2016
HSC
ASSESSMENT TASK 4

Trial HSC Examination

Mathematics

Examiners ~ Mrs P. Biczo, Mr J. Dillon, Mrs M. Sabah, Mrs D. Crancher, Mrs T. Tarannum, Mr B. Shenouda.

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using a blue or black pen.
- Board approved calculators and mathematical templates and instruments may be used.
- Show all necessary working in Questions 11 – 16.
- This examination booklet consists of 13 pages including a multiple choice answer sheet.

Total marks (100)

Section I

Total marks (**10**)

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided.
- Allow 15 minutes for this section.

Section II

Total marks (**90**)

- Attempt questions 11 – 16
- Answer each question in the writing booklets provided.
- Start a new booklet for each question with your name and question number at the top of the page.
- All necessary working should be shown for every question.
- Allow 2 hours 45 minutes for this section.

Name: ______________________________

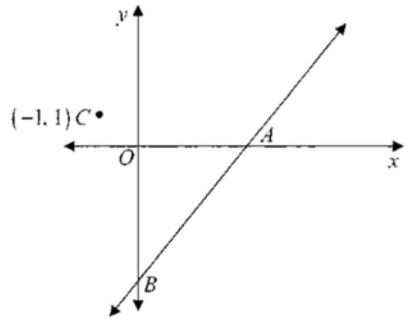
Teacher: ______________________________

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

1.



The line AB above has equation $3x-y-6=0$. What is the perpendicular distance of the point $C(-1,1)$ to the line AB?

(A) 10 units (B) $\sqrt{10}$ units (C) $5\sqrt{2}$ units (D) $\frac{\sqrt{10}}{5}$ units

2. $\int\left(x^2+\frac{1}{x^2}\right)dx$ is given by which of the following expressions?

(A) $\frac{x^3}{3}-\frac{1}{x}+C$ (B) $\frac{x^3}{3}+\frac{1}{x}+C$

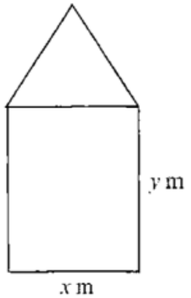
(C) $\frac{x^3}{3}-\frac{1}{2x}+C$ (D) $\frac{x^3}{3}+\frac{1}{2x}+C$

3. What is the derivative of $\ln(\cos x+\sin x)$?

(A) $\frac{1}{\cos x+\sin x}$ (B) $\frac{\cos x-\sin x}{\cos x+\sin x}$

(C) $\frac{\cos x+\sin x}{\cos x-\sin x}$ (D) $\cos x-\sin x$

4.



The diagram shows a window consisting of two sections. The top section is an equilateral triangle of side x m. The bottom section is a rectangle of width x m and height y m. The entire frame of the window, including the piece that separates the two sections, is made using 8 m of thin timber. The area of the glass used in terms of x can be expressed by which of the following expressions?

(A) $A = x\left\{4+\frac{x}{4}\left(8-\sqrt{3}\right)\right\}$ (B) $A = x\left\{4-\frac{x}{4}\left(4+\sqrt{3}\right)\right\}$

(C) $A = x\left\{4+\frac{x}{4}\left(4+\sqrt{3}\right)\right\}$ (D) $A = x\left\{4-\frac{x}{4}\left(8-\sqrt{3}\right)\right\}$

5. The table below shows the values of a function $f(x)$ for five values of x.

x	2	2.5	3	3.5	4
$f(x)$	4	1	−2	3	8

Which of the following values is an estimate for $\int_2^4 f(x)\,dx$ using Simpson's Rule with these five values?

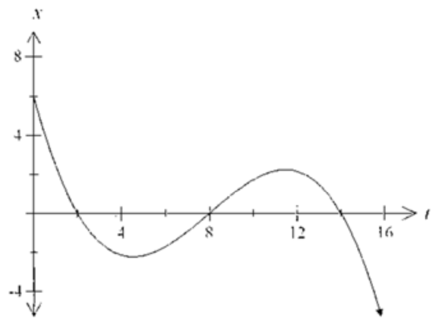
(A) 4 (B) 6 (C) 8 (D) 12

6. It is assumed that the number $N(t)$ of ants in a certain nest at time $t \geq 0$ is given by $N(t) = \frac{A}{1+e^{-t}}$ where A is a constant and t is measured in months.
At time $t = 0$, $N(t)$ is estimated at 2×10^5 ants. What is the value of A?

(A) 2×10^5 (B) 2×10^{-5}

(C) 4×10^5 (D) 4×10^{-5}

7. The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph.



When was the particle at rest?

(A) $t = 4.5$ and $t = 11.5$ (B) $t = 0$

(C) $t = 2$, $t = 8$ and $t = 14$ (D) $t = 1.5$ and $t = 8$

8. What is the value of $\int\left(\sec^2 \pi x\right)dx$?

(A) $\frac{1}{\pi}\tan \pi x + C$ (B) $\tan \pi x + C$ (C) $\pi \tan \pi x + C$ (D) $\tan^2 \pi x + C$

9. There are two prizes in a raffle in which 50 tickets are sold. The first prize is obtained by drawing a ticket at random and this ticket is not replaced for the draw of the second prize. Deepti buys two tickets in the raffle.
What is the probability that she does not win a prize?

(A) $\frac{576}{625}$ (B) $\frac{47}{50}$ (C) $\frac{1128}{1225}$ (D) $\frac{1152}{1225}$

10. Which of the following would represent $f'(x)$ if $f(x) = \frac{g(x)}{h(x)}$?

(A) $f'(x) = h(x)\times g'(x) + g(x)\times h'(x)$

(B) $f'(x) = \frac{h(x)\times g'(x) + g(x)\times h'(x)}{\left(h(x)\right)^2}$

(C) $f'(x) = h(x)\times g'(x) - g(x)\times h'(x)$

(D) $f'(x) = \frac{h(x)\times g'(x) - g(x)\times h'(x)}{\left(h(x)\right)^2}$

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) Start a new booklet **Marks**

(a) Find the value of a and b if: $a+b\sqrt{2}=\dfrac{1}{5+2\sqrt{2}}$ **2**

(b) Find values of m for which the quadratic equation $x^2-2mx+m=0$ has real and different roots. **2**

(c) The roots of the quadratic equation $x^2+5x+k=0$ are α and β. Find the value of k given $\alpha^2\beta+\alpha\beta^2=20$. **2**

(d) Consider the function $g(x)=\dfrac{2}{x^2-1}$.

(i) State the domain of $y=g(x)$. **1**

(ii) Show that $g(x)$ is an even function. **1**

(e) Graph the region where the inequalities hold simultaneously: $y>x^2$

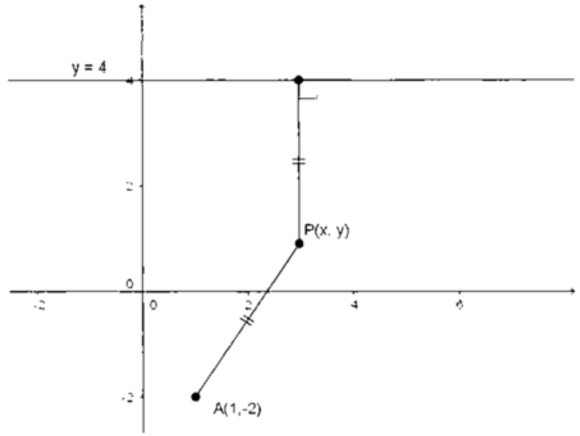
$y\geq x+6$

Your graph should be approximately one third of a page and should clearly indicate which points are included in the region. **3**

Question 11 continued over page

Marks

(f)



The diagram shows the point $P(x,y)$ equidistant from the point $A(1,-2)$ and the line $y=4$.

(i) The locus of P is a parabola. Show that the equation of the locus is given by $x^2-2x+12y-11=0$. **2**

(ii) Find the co-ordinates of the vertex of the parabola. **2**

Question 12 (15 marks) Start a new booklet **Marks**

(a) Find the equation of the curve passing through the point $(0,2)$ if its gradient function is given by $\frac{dy}{dx}=24x+9x^2-4x^3$ **2**

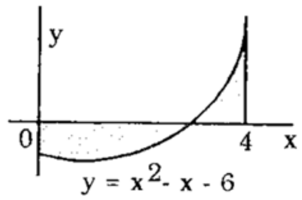
(b) Draw a neat sketch (about one quarter of a page) of the continuous curve $y=f(x)$ which has <u>all</u> of the following properties:

$y'>0$ for $x<-1$
$y'=0$ at $(-1,4)$
$y'<0$ for $-1<x<3$
$y'=0$ at $(3,-2)$
$y'<0$ for $x>3$

2

(c) A sheet of cardboard 3 metres by 4 metres is to be made into a box by cutting equal-sized squares from each corner and folding up the four edges. By letting x be the length of one edge of the square cut from each corner of the sheet of cardboard, find the value of x, that will give the greatest volume. **4**

(d)



(i) Write down an expression using integrals to describe the shaded area. **2**

(ii) Find the shaded area. **2**

(e) The area bounded by the curve $y=x^3$, $x=1$, $x=2$ and the x-axis is rotated about the x-axis. Find the volume of the solid of revolution. **3**

Question 13 (15 marks) Start a new booklet **Marks**

(a) (i) Prove that $\sec^2\theta-2\tan\theta=(\tan\theta-1)^2$. **2**

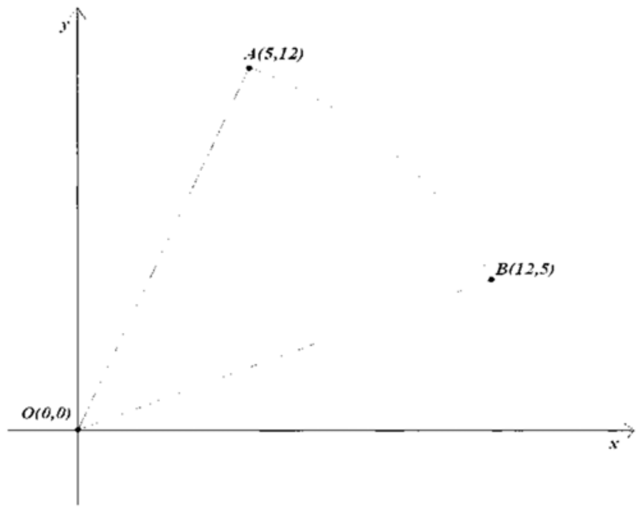
(ii) Hence, or otherwise, solve $\sec^2\theta-2\tan\theta=0$ for $0\le\theta\le2\pi$ **2**

(b) (i) Show that $x=\frac{2\pi}{3}$ is a solution of $\cos x=\cos 2x$. **1**

(ii) On the same set of axes, sketch the graphs of $y=\cos x$ and $y=\cos 2x$ for $0\le x\le\pi$, showing the x coordinate of all points of intersection. **2**

(iii) Find the exact area of the region bounded by the curves $y=\cos x$ and $y=\cos 2x$ over the interval $0\le x\le\frac{2\pi}{3}$. **3**

(c) The figure below shows a sector OAB of a circle formed by joining the centre $O(0,0)$ and the points $A(5,12)$ and $B(12,5)$ on a circle.



(i) Find the value of one radian in degrees. Give your answer correct to the nearest minute. **1**

(ii) Show that the size of $\angle AOB$ is $0\cdot78$ radians, correct to 2 decimal places. **2**

(iii) Calculate the perimeter of sector OAB, correct to 2 decimal places. **2**

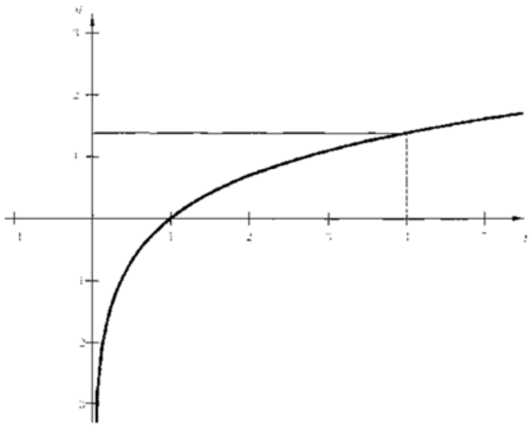
Question 14 (15 marks) Start a new booklet **Marks**

(a) If $\log_{10} 7 = a$, find the value of $\log_{10}\left(\frac{1}{70}\right)$ **2**

(b) Solve for x: $\frac{1}{2}\ln(2x+3) = \ln x$ **3**

(c) Differentiate $x^2 \log_e x$ **2**

(d)



The diagram shows the graph of $y = \ln x$. The shaded region, bounded by $y = \ln x$, the line $y = \ln 4$ and both the x and y axes, is rotated about the y-axis to form a solid.

(i) Show that the volume of the solid is given by **2**

$$V = \pi \int_0^{\ln 4} e^{2y}\ dy$$

(ii) Hence find the volume of the solid. **2**

(e) Find $\int \frac{1}{2x+1}\, dx$ **2**

(f) Given that $\frac{d}{dx}\left(e^{2x^2}\right) = 4xe^{2x^2}$, evaluate $\int_0^1 xe^{2x^2}\, dx$ **2**

Question 15 (15 marks) Start a new booklet **Marks**

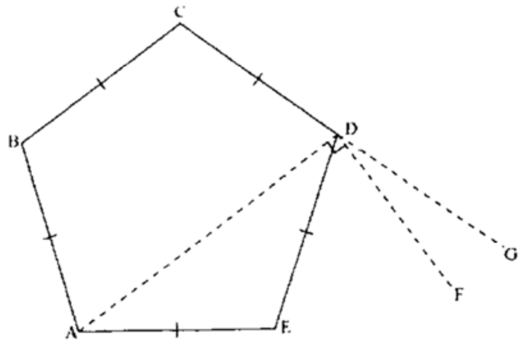
(a) Danny borrows \$ 18 000 to buy a new car for Nina. He is charged interest at 12% p.a. compounded monthly, on the balance owing. The loan is to be repaid in equal monthly instalments over 5 years. Let A_n be the amount owing after the nth monthly repayment M has been made.

(i) Find expressions for A_1, and A_n. **2**

(ii) Calculate her monthly instalments. **3**

(iii) Calculate the equivalent simple interest rate to two decimal places. **1**

(b) In the diagram below, $ABCDE$ is a regular pentagon. CD is produced to G and DF is drawn perpendicular to AD as shown. Find the size of $\angle FDG$, giving reasons. **3**



(c) If 3% of coins produced are faulty in some way, what is the probability that a sample of 3 coins taken from the production line will have at least two faulty coins? **2**

(d) The points $A(-1,11)$, $B(3,-1)$ and $C(a,-7)$ are collinear. Find the value of a. **2**

(e) A bell rings at 6:32 am and then every 3 minutes until it last rings at 10:14 am. Using an arithmetic sequence, calculate the number of times the bell rings. **2**

Question 16 (15 marks) Start a new booklet **Marks**

(a) When a valve is released, a chemical flows into a large tank that is initially empty. The volume, V litres, of chemical in the tank increases at the rate

$$\frac{dV}{dt} = 2e^{t} + 2e^{-t}$$

where t is measured in hours from the time the valve is released.

(i) At what rate does the chemical initially enter the tank? **1**

(ii) Use integration to find an expression for V in terms of t. **2**

(iii) Show that $2e^{2t} - 3e^{t} - 2 = 0$ when $V = 3$. **1**

(iv) Find t, to the nearest minute, when $V = 3$. **2**

(b) In November 1923, 18 koalas were introduced on Kangaroo Island. By November 1993, the number of koalas had increased to 5000.

Assume that the number N of koalas is increasing exponentially and satisfies an equation of the form $N = N_0 e^{kt}$, where N_0 and k are constants and t is measured in years from November 1923.

(i) Find the values of N_0 and k. **2**

(ii) Predict the number of koalas that will be present on Kangaroo Island in November 2016. **2**

(c) A particle moves in a straight line so that its displacement, in metres, is given by

$$x = \frac{t-2}{t+2}$$

where t is measured in seconds.

(i) What is the displacement when $t = 0$? **1**

(ii) Find expressions for the velocity and acceleration in terms of t. **2**

(iii) Is the particle ever at rest? Justify your answer. **1**

(iv) What is the limiting velocity of the particle as t increases indefinitely? **1**

End of Examination

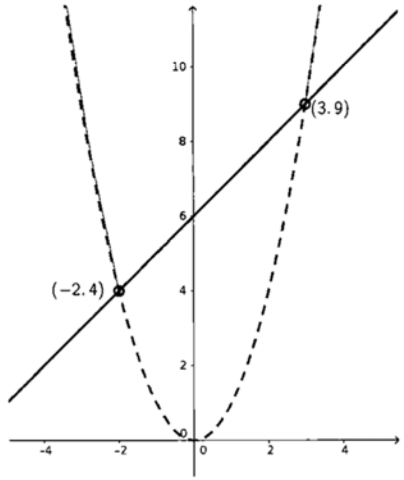
Outcomes Addressed in this Question

P3 Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.

P4 Chooses and applies appropriate arithmetic algebraic, graphical, trigonometric and geometric techniques.

P5 Understands the concept of a function and the relationship between a function and its graph.

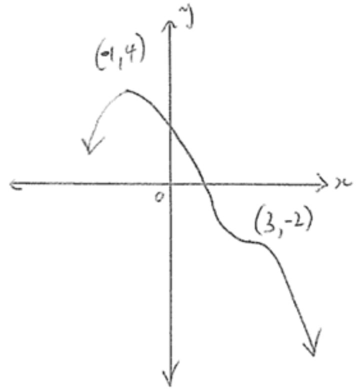
Outcome	Solutions	Marking Guidelines
P3	(a) $\frac{1}{5+2\sqrt{2}} = \frac{1}{5+2\sqrt{2}} \times \frac{5-2\sqrt{2}}{5-2\sqrt{2}}$ $= \frac{5-2\sqrt{2}}{25-8}$ $\therefore$ if $a+b\sqrt{2} = \frac{5}{17} - \frac{2\sqrt{2}}{17}$, $a = \frac{5}{17}, b = -\frac{2}{17}$.	2 marks: correct solution 1 mark: substantial progress towards correct solution
P4	(b) Given $x^2 - 2mx + m = 0$, $\Delta = (-2m)^2 - 4m$ For real and different roots $\Delta > 0$. Solving $4m^2 - 4m > 0$, $4m(m-1) > 0$. Graphing this concave up parabola which cuts the x axis at 0, 1 gives $m < 0$ *and* $m > 1$.	2 marks: correct solution 1 mark: substantial progress towards correct solution
P4	(c) Given $x^2 + 5x + k = 0$, $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$ and $\alpha + \beta = \frac{-b}{a} = \frac{-5}{1} = -5$. Since $\alpha^2\beta + \alpha\beta^2 = 20$, $\alpha\beta(\alpha+\beta) = 20$. $\therefore k \times -5 = 20$, $\therefore k = -4$.	2 marks: correct solution 1 mark: substantial progress towards correct solution
P5	(d) (i) Domain is all real x except ± 1. (ii) $g(-x) = \frac{2}{(-x)^2 - 1}$ $= \frac{2}{x^2-1}$ $= g(x)$, $\therefore g(x)$ is an even function.	1 mark: correct answer 1 mark: correct solution.

P5	(e) 	3 marks: correct solution 2 marks: partly correct solution 1 mark: substantial progress towards correct solution
P4	(f) (i) Distance of $P(x,y)$ to the line $y = 4$ is distance of P to the point $(x,4)$. Given distance $PA = d(x,y),(x,4)$, $\sqrt{(x-1)^2+(y+2)^2} = \sqrt{(x-x)^2+(y-4)^2}$. $\therefore \sqrt{(x-1)^2+(y+2)^2} = \sqrt{(x-x)^2+(y-4)^2}$, $\therefore (x-1)+(y+2)^2 = (y-4)^2$. Locus is $x^2-2x+1+y^2+4y+14 = y^2-8y+16$. $\therefore x^2-2x+12y-11=0$.	2 marks: correct solution 1 mark: substantial progress towards correct solution
	(ii) Completing the square on $x^2-2x+12y-11=0$, $x^2-2x+1-1+12y-11=0$, $(x-1)^2 = -12y+12$. $(x-1)^2 = -12(y-1)$, which has vertex $(1,1)$.	2 marks: correct solution 1 mark: substantial progress towards correct solution
	ANSWERS TO MULTIPLE CHOICE: **1. B 2. A 3. B 4. D 5. A 6. C 7. A 8. A 9. C 10. D**	

Year 12 2016	Mathematics	Task 4 HSC Trial
Question No. 12	Solutions and Marking Guidelines	

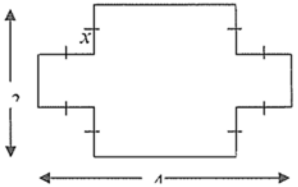
Outcomes Addressed in this Question

H5 – applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
H6 – uses the derivative to determine the features of the graph of a function
P6 – relates the derivative of a function to the slope of its graph
H7 – uses the features of a graph to deduce information about the derivative
P7 – determines the derivative of a function through routine application of the rules of differentiation
H8 – uses techniques of integration to calculate areas and volumes
P8 – understands and uses the language and notation of calculus
H9 – communicates using mathematical language, notation, diagrams and graphs

Outcome	Solutions	Marking Guidelines
H5,P8,H9	a). $y = \int (24x+9x^2-4x^3)dx$ $y = \frac{24x^2}{2}+\frac{9x^3}{3}-\frac{4x^4}{4}+C$ $y = 12x^2+3x^3-x^4+C$ When x=0 and y=2 $2 = 12(0)^2+3(0)^3-(0)^4+C$ $2 = C$ Equation is: $y = 12x^2+3x^3-x^4+2$	2 marks for correct solution 1 mark for integrating correctly
H5,H6,H7,P8,H9	b). 	2 marks for correct graph 1 mark for substantial working that could lead to a correct graph

H5,P7,P8,H9

c).



Length (l) of box $= 4-2x$

Width (b) of box $= (3-2x)$

Height (h) of box $= x$

Volume (V) of box $= x(4-2x)(3-2x)$

$= 4x^3 - 14x^2 + 12x$

$\therefore V = 4x^3 - 14x^2 + 12x$

$\frac{dV}{dx} = 12x^2 - 28x + 12$

$12x^2 - 28x + 12 = 0$

$3x^2 - 7x + 3 = 0$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(3)}}{2(3)}$

$x = \frac{7 \pm \sqrt{13}}{6}$

$\frac{d^2V}{dx^2} = 24x - 28$

when $x = \frac{7+\sqrt{13}}{6}$

$\frac{d^2V}{dx^2} = 24\left(\frac{7+\sqrt{13}}{6}\right) - 28 = 14.422205... > 0$

when $x = \frac{7-\sqrt{13}}{6}$

$\frac{d^2V}{dx^2} = 24\left(\frac{7-\sqrt{13}}{6}\right) - 28 = -14.422205... < 0$

Since $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} < 0$ when $x = \frac{7-\sqrt{13}}{6}$,

there is a maximum volume when $x = \frac{7-\sqrt{13}}{6}$

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4 marks for correct solution

3 marks for substantial working that could lead to a correct solution with only one error

2 marks for substantial working that could lead to a correct solution

1 mark for finding an equation for the volume

Continued from previous page...

Therefore, the dimensions of the box to make the greatest volume will be:

Length $= 4 - 2\left(\frac{7-\sqrt{13}}{6}\right) = 2.87$ (to 2 d.p.)

Width $= 3 - 2\left(\frac{7-\sqrt{13}}{6}\right) = 1.87$ (to 2 d.p.)

Height $= \left(\frac{7-\sqrt{13}}{6}\right) = 0.57$ (to 2 d.p.)

H5,H8,P8H9

d).

(i)

$A = \left|\int_0^3 (x^2 - x - 6)\,dx\right| + \int_3^4 (x^2 - x - 6)\,dx$

2 mark for correct solution

1 mark for substantial working that could lead to a correct solution

H5,H8,P8,H9

(ii)

$A = \left|\int_0^3 (x^2 - x - 6)\,dx\right| + \int_3^4 (x^2 - x - 6)\,dx$

$= \left|\left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_0^3\right| + \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_3^4$

$= \left|\left(\frac{(3)^3}{3} - \frac{(3)^2}{2} - 6(3)\right) - \left(\frac{(0)^3}{3} - \frac{(0)^2}{2} - 6(0)\right)\right|$

$+ \left(\frac{(4)^3}{3} - \frac{(4)^2}{2} - 6(4)\right) - \left(\frac{(3)^3}{3} - \frac{(3)^2}{2} - 6(3)\right)$

$= \left|-\frac{27}{2} - 0\right| - \frac{32}{3} + \frac{27}{2}$

$= \frac{27}{2} - \frac{32}{3} + \frac{27}{2}$

$= \frac{49}{3}$ or $16\frac{1}{3}$

2 marks for correct solution

1 mark for substantial working that could lead to a correct solution

H5,H8,P8,H9

e).

$V = \pi\int_1^2 y^2\,dx$

$= \pi\int_1^2 (x^3)^2\,dx$

$= \pi\int_1^2 (x^6)\,dx$

$= \pi\left[\frac{x^7}{7}\right]_1^2$

$= \pi\left(\frac{(2)^7}{7} - \frac{(1)^7}{7}\right)$

$= \frac{127}{7}\pi$ or 18.14(to 2 d.p.)

3 marks for correct solution

2 marks for substantial working that could lead to a correct solution

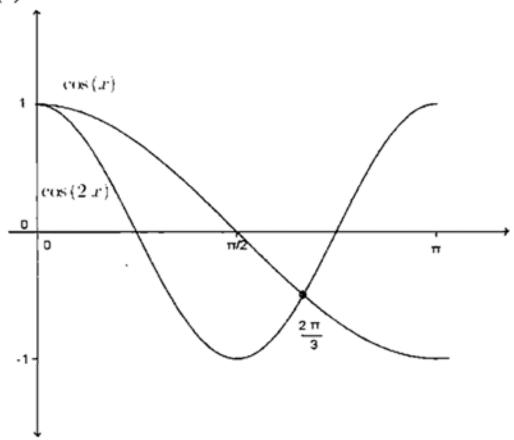
1 mark for working that could lead to a correct solution

Year 12 Mathematics Trial 2016

Question No. 13 Solutions and Marking Guidelines

Outcomes Addressed in this Question

Applies appropriate techniques from the study of trigonometry

Outcome	Solutions	Marking Guidelines
	(a) **(i)** $LHS = \sec^2\theta - 2\tan\theta$ $= 1 + \tan^2\theta - 2\tan\theta$ $= (\tan\theta - 1)^2 = RHS$	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution
	(ii) $\sec^2\theta - 2\tan\theta = 0$ $(\tan\theta - 1)^2 = 0$ $\tan\theta - 1 = 0$ $\tan\theta = 1$ $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution
	(b) **(i)** Substitute $x = \frac{2\pi}{3}$ into LHS and RHS $LHS = \cos x = \cos\frac{2\pi}{3} = -0.5$ $RHS = \cos 2x = \cos\left(2\times\frac{2\pi}{3}\right) = -0.5$ $LHS = RHS$	**Award 1** mark for the correct answer.
	(ii)  	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution
	(iii) $Area = \int_0^{\frac{2\pi}{3}} (\cos x - \cos 2x)dx$ $= \left[\sin x - \frac{\sin 2x}{2}\right]_0^{\frac{2\pi}{3}}$ $= \left[\sin\left(\frac{2\pi}{3}\right) - \frac{\sin\left(\frac{4\pi}{3}\right)}{2}\right] - [\sin 0 - 2\sin 0]$ $= \frac{\sqrt{3}}{2} - \left(\frac{1}{2}\times\left(-\frac{\sqrt{3}}{2}\right)\right) - 0 = \frac{3\sqrt{3}}{4}u^2$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.
	(c) **(i)** $1 \text{ radian} = 1\times\frac{180^o}{\pi}$ $= 57.29577951...$ $= 57^o18'$ (to the nearest minute)	**Award 1** mark for the correct answer.
	(ii) Let α be the angle subtended by the line OB with the positive direction of x-axis and β be the angle subtended by the line OA with the positive direction of x-axis $\tan\alpha = \frac{5}{12}$ and $\tan\beta = \frac{12}{5}$ $\angle AOB = \beta - \alpha$ $= \tan^{-1}\left(\frac{12}{5}\right) - \tan^{-1}\left(\frac{5}{12}\right)$ $= 0.781214.... = 0.78$ radians (correct to 2 dp)	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution
	(iii) Perimeter $=$ length of $OA +$ length of $OB +$ arc length of AB length of $OA =$ length of $OB =$ radius(r) $= \sqrt{(0-5)^2 + (0-12)^2} = 13$ arc length of $AB = r\theta = 13\times 0.78$ Perimeter $= 13 + 13 + 10.14 = 36.14$ *units*	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution

Year 12 Mathematics Trial HSC Examination 2016

Question No. 14 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H3 **manipulates algebraic expressions involving logarithmic and exponential functions**

H5 **applies appropriate techniques from the study of calculus**, geometry, probability, trigonometry and series **to solve problems**

H8 **uses techniques of integration to calculate areas and volumes**

Outcome	Solutions	Marking Guidelines
H3	(a) $\log_{10}\left(\frac{1}{70}\right)=\log_{10}(10\times 7)^{-1}$ $=-(\log_{10}10+\log_{10}7)$ $=-(1+a)$ since $\log_{10}7=a$ $=-1-a$	**2 marks** Correct solution. **1 mark** Demonstrates some knowledge of laws of logarithms.
H3	(b) $\frac{1}{2}\log(2x+3)=\log x$ $\log(\sqrt{2x+3})=\log x$ $\therefore \sqrt{2x+3}=x$ $x^2=2x+3$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3,-1$ but, when $x=-1$, $\log x$ does not exist $\therefore x=3$ is the only solution	**3 marks** Correct solution. **2 marks** Both possible solutions for x given but $x=-1$ is not excluded. **1 mark** Forms correct quadratic equation to find solutions.
H3	(c) $\frac{d}{dx}x^2\log x=x^2.\frac{1}{x}+2x.\log x$ using the product rule $=x+2x\log x$	**2 marks** Correct solution. **1 mark** Clearly demonstrates a knowledge that the product rule for differentiation is required to be used.
H8	(d) (i) $y=\log x \Leftrightarrow x=e^y$ when $x=4$, $y=\log 4$ Volume of solid: $V=\pi\int_a^b x^2\,dy$ $=\pi\int_0^{\log 4}(e^y)^2\,dy$ $=\pi\int_0^{\log 4}e^{2y}\,dy$ as required	**2 marks** Correct solution. **1 mark** Shows the logarithmic and exponential equivalence and finds limits of the integration.
H8	(ii) $V=\pi\int_0^{\log 4}e^{2y}\,dy$ $=\pi\left[\frac{e^{2y}}{2}\right]_0^{\log 4}$ $=\pi\left(\frac{e^{2\log 4}}{2}-\frac{e^0}{2}\right)$ $=\pi\left(\frac{16}{2}-\frac{1}{2}\right)$ $=\frac{15\pi}{2}$ units^3	**2 marks** Correct solution. **1 mark** Finds correct primitive in making substantial progress towards a correct solution.
H3, H5	(e) $\int\frac{1}{2x+1}\,dx=\frac{1}{2}\log(2x+1)+c$	**2 marks** Correct solution. **1 mark** Recognises the primitive as a logarithmic function.
H3, H5	(f) $\frac{d}{dx}e^{2x^2}=4xe^{2x^2}$ $e^{2x^2}=\int 4xe^{2x^2}\,dx$ $=4\int xe^{2x^2}\,dx$ $\int_0^1 xe^{2x^2}\,dx=\left[\frac{e^{2x^2}}{4}\right]_0^1$ from the above result $=\frac{e^2-1}{4}$	**2 marks** Correct solution. **1 mark** Finds the correct primitive using the given result.

Year 12 Trial Question No. 15	Mathematics Solutions and Marking Guidelines	Examination 2016

Outcomes Addressed in this Question

Applies appropriate techniques from the study of series, probability and geometry to solve problems

Part	Solutions	Marking Guidelines
a.		
i.	Loan = \$18000; Rate = 12%pa = 1% per month; 5 years = 60 months $A_n =$ Amount owing after n[th] payment; $M =$ monthly payment $A_1 = 18000 + (0.01\times 18000) - M$ $= 18000(1.01) - M$ $A_2 = A_1(1.01) - M$ $A_2 = \left(18000(1.01) - M\right)(1.01) - M$ $A_2 = 18000(1.01)^2 - M\left\{1 + (1.01)\right\}$ $A_3 = A_2(1.01) - M$ $A_3 = \left[18000(1.01)^2 - M\left\{1+(1.01)\right\}\right](1.01) - M$ $A_3 = 18000(1.01)^3 - M\left\{1 + (1.01) + (1.01)^2\right\}$ $A_n = 18000(1.01)^n - M\left\{1 + (1.01) + (1.01)^2 + (1.01)^3 + \dots\dots\dots + (1.01)^n\right.$ $= 18000(1.01)^n - M\dfrac{(1.01^n - 1)}{(1.01 - 1)}$ $= 18000(1.01)^n - 100M(1.01^n - 1)$	**Award 2~** Correct Solution **Award 1 ~** Makes substantial progress towards solution
ii	When $n = 60$: $A_{60} = 0$ *i.e.* After 60^{th} payment there is zero owing $0 = 18000(1.01)^{60} - M\underbrace{\left\{1 + 1.01 + (1.01)^2 + (1.01)^3 + \dots\dots\dots + (1.01)^{59}\right\}}_{\text{G.P. } a=1;\ r=1.01;\ n=60}$ $S_n = \dfrac{a(r^n - 1)}{r - 1}$ $S_{60} = \dfrac{1((1.01)^{60} - 1)}{1.01 - 1}$ $= \dfrac{(1.01)^{60} - 1}{0.01}$ $= 81.66966985$ $0 = 18000(1.01)^{60} - M(81.66966985)$ $M(81.66966985) = 8000(1.01)^{60}$ $M = \dfrac{18000(1.01^{60})}{81.66966985}$ $M = 400.40$ $\therefore$ Monthly instalment is \$400.40	**Award 3~** Correct Solution **Award 2 ~** Makes substantial progress towards solution **Award 1 ~** Makes limited progress towards solution
iii	Total repayments $= \$400.40\times 60$ $= 24024.00$ Total interest $= \$24024 - \$18000$ $= \$6024$ Using Simple Interest formula $I = \dfrac{PRT}{100}$ $\therefore R = \dfrac{100I}{PT}$ $= \dfrac{100\times 6024}{18000\times 5}$ $\therefore R = 6.69\%$	**Award 1~** Correct Solution
b.	$\angle AED = \left[(5-2)\times 180°\right]\div 5$ $\therefore \angle AED = 108°$ (angle of a regular pentagon) ΔADE is isosceles (since $AE = DE$) $\therefore \angle ADE = \dfrac{180° - 108°}{2} = 36°$ $\therefore \angle EDF = 90° - 36°$ $\therefore \angle EDF = 54°$ $\angle EDG = 180° - 108° = 72°$ (angles on a straight line add to 180°) $\therefore \angle FDG = 72° - 54° = 18°$	**Award 3~** Correct Solution **Award 2 ~** Makes substantial progress towards solution **Award 1 ~** Makes limited progress towards solution
c.	Faulty (F) = 3% = $\dfrac{3}{100}$ Not Faulty (NF) = 97% = $\dfrac{97}{100}$ $P(2\text{F}) = P(\text{F, F, NF}) + P(\text{F, NF, F}) + P(\text{NF, F, F})$ $= \left(\dfrac{3}{100}\times\dfrac{3}{100}\times\dfrac{97}{100}\right) + \left(\dfrac{3}{100}\times\dfrac{97}{100}\times\dfrac{3}{100}\right) + \left(\dfrac{97}{100}\times\dfrac{3}{100}\times\dfrac{3}{100}\right)$ $= \dfrac{2619}{1000000}$ $P(3\text{F}) = P(\text{F, F, F})$ $= \left(\dfrac{3}{100}\times\dfrac{3}{100}\times\dfrac{3}{100}\right)$ $= \dfrac{27}{1000000}$ $P(\text{atleast 2F}) = P(2\text{F}) + P(3\text{F})$ $= \dfrac{2619}{1000000} + \dfrac{27}{1000000}$ $= \dfrac{2646}{1000000}$ $= \dfrac{1323}{500000} = 0.002646$	**Award 2~** Correct Solution **Award 1 ~** Makes substantial progress towards solution

c.	If the points are collinear then the gradients of any points are equal $m=\frac{y_2-y_1}{x_2-x_1}$ $m_{AB}=\frac{-1-11}{3-(-1)}$ $m_{AB}=-3$ $m_{BC}=\frac{-7--1}{a-3}$ $\therefore\ =\frac{-6}{a-3}=-3$ $-6=-3(a-3)$ $-15=-3a$ $a=5$	**Award 2**~ Correct Solution **Award 1** ~ Makes substantial progress towards solution
d.	From 6:32am to 10:14am = 222 minutes Working from 32 minutes past 6 to 254 minutes past 6 we get the sequence 32, 35, 38, ... 254. A.P $a=32;\ d=3$ $T_n=a+(n-1)d$ $254=32+(n-1)(3)$ $254=32+3n-3$ $254=29+3n$ $225=3n$ $n\ =\ 75$	**Award 2**~ Correct Solution **Award 1** ~ Makes substantial progress towards solution

Year 12 Mathematics Task 4 (Trial HSC) 2016

Question 16 Solutions and Marking Guidelines

Outcome Addressed in this Question

H4 expresses practical problems in mathematical terms based on simple given models

Part	Solutions	Marking Guidelines
(a) (i)	When $t=0,\ \frac{dV}{dt}=2e^0+2e^0=4$ litres / min	**Award 1** for correct answer
(ii)	$V=2e^t-2e^{-t}+c$ $t=0,V=0\Rightarrow 0=2e^0-2e^0+c$ $\therefore c=0$ $\therefore V=2e^t-2e^{-t}$	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(iii)	When $V=3$, $2e^t-2e^{-t}=3$ $2e^t-\frac{2}{e^t}=3$ $2e^t.e^t-\frac{2}{e^t}.e^t=3.e^t$ $\therefore 2e^{2t}-2=3.e^t$ $\therefore 2e^{2t}-3.e^t-2=0$ (as required)	**Award 1** for correct solution
(iv)	If $2e^{2t}-3.e^t-2=0$ then $(2e^t+1)(e^t-2)=0$ $\therefore e^t=-\frac{1}{2}$ (which has no (real) solution) *or* $e^t=2\Rightarrow t=\ln 2=0.6931471806$ hours $=41.58883083$ minutes ≈ 42 minutes	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(b) (i)	$t=0,\ N=18\Rightarrow 18=N_0e^0$ $\therefore N_0=18$ $t=70,\ N=5000\Rightarrow 5000=18e^{70k}$ $\therefore e^{70k}=\frac{5000}{18}=\frac{2500}{9}$ $\therefore k=\frac{1}{70}\ln\left(\frac{2500}{9}\right)\approx 0.08038316334$	**Award 2** for both correct values **Award 1** for only one correct value (or substantial progress towards solution)

(ii)	$2016 \Rightarrow N = 93$ $\therefore N = 18e^{\frac{1}{70}\ln\left(\frac{2500}{9}\right)\times 93}$ $= 31761.36675$ ≈ 31761 koalas	**Award 2** for correct answer **Award 1** for substantial progress towards solution
(c) (i)	$t = 0, x = \frac{0-2}{0+2} = -1$	**Award 1** for correct answer
(ii)	$\dot{x} = \frac{(t+2).1-(t-2).1}{(t+2)^2} = \frac{4}{(t+2)^2} = 4(t+2)^{-2}$ $\ddot{x} = -8(t+2)^{-3} = -\frac{8}{(t+2)^3}$	**Award 2** for both correct expressions **Award 1** for only one correct expression (or substantial progress towards expressions for both)
(iii)	To be at rest, $\dot{x}$ must be 0 $\therefore \dot{x} = 0 \Rightarrow \frac{4}{(t+2)^2} = 0$ which has no solution. $\therefore$ Particle is never at rest.	**Award 1** for correct answer with justification provided
(iv)	As $t \to \infty$, $(t+2)^2 \to \infty$ $\therefore \frac{4}{(t+2)^2} \to 0$ $\therefore$ The limiting velocity of the particle is zero.	**Award 1** for correct answer