark)** correct solution
	(ii) $P(H,H,\_) + P(H,\_,H) + P(\_,H,H)$ $= 3 \times (0.3^2 \times 0.7)$ $= 0.189$	**(1 mark)** correct solution
	(c) $P(\text{atleast one}) = 1 - P(\text{none win})$ $= 1 - \left(\frac{8}{9}\right)^3 \times \left(\frac{15}{16}\right)^3$ $= \frac{91}{216}$	**(2 marks)** correct solution **(1 mark)** some progress towards correct solution
	(d) (i) 60, 72, 86.4, 103.68, ... $T_n = ar^{n-1}$ $T_8 = 60(1.2)^7$ $= 214.99$ $= 215$ mm (nearest mm)	**(2 marks)** correct solution **(1 mark)** some progress towards correct solution

H5	**(ii)** $$T_n = ar^{n-1}$$ $$T_n > 400$$ $$ar^{n-1} > 400$$ $$60(1.2)^{n-1} > 400$$ $$(1.2)^{n-1} > \frac{20}{3}$$ $$\log(1.2)^{n-1} > \log\left(\frac{20}{3}\right)$$ $$n-1 > \frac{\log\left(\frac{20}{3}\right)}{\log(1.2)}$$ $$n > 11.405$$ $$\therefore n = 12$$	**(2 marks)** correct solution **(1 mark)** some progress towards correct solution
	(e) **(i)** limiting sum if $\lvert r \rvert < 1$ $$r = \frac{10x}{5} = \frac{20x^2}{10x}$$ $$r = 2x$$ $$\lvert 2x \rvert < 1$$ $$2x < 1 \quad \text{or} \quad -2x < 1$$ $$x < \frac{1}{2} \quad \text{or} \quad x > \frac{-1}{2}$$ $$\therefore \frac{-1}{2} < x < \frac{1}{2}$$	**(2 marks)** correct solution **(1 mark)** some progress towards correct solution
	(ii) Limiting sum: $$S_\infty = \frac{a}{1-r}$$ $$100 = \frac{5}{1-2x}$$ $$100 - 200x = 5$$ $$200x = 95$$ $$x = \frac{19}{40} \text{ or } 0.475$$	**(2 marks)** correct solution **(1 mark)** some progress towards correct solution

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H3 manipulates algebraic expressions involving logarithmic and exponential functions
H5 applies appropriate techniques from the study of calculus to solve problems
H8 uses techniques of integration to calculate areas and volumes</td></tr>
<tr><th>Outcome</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>H3, H5</td><td>(a)
$y = 7^x$
$x = \log_7 y$ (written in equivalent logarithmic form)
Changing the base to 'e'
$x = \dfrac{\ln y}{\ln 7}$
$\dfrac{dx}{dy} = \dfrac{1}{\ln 7}.\dfrac{1}{y}$
$\dfrac{dy}{dx} = \dfrac{1}{\dfrac{1}{\ln 7}.\dfrac{1}{y}}$
$= \ln 7.y$
$= \ln 7.7^x$ (as required)</td><td>2 marks
Correct solution showing each step
1 mark
Demonstrates ability to express exponential equation in logarithmic form.</td></tr>
<tr><td>H3, H5</td><td>(b)
$f''(x) = -\dfrac{3}{x^2}$
$f'(x) = \dfrac{3}{x} + c$
But $f'\left(\dfrac{1}{2}\right) = 6$
Substituting gives $6 = \dfrac{3}{\frac{1}{2}} + c$
$c = 0$
$\therefore f'(x) = \dfrac{3}{x}$
$f(x) = 3\ln x + c$
But $f\left(\dfrac{1}{2}\right) = 1$
Substituting gives $1 = 3\ln 2^{-1} + c$
$c = 1 + 3\ln 2$
$\therefore f(x) = 3\ln x + 3\ln 2 + 1$
$= 3\ln 2x + 1$</td><td>3 marks
Correct solution showing each step
2 mark
Substantial progress towards correct solution demonstrating the abilty to find a primitive function and evaluate the constant as required.
1 mark
Some progress towards correct solution.</td></tr>
<tr><td>H3, H5</td><td>(c)
$y = xe^x$
$y' = xe^x + e^x$
$= e^x(x+1)$
Gradient of curve when $x = \ln 2$
$m = e^{\ln 2}(\ln 2 + 1)$
$= 2\ln 2 + 2$
$= 2(\ln 2 + 1)$</td><td>2 marks
Correct solution showing each step
1 mark
Differentiates the function correctly.</td></tr>
<tr><td>H3</td><td>(d) (i)
$e^{x-1} - 2 = 0$
$e^{x-1} = 2$
$x - 1 = \ln 2$
$x = \ln 2 + 1$
$= 1.69$ (2 dec. pl.)</td><td>1 mark
Correct solution</td></tr>
</table>

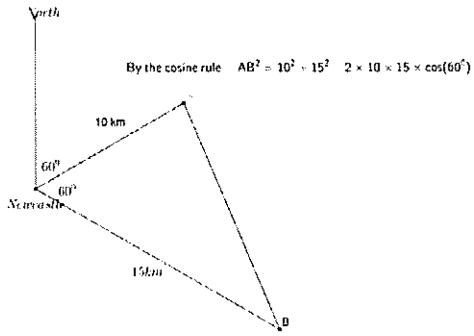
H3, H5	**(ii)** $$\int_1^2 e^{x-1} - 2\,dx = \left[e^{x-1} - 2x\right]_1^2$$ $$= (e-4) - (1-2)$$ $$= e - 4 + 1$$ $$= e - 3$$	**2 marks** Correct solution showing each step **1 mark** Correctly finds the integral but does not proceed to correct solution.
H8	**(iii)** The curve $y = e^{x-1} - 2$ cuts the x-axis at 1.69. Hence, some of the curve will be above the x-axis, a "positive" area, and some below the x-axis, a "negative" area. Area under the curve should be the magnitude of these areas summed. The integral is the sum of the "signed" area.	**1 mark** Statement demonstrates a clear understanding of the relationship between an integral and an area.
H2	**(e) (i)** In Δ's ABE and CED $\angle ABE = \angle CDE$ (alternate angles equal, $AB \parallel CD$) $\angle AEB = \angle CED$ (vertically opposite angles equal) $AE = CE$ (given) $\therefore \Delta ABE \equiv \Delta CED$ (AAS Test)	**2 marks** Correct solution showing all reasoning. **1 mark** Substantial progress towards correct solution or correct solution with deficient reasoning.
H2	**(ii)** Now, $BE = DE$ (matching sides in congruent triangles) Since $AE = CE$ (given) And $BE = DE$ (proven) Diagonals of quadrilateral $ABCD$ bisect each other, $\therefore$ $ABCD$ is a parallelogram (diagonal properties of a parallelogram.) $\therefore$AD $\parallel$ BC (opposite sides of a parallelogram)	**2 marks** Correct solution showing all reasoning. **1 mark** Substantial progress towards correct solution or correct solution with deficient reasoning.

Year 12 Trial Higher School Certificate Extension 1 Mathematics 2013	
Question No. 16	**Solutions and Marking Guidelines**

Outcomes Addressed in this Question
H3 - manipulates algebraic expressions involving logarithmic and exponential functions P3 - performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities H5 - applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems H9 - communicates using mathematical language, notation, diagrams and graphs

Question 16:

H3 - manipulates algebraic expressions involving logarithmic and exponential functions
P3 - performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
H5 - applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
H9 - communicates using mathematical language, notation, diagrams and graphs

	(a) $\frac{200}{360}\times 2\pi = 3\cdot 491$	Correct answer only 1 mark
	(b) Quadrants 1 & 3. Related angle $\approx 0\cdot 6155$ (retain full value in memory) $\theta = 0\cdot 6155$ or $-2\cdot 5261$	Both answers 2 marks
H5	(c) (i)  (*ii*) Distance $= 5\sqrt{7}$ km $\approx 13\cdot 23$ km	1 mark for a neat, labelled diagram 2 marks for complete solution
H5	(d) In right triangle ABC BC$=10\sqrt{3}$ m Area of ABC$= \frac{1}{2}\times 10\times 10\sqrt{3}$ $\therefore$ $50\sqrt{3}$ m^2 Area of sector ABD$= \frac{\pi\times 10^2}{6}$ $\therefore$ $\frac{50\pi}{3}$ m^2 Exact area $= (50\sqrt{3} - \frac{50\pi}{3})$ m^2	1 mark each for the area of the triangle and the sector. 3 marks for the complete solution.

H5	(e) $\dfrac{d(x^2\tan^2(x^2))}{dx} = 2x \times \tan^2(x^2) + x^2 \times \dfrac{d(\tan^2(x^2))}{dx}$ Using the chain rule and letting $u=\tan(v)$ where $v=x^2$ $\dfrac{d(\tan^2(x^2))}{dx} = \dfrac{du^2}{du} \times \dfrac{du}{dv} \times \dfrac{dv}{dx} = 2u \times \sec^2 v \times 2x$ $= 2\tan(x^2) \times \sec^2(x^2) \times 2x$ $= 4x\tan(x^2)\sec^2(x^2)$ Hence: $\dfrac{d(x^2\tan^2(x^2))}{dx} = 2x\tan^2(x^2) + 4x^3\tan(x^2)\sec^2(x^2)$ $= 2x\tan(x^2)\{\tan(x^2) + 2x^2\sec^2(x^2)\}$	1 mark for the product rule or correct use. 1 mark for substantial progress. 3 marks for the complete solution.
H3	(f) (i) $\dfrac{d\ln(\sin x)}{dx} = \cot x$ (ii) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2}{3}\cot x\,dx = \frac{2}{3}\{\ln(\sin x)\}_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \dfrac{2}{3}\{\ln\dfrac{\sqrt{3}}{2} - \ln\dfrac{1}{\sqrt{2}}\}$ $= \dfrac{2}{3}\{\ln\dfrac{\sqrt{3}}{2} + \ln\sqrt{2}\}$ $= \dfrac{2}{3}\ln\dfrac{\sqrt{6}}{2}$ ← acceptable progress for full marks $= \dfrac{2}{3}\{\ln 6^{\frac{1}{2}} - \ln 2\}$ $= \dfrac{2}{3} \times \dfrac{1}{2}\ln 6 - \dfrac{2}{3}\ln 2$ $= \dfrac{\ln 6 - 2\ln 2}{3}$ $(2\ln 2 = \ln 4)$ $= \dfrac{1}{3}\ln(\dfrac{3}{2})$	1 mark 1 mark for correct integration and 1 mark for substantial work towards the complete solution 2 marks for the complete solution.

Note: Part (e) was, in some cases, expressed as
$2x(2x^2+1)\tan^2(x^2) + 4x^3\tan(x^2)$, after having replaced $\sec^2(x^2)$ by $1+\tan^2(x^2)$
and collecting the $\tan^2(x^2)$