HURLSTONE AGRICULTURAL HIGH SCHOOL

2014

MATHEMATICS

HSC TRIAL EXAMINATION

(ASSESSMENT TASK 4)

EXAMINERS ~ S. FAULDS, S. HACKETT, P. BICZO, G. HUXLEY, D. CRANCHER

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 3 hours.
- Attempt **all** questions.
- Board approved calculators and mathematical templates may be used.
- This examination must **NOT** be removed from the examination room.
- Question 1 – 10 are to be completed on the Multiple Choice Answer Sheet.
- Show all necessary working in Questions 11 – 16.
- **Start each question in a separate answer booklet.**
- Put your student number on each booklet.
- A table of standard integrals is on the back of the Multiple Choice Answer Sheet.

Total marks – 100

Section I

10 marks

- **Attempt Questions 1 – 10.**
- Allow about 15 minutes for this section.
- Fill in your answers on the multiple choice answer sheet provided.

Section II

90 marks

- **Attempt Questions 11 – 16.** Each of these six (6) questions worth 15 marks. Allow about 2 hours and 45 minutes for this section. **Each question is to be started in a new answer booklet.** Additional booklets are available if required.

STUDENT NAME: ____________________

CLASS TEACHER: ____________________

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2014 HSC Mathematics Examination.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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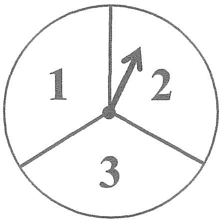
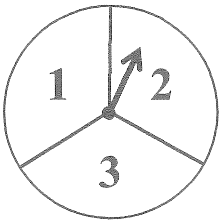
1. $|-6|-|-12|=$

A: 6　　**B:** –6　　**C:** –18　　**D:** 18

2.





Two identical spinners, containing the values 1, 2, and 3 are spun and the results on each are multiplied together. What is the probability that the resulting ~~sum~~ product is either an even number **<u>or</u>** a number greater than 6?

A: $\frac{1}{3}$　　**B:** $\frac{5}{9}$　　**C:** $\frac{4}{9}$　　**D:** $\frac{2}{3}$

3. $\dfrac{\log_3 8}{\log_3 2}=$

A: $2\log_3 2$　　**B:** $\log_3 6$　　**C:** 4　　**D:** 3

4. Fully factorised, $16x^3-54=$

A: $2(2x-3)(4x^2+12x+9)$　　**B:** $2(2x-3)(4x^2+6x+9)$

C: $2(2x-3)(4x^2-6x+9)$　　**D:** $2(2x-3)(4x^2-12x+9)$

5. If $2\sqrt{80}+\sqrt{45}=a\sqrt{b}$, then

A: $a=11,\ b=5$　　**B:** $a=5,\ b=5$　　**C:** $a=7,\ b=5$　　**D:** $a=17,\ b=5$

6.

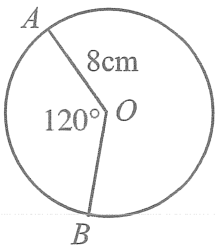


DIAGRAM
IS NOT TO
SCALE

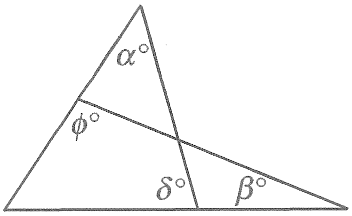
Which of these calculations would NOT give the correct area of sector ***AOB***?

A: $\frac{1}{2}\times 8^2\times\frac{2\pi}{3}$ **B:** $\frac{120}{360}\times\pi\times 8^2$ **C:** $\frac{1}{2}\times 8^2\times 120$ **D:** $\frac{1}{2}\times 8^2\times\frac{120}{180}\times\pi$

7. A primitive function of $(3x-2)^3$ is:

A: $9(3x-2)^2$ **B:** $\frac{(3x-2)^4}{12}$ **C:** $\frac{(3x-2)^4}{4}$ **D:** $9(3x-2)^4$

8. In the diagram below:



$\alpha°+\delta°=$

A: $\beta°$ **B:** $180°-(\beta°+\phi°)$ **C:** $\phi°$ **D:** $\beta°+\phi°$

9. The exact value of $\sin\frac{4\pi}{3}=$

A: $\frac{1}{2}$ **B:** $-\frac{1}{2}$ **C:** $\frac{\sqrt{3}}{2}$ **D:** $-\frac{\sqrt{3}}{2}$

10. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\sin 2x\,dx=$

A: 4 **B:** –4 **C:** 0 **D:** 2

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a new answer booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Start a new answer booklet.** **Marks**

(a) Solve: $|x-5| \geq 2$ **2**

(b) Find the exact value (in simplest form) of $\sqrt{p^4 - 2p^2}$ when $p = 2\sqrt{5}$. **2**

(c) State the range of $f(x) = (x+2)^2 - 3$. **1**

(d) Evaluate $\sqrt{\frac{30}{7} - \sqrt{12}}$ correct to 3 significant figures. **1**

(e) Express $0.6\dot{4}$ as a fraction in simplest form, showing all working. **3**

(f) Solve: $\frac{x}{8} - \frac{x-6}{4} = 2$ **3**

(g) Given $5^m = 4$, evaluate 5^{1-2m}. **3**

Question 12 (15 marks) **Start a new answer booklet.** **Marks**

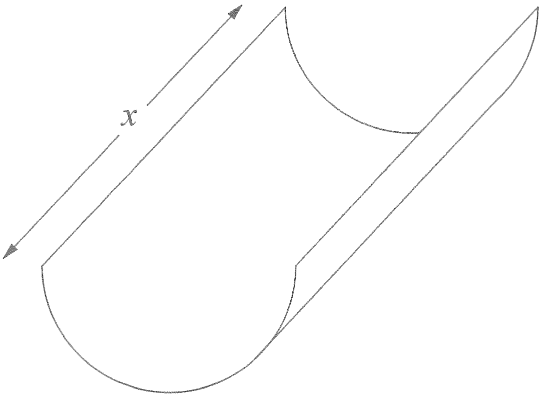
(a) Use the definition of the derivative, $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, to find $f'(x)$ when $f(x)=\dfrac{1}{x}$. **2**

(b) (i) Use the quotient rule to show that if $y=\dfrac{x}{(2x-3)^3}$, $\dfrac{dy}{dx}=\dfrac{-4x-3}{(2x-3)^4}$. **2**

(ii) Hence, find the equation of the normal to the curve $y=\dfrac{x}{(2x-3)^3}$, at the point $(1,-1)$. **2**

(c) Given $f'(x)=x(x-3)(x+1)$ and $f(0)=0$, find the function $f(x)$. **2**

(d)



The diagram above, shows a half-pipe, which is to be made from a rectangular piece of metal of length x ~~cm~~ m. The metal sheet is to be fabricated to form the open half cylinder shown. The perimeter of the rectangular sheet is 60 m.

(i) By using Calculus, find the dimensions of the rectangle that will give the maximum surface area. **2**

(ii) For the dimensions giving this maximum area, find the height from the ground up to the top of the half-pipe. **1**

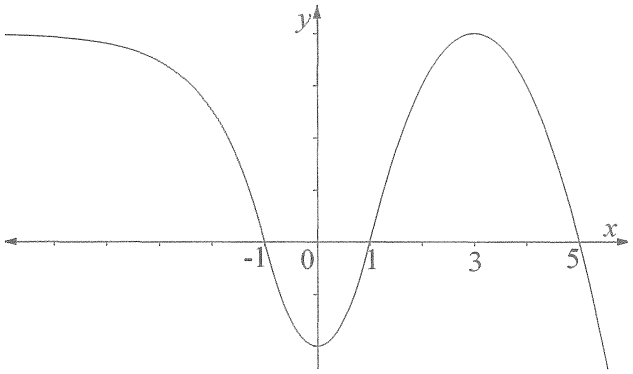
Question 12 continues on the next page…

(e) A continuous curve, $y = f(x)$ has the following properties for the interval $a \leq x \leq b$. **2**

$$f(x) < 0,\ f'(x) < 0,\ f''(x) > 0$$

Sketch a curve satisfying these conditions for $a \leq x \leq b$.

(f) **2**



Sketch the derivative function of the curve above.

Question 13 (15 marks) **Start a new answer booklet.** **Marks**

(a) Solve: $\sin^2 x - \sin x - 2 = 0$ for $-\pi \le x \le \pi$. **3**

(b) Show that: $\dfrac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta} = \cot\theta$. **3**

(c) (i) Sketch the curve $y = 3\sin 2x$ in the domain $0 \le x \le 2\pi$ showing the main features of the graph. **2**

(ii) Use your graph to find the number of solutions to the equation $3\sin 2x - 1 = 0$ for $0 \le x \le 2\pi$. **1**

(d) Find: $\dfrac{d}{dx}\left(\tan^2\left(\dfrac{x}{3}\right)\right)$ **2**

(e) Ship ***A*** sails from port ***P*** on a bearing of 060° for 56 nautical miles. Ship ***B*** sails from port ***P*** on a bearing of 110° for 48 nautical miles. Draw a diagram to show this information.

(i) Calculate the distance between the ships, correct to one decimal place. **2**

(ii) Find the bearing of ship ***A*** from ship ***B***. **2**

Question 14 (15 marks) **Start a new answer booklet.** **Marks**

(a) Find the values of m for which the equation $x^2 + (m - 2)x + 4 = 0$ has no real roots. **2**

(b) The quadratic equation $2x^2 - (m + 2)x + m = 0$ has one root which is twice the other. Find the values of m. **2**

(c) Differentiate with respect to x: $e^x \sin x$ **1**

(d) Find the primitive of the function $g(x) = \dfrac{x}{x^2 - 11}$. **2**

(e)

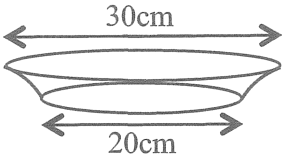
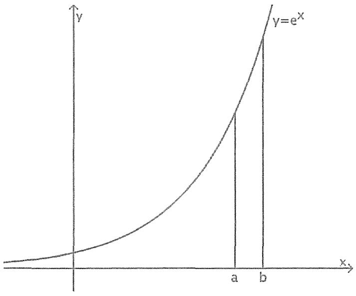


Diagram is not to scale

An artist produces a brass bowl as shown above. The artist made the following comment: "I received inspiration from the mathematical curve $y = e^x$. If the curve is rotated about the x-axis, the volume formed will be exactly that of the bowl."

(i) If the curve $y = e^x$ is rotated about the x-axis, the domain of the section of curve that is rotated to form the bowl is $a \le x \le b$. **2**



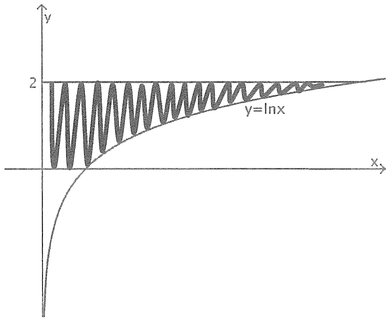
Graph not to scale

Find values for a and b to provide a bowl with the given dimensions.

(ii) What will be the ***exact*** capacity of the bowl in mL? **3**

Question 14 continues on the next page...

(f) Simpson's Rule is to be used to approximate the area enclosed by the y-axis, the curve $y = \ln x$ and the lines $y = 0$ and $y = 2$ as shown below:



(i) Copy and complete the table below in your answer booklet. Write your calculated values to 1 decimal place. **1**

y	0	0.5	1	1.5	2
x	1	1.6	2.7		

(ii) Use two applications of Simpson's Rule to find an approximation for the area described. Give your answer correct to one decimal place. **2**

Question 15 (15 marks) **Start a new answer booklet.** **Marks**

(a) (i) A point $P(x, y)$ is equidistant from the points $A(-7, 4)$ and $B(-1, 12)$. Show that the locus of the point P is $3x + 4y - 20 = 0$. **2**

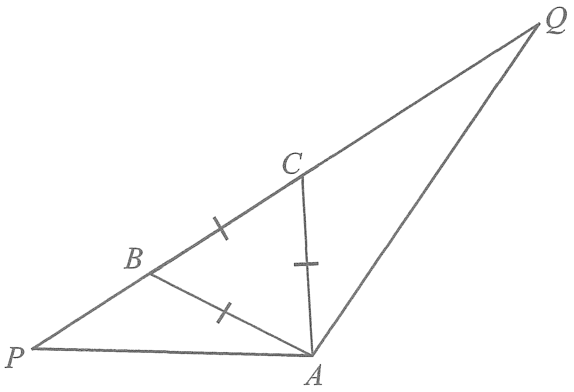
(ii) Show that the equation of the locus found in (i) is also the equation of a focal chord of the parabola $(x - 4)^2 = 4(y - 1)$. **3**

(iii) Show that the focal chord and the parabola intersect on the y-axis at a point G. **2**

(iv) Show that the equation of the line perpendicular to the focal chord which passes through the focus, S, is $4x - 3y - 10 = 0$ and find the point, H, where it crosses the x-axis. **2**

(v) Hence, find the area of ΔGSH. **2**

(b)



In the diagram, ΔABC is equilateral and $\angle PAQ = 120°$.

(i) Prove $\Delta PBA \,|||\, \Delta PAQ$. **3**

(ii) Hence, show $PA^2 = PQ.PB$, giving a reason for your answer. **1**

Question 16 (15 marks) **Start a new answer booklet.** **Marks**

(a) A banksia bush was planted when it was 50cm tall.
At the end of the first year after planting, the banksia was 90cm tall.
Each year's growth was then 80% of the previous year's growth.
What is the limiting maximum height of the banksia? **1**

(b) The second term of an arithmetic series is 37 and the sixth term is 17.
What is the sum of the first ten terms of the series? **2**

(c) Kylie invests $\$P$ at 9% per annum compounded annually.
She plans to withdraw \$5000 at the end of each year for six years to cover university fees.

(i) Write down an expression for the amount $\$A_1$ remaining in the account following the withdrawal of the first \$5000. **1**

(ii) Find an expression for the amount $\$A_3$ remaining in the account after the third withdrawal. **2**

(iii) How much does Kylie need to invest if the account balance is to be \$0 at the end of the six years? **2**

(d) A game is played in which two coloured dice are thrown once.
The six faces of the red die are numbered 1, 3, 5, 7, 9 and 11.
The six faces of the white die are numbered 2, 4, 6, 8, 10 and 12.
The player wins if the number on the white die is larger than the number on the red die.

(i) Show that the probability of the player winning a game is $\frac{7}{12}$. **1**

(ii) What is the probability that the player wins exactly once in two successive games? **2**

(iii) What is the probability that the player wins at least once in two successive games? **1**

(e) There is a 75% chance that Lloyd will pass his driving test and a 40% chance that Harry will pass his.

(i) Draw a probability tree to represent this information, including the list of possible outcomes. **2**

(ii) Find the probability that only one of Lloyd and Harry pass the test. **1**

END OF EXAMINATION

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HURLSTONE AGRICULTURAL HIGH SCHOOL

Year 12 Mathematics 2014
HSC Trial Examination
(Assessment Task 4)

NAME.......................................

CLASS.................

EXAMPLE 2 + 4 =

A 2 **B** 4 **C** 6 **D** 8 Ⓐ Ⓑ ● Ⓓ

ATTEMPT ALL QUESTIONS

1	Ⓐ Ⓑ Ⓒ Ⓓ
2	Ⓐ Ⓑ Ⓒ Ⓓ
3	Ⓐ Ⓑ Ⓒ Ⓓ
4	Ⓐ Ⓑ Ⓒ Ⓓ
5	Ⓐ Ⓑ Ⓒ Ⓓ
6	Ⓐ Ⓑ Ⓒ Ⓓ
7	Ⓐ Ⓑ Ⓒ Ⓓ
8	Ⓐ Ⓑ Ⓒ Ⓓ
9	Ⓐ Ⓑ Ⓒ Ⓓ
10	Ⓐ Ⓑ Ⓒ Ⓓ

This sheet should be removed from the question booklet and handed in with your answer booklets.

STANDARD INTEGRALS

$$\int x^n dx \qquad = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \qquad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx \qquad = \ln x, \qquad x > 0$$

$$\int e^{ax} dx \qquad = \frac{1}{a} e^{ax}, \qquad a \neq 0$$

$$\int \cos ax dx \qquad = \frac{1}{a} \sin ax, \qquad a \neq 0$$

$$\int \sin ax dx \qquad = -\frac{1}{a} \cos ax, \qquad a \neq 0$$

$$\int \sec^2 ax dx \qquad = \frac{1}{a} \tan ax, \qquad a \neq 0$$

$$\int \sec ax \tan ax dx \qquad = \frac{1}{a} \sec ax, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx \qquad = \frac{1}{a} \tan^{-1} \frac{x}{a}, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx \qquad = \sin^{-1} \frac{x}{a}, \qquad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx \qquad = \ln\left(x + \sqrt{x^2 - a^2}\right), \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx \qquad = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

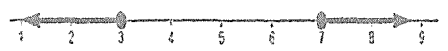
NOTE : $\ln x = \log_e x, \quad x > 0$

Year 12 Mathematics Trial HSC (Task 4) 2014

Question No's. 1 – 10 Multiple Choice Answers

	Answers	
	1. B	
	2. D	
	3. D	
	4. B	
	5. A	
	6. C	
	7. B	
	8. D	
	9. D	
	10. C	

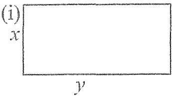
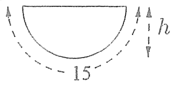
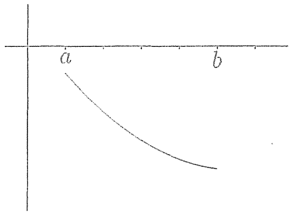
QUESTION 11 –Advanced Mathematics Trial HSC Exam 2014

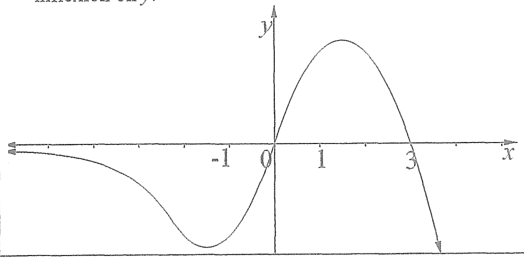
	SAMPLE SOLUTION	
	a)  $x \le 3, x \ge 7$	**2 marks** – correct answer clearly showing how the answer was reached **1 mark** – substantial progress towards correct answer *This part was generally done well. A number of students found the critical values x=3 and x=7, and then tested them in the original inequality - this step achieved nothing. The best responses utilised a number line to find all points more than 2 units away from 5.*
	b) $\sqrt{p^4-2p^2} = \sqrt{(2\sqrt{5})^4-2(2\sqrt{5})^2}$ $=\sqrt{400-2\times 20}$ $=\sqrt{360}$ $=6\sqrt{10}$	**2 marks** – correct answer in simplest form **1 marks** – correct answer unsimplified *A range of errors were made in the simplification of the surds in this question. The question stated "in simplest form" yet many students failed to simplify their answer.*
	c) *All real* $y, y \ge -3$	**1 mark** – correct answer *The best responses sketched the parabola, using the transformation rules, then read the range from their graph.*
	d) 0.906	**1 mark** – correct answer to 3 sig figs
	e) *Let* $x = 0.6\dot{4}$ $x = 0.6444...$ $10x = 6.444...$ (1) $100x = 64.444...$ (2) $(2)-(1)$ $90x = 58$ $x = \frac{58}{90} = \frac{29}{45}$ $\therefore 0.6\dot{4} = \frac{29}{45}$	**3 marks** – correct answer in simplest form, showing all steps using a valid method **2 marks** – correct answer NOT in simplest form, showing all steps using a valid method **1 mark** – substantial progress towards correct answer *Many students failed to simplify the fraction or show all working, even though both of these were specified in the question.*
	f) $\frac{x}{8} - \frac{x-6}{4} = 2$ $(\times 8)$ $x-2(x-6)=16$ $x-2x+12=16$ $-x = 4$ $x = -4$	**3 marks** – correct solution **2 marks** – substantial progress towards correct answer **1 mark** – limited progress towards correct answer *This question was done well by most students.*
	g) $5^{1-2m} = \frac{5^1}{5^{2m}}$ $=\frac{5}{(5^m)^2}$ $=\frac{5}{4^2}$ $=\frac{5}{16}$	**3 marks** – correct solution **2 marks** – substantial progress towards correct answer **1 mark** – limited progress towards correct answer *Many students complicated this question by using logarithms – often unsuccessfully.*

Year 12 Trial Question No.12	Mathematics Solutions and Marking Guidelines	Examination 2014

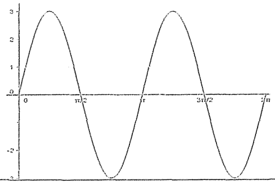
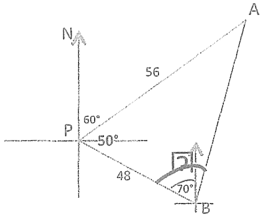
Outcomes Addressed in this Question

P6 relates the derivative of a function to the slope of its graph
P7 determines the derivative of a function through routine application of the rules of differentiation
H5 applies appropriate techniques from the study of **calculus**, **geometry**, probability, trigonometry and series to solve problems
H6 uses the derivative to determine features of the graph of a function
H7 uses the features of a graph to deduce information about the derivative

Outcome	Solutions	Marking Guidelines
P7	(a) $f'(x)=\lim\limits_{h\to 0}\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$ where $f(x)=\dfrac{1}{x}$. $=\lim\limits_{h\to 0}\dfrac{\frac{x-(x+h)}{x(x+h)}}{h}$ $=\lim\limits_{h\to 0}\dfrac{-h}{x(x+h)}\times\dfrac{1}{h}$ $=\lim\limits_{h\to 0}\dfrac{-1}{x(x+h)}$ $=\dfrac{-1}{x(x+0)}\quad=\dfrac{-1}{x^2}$	2 marks : correct solution 1 mark : substantial progress towards correct solution
P7	(b)(i) $y=\dfrac{x}{(2x-3)^3}$ $\dfrac{dy}{dx}=\dfrac{(2x-3)^3\times 1-x.3(2x-3)^2.2}{(2x-3)^6}$ $\dfrac{dy}{dx}=\dfrac{(2x-3)^2((2x-3)-6x)}{(2x-3)^6}$ $\therefore\dfrac{dy}{dx}=\dfrac{-4x-3}{(2x-3)^4}$	2 marks : correct solution 1 mark : substantial progress towards correct solution
P6	(ii) Gradient of tangent at $x=1$ is $\dfrac{dy}{dx}=\dfrac{-4-3}{(2-3)^4}=-7$ $\therefore$ gradient of normal at $x=1$ is $\dfrac{1}{7}$. $\therefore$ equation of normal is $y+1=\dfrac{1}{7}(x-1)$ Equation is $x-7y-8=0$.	2 marks : correct solution 1 mark : substantial progress towards correct solution
H5	(c) $f'(x)=x(x-3)(x+1)$ $=x(x^2-2x-3)$ $=x^3-2x^2-3x$ $f(x)=\int(x^3-2x^2-3x)dx$ $f(x)=\dfrac{x^4}{4}-\dfrac{2x^3}{3}-\dfrac{3x^2}{2}+c$ $f(0)=0,\therefore\quad 0=0-0-0+c$ $f(x)=\dfrac{x^4}{4}-\dfrac{2x^3}{3}-\dfrac{3x^2}{2}$	2 marks : correct solution 1 mark : substantial progress towards correct solution
H5	(d)(i)  $2x+2y=60$ $x+y=30$ $y=30-x$ $A=x(30-x)$ $A=30x-x^2$ $\dfrac{dA}{dx}=30-2x=0$ for maximum area. $\dfrac{d^2A}{dx^2}=-2$ Since $\dfrac{d^2A}{dx^2}<0$, $\therefore$ maximum when $x=15$ $\therefore$ maximum surface area when the rectangle is 15 m by 15 m.	2 marks : correct solution 1 mark : substantial progress towards correct solution
H5	(ii)  Height from the ground equals the radius of the semicircle. Half the circumference = 15 $\therefore \pi r=15$ $\therefore$ radius $=\dfrac{15}{\pi}$ Height is $\dfrac{15}{\pi}$ or 4.77 m	1 mark: correct answer
H6	(e) $f(x)<0$: y values negative $f'(x)<0$: decreasing $f''(x)>0$: concave up 	2 marks : correct solution 1 mark : displays graph with two of the given properties correct

H7	(f) At stationary points derivative = 0. $\therefore y' = 0$ at $x = 0, 3$. Curve decreasing when $x < 0$ and $x > 3$, and increasing when $0 < x < 3$. $\therefore y'$ is negative when $x < 0$ and $x > 3$, and y' positive when $0 < x < 3$. Maximum, minimum values of y' occur at points of inflexion on y. 	2 marks : correct solution 1 mark : substantial progress towards correct solution

QUESTION 13 –Advanced Mathematics Trial HSC Exam 2014

	SAMPLE SOLUTION	
	a) $\sin^2 x - \sin x - 2 = 0$ $(\sin x - 2)(\sin x + 1) = 0$ $\sin x = 2 \qquad \sin x = -1$ *no solution* $\qquad x = -\frac{\pi}{2}$ $\therefore x = -\frac{\pi}{2}$	**3 marks** – correct solution **2 marks** – solution written in incorrect form (eg $-90°$) or outside of the given domain (eg $\frac{3\pi}{2}$) **1 mark** – quadratic formed and correctly solved *Many students made errors in solving the quadratic or obtaining the correct answer in the given domain. The best responses used the unit circle or a graph to obtain the correct boundary angle.*
	b) $LHS = \frac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta}$ $= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)}$ $= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - (1 - \cos^2\theta))}$ $= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(2\cos^2\theta - 1)}$ $= \frac{\cos\theta}{\sin\theta}$ $= \cot\theta$ $= RHS$	**3 marks** – correct solution clearly showing all steps **2 marks** – substantial progress towards correct solution **1 mark** – original expression correctly factorised *Some students failed to show all steps in their proof. All steps are essential in any "show that" response.*
	c) (i) 	**2 marks** – correct graph **1 mark** – graph with correct shape and correct period OR correct amplitude
	c) (ii) *4 solutions*	**1 mark** – correct answer *The best responses drew the line y=1 on their graph from part (i) to see how many points of intersection there were.*
	d) $\frac{d}{dx}\left(\tan^2\left(\frac{x}{3}\right)\right) = 2\tan\left(\frac{x}{3}\right)\sec^2\left(\frac{x}{3}\right)\frac{1}{3}$ $= \frac{2}{3}\tan\left(\frac{x}{3}\right)\sec^2\left(\frac{x}{3}\right)$	**2 marks** – correct solution **1 marks** – substantial progress towards correct solution *MANY students did not use the function of a function rule correctly, omitting one or more parts of the solution.*
	e)  i) $AB^2 = 56^2 + 48^2 - 2 \times 56 \times 48 \times Cos50°$ $= 1984.37$ $AB = 44.5\ nm$ ii) $\frac{\sin\theta}{56} = \frac{\sin 50°}{44.5}$ $\sin\theta = \frac{56\sin 50°}{44.5} = 0.964$ $\theta = 74°35'$ $Bearing = 74°35' - 70° = 004°35'$	**2 marks** –correct solution **1 marks** – substantial progress towards correct solution *Many students drew incorrect diagrams.* *Some misquoted the cosine rule.* *Students did not check their answers to ensure they made sense in the context of the question.*

Year 12 2014	Mathematics	Task 4 Trial HSC
Question No. 14	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations
H4 - expresses practical problems in mathematical terms based on simple given models
HE4 - uses the relationship between functions, inverse functions and their derivatives.
PE5 - determines derivatives which require the application of more than one rule of differentiation

Outcome	Solutions	Marking Guidelines
PE3	Question 14 a) $x^2+(m-2)x+4=0$ for no real roots Δ<0 $(\text{m-2})^2-4(1)(4)<0$ $(\text{m-2})^2-4(1)(4)<0$ $(\text{m-2})^2-4(1)(4)<0$ $m^2-4m+4-16<0$ $m^2-4m-12<0$ $(\text{m-6})(m+2)<0$ -2 6 $-2<m<6$	1 mark for stating "for no real roots Δ<0" 1 mark for complete correct solution
PE3	b) $2x^2-(m+2)x+m=0$ let the roots be α and β such that $\beta=2\alpha$ $\alpha+\beta=\frac{m+2}{2}$ $\therefore \alpha+2\alpha=\frac{m+2}{2}$, since $\beta=2\alpha$ $\therefore 3\alpha=\frac{m+2}{2}$ $\therefore \alpha=\left(\frac{m+2}{6}\right)$..........................(A) $\alpha\beta=\frac{m}{2}$ $\therefore \alpha(2\alpha)=\frac{m}{2}$, since $\beta=2\alpha$ $\therefore 2\alpha^2=\frac{m}{2}$ $\therefore \alpha^2=\frac{m}{4}$................................(B) *Continued next page...*	1 mark for establishing A and B in any equivalent form.
	Continued from previous page... Solve simultaneously A and B Sub A into B to find m $\left(\frac{m+2}{6}\right)^2=\frac{m}{4}$ $\frac{m^2+4m+4}{36}=\frac{m}{4}$ $m^2+4m+4=\frac{36m}{4}$ $m^2+4m+4=9m$ $m^2-5m+4=0$ $(m-1)(m-4)=0$ $\therefore m=1, m=4$	1 mark for $m=1, m=4$
PE5 **HE4**	c) $\frac{d(e^x.\sin x)}{dx}=e^x.\cos x+\sin x.e^x$ or $e^x(\cos x+\sin x)$	1 mark for correct differentiation
HE4	d) The primitive of the function $g(x)$ is: $\int g(x)\,dx$ $=\int\frac{x}{x^2-11}dx$ $=\frac{1}{2}\int\frac{2x}{x^2-11}dx$ $=\frac{1}{2}\ln(x^2-11)+C$	2 marks for complete correct solution 1 mark for partial correct solution
H4	e) (i) When $x=a$, $y=10$ $\therefore 10=e^a$ $\therefore a=\ln 10$ When $x=b$, $y=15$ $\therefore 15=e^b$ $\therefore b=\ln 15$ The values are: $a=\ln 10$, $b=\ln 15$.	2 marks for obtaining both a and b correctly 1 mark for obtaining either a or b correctly

H4
HE4
PE5

(ii)

$$\begin{aligned}V&=\pi\int_a^b y^2dx\\&=\pi\int_{\ln 10}^{\ln 15}\left(e^x\right)^2dx\\&=\pi\int_{\ln 10}^{\ln 15}e^{2x}dx\\&=\pi\left[\frac{e^{2x}}{2}\right]_{\ln 10}^{\ln 15}\\&=\pi\left(\frac{e^{2.\ln 15}}{2}-\frac{e^{2.\ln 10}}{2}\right)\\&=\pi\left(\frac{e^{\ln(15)^2}}{2}-\frac{e^{\ln(10)^2}}{2}\right)\\&=\pi\left(\frac{225}{2}-\frac{100}{2}\right)\\&=\frac{125\pi}{2}cm^3\\&=\frac{125\pi}{2}ml\end{aligned}$$

3 marks for complete correct solution

2 marks for substantial progress to solution

1 marks for limited progress to solution

f)
(i)

H4

y	0	0.5	1	1.5	2
x	1	1.6	2.7	4.5	7.4

1 mark for both correct answers

H4
HE4

(ii)

Two applications of Simpson's rule:

$$\begin{aligned}A&\approx\frac{\left(\frac{1}{2}\right)}{3}\left[f(0)+4f(0\cdot 5)+f(1)\right]+\frac{\left(\frac{1}{2}\right)}{3}\left[f(1)+4f(1\cdot 5)+f(2)\right]\\&\approx\frac{1}{6}\left[1+4(1\cdot 6)+2\cdot 7\right]+\frac{1}{6}\left[1+4(4\cdot 5)+7\cdot 4\right]\\&\approx 6\cdot\dot{3}\\&\approx 6\cdot 4\text{ units}^2\text{ (to one decimal place)}\end{aligned}$$

2 marks for complete correct solution [must apply Simpson's rule twice]

1 mark for partial correct solution

Year 12 Mathematics Trial HSC (Task 4) 2014

Question No. 15 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

H2 constructs arguments to prove and justify results

Outcome	Solutions	Marking Guidelines

H5

(a) (i)

$$\begin{aligned}PA&=PB\\(x+7)^2+(y-4)^2&=(x+1)^2+(y-12)^2\\x^2+14x+49+y^2-8y+16&=x^2+2x+1+y^2-24y+144\\12x+16y-80&=0\end{aligned}$$

Eqn.of locus: $3x+4y-20=0$ as required.

2 marks
Correct solution
1 mark
Substantial progress towards correct solution.

H5

(ii) Parabola $(x-4)^2=4(y-1)$

Vertex (4, 1)
Concave up
Focal length = 1
∴ Focus (4, 2)

For focal chord, line must pass through focus
Sub. (4, 2) into equation of line from (i)

$$\begin{aligned}3.4+4.2-20&=12+8-20\\&=0\end{aligned}$$

∴ Line passes through (4, 2), hence a focal chord.

3 marks
Correct solution
2 mark
Substantial progress towards correct solution including description of parabola.
1 mark
Some progress towards a correct solution

H5

(iii) To find y-intercepts, let $x=0$

y-intercept of line $4y-20=0$
$y=5$

y-intercept of parabola $(-4)^2=4(y-1)$
$16=4y-4$
$y=5$

∴ Line and parabola meet on the y-axis at G when $y=5$.

2 marks
Correct solution.
1 mark
Substantial progress towards correct solution.

H5

(iv) Gradient of focal chord $3x+4y-20=0$: $m=-\frac{3}{4}$

Gradient of perpendicular: $m=\frac{4}{3}$

Perpendicular passes through the point (4, 2) (ie. the focus)
∴ Equation of normal:

$$\begin{aligned}y-2&=\frac{4}{3}(x-4)\\3y-6&=4x-16\\4x-3y-10&=0\end{aligned}$$

For x-intercept let $y=0$

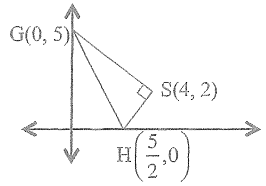
$$\begin{aligned}4x&=10\\x&=\frac{5}{2}\end{aligned}$$

ie. H is the point $\left(\frac{5}{2},0\right)$

2 marks
Correct solution.
1 mark
Substantial progress towards correct solution including equation of normal.

H5

(v)



2 marks
Correct solution.
1 mark
Substantial progress towards finding correct area.

Using Pythagoras' Theorem:

$$GS = 5\text{units}$$

$$HS = \frac{5}{2}\text{units}$$

$$\therefore \text{Area } \Delta GSH = \frac{1}{2}\times\frac{5}{2}\times 5$$

$$= \frac{25}{4}\text{units}$$

H2, H5

(b) (i)

$\angle CBA = 60°$ (Angle in equilateral ΔABC)

$\angle PBA = 180° - \angle CBA$ (angles on a straight line)

$= 180° - 60°$

$= 120°$

In Δ's PBA and PAQ

$\angle P$ is common

$\angle PAQ = 120°$ (given)

$= \angle PBA$

$\therefore \Delta PBA ||| \Delta PAQ$ (equiangular)

3 marks
Correct solution including full reasoning.
2 marks
Substantial progress towards correct solution including full reasoning.
1 mark
Some progress towards correct solution.

H2, H5

(ii)

Now, $\frac{PA}{PQ} = \frac{PB}{PA}$ (corresponding sides in similar triangles are in the same ratio)

$\therefore PA^2 = PQ.PB$

1 marks
Correct solution.

Year 12 HSC Mathematics Trial Examination 2014
Question No. 16 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

Outcome	**Solutions**	**Marking Guidelines**
H5		

(a) $\text{Max. height} = 50 + \frac{40}{1-0\cdot 8} = 250\,\text{cm}$

(a) **1 mark** : correct answer.

(b) Common difference $= -5$
Therefore $T_1 = 42$ and $T_{10} = -3$

$$S_{10} = \frac{10}{2}(42-3) = 195$$

(b) **2 marks** : correct solution

1 mark relevant progress toward correct solution

(c) (i) $A_1 = P(1\cdot 09) - 5000$

(ii) $A_3 = P(1\cdot 09)^3 - 5000\left(1 + 1\cdot 09 + 1\cdot 09^2\right)$

(iii)

$$A_6 = 0 = P(1\cdot 09)^6 - 5000\left(1 + 1\cdot 09 + 1\cdot 09^2 + \ldots. + 1\cdot 09^5\right)$$

$$P = \frac{5000\left(1\cdot 09^6 - 1\right)}{0\cdot 09\left(1.09^6\right)}$$

$$\$P = \$22429\cdot 59$$

(c) (i) **1 mark** correct expression.
(ii) **2 marks:** correct expression or equivalent.
1 mark: Significant progress.
(iii) **2 marks:** Correct solution.
1 mark: Relevant progress, correct expression for P.

(d) (i)

	1	**3**	**5**	**7**	**9**	**11**
2	Win	Lose	L	L	L	L
4	W	W	L	L	L	L
6	W	W	W	L	L	L
8	W	W	W	W	L	L
10	W	W	W	W	W	L
12	W	W	W	W	W	W

$$\text{P(Win)} = \frac{21}{36} = \frac{7}{12}$$

(d) (i) **1 mark**: Using a method to show that the event occurs 21 ways out of 36

(ii) $\text{P(Exactly one win from two)} = \frac{7}{12}\times\frac{5}{12} + \frac{5}{12}\times\frac{7}{12}$

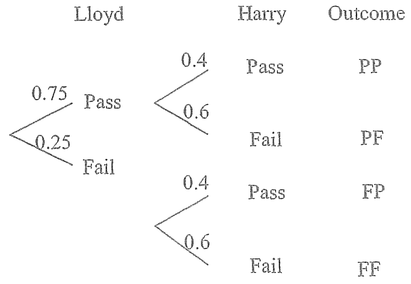
$$= \frac{70}{144} \text{ or } \frac{35}{72}$$

(iii) $\text{P(Win at least once)} = 1 - \text{P(lose both)}$

$$= 1 - \frac{5}{12}\times\frac{5}{12}$$

$$= \frac{119}{144}$$

(ii) **2 marks:** Correct solution.
1 mark: significant progress.
(iii) **1 mark:** Correct answer..

	(e) (i)  **(ii)** P(Only 1 passes) = P(PF) + P(FP) $=0\cdot75\times0\cdot6+0\cdot25\times0\cdot4$ $=0\cdot55$ or 55%	**(e) (i) 2 marks:** Weighted tree with branches labelled and outcomes listed. **1 mark:** Tree with feature excluded.