STUDENT NAME: ________________________

TEACHER: ________________________________

HURLSTONE
AGRICULTURAL
HIGH
SCHOOL

2020	TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Assessment Task 4

Examiners

- Ms. L. Yuen, Mr. D. Potaczala, Ms. T. Tarannum

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA-approved calculators may be used
- A Reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 26)

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

SECTION I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10

Question 1

What is the value of $f'(x)$ if $f(x)=3x^4(4-x)^3$?

A. $3x^3(4-x)^3(7x-16)$

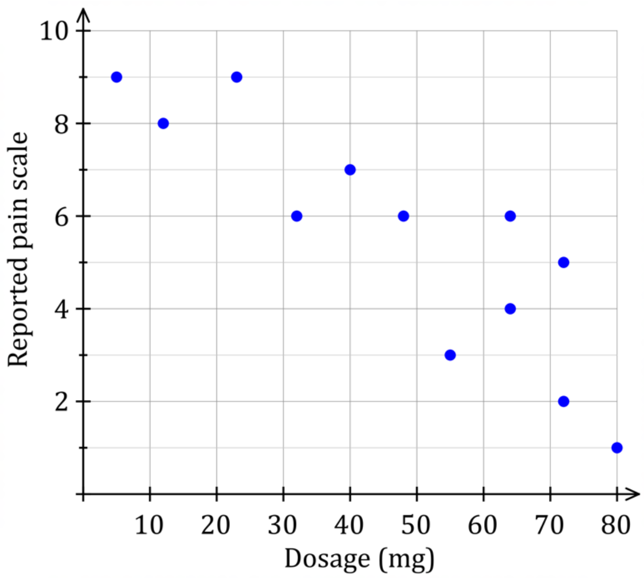
B. $3x^3(4-x)^3(16-7x)$

C. $3x^3(4-x)^2(7x-16)$

D. $3x^3(4-x)^2(16-7x)$

Question 2

A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



How could you describe the correlation between the pain and the dosage?

A. A moderate negative correlation

B. A moderate positive correlation

C. A weak positive correlation.

D. No correlation.

Question 3

For what values of x is the curve $f(x)=2x^3+x^2$ concave down?

A. $x<-\frac{1}{6}$

B. $x>-\frac{1}{6}$

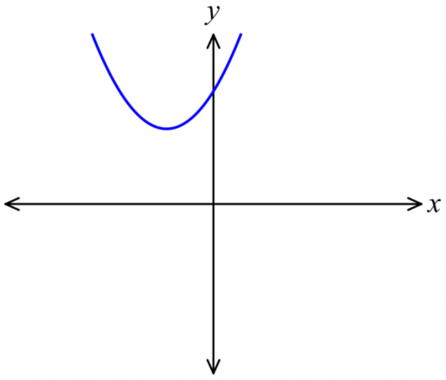
C. $x<-6$

D. $x>6$

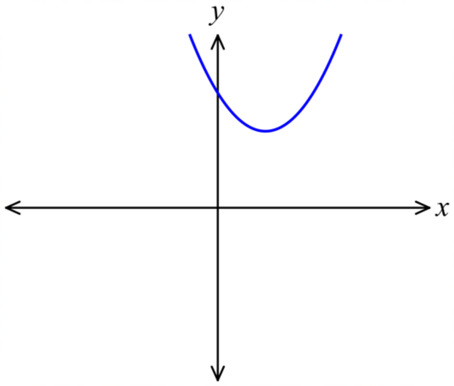
Question 4

Which diagram best shows the graph of the parabola $y=2-(x+1)^2$?

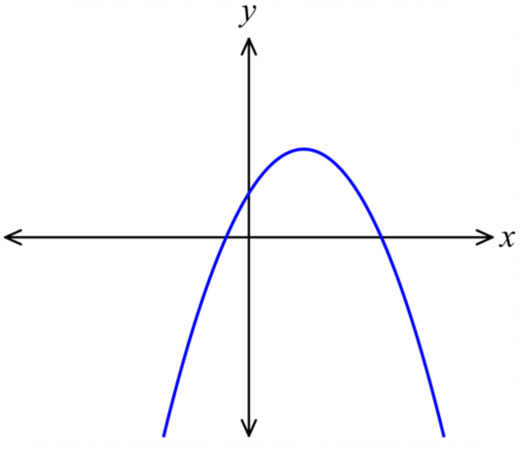
A.



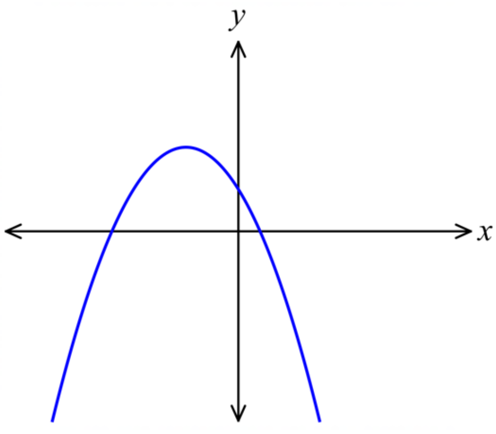
B.



C.



D.



Question 5

For what the value of x is $\log x - \log 3 = \log\left(\frac{x-3}{2}\right)$?

A. $x = 3$

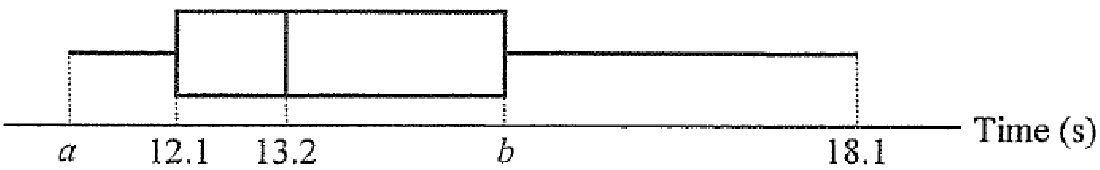
B. $x = 5$

C. $x = 7$

D. $x = 9$

Question 6

The box-and-whisker diagram below shows the distribution of the times taken by a large group of students of an athletic club to finish a 100m race:



The inter-quartile range and the range of the distribution are 3.2s and 6.8s respectively. What are the values of a and b?

A. $a = 11$, $b = 15.65$

B. $a = 11$, $b = 14.3$

C. $a = 11.3$, $b = 15.3$

D. $a = 11.3$, $b = 15.65$

Question 7

The continuous random variable X has a normal distribution with a mean of 11 and standard deviation of 2. If the random variable Z has the standard normal distribution, then the probability that X is greater than 16 is equal in value to which expression below?

A. $P(Z > 2)$

B. $P(Z < -2.5)$

C. $P(Z < 2.5)$

D. $P(Z > 5)$

Question 8

What is the period and amplitude for the curve $y = \sin \pi x$?

A. Period = 2; Amplitude = 1

B. Period = 2; Amplitude = π

C. Period = 2π ; Amplitude = 1

D. Period = 2π ; Amplitude = π

Question 9

What are the solutions to the equation $2\sin x + \sqrt{3} = 0$ where $\{x : 0 \le x \le 2\pi\}$?

A. $\frac{\pi}{3}, \frac{2\pi}{3}$

B. $\frac{2\pi}{3}, \frac{5\pi}{3}$

C. $\frac{4\pi}{3}, \frac{5\pi}{3}$

D. $\frac{7\pi}{3}, \frac{11\pi}{3}$

Question 10

At time t a particle has displacement and acceleration functions $x(t)$ and $a(t)$.
For which of the following functions is $x(t) \equiv a(t)$?

A. $x(t) = 3\sin(t) - e^{t}$

B. $x(t) = 3\cos(t) - e^{-t}$

C. $x(t) = e^{t} - e^{-t}$

D. $x(t) = 3\sin(t) - 3\cos(t)$

SECTION II

90 marks

Attempt all questions

Allow about 2 hours 45 minutes for this section.

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your response should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available. If you use extra writing paper, clearly indicate which question you are answering.

Question 11 (2 marks)

Find the anti-derivative of $\frac{1}{1-2x}$ with respect to x. **2**

Question 12 (3 marks)

Differentiate $e^{x\sin x}$. **3**

Question 13 (3 marks)

The random variable X has this probability distribution.

X	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Find $P(1 < X \leq 3)$. **1**

(b) Find the variance of X. **2**

Question 14 (2 marks)

Find $\int 6x^2 + 2 + x^{\frac{-1}{2}}\,dx$, giving each term in its simplest form. **2**

Question 15 (2 marks)

Find the number of terms in the geometric sequence 128, 192, ..., 972. **2**

Question 16 (3 marks)

3

Find the exact value of $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$.

(Give your answer with a rational denominator.)

Question 17 (5 marks)

Given that $f(x) = (x^2 - 6x)(x - 3) + 2x$.

(a) Express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants. **2**

(b) Hence factorise $f(x)$ completely. **1**

(c) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes. **2**

Question 18 (8 marks)

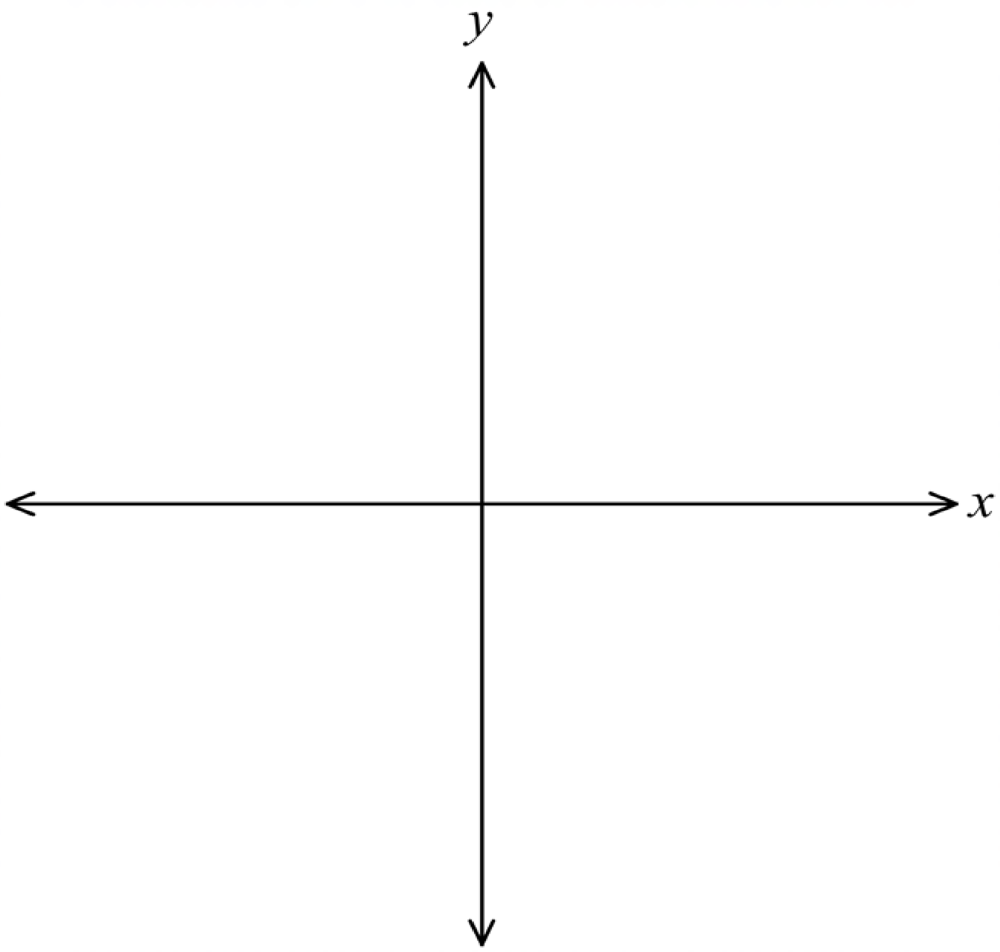
Let $f(x) = (x - 2)(x^2 + 1)$

(a) Find the points where the graph of $y = f(x)$ cuts the *x-axis* and *y*-axis. **2**

(b) Find the coordinates of the stationary points on the curve with the equation $y = f(x)$ and determine their nature. **3**

Question 18 continues on page 11

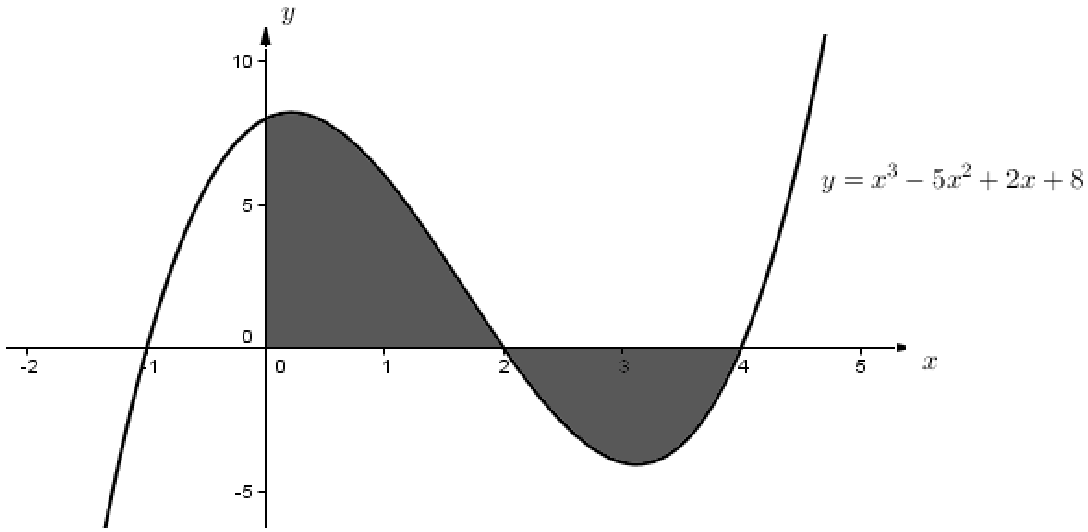
(c) Sketch the graphs of $y = f(x)$ and $y = -f(x)$ on the same diagram. Clearly label each graph and showing all important features. **3**



Question 19 (2 marks)

Differentiate $\dfrac{\sin x}{x^2}$ with respect to x. **2**

Question 20 (4 marks)



Find the shaded area enclosed by the curve $y = x^3 - 5x^2 + 2x + 8$ and the axes. **4**

Question 21 (3 marks)

The probability density function for the continuous random variable X is given by:

$$f(x) = \begin{cases} |1 - x| & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(X \le 1.5)$. **2**

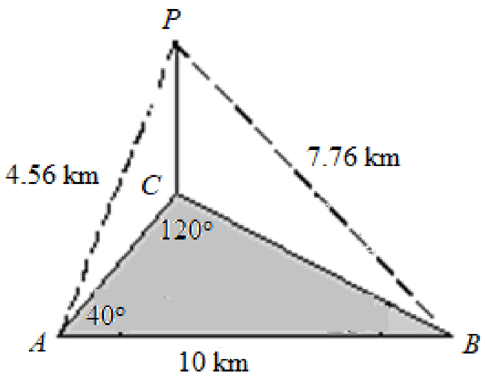
(b) Find $P(1 \le X \le 2)$. **1**

Question 22 (2 marks)

If $\tan\theta = \dfrac{2}{3}$, and θ is acute, find the exact value of $\sin\theta$. **2**

Question 23 (6 marks)

A, B and C are three points on the horizontal ground. P is a point vertically above C. Given $AB = 10$ km, $AP = 4.56$ km, $PB = 7.76$ km, $\angle CAB = 40°$ and $\angle ACB = 120°$.



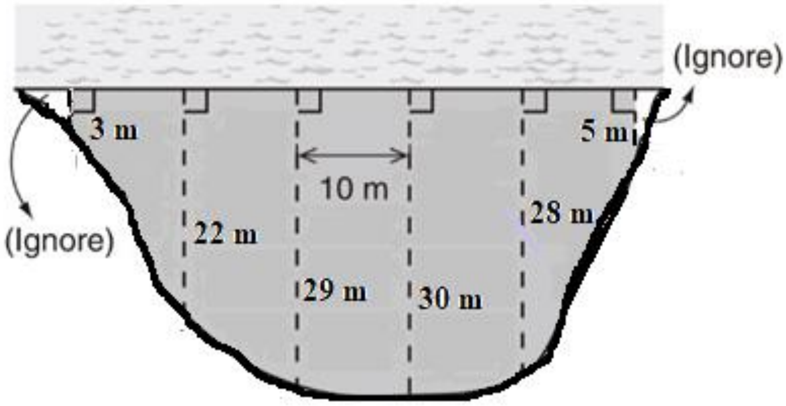
(a) Find the length of BC correct to the nearest 0.1 km. **2**

(b) Hence, find the angle of elevation of P from B, in radians. (Give your answer correct to one decimal place.) **2**

(c) Find the angle between PA and PB. **2**

Question 24 (2 marks)

A surveyor takes several measurements to estimate the area of a piece of land by a river. The river is 50 metres wide. The depth of the river, in metres, has been measured at 10 metre intervals. The cross-section is shown below.



Use the trapezoidal rule to estimate the area by ignoring the two small regions at both ends. **2**

Question 25 (6 marks)

A curve with the equation $y = f(x)$, has $\dfrac{dy}{dx} = x^3 + 2x - 7$.

(a) Find $\dfrac{d^2y}{dx^2}$. **1**

(b) Show that $\dfrac{d^2y}{dx^2} \geq 2$ for all values of x. **1**

(c) The point $P(2, 4)$ lies on the curve. Find y in terms of x. **2**

Question 25 continues on page 17

(d) Find an equation for the normal to the curve at P, in the form $ax + by + c = 0$, where a, b and c are integers. **2**

Question 26 (3 marks)

The students in a school were surveyed on the number of hours of sleep per week. The results were normally distributed with a mean of 48 hours.

(a) The survey indicated that 95% of students had between 42 and x hours of sleep per week. Determine the value of x. **1**

(b) What was the standard deviation? **1**

(c) What percentage of students would have indicated they had between 51 and 57 hours of sleep per week? **1**

Question 27 (2 marks)

The probability density function for the continuous random variable X is: **2**

$$f(x) = \begin{cases} x^3 - x + 4 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

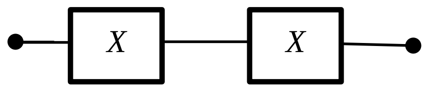
Find E(X).

Question 28 (5 marks)

An alarm system is made up of a number of identical components.

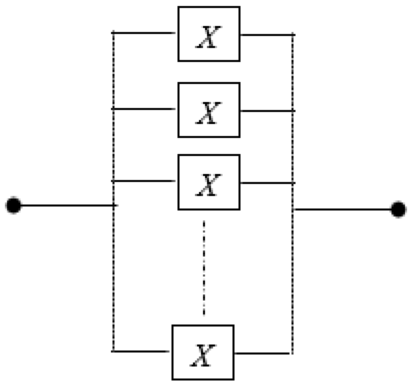
The probability that component X fails is 0.05.

(a) (i) The alarm system below is wired in series, i.e. the system works only when both components operate properly. Find the probability that the system does not fail. (Give your answer correct to three decimal places.) **1**



Question 28 continues on page 19

(ii) Another alarm system below is wired in parallel i.e. only one component needs to work for the circuit to work. What is the minimum number of components needed so that the probability of the system working is greater than 99.999%? Clearly justify your working. **3**



(b) If a coin is tossed n times, where $n > 1$, find the probability of obtaining at least one head and at least one tail. **1**

Question 29 (4 marks)

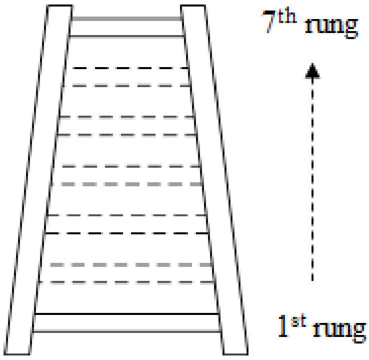
A viable nest of an insect has between 100 000 to 500 000 insects. The population P of a nest of this insect grows exponentially so that: **4**

$$\frac{dp}{dt} = 1200e^{0.3t}$$

A nest of these insects has a population of 5000 after one month. Determine how long it will take the nest to reach the viable stage (i.e. when the population has reached 100 000). Give your answer correct to the nearest month.

Question 30 (4 marks)

John wishes to construct a ladder with 7 rungs which diminish uniformly from 48 cm (the length of the lowest rung) to 33 cm (the length of the highest rung).



(a) Find the common difference of the length between each rung. **1**

(b) Hence, find the length of the fifth rung. **1**

(c) If John wants to extend his ladder to a maximum number of rungs, how many more rungs he can have? **2**

Question 31 (6 marks)

A particle moves in a straight line. At time t seconds, its displacement from the origin is x metres and is given by $x = 1 - \cos 2t$.

(a) Sketch the graph of x as a function of t for $0 \le t \le \pi$ **2**

(b) Using your graph, or otherwise, find the times when the particle is at rest and the position of the particle at these times. **2**

(c) Find the velocity of the particle when $t = \dfrac{\pi}{4}$. **1**

Question 31 continues on page 23

(d) When is the particle's velocity greater than 1 m/s? **1**

Question 32 (3 marks)

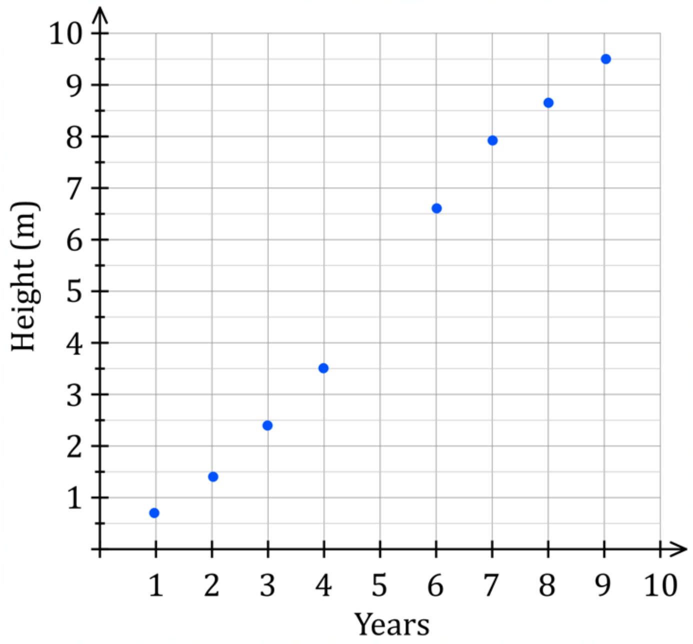
For the curve $y = 2x^2 - \ln\left(\frac{x}{2}\right) - 4$, find the coordinates of the minimum stationary point. Give your answer in exact form. **3**

Question 33 (5 marks)

Hayden is an agricultural scientist studying the growth of a particular tree over several years. The data he recorded is shown in the table below.

Years since planting, t	1	2	3	4	5	6	7	8	9
Height of tree, h metres	0.7	1.4	2.4	3.5		6.6	7.9	8.7	9.5

A scatterplot of the data is shown below.



(a) What is Pearson's correlation coefficient? Answer correct to 4 decimal places. **1**

(b) Find the equation of the least-squares line of best fit in terms of years (t) and height (h). Answer correct to 2 decimal places. **1**

Question 33 continues on page 25

(c) Hayden did not record the tree's height after five years. Predict the height after five years, correct to one decimal place. **1**

(d) Use algebra to estimate how many years it will take for the tree to reach a height of 20 metres. Answer correct to 1 decimal place. **1**

(e) Comment on the reliability of your answers in (c) and (d). **1**

Question 34 (2 marks)

By expressing $\sec\theta$ and $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$, show that **2**

$$\sec^2\theta - \tan^2\theta = 1$$

Question 35 (3 marks)

The Pareto chart shows complaints about the delivery of pizzas from a new pizza store.

Use the graph to answer the following questions:

(a) What percentage of customers complained about late delivery of pizzas? **1**

(b) Explain how the Pareto principle can be used by the pizza company to direct their efforts to improve the customer experience. **2**

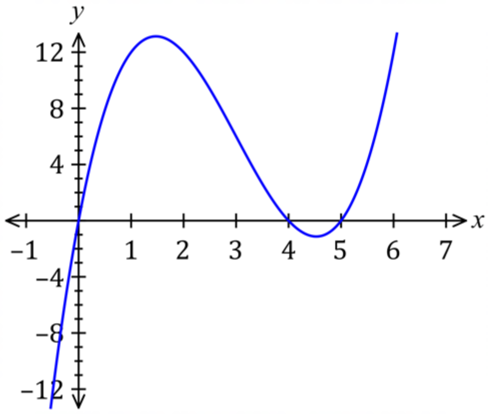
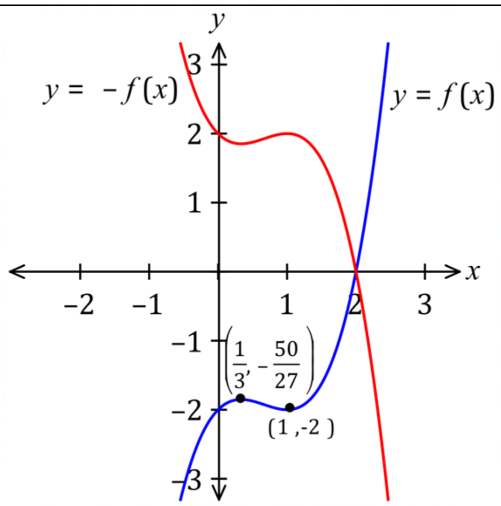
~~ End of paper~~

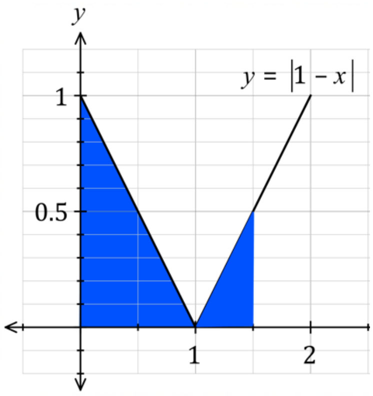
HAHS Year 12 2020 Mathematics Advanced Trial Solutions and Marking Guidelines
Outcomes Addressed
MA12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts
MA12-3 applies calculus techniques to model and solve problems
MA12-4 applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems
MA12-5 applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs
MA12-6 applies appropriate differentiation methods to solve problems
MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems
MA12-8 solves problems using appropriate statistical processes
MA12-10 constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

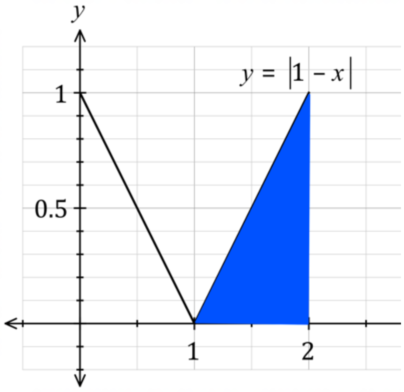
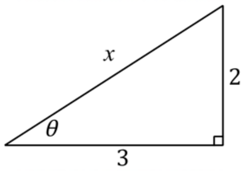
Outcome	Question	Solution	Marks
MA12-6	1	$f(x)=3x^4(4-x)^3$ $f'(x)=3x^4\times\frac{d(4-x)^3}{dx}+(4-x)^3\times\frac{d(3x^4)}{dx}$ $=3x^4\times-3(4-x)^2+(4-x)^3\times12x^3$ $=3x^3(4-x)^2\left[-3x+4(4-x)\right]$ $=3x^3(16-7x)(4-x)^2$	D
MA12-8	2	Moderate negative correlation	A
MA12-3	3	$f(x)=2x^3+x^2$ $f'(x)=6x^2+2x$ $f''(x)=12x+2$ $f''(x)<0$ when $12x+2<0$ $x<-\frac{1}{6}$	A
MA12-1	4	Graph of $y=2-(x+1)^2$ is graph of $y=(x+1)^2$ reflected in x axis , hence concave down and shifted up 2 units. $y=(x+1)^2$ touches x axis at $x=-1$.	D
MA12-1	5	$\log x-\log 3=\log\left(\frac{x-3}{2}\right)$ $\log\frac{x}{3}=\log\left(\frac{x-3}{2}\right)$ $\frac{x}{3}=\frac{x-3}{2}$ $2x=3x-9$ $x=9$	D
MA12-8	6	$b-12.1=3.2$ and $18.1-a=6.8$ $b=15.3$ and $a=11.3$	C
MA12-8	7	For $\mu=11$ and $\sigma=2$, 16 is 2.5 standard deviations above the mean. $P(X>16)=P(Z>2.5)=P(Z<-2.5)$ as the normal curve is symmetrical	B

MA12-5	8	Period of $y = a\sin bx$ is $\dfrac{2\pi}{b}$ and amplitude is a $\therefore\ y = \sin\pi x$ has amplitude 1 and period $\dfrac{2\pi}{\pi} = 2$	A
MA12-5	9	$2\sin x + \sqrt{3} = 0$ $\sin x = -\dfrac{\sqrt{3}}{2}$ Sin negative quadrants 3, 4; basic angle $\dfrac{\pi}{3}$ $\therefore\ x = \pi + \dfrac{\pi}{3}, 2\pi - \dfrac{\pi}{3}$ $= \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$	C
MA12-3	10	$x = 3\sin t - e^t$ $\quad x = 3\cos t - e^{-t}$ $\quad x = e^t - e^{-t}$ $\quad x = 3\sin t - 3\cos t$ $x' = 3\cos t - e^t$ $\quad x' = -3\sin t + e^{-t}$ $\quad x' = e^t + e^{-t}$ $\quad x' = 3\cos t + 3\sin t$ $x'' = -3\sin t - e^t$ $\quad x'' = -3\cos t - e^{-t}$ $\quad x'' = e^t - e^{-t}$ $\quad x'' = -3\sin t + 3\cos t$ Displacement and acceleration functions equal when $x = e^t - e^{-t}$	C
MA12-3	11	$\int \dfrac{1}{1-2x}\,dx = -\dfrac{1}{2}\ln\lvert 1-2x\rvert$	**2** Marks: Correct answer. 1 Mark: Finds the integral as a log function.
MA12-6	12	$\dfrac{d}{dx}\left(e^{x\sin x}\right) = e^{x\sin x} \times \dfrac{d}{dx}(x\sin x)$ $= e^{x\sin x} \times (\sin x + x\cos x)$	**3** Marks: Provides correct solution 2 Marks: Differentiates the exponential expression but provides a partially correct product rule 1 Mark: Only uses the product rule
MA12-8	13a	$P(1 < X \le 3) = 0.4 + 0.2$ $= 0.6$	**1** Mark: Correct answer.
	b	$\mu = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1$ $= 2$ $Var(X) = E(X^2) - \mu^2$ $= 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.2 + 4^2 \times 0.1 - 2^2$ $= 1.2$ $\therefore$ Variance is 1.2	**2** Marks: Correct answer. 1 Mark: Shows some understanding

MA12-3	14	$\int 6x^2 + 2 + x^{-\frac{1}{2}}\,dx = \frac{6x^3}{3} + 2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$ $= 2x^3 + 2x + 2x^{\frac{1}{2}} + C$	**2** Marks: Correct answer. 1 Mark: Integrates one term correctly
MA12-4	15	The first term = 128 The common ratio $= \frac{192}{128} = \frac{3}{2}$ Suppose there are altogether k terms in the geometric sequence. Then $T_k = 972$. Substituting $a = 128$, $r = \frac{3}{2}$, $n = k$ into the formula $T_n = ar^{n-1}$, then $972 = 128 \times \left(\frac{3}{2}\right)^{k-1}$ $\left(\frac{3}{2}\right)^{k-1} = \frac{243}{32}$ $\left(\frac{3}{2}\right)^{k-1} = \left(\frac{3}{2}\right)^{5}$ $\therefore \quad k - 1 = 5$ $k = 6$ $\therefore$ There are 6 terms in the geometric sequence.	**2** Marks: Correct answer. 1 Mark: Shows some understanding
MA12-7	16	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right)$ $= 1 - \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{2} - 1}{\sqrt{2}}$ $= \frac{2 - \sqrt{2}}{2}$	**3** Marks: Correct answer. 2 Marks: Correctly integrates and sub 1 Mark: Shows some understanding
MA12-1	17a	$f(x) = (x^2 - 6x)(x - 3) + 2x$ $= x[(x - 6)(x - 3) + 2]$ $= x(x^2 - 9x + 20)$	**2** Marks: Correct answer. 1 Mark: Makes some progress.
	b	$f(x) = x(x^2 - 9x + 20)$ $= x(x - 4)(x - 5)$	**1** Mark: Correct answer.
	c		**2** Marks: Correct answer. 1 Mark: Shows the general shape of the curve or finds the intercepts.

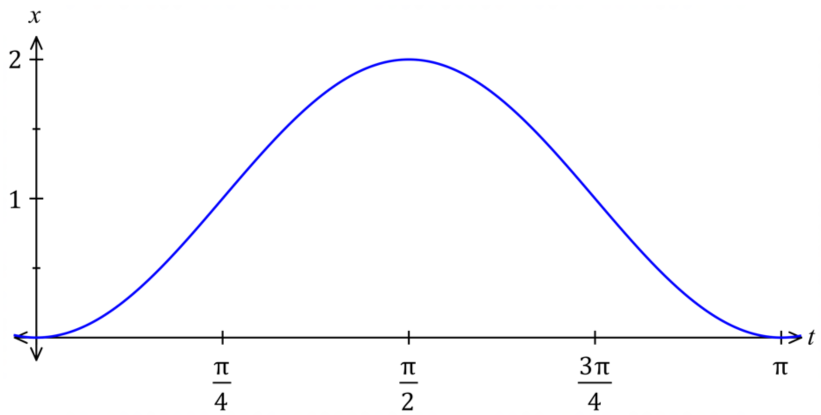
			
MA12-6	18a	x-intercept then $y = 0$ $0 = (x-2)(x^2+1)$ $x = 2$ y-intercept then $x = 0$ $y = (0-2)(0^2+1)$ $\quad = -2$ $\therefore$ Intercepts are (2, 0) and (0,–2)	**2** Marks: Correct answer. 1 Mark: Finds either the x or y intercept
	b	$f(x) = (x-2)(x^2+1)$ $f'(x) = (x-2)2x + (x^2+1) \times 1$ $\quad = 3x^2 - 4x + 1$ $f''(x) = 6x - 4$ Stationary points occur when first derivative is equal to zero. $3x^2 - 4x + 1 = 0$ $(3x-1)(x-1) = 0$ $x = \frac{1}{3}$ or $x = 1$ When $x = \frac{1}{3}$ then $y = -\frac{50}{27}$ and when $x = 1$ then $y = -2$ $\therefore$ Stationary points $(\frac{1}{3}, -\frac{50}{27})$ (1, –2) At $(\frac{1}{3}, -\frac{50}{27})$ $f''(x) = 6 \times \frac{1}{3} - 4 = -2 < 0$ Max At (1, –2) $f''(x) = 6 \times 1 - 4 = 2 > 0$ Min	**3** Marks: Correct answer. 2 Marks: Finds the stationary points. 1 Mark: Finds the derivative.
	c		**3** Marks: Correct answer. 2 Marks: Finds $y = f(x)$ or $y = -f(x)$. 1 Mark: Finds the general shape of $y = f(x)$ or shows some understanding.

MA12-3	19	$\frac{d}{dx}\left(\frac{\sin x}{x^2}\right)=\frac{x^2\cos x-\sin x(2x)}{\left(x^2\right)^2}$ $=\frac{x\cos x-2\sin x}{x^3}$	**2** Marks: Correct answer. 1 Mark: Uses the quotient rule correctly
MA12-7	20	$Area=\int_0^2(x^3-5x^2+2x+8x)dx+\left\|\int_2^4(x^3-5x^2+2x+8x)dx\right\|$ $=\left[\frac{1}{4}x^4-\frac{5}{3}x^3+x^2+8x\right]_0^2+\left\|\left[\frac{1}{4}x^4-\frac{5}{3}x^3+x^2+8x\right]_2^4\right\|$ $=\left[\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right]_0^2-0+$ $\left\|\left[\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)\right]_2^4\right\|$ $=\frac{32}{3}+\frac{16}{3}$ $=16\text{ units}^2$	**4** Marks: Correct solution 3 Marks: Substantially correct solution 2 Marks: Significant progress 1 Mark: shows some understanding
MA12-8	21	$P(X\le 1.5)$ $=\frac{1}{2}\times 1\times 1+\frac{1}{2}\times 0.5\times 0.5$ $=0.625$ 	**2** Marks: Correct solution 1 Mark: Sketches the function or shows some understanding.
		$P(1\le X\le 2)$ $=\frac{1}{2}\times 1\times 1$ $=0.5$	**1** Mark: Correct answer. .

			
MA12-5	22	$x^2 = 2^2 + 3^2$ $x = \sqrt{13}$ $\sin\theta = \frac{2}{\sqrt{13}}$ 	**2** Marks: Correct answer. 1 Mark: Finds the value of *x* or makes significant progress.
MA12-5	23a	Using the sine rule in ΔABC, $\frac{BC}{\sin 40°} = \frac{10}{\sin 120°}$ $BC = \frac{10\sin 40°}{\sin 120°}$ $= 7.4$ km, *correct to the nearest 0.1 km* [7.422]	**2** Marks: Correct solution 1 Mark: shows significant understanding.
	b	Consider ΔCBP. $\cos\angle CBP = \frac{7.4}{7.76}$ $\angle CBP = 0.2967$ $= 0.3$ radians (to 1 d.p.)	**2** Marks: Correct answer. 1 Mark: shows significant understanding.
	c	Consider ΔAPB. By the cosine rule, $\cos\angle APB = \frac{4.560^2 + 7.764^2 - 10^2}{2(4.560)(7.764)}$ $\angle APB = 105°34'$ or 1.84 radians (to 2 d.p.)	**2** Marks: Correct solution. 1 Mark: shows significant understanding.
MA12-7	24	Let the equation of the boundary of the piece of land be $y = f(x)$. When $n = 5$, $h = 10$. The end-points of the sub-intervals: 0, 10, 20, 30, 40, 50. The area	**2** Marks: Correct solution. 1 Mark: shows significant understanding.

		$= \int_0^{50} f(x)\, dx$ $\approx \frac{h}{2}[f(0)+2\{f(10)+f(20)+f(30)+f(40)\}+f(50)]$ $= \frac{10}{2}(3+2\times 22+2\times 29+2\times 30+2\times 28+5)$ $=1130 \text{ m}^2$	
MA12-6	25a	$\frac{dy}{dx} = x^3 + 2x - 7$ $\frac{d^2y}{dx^2} = 3x^2 + 2$	**1** Mark: Correct answer.
	b	Since $x^2 \geq 0$ (for all values of x) then $3x^2 \geq 0$ Hence $3x^2 + 2 \geq 2$ $\therefore \frac{d^2y}{dx^2} \geq 2$ **Note: as 'show that' question, you cannot start by assuming the result is true**	**1** Mark: Correct solution.
	c	Finding the anti-derivative. $y = \frac{x^4}{4} + x^2 - 7x + C$ $P(2, 4)$ is on the curve and satisfies the equation. $4 = \frac{2^4}{4} + 2^2 - 14 + C$ $C = 10$ $\therefore y = \frac{x^4}{4} + x^2 - 7x + 10$	**2** Marks: Correct solution 1 Mark: Finds the anti-derivative or correctly finds c.
	d	Gradient of the tangent at $P(2, 4)$ $m = \frac{dy}{dx} = 2^3 + 2 \times 2 - 7 = 5$ Gradient of the normal at $P(2, 4)$ $m_1 m_2 = -1$ $m = -\frac{1}{5}$ Equation of the normal at $P(2, 4)$ $y - y_1 = m(x - x_1)$ $y - 4 = -\frac{1}{5}(x - 2)$ $\therefore x + 5y - 22 = 0$ **Note: question stated answer was to be in form $ax + by + c = 0$, where *a*, *b* and *c* are integers.** **Re-read questions after you have found the answer to ensure you answer them in the method stated**	**2** Marks: Correct solution 1 Mark: Significant progress
MA12-8	26a	95% of the data lie within two standard deviations of the mean. 42 hours is a *z*-score of –2 and 54 hours has a *z*-score of 2. The mean 48 is midway between 42 and x hours. $\therefore$x is 54 hours.	**1** Mark: Correct answer.
	b	There are 4 standard deviations between 42 and 54 hours. Standard deviation $= \frac{54 - 42}{4}$ $= 3$ hours	**1** Mark: Correct answer.

	c	To find the *z*-score of 51 and 57 $$z = \frac{x-\bar{x}}{s} = \frac{51-48}{3} = 1$$ $$z = \frac{x-\bar{x}}{s} = \frac{57-48}{3} = 3$$ $$\text{Percentage} = \frac{99.7\%}{2} - \frac{68\%}{2}$$ $$= 15.85\%$$ $\therefore$ 15.85% have sleep between 51 and 57 hours of sleep per week	**1** Mark: Correct answer.
MA12-8	27	**To find the expected value or mean** $\int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x(x^3 - x + 4)\,dx$ $= \int_0^1 x^4 - x^2 + 4x\,dx$ $= \left[\frac{x^5}{5} - \frac{x^3}{3} + 2x^2\right]_0^1$ $= \frac{28}{15}$	**2** Marks: Correct solution. 1 Mark: shows significant understanding.
MA12-10	28a	(1 – 0.05)(1 – 0.05) = 0.9025 =0.903	**1** Mark: Correct answer.
	b	Let the number of components be *n* P(the system works) = P(all least one component works) $=1-(0.05)^n$ Require $1 - (0.05)^n > 0.99999$ $0.00001 > (0.05)^n$ $\log 0.00001 > n\log(0.05)$ $3.843 < n$ (inequality sign reversed as log(0.05) is negative) $\therefore$ minimum number of components needed is 4	**3** Marks: Correct solution. **2** Marks correct finding of n with incorrect sign 1 Mark: shows some understanding.
	c	P(at least one head and at least one tail) $1 - P(\text{no heads}) - P(\text{no tails})$ $= 1 - \frac{1}{2^n} - \frac{1}{2^n}$ $= 1 - \frac{2}{2^n}$ $= 1 - 2^{1-n}$	**1** Mark: Correct answer.
MA12-3	29	$\frac{dP}{dt} = 1200e^{0\cdot 3t}$ $P = \frac{1200}{0.3}e^{0\cdot 3t} + C$ $P = 4000e^{0\cdot 3t} + C$ Given *P* = 5000 when *t* = 1 $5000 = 4000e^{0\cdot 3} + C$	**4** Marks: Correct solution **3** Marks substantially correct solution 2 Marks: Makes significant progress.

		$C = 5000 - 4000e^{0\cdot 3}$ $\therefore P = 4000e^{0.3t} + 5000 - 4000e^{0.3}$ When $P = 100\,000$, $100\,000 = 4000e^{0\cdot 3t}$ $+ 5000 - 4000e^{0.3}$ $$e^{0.3t} = \frac{95000 + 4000e^{0.3}}{4000}$$ $\ln 25.0998\ldots = 0.3t$ $t = 10.7428\ldots$ ≈ 11 $\therefore$ The nest reaches a viable stage after 11 months. **Note: Many students rounded off early in this question. If this made the question significantly easier, marks were deducted.** **You should not round until the final answer**	1 Mark: Correctly finds the anti-derivative or correctly demonstrates use of appropriate log laws
MA12-4	30a	Substituting $a = 48$ and $n = 7$ into the formula $T_n = a + (n-1)d$, we get $T_7 = 48 + (7-1) \times d = 33$ $48 + 6d = 33$ $6d = -15$ $d = -2.5$	**1** Mark: Correct answer.
	b	$T_5 = 40.5 - 2.5 = 38$	**1** Mark: Correct answer.
	c	$T_n = a + (n-1)d$ $\therefore 48 + (n-1)(-2.5) > 0$ $50.5 > 2.5n$ $n < 20.2$ $\therefore n = 20$ $\therefore$ There should be 13 more rungs.	**2** Marks: Correct solution. **1** States equation or inequality to be solved or equivalent
MA12-5	31a	Amplitude = 1 Period $= \dfrac{2\pi}{1} = 2\pi$  **Note: rulers should always be used in diagrams and to mark the scale. Many students lost marks unnecessarily due to not drawing the correct shape of the curve. This is essential.**	**2** Marks: Correct answer and shape. 1 Mark: makes significant progress.
	b	The particle is a rest when $v = 0$ or $\frac{dx}{dt} = 0$	**2** Marks: Correct solution

		$\therefore t = 0, \frac{\pi}{2}, \pi$ ∴ Position of the particle at these times: $x = 0, 2, 0$	1 Mark: Finds at least two of the required times or positions
	c	$x = 1 - \cos 2t$ $v = \frac{dx}{dt} = 2\sin 2t$ At $t = \frac{\pi}{4}$. $v = 2\sin\left(2 \times \frac{\pi}{4}\right)$ $\quad = 2 \text{ m/s}$ ∴ Velocity of the particle is 2 m/s.	**1** Mark: Correct answer.
	d	$2\sin 2t = 1$ $\sin 2t = \frac{1}{2}$ $2t = \frac{\pi}{6}, \frac{5\pi}{6}$ $t = \frac{\pi}{12}, \frac{5\pi}{12}$ Using the graph $\therefore \frac{\pi}{12} < t < \frac{5\pi}{12}$	**1** Mark: Correct answer.
MA12-6	32	Minimum value occurs when $\frac{dy}{dx} = 0$ $\frac{dy}{dx} = 4x - \frac{0.5}{\frac{x}{2}}$ $0 = 4x - \frac{1}{x}$ $4x^2 = 1$ $x^2 = \frac{1}{4}$ $x = \pm\frac{1}{2}$ Since x cannot take a negative value, $x = 0.5$. $y = 2(0.5)^2 - \ln\left(\frac{0.5}{2}\right) - 4 = \ln 4 - 3\frac{1}{2}$ $\therefore \left(\frac{1}{2}, \ln 4 - 3\frac{1}{2}\right)$ Check if it is a minimum $\frac{d^2y}{dx^2} = 4 + \frac{1}{x^2}$ When $x = 0.5$ $\frac{d^2y}{dx^2} = 4 + \frac{1}{0.5^2} = 8 > 0$ Minima	**3** Marks: Correct solution 2 Marks: Find the minimum value at $\left(\frac{1}{2}, \ln 4 - 3\frac{1}{2}\right)$ without testing for a minima or equivalent 1 Mark: Finds the derivative or equivalent

MA12-10	33a	$r = 0.995193611\ldots$ ≈ 0.9952	**1** mark: Correct answer.
	b	$y = mx + c$ $= BX + A$ $\mathrm{H} = 1.19t - 0.85$	**1** mark: Correct answer.
	c	When $t = 5$ years $H = 1.19t - 0.85$ $= 1.19 \times 5 - 0.85$ ≈ 5.1 m $\therefore$ Height of the tree after 5 years is 5.1 metres.	**1** mark: Correct answer.
	d	When $H = 20$ m $H = 1.19t - 0.85$ $20 = 1.19 \times t - 0.85$ $1.19t = 20.85$ $t \approx 17.5$ years $\therefore$ It takes 17.5 years for the tree to reach a height of 20 metres.	**1** mark: Correct answer.
	e	Strong positive linear association Question (C) involves interpolation. Very reliable. Question (D) involves extrapolation. Less reliable.	**1** mark: Correct answer.
MA12-10	34	$\sec^2\theta - \tan^2\theta = 1$ $\sec\theta = \dfrac{1}{\cos\theta} \qquad \tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\therefore \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{\cos^2\theta}{\cos^2\theta}$ $= 1$	**2** Marks: Correct solution 1 Mark: for some progress
MA12-10	35a	55% - 63% (acceptable range)	**1** mark: Correct answer.
	b	The Pareto Principal states that 80% of issues come from 20% of problems. This would direct the business to focus improving delivery times and pizza temperature as they both account for 80% of complaints made and are close to 20% of all complaint types.	**2** Marks: Correct answer. 1 Mark: for some progress