International Grammar School

Student Name: ....

Maths Teacher: ....

2008

Mathematics

Trial Examination

HSC Assessment Task 4

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen.
- Board-approved calculators may be used.
- All necessary working should be shown in every question
- START EACH QUESTION ON A NEW SHEET OF PAPER AND WRITE YOUR NAME ON THE SHEET.
- A table of Standard Integral has been provided at the back of the examination booklet.
- Students may use a curve drawing template which does not contain printed formulae other than equations of simple curves that may be drawn using the template.

Total Marks - 120

- Attempt all questions from 1 – 10
- All questions are of equal value

STUDENT NUMBER/NAME: ....

Total marks – 120 marks
Attempt Questions 1 – 10
All questions are of equal value

Answer the questions on your own paper or writing booklet, if provided. Start each question on a new page.

Question 1 (12 marks) Use a SEPARATE page or writing booklet **Marks**

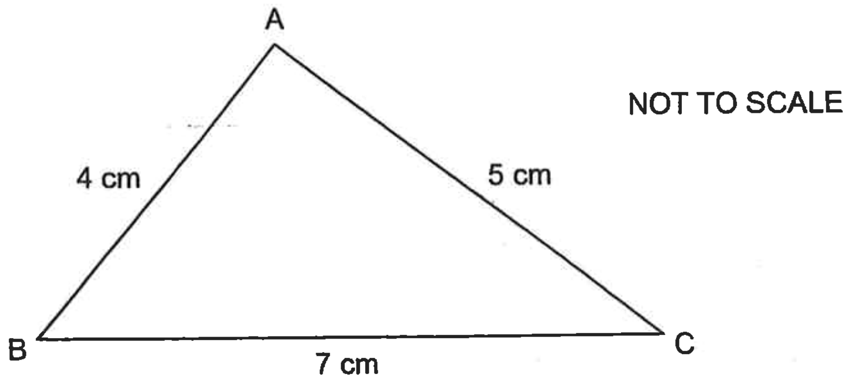
(a) Evaluate: $\sqrt[3]{e^{2.4}-1}$ correct to 5 significant figures. **2**

(b) Given: $\dfrac{5}{\sqrt{3}-1}=a\sqrt{3}+b$, find the values of a and b. **2**

(c) Solve, giving your answer(s) in exact form: $2x^2-5x-4=0$. **2**

(d) Find a primitive of: $\dfrac{1}{x^2}+\dfrac{1}{x}$. **2**

(e)



In the diagram above, find the size of the largest angle.
Give your answer correct to the nearest degree. **2**

(f) Simplify: $\dfrac{5}{m-2}-\dfrac{2}{m-3}$. **2**

STUDENT NUMBER/NAME: ....

Question 2 (12 marks) Use a SEPARATE page or writing booklet — **Marks**

(a) Differentiate with respect to x:

(i) $\dfrac{\cos x}{x-1}$. — **2**

(ii) $\left(3x^2-7\right)^5$. — **2**

(b) Solve: $|2x-3|<1$. — **2**

(c) (i) Find: $\int \dfrac{x}{x^2+2}\,dx$. — **2**

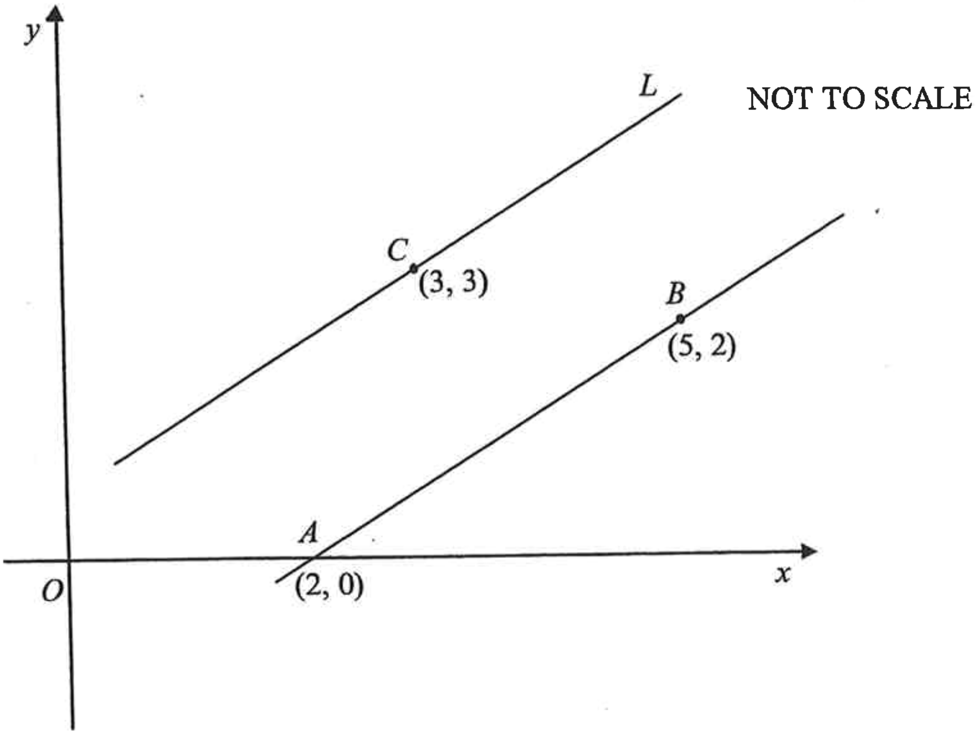
(ii) Evaluate: $\int_0^{\frac{2\pi}{3}} \sin 2x\,dx$. — **3**

(d) Use the change of base rule to evaluate: $\log_8 4$. — **1**

STUDENT NUMBER/NAME: ....

Question 3 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a)



In the diagram above the points A(2,0), B(5,2) and C(3,3) are shown.
Copy or trace the diagram onto your worksheet.

(i) Find the exact length of AB. **1**

(ii) SHOW that the equation of AB is $2x-3y-4=0$. **1**

(iii) Find the exact perpendicular distance from C to AB. **1**

(iv) The line L passing through C has equation $2x-3y+3=0$. Show that L is parallel to AB. **2**

(v) D is a point on L such that the length of DC is $\frac{\sqrt{13}}{2}$ units. What type of quadrilateral is ABCD? Give reasons. **1**

(vi) Calculate the area of ABCD. **1**

(b) Solve: $2\cos A = -\sqrt{3},\ for\ 0 \le A \le 2\pi$. **2**

(c) Find the equation of the tangent to the curve $y = x^2 \ln x$ at the point P on it where $x = e$. **3**

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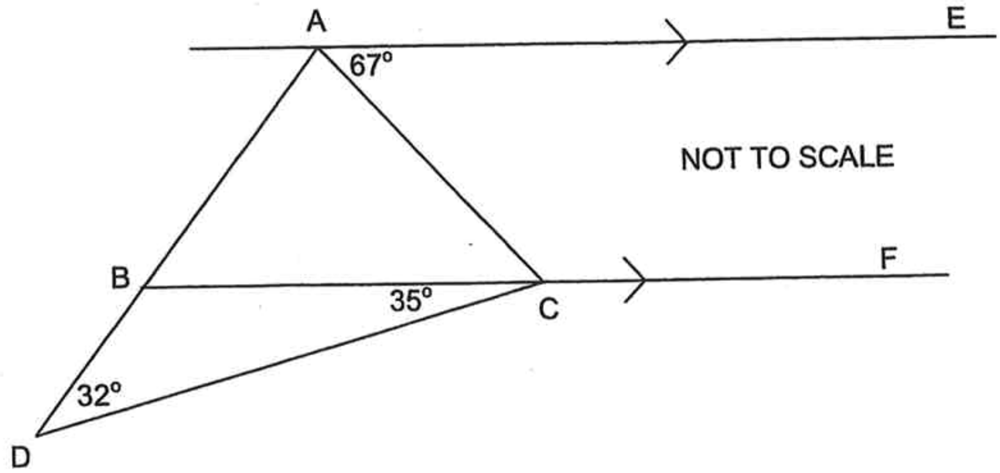


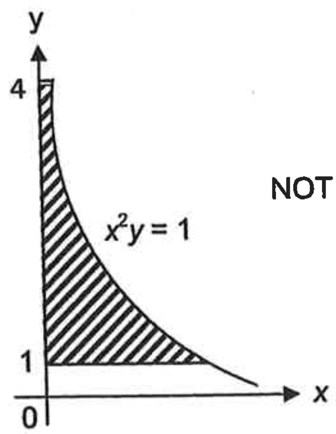
In the diagram above AE is parallel to BF.

$\angle ADC = 32°$, $\angle BCD = 35°$ and $\angle CAE = 67°$.

(i) Show that ΔABC is isosceles. 3

(ii) Find the size of $\angle BAC$. 1

(b)



The shaded region above shows the area bounded by the graph $x^2 y = 1$, $(x > 0)$, the y-axis and the lines $y = 1$ and $y = 4$.

Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis. Give your answer in exact form. 3

Question 4 continues on the next page

Question 4 (continued) **Marks**

(c) During the drought of the last few years, the water level in the local dam in the township of Wallaville was reduced to 2.5% of its capacity.

In the first week of drought breaking rain, the inflow added 3% of capacity to the amount of water in the dam (i.e. the dam was 5.5% full).

In the next week the inflow added 3.5% of capacity to the amount of water in the dam.

In the third week 4% of capacity was added.

This pattern continued so that each week an extra 0.5% of capacity was added to the dam until it was full.

(i) What percentage of capacity was added to the dam in the 10^{th} week? **1**

(ii) What percentage of capacity was in the dam after 10 weeks? **2**

(iii) How many weeks would it have taken to fill the dam? **2**

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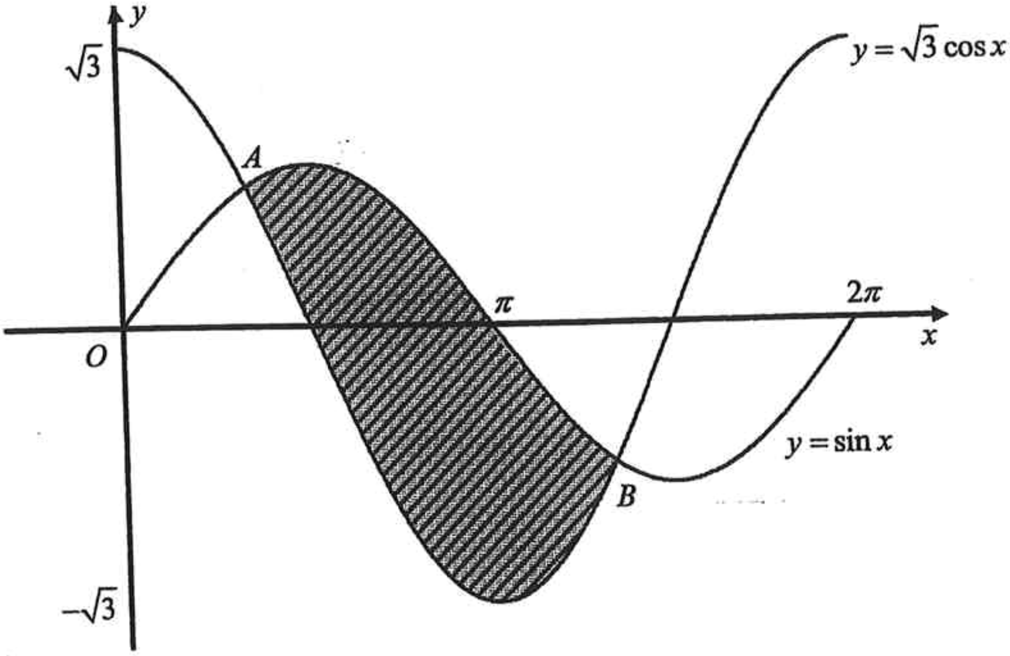

The diagram shows the graphs $y = \sin x$ and $y = \sqrt{3}\cos x$, $0 \le x \le 2\pi$.
The graphs intersect at points A and B.

(i) Show that point A has coordinates $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$ and find the coordinates of B. **1**

(ii) Find the area enclosed by the two graphs. **3**

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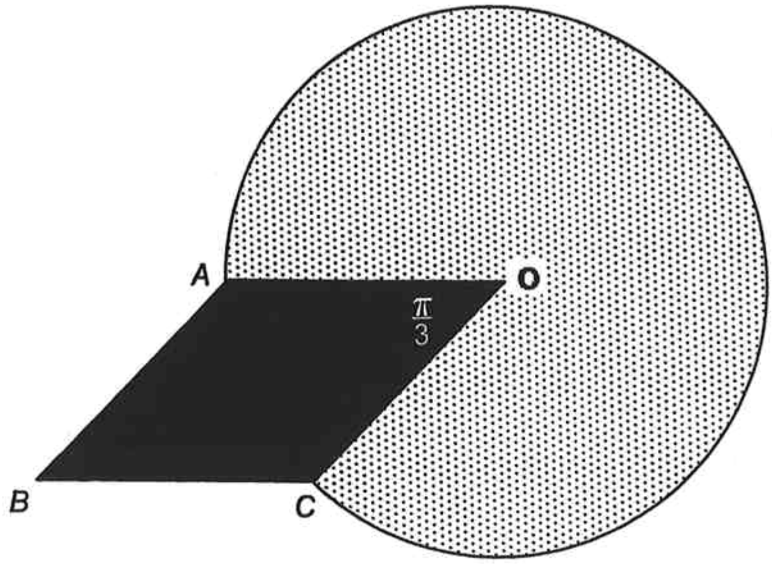
Question 5 (continued) **Marks**

(b) A concrete viewing platform is to be built at a mountain lookout. The platform is formed from a rhombus $AOCB$ with side $AO = 5$m and $\angle AOC = \frac{\pi}{3}$, and the major sector of a circle centre O, radius AO. The concrete is 200mm thick. The platform is illustrated in the diagram below.



NOT TO SCALE

(i) Show that reflex $\angle AOC = \frac{5\pi}{3}$. **1**

(ii) Calculate the area of the platform. **3**

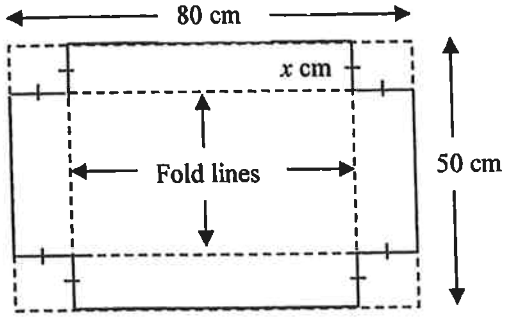
(iii) Find the volume of concrete used to make the platform. **1**

(c) Given $\tan A = \frac{\sqrt{15}}{7}$ and $\pi \le A \le 2\pi$, find the exact value of $\operatorname{cosec} A$. **3**

STUDENT NUMBER/NAME: ....

Question 6 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) A piece of metal 80 cm long and 50 cm wide will be used to make an open box. A square of side x cm will be cut from each corner and the sides folded up to form a box.



(i) What is the limit to the maximum value of x? Justify your answer. **1**

(ii) Show that the volume of the box is given by $V = 4000x - 260x^2 + 4x^3$. **2**

(iii) Find the value of x for which the box will have a maximum volume. **3**

(iv) Find the maximum volume of the box. **1**

(b) Evaluate: $\sum_{x=0}^{4}\left(\sin\frac{\pi x}{4}\right)$ **2**

(c) Given the infinite series: $\frac{x}{3}+\frac{2x^2}{9}+\frac{4x^3}{27}+\ldots$:

(i) Show that it is a geometric series. **1**

(ii) Find the values of x such that the series has a limiting sum and find the sum (in terms of x). **2**

Question 7 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) A function is defined as $f(x) = x^3 - 3x^2$.

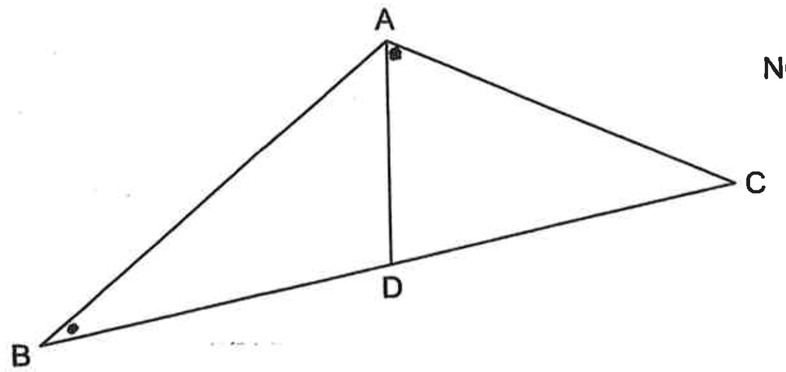
(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion. **1**

(iii) Sketch the graph of $y = f(x)$ indicating clearly the stationary points, the point of inflexion and the x-intercepts. **3**

(iv) Find the minimum value of the function in the interval $-2 \le x \le 3$. **1**

(b) In the diagram $\angle CAD = \angle ABC$.



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Copy or trace the diagram onto your worksheet.

(i) Prove that ΔCAD is similar to ΔCBA. **3**

(ii) Hence or otherwise show that $AC^2 = CD.CB$. **1**

Question 8 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) The number of worms, N, in a worm farm at time t weeks, is given by the formula:

$N = N_0 e^{kt}$ where N_0 and k are constants.

Initially there were 200 worms placed in the worm farm. After 2 weeks the number of worms had doubled.

(i) Find the value of N_0 and show that the value of k is 0.3466. **2**

(ii) How many worms were in the farm after 10 weeks? **1**

(iii) Find the rate of increase in the number of worms at 10 weeks. **2**

(iv) How many weeks would it take for the number of worms to increase by 900%? **2**

(b) Show that the quadratic equation $x^2 + (p-3)x - (2p+1) = 0$, where p is real, has real distinct roots **3**

(c) If $A = \sin\beta$ express $1 + \cot^2\beta$ in terms of A. **2**

STUDENT NUMBER/NAME: ....

Question 9 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) (i) Copy and complete the table below for the function $f(x)=(x-1)^{-2}$, giving the values correct to 3 significant figures. **1**

x	2	2.5	3	3.5	4
$f(x)$					

(ii) Using Simpson's Rule with 5 function values, find an approximate value for:

$$\int_2^4 (x-1)^{-2}\,dx.$$ **2**

(b) The local swimming pool has been closed and is being drained for repairs. The rate, at which the amount of water in the pool, (P kilolitres) at time t hours after draining has commenced, is decreasing is given by:

$$\frac{dP}{dt}=-30(20-t)$$

Initially the pool held 6000 kilolitres.

(i) Express P as a function of t. **2**

(ii) Find how much water was in the pool after 5 hours. **1**

(iii) How long does it take to empty the pool? **2**

(c) The equation of a parabola is: $2y=x^2-4x+6$.

(i) Find the coordinates of the vertex, V, and the focus, S. **2**

(ii) Find the equation of the directrix. **1**

(iii) Draw a neat sketch of the graph of this parabola showing the information obtained in (i) and (ii) above. **1**

STUDENT NUMBER/NAME: ....

Question 10 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) Alana has borrowed \$17 000 to buy a new car. The interest on the loan is 18% per annum paid monthly. The loan is to be repaid in equal monthly instalments of $\$P$ over a term of 5 years.

Let the amount owing on the loan after n months be $\$A_n$.

(i) Show that the amount $\$A_1$ owing after one month is given by: **1**
$A_1 = \$\{(17000 \times 1.015) - P\}$.

(ii) Show that the amount $\$A_3$ owing after 3 months is given by: **2**
$A_3 = \$\{(17000 \times 1.015^3) - P(1 + 1.015 + 1.015^2)\}$.

(iii) Write down a similar expression for the amount owing after 5 years (60 months). **1**

(iv) Calculate the monthly instalment $\$P$ paid on the loan. **2**

(v) How much would Alana have saved by paying cash for the car? **1**

(b) A particle is moving in a straight line so that at time t seconds its displacement from the origin is x metres. Initially the particle is 1 metre to the left of the origin.

The velocity of the particle is given by $v = 2\cos t - 1$.

(i) Express the displacement x as a function of t. **2**

(ii) At what time is the particle first at rest? **2**

(iii) Find the position of the particle at this instant. **1**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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(a) Evaluate: $\sqrt[3]{e^{2.4}-1}$ correct to 5 significant figures. **2**

(b) Given: $\frac{5}{\sqrt{3}-1}=a\sqrt{3}+b$, find the values of a and b. **2**

(c) Solve, giving your answer(s) in exact form: $2x^2-5x-4=0$. **2**

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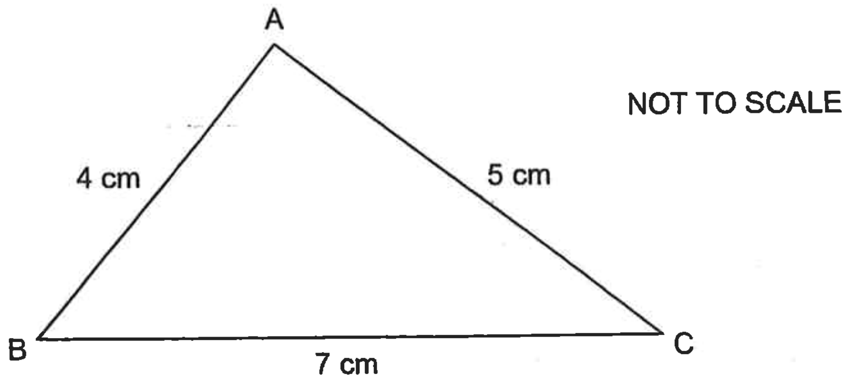

In the diagram above, find the size of the largest angle.
Give your answer correct to the nearest degree. **2**

(f) Simplify: $\frac{5}{m-2}-\frac{2}{m-3}$. **2**

STUDENT NUMBER/NAME: ....

Question 2 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) Differentiate with respect to x:

(i) $\dfrac{\cos x}{x-1}$. **2**

(ii) $\left(3x^2-7\right)^5$. **2**

(b) Solve: $|2x-3|<1$. **2**

(c) (i) Find: $\int \dfrac{x}{x^2+2}\,dx$. **2**

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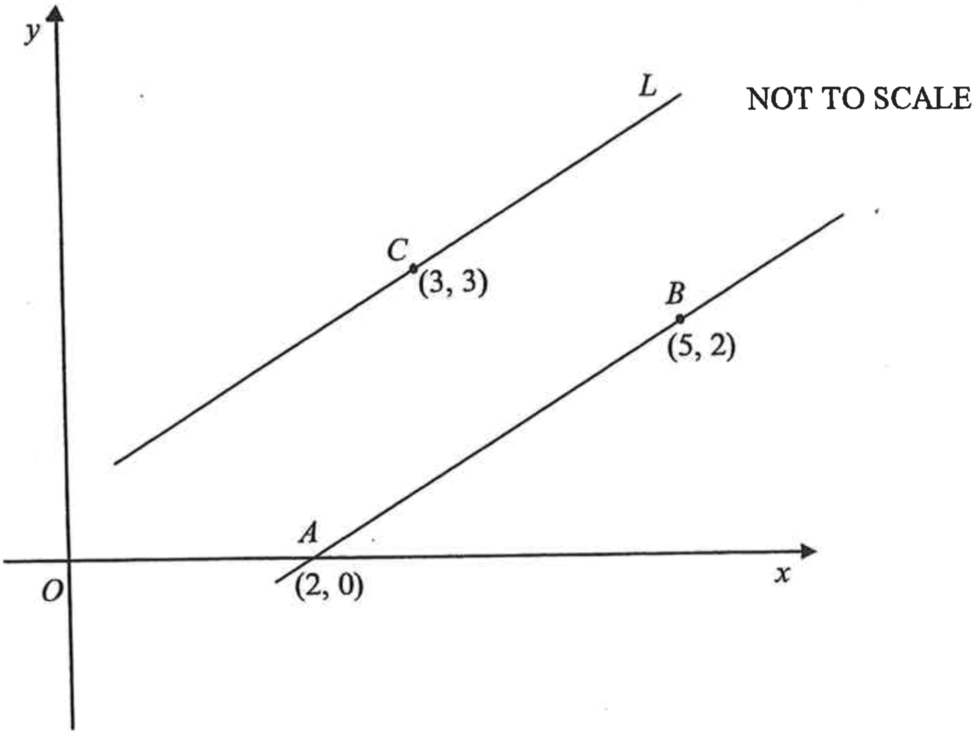

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(v) D is a point on L such that the length of DC is $\frac{\sqrt{13}}{2}$ units. What type of quadrilateral is ABCD? Give reasons. **1**

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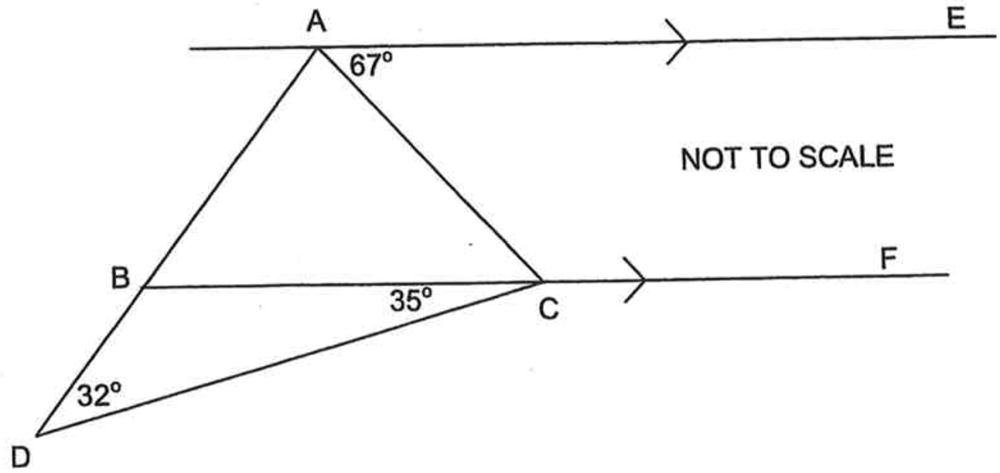

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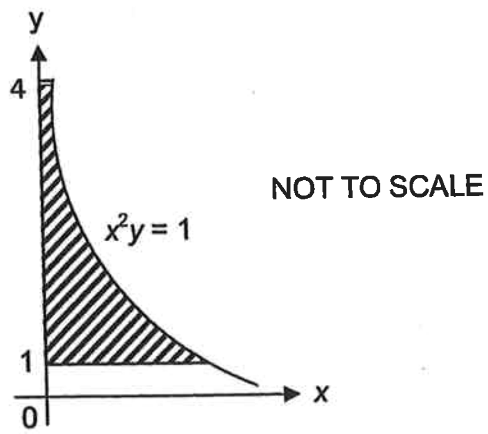

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Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis. Give your answer in exact form. 3

Question 4 continues on the next page

Question 4 (continued) **Marks**

(c) During the drought of the last few years, the water level in the local dam in the township of Wallaville was reduced to 2.5% of its capacity.

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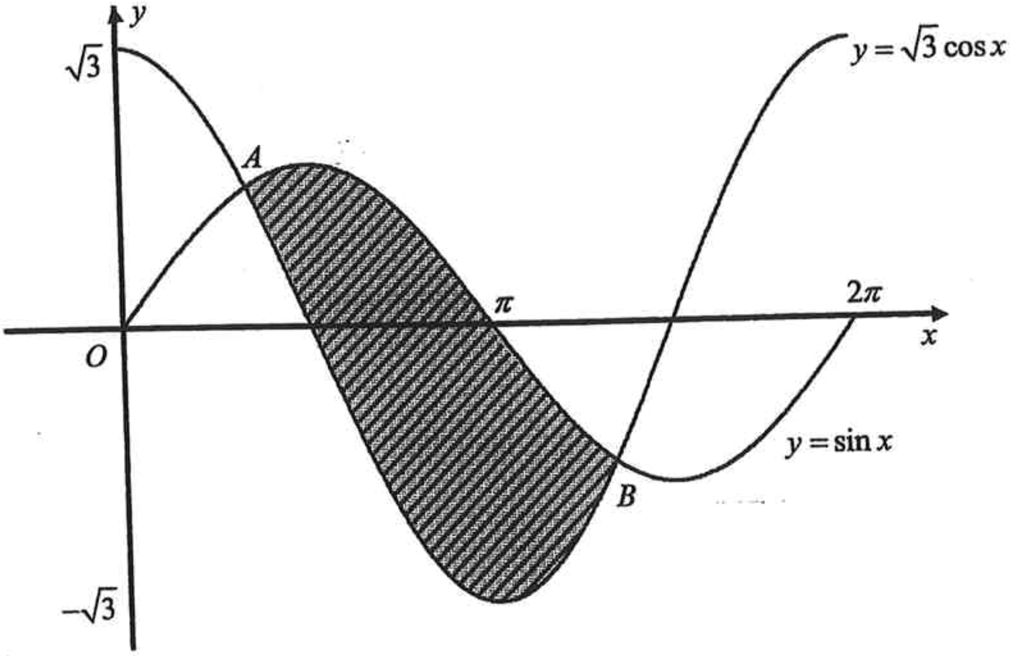

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Question 5 (continued) **Marks**

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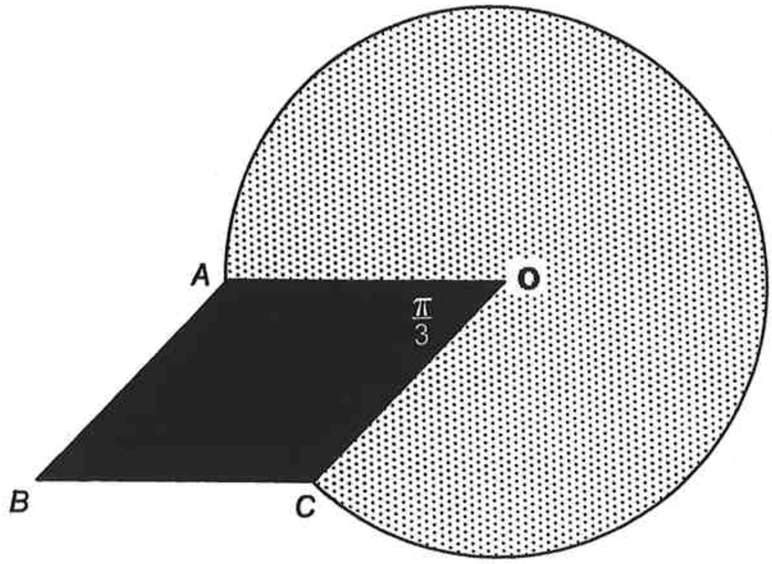

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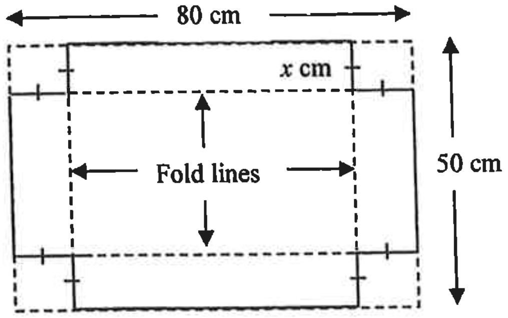

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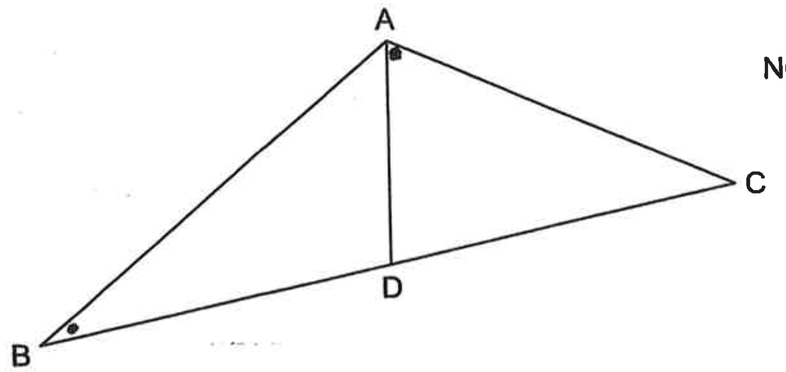

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(iii) How long does it take to empty the pool? **2**

(c) The equation of a parabola is: $2y=x^2-4x+6$.

(i) Find the coordinates of the vertex, V, and the focus, S. **2**

(ii) Find the equation of the directrix. **1**

(iii) Draw a neat sketch of the graph of this parabola showing the information obtained in (i) and (ii) above. **1**

STUDENT NUMBER/NAME: ....

Question 10 (12 marks) Use a SEPARATE page or writing booklet **Marks**

(a) Alana has borrowed \$17 000 to buy a new car. The interest on the loan is 18% per annum paid monthly. The loan is to be repaid in equal monthly instalments of \$$P$ over a term of 5 years.

Let the amount owing on the loan after n months be \$$A_n$.

(i) Show that the amount \$$A_1$ owing after one month is given by: $A_1 = \$\{(17000 \times 1.015) - P\}$. **1**

(ii) Show that the amount \$$A_3$ owing after 3 months is given by: $A_3 = \$\left\{\left(17000 \times 1.015^3\right) - P\left(1 + 1.015 + 1.015^2\right)\right\}$. **2**

(iii) Write down a similar expression for the amount owing after 5 years (60 months). **1**

(iv) Calculate the monthly instalment \$$P$ paid on the loan. **2**

(v) How much would Alana have saved by paying cash for the car? **1**

(b) A particle is moving in a straight line so that at time t seconds its displacement from the origin is x metres. Initially the particle is 1 metre to the left of the origin.

The velocity of the particle is given by $v = 2\cos t - 1$.

(i) Express the displacement x as a function of t. **2**

(ii) At what time is the particle first at rest? **2**

(iii) Find the position of the particle at this instant. **1**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$