International Grammar School

Mathematics
Trial Higher School Certificate Examination
2010

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page of this question paper
- All necessary working should be shown in every question

.

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value
- Start each question in a new writing booklet
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Total Marks – 120
Attempt Questions 1-10
All Questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 **(12 Marks)** Use a Separate Booklet

Marks

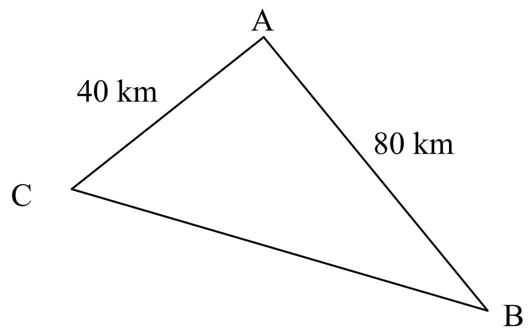
a) Evaluate $\dfrac{\sqrt{5^2+144}}{13-6}$ to two decimal places. **1**

b) Evaluate $\displaystyle\int_1^5 (3x-7)\,dx$ **1**

c) Rationalise the denominator and simplify: **2**

$$\frac{1}{2-\sqrt{3}}-\frac{1}{2+\sqrt{3}}$$

d) Three towns form a triangle. Town A is 80 km from Town B and Town C is 40 km from Town A as shown below:



The bearing of Town B from Town A is $130°$. The bearing of Town C from Town A is $240°$.

i) Find the area enclosed by the 3 towns **2**

ii) Using the cosine rule, find the distance to the nearest kilometre between Town B and Town C **2**

e) Express the following as a single fraction

$$\frac{5}{2a+6}+\frac{a}{a^2-9}$$ **2**

f) Solve $|2x+5|<3$ **2**

End of Question 1

Question 2 **(12 Marks)** Use a Separate Booklet **Marks**

a) Differentiate with respect to x:

i) $3x^2 + 7 \quad ^6$ **2**

ii) $4x^2e^{3x^3}$ **2**

iii) $\dfrac{\pi \cos x}{x^2}$ **2**

b) Find $\displaystyle\int \frac{dx}{3x+5}$ **2**

c) On a diagram, indicate the region where the following inequalities hold simultaneously: **2**

$$y + 1 \geq 0, \qquad x + y - 2 \leq 0 \qquad and\ x \geq 2$$

d) Find the obtuse angle in degrees and minutes, that a line with gradient -2.5 makes with the positive x axis. **2**

End of Question 2

Question 3 (12 Marks) Use a Separate Booklet **Marks**

a) Find $\lim_{x \to -3} \frac{x^2 + 8x + 15}{x + 3}$ **2**

b) Evaluate $\int_0^{\frac{\pi}{6}} (x^2 + \sin 2x)\, dx$. Leave your answer in exact form. **2**

c) i) On the same set of axes, sketch the functions $y = 4x - x^2$ and $y = 3$ **2**

ii) Find the area contained between these two curves **2**

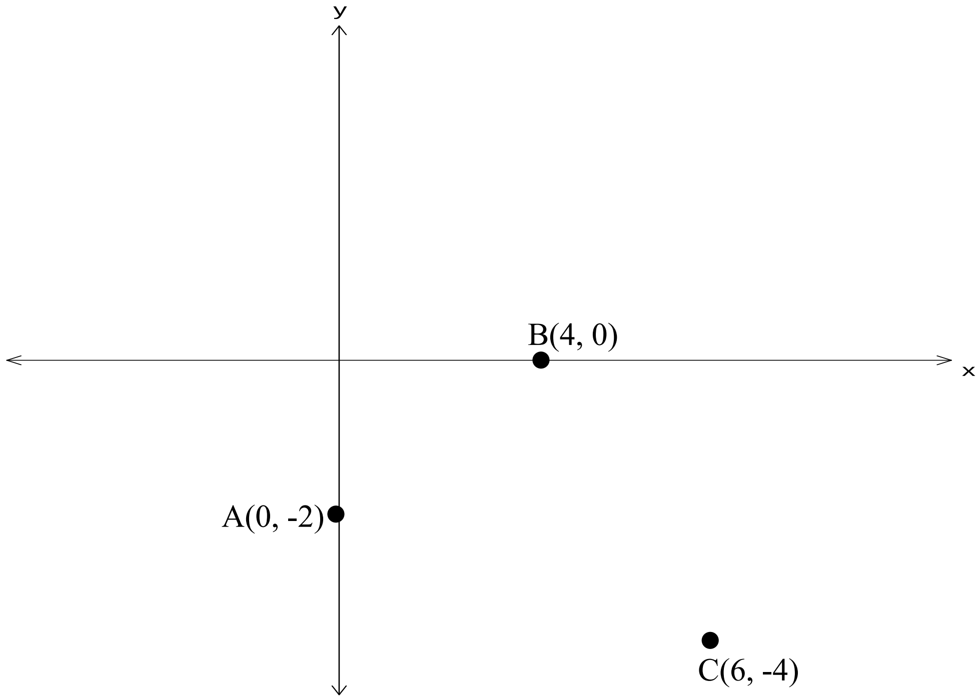
d) Determine if the line $x + y + 3 = 0$ is a tangent to the parabola $y = 2x^2 + 3x - 1$ **2**

e) For the curve $y = \sin \pi x$, state the period and amplitude **2**

End of Question 3

Question 4 **(12 Marks)** Use a Separate Booklet **Marks**

a) The coordinates of the points *A*, *B* and *C* are , (0, -2), (4, 0) and (6, -4) respectively.



(i) Find the length *AB*, and the gradient of *AB*. **2**

(ii) Show that the equation of the line *L*, drawn through *C* parallel to *AB*, is $x - 2y - 14 = 0$. **1**

(iii) Find the coordinates of *D*, the point where *L* intersects the *x*-axis. **1**

(iv) Find the perpendicular distance of the point *B* from the line *L*. **1**

(v) Find the area of the quadrilateral *ABDC*. **2**

b) For the arithmetic sequence

2, 7, 12, 17, ………..

i) Find the general term T_n **1**

ii) Find the 23^{rd} term **1**

iii) Find the sum of the first 47 terms **1**

c) Find the exact value of x such that $secx + 1 = 3 \quad where \quad 0 \leq x \leq \frac{\pi}{2}$ **2**

End of Question 4

Question 5 (12 Marks) Use a Separate Booklet **Marks**

a) Calculate the area of the region enclosed by the graph of $y = \cos 2x$ the x axis and the lines $x = 0$ and $x = \dfrac{\pi}{4}$ **2**

b) The roots of the equation $2x^2 - 7x + 12 = 0$ are α and β
Find:

i) $\alpha + \beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **2**

iv) $\alpha^2 + \beta^2$ **2**

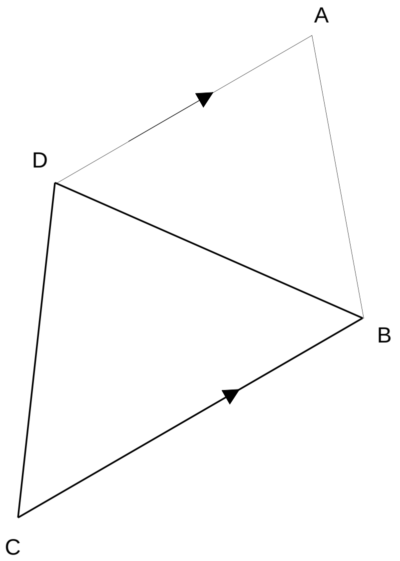
c) A pendulum consisting of a bob and a long string attached to a fixed point is set swinging with an initial arc of 40cm. If each subsequent oscillation is $\frac{5}{6}$ of the preceding one, Find the total distance travelled by the bob before it comes to rest. **2**

d) The gradient function of a curve is $y' = \dfrac{4x}{x^2 + 1}$ and the curve passes through the point (0 , e). Find the equation of the curve. **2**

End of Question 5

Question 6 (12 Marks) Use a Separate Booklet **Marks**

a) **3**



Given AD = AB, DB = DC, AD $\|$ BC and $\angle DAB = 100°$.
Copy or trace the diagram into your answer booklet.

Find $\angle BDC$ giving reasons for each step.

b) Let $f(x) = x^3 - 6x^2$

i) Find the coordinates where the curve crosses the axes. **2**

ii) Find the coordinates of any stationary points and determine their nature. **2**

iii) Find the coordinates of any points of inflexion. **1**

iv) Sketch the curve $y = f(x)$, indicating clearly the intercepts and any stationary points and points of inflexion. **3**

v) For what values of x is $y = f(x)$ increasing. **1**

End of Question 6

Question 7 (12 Marks) Use a Separate Booklet **Marks**

a) i) Given $f(x) = \sqrt{4 - x^2}$, copy and complete the table of values to 3 decimal places. **1**

x	0	0.5	1	1.5	2
$f(x)$					

ii) Hence evaluate an approximation for $\int_0^2 \sqrt{4 - x^2}\,dx$ using Simpson's rule with 5 function values. **2**

b) A pendulum on a grandfather clock is 50 cm long. When it swings the maximum length of the arc it makes is 40 cm.

i) In radians find the angle through which the pendulum swings. **1**

ii) Find the shortest distance between the maximum positions of the pendulum. **2**

c) The number of bacteria N a person has after being infected with a virus after t hours is given by:

$$N = 10000e^{0.05t}$$

i) Find the number of bacteria after 10 hours **1**

ii) Find the time required for the number of bacteria to reach 100000 **2**

iii) At what rate is the bacteria increasing after 1 day **1**

d) The area bounded by $y^2 = 3 - 2x - x^2$, $y \geq 0$ and between $x = -3$ and $x = 1$ is revolved about the x axis. Calculate the volume of the solid formed if this area is rotated about the x axis. **2**

End of Question 7

Question 8 **(12 Marks)** Use a Separate Booklet **Marks**

a) Tiarn borrows \$500 000 to buy a house. An interest rate of 9% p.a. compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly instalements (R) over a 25 year period.

i) Show the amount owing after 3 months is: **2**

$$A_3 = 500000\ 1.0075^{\ 3} - R\left[1 + 1.0075 + 1.0075^2\right]$$

ii) Assuming this pattern continues the monthly repayment can be calculated using: **2**

$$A_n = 500000\ 1.0075^{\ n} - R\left[1 + 1.0075 + 1.0075^2 + \dots\dots\dots + 1.0075^{n-1}\right]$$

How much should Tiarn be paying each month?

iii) How much interest does Tiarn pay over the 25 years? **1**

iv) What is the equivalent simple interest rate of this loan? **1**

b) i) Sketch the Parabola, whose focus is the point $(2,5)$ and whose directrix is the line $y = -3$. Indicate on your diagram the vertex and its coordinates **2**

ii) Find the equation of the parabola. **1**

c) If $f(x) = 4 - 2^{-x}$ find:

i) $f(x^2)$ **1**

ii) $f(x)^{\ 2}$ **1**

iii) Is $f(x)$ even, odd or neither **1**

End of Question 8

Question 9 **(12 Marks)** Use a Separate Booklet **Marks**

a) The acceleration a ms^{-2} of a moving particle is given after t seconds by $a = -2$. Initially the particle is located at $x = -3$ and its velocity is 4 ms^{-1}

i) Find the velocity(v) and displacement (x) as functions of time (t) **2**

ii) Determine when the particle is at rest. **2**

iii) When will the particle first be at the origin? **2**

iv) Sketch displacement (x) as a function of time (t) **2**

b) i) Differentiate $y = 3^{4x-2}$ with respect to x **3**

ii) Hence find: $\int 3^{4x-2}\, dx$ **1**

End of Question 9

Question 10 (12 Marks) Use a Separate Booklet **Marks**

a) A swimming pool is to be emptied for maintenance. The quantity of water Q litres, remaining in the pool at anytime, t minutes, after it starts to empty is given by:

$$Q(t) = 2000(25-t)^2, \quad t \geq 0$$

i) At what rate is the pool being emptied at any time (t) **1**

ii) How long will it take to half empty the pool to the nearest minute? **2**

iii) At what time is the water flowing out at 20 kL / minute. **2**

iv) What is the average water flow in the first 10 minutes in litres? **2**

b) Adam is on a paddle board in the ocean 3 kilometres from the nearest point O on a straight beach. He needs to meet his friend Josh who is 6 kilometres along the beach from O. Adam is able to paddle at a rate of 4km/h and walk at a rate of 5km/h.

i) Draw a diagram to represent this information. **1**

ii) Show the total time T(x) hours, for Adam to reach Josh is given by: **2**

$$T(x) = \frac{\sqrt{x^2+9}}{4} + \frac{6-x}{5}$$

iii) Find the minimum time for Adam to reach Josh on the beach. **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Mathematics

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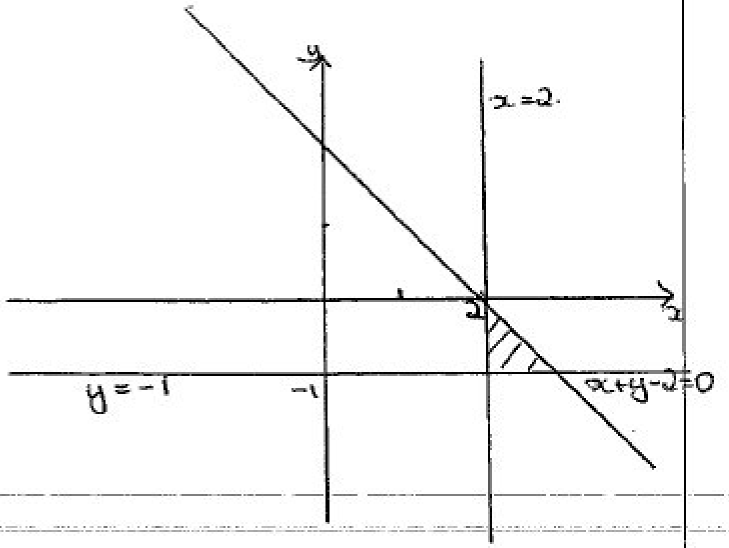
SOLUTIONS

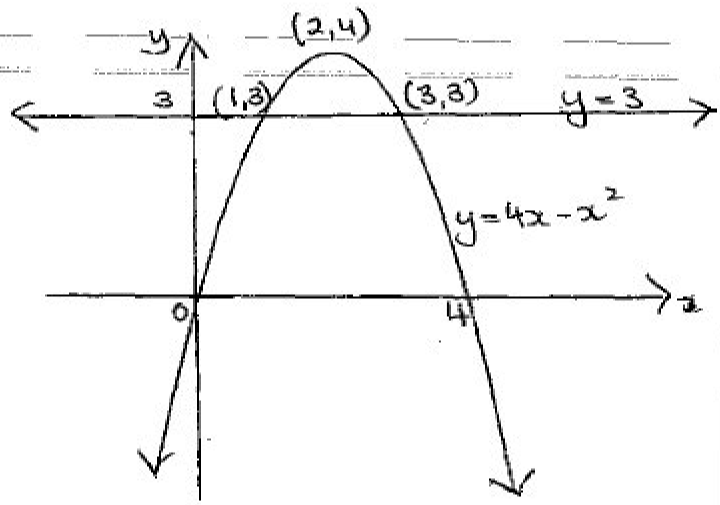
Question 1	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$\dfrac{\sqrt{5^2+144}}{13-6} = 4.9135\ldots$ $= 4.91 \text{ 2dp}$	1	
b)	$\int_1^5 (3x-7)dx$ $= \left[\dfrac{3x^2}{2} - 7x\right]_1^5$ $= (\dfrac{75}{2} - 35)(\dfrac{3}{2} - 7)$ $= 8$	1	
c)	$\dfrac{1}{2-\sqrt{3}} + \dfrac{1}{2+\sqrt{3}}$ $= \dfrac{(2+\sqrt{3}) + (2-\sqrt{3})}{(2-\sqrt{3})+(2+\sqrt{3})}$ $= \dfrac{2+\sqrt{3}+2-\sqrt{3}}{4-3}$ $= 4$	2	
d)	i) $A = \dfrac{1}{2}ab\sin C$ $A = \dfrac{1}{2} \times 40 \times 80 \times \sin 110°$ $= 1503.5km^2$ ii) $a^2 = b^2 + c^2 - 2bc\cos A$ $a^2 = 40^2 + 80^2 - 2 \times 40 \times 80 \times \cos 110°$ $a^2 = 10188.93$ $a = 100.9$ $a = 101km$	4	1 correct use sinerule with correct angle 1 correct answer 1 correct use cosine rule 1 correct rounded answer

Question 1	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
e)	$\frac{5}{2a+6}+\frac{a}{a^2-9}$ $=\frac{5}{2(a+3)}+\frac{a}{(a+3)(a-3)}$ $=\frac{5(a-3)+2a}{2(a+3)(a-3)}$ $=\frac{5a-15+2a}{2(a+3)(a-3)}$ $=\frac{7a-15}{2(a^2-9)}$	2	1 1
f)	$\|2x+5\|<3$ $2x+5<3$ or $2x+5>-3$ $2x<-2$ $\quad$ $2x>-8$ $x<-1$ $\quad$ $x>-4$ *check* $\therefore -4<x<-1$	2	1 1
		/12	

<table>
<tr><th>Question 2</th><th colspan="2">Trial HSC Examination - Mathematics</th><th>2010</th></tr>
<tr><th>Part</th><th>Solution</th><th>Marks</th><th>Comment</th></tr>
<tr><td>a)</td><td>i) $\dfrac{d}{dx}(3x^2+7)^6$
$=6\times 6x(3x^2+7)^5$
$=36x(3x^2+7)^5$</td><td>2</td><td>1
1</td></tr>
<tr><td></td><td>ii) $4x^2e^{3x^3}$
$u=4x^2 \quad v=e^{3x^3}$
$u'=8x \quad v'=9x^2e^{3x^3}$
$\dfrac{d}{dx}=8xe^{3x^3}+4x^2\times 9x^2e^{3x^3}$
$=8xe^{3x^3}+36x^4e^{3x^3}$
$=4xe^{3x^3}\left[2+9x^3\right]$</td><td>2</td><td>1
1</td></tr>
<tr><td></td><td>iii) $\dfrac{\pi\cos x}{x^2}$
$u=\pi\cos x \quad v=x^2$
$u'=-\pi\sin x \quad v'=2x$
$\dfrac{d}{dx}=\dfrac{-\pi x^2\sin x-2\pi x\cos x}{(x^2)^2}$
$=\dfrac{\pi x(-x\sin x-2\cos x)}{x^4}$
$=\dfrac{\pi(-x\sin x-2\cos x)}{x^3}$</td><td>2</td><td>1
1</td></tr>
<tr><td>b)</td><td>$\int\dfrac{dx}{3x+5}$
$=\dfrac{1}{3}\int\dfrac{3}{3x+5}dx$
$=\dfrac{1}{3}\ln(3x+5)+C$</td><td>2</td><td>1
1</td></tr>
</table>

Question 2	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
c)	$y+1\geq 0$ $x+y-2\leq 0$ $x\geq 2$ $y\geq -1$ $y\leq 2-x$ 	2	1 correct lines 1 correct region.
d)	$m=\tan\theta$ $-2.5=\tan\theta$ $\theta=-68°12'$ $\therefore\theta=180°-68°12'$ $=111°48'$	2	1 1
		/12	

Question 3	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$\lim\limits_{x \to -3} \frac{x^2+8x+15}{x+3}$ $= \lim\limits_{x \to -3} \frac{(x+\cancel{3})(x+5)}{(x\cancel{+}3)}$ $= \lim\limits_{x \to -3} x+5$ $= 2.$	2	1 1
b)	$\int_0^{\frac{\pi}{6}} x^2 + \sin 2x \, dx$ $= \left[\frac{x^3}{3} - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$ $= \left[\frac{(\frac{\pi}{6})^3}{3} - \frac{1}{2}\cos(\frac{2\pi}{6})\right] - \left[-\frac{1}{2}\cos 0\right]$ $= \frac{\pi^3}{648} - \frac{1}{4} + \frac{1}{2}$ $= 0.298$	2	1 1

Question 3	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
c)	i) $y = 4x - x^2 \quad y = 3$ Pts of intersection: $4x - x^2 = 3$ $x^2 - 4x + 3 = 0$ $(x-1)(x-3) = 0$. $x = 1 \quad x = 3$ at $x = 1$, $y = 4.1 - 1 = 3 \quad (1,3)$ $x = 3$, $y = 12 - 9 = 3 \quad (3,3)$ 	2	1 correct diagram 1 all intercepts
	ii) $A = \int_1^3 (4x - x^2 - 3)\,dx$ $= \left[2x^2 - \frac{x^3}{3} - 3x\right]_1^3$ $= \left(2.9 - \frac{27}{3} - 3.3\right) - \left(2.1 - \frac{1}{3} - 3\right)$ $= (18 - 9 - 9) - \left(-1 - \frac{1}{3}\right)$ $= \frac{4}{3}\,u^2$	2	1 correct integral 1 correct answer

Question 3	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
d)	$x + y + 3 = 0 \Rightarrow y = -x - 3$. $y = 2x^2 + 3x - 1$. If tangent, one point intersection. $2x^2 + 3x - 1 = -x - 3$ $2x^2 + 4x + 2 = 0$ $x^2 + 2x + 1 = 0$ $(x+1)^2 = 0$ $x = -1$ Since there is one point of intersection, $x + y + 3 = 0$ is a tangent to $2x^2 + 3x - 1 = 0$ at $x = -1$.	2.	1 1
e)	Amplitude = 1 Period = $\frac{2\pi}{\pi} = 2$	2	1 1
		/12	

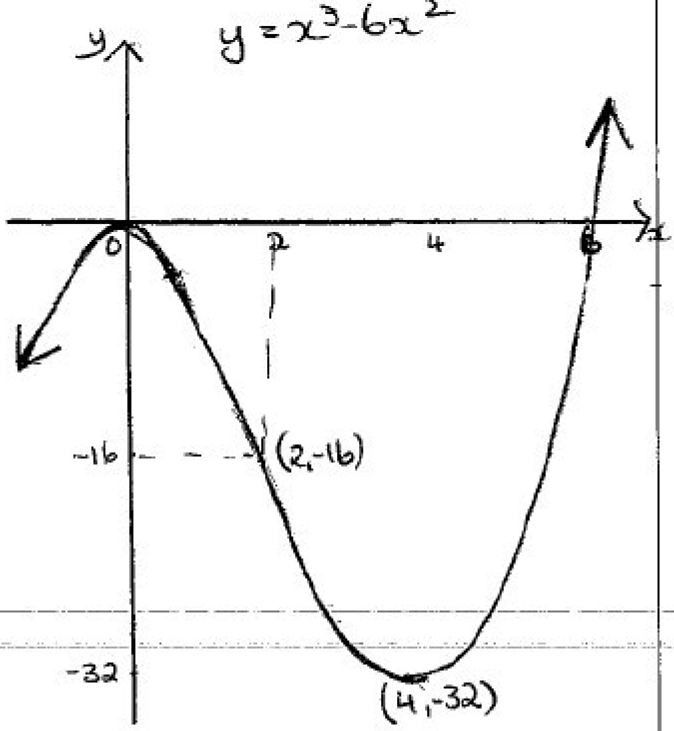
Question 4	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	i) $AB = \sqrt{(4-0)^2 + (0+2)^2}$ $= \sqrt{16+4}$ $= \sqrt{20} = 2\sqrt{5}$ u. $m_{AB} = \frac{0--2}{4-0}$ $= \frac{2}{4} = \frac{1}{2}$	2	1 1
	ii) L parallel to AB so $m_L = \frac{1}{2}$. $y--4 = \frac{1}{2}(x-6)$ $2y+8 = x-6$ $x-2y-14=0$	1	1
	iii) For D, $y=0$. $x-14=0$ $x=14$ D(14, 0)	1	1
	iv) B(4, 0) L: $x-2y-14=0$ $d = \left\|\frac{4.1+0.-2-14}{\sqrt{1+(-2)^2}}\right\| = \left\|\frac{4-14}{\sqrt{5}}\right\|$ $= \frac{10}{\sqrt{5}}$ or $2\sqrt{5}$u.	1	1
	v) Area ABDC (trapezium) $= \frac{h}{2}(a+b)$. Need $CD = \sqrt{(6-14)^2+(-4-0)^2}$ $= \sqrt{8^2+4^2}$ $= \sqrt{80}$ or $4\sqrt{5}$ $A = \frac{2\sqrt{5}}{2}(2\sqrt{5}+4\sqrt{5}) = \sqrt{5}.6\sqrt{5} = 30u^2$	2	1 1
b)	i) 2,7,12,17,.... $T_n = 5n-3$	3	1
	ii) $T_{23} = 5\times 23 - 3$ $=112$		1
	iii) $S_n = \frac{n}{2}[2a+(n-1)d]$ $= \frac{47}{2}[2\times 2 + (47-1)5]$ $= 5499$		1

Question 4	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
c)	$\sec x + 1 = 3 \qquad 0 \le x \le \frac{\pi}{2}$	2	
	$\sec x = 2$ Q1		1
	i.e. $\frac{1}{\cos x} = 2$.		
	$\cos x = \frac{1}{2}$		
	$x = \pi/3 \quad 0 \le x \le \frac{\pi}{2}$		1

Question 5	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	$\int_0^{\frac{\pi}{4}} \cos 2x$ $=\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$ $=\frac{1}{2}\sin\frac{\pi}{2}-\frac{1}{2}\sin 0$ $=\frac{1}{2}\times 1$ $=\frac{1}{2}unit^2$	2	1 1
b)	$2x^2-7x+12=0$ $\alpha+\beta=\frac{-b}{a} \quad \alpha\beta=\frac{c}{a}$		
	i) $\alpha+\beta=\frac{-b}{a}=\frac{--7}{2}$ $=\frac{7}{2}$ ii) $\alpha\beta=\frac{c}{a}=\frac{12}{2}$ $=6$ iii) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ $=\frac{\left(\frac{7}{2}\right)}{6}$ $=\frac{7}{12}$	1 1 2	1 1
	iv) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{7}{2}\right)^2-2\times 6$ $=\frac{1}{4}$	2	1 1

Question 5	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
c)	$a = 40 \quad r = \frac{5}{6}. \qquad \lvert r \rvert < 1$ G.P. $S_\infty = \frac{a}{1-r}$ $= \frac{40}{1-\frac{5}{6}}$ $= \frac{40}{1/6}$ $= 240$ cms	2	 1 1
d)	$y' = \frac{4x}{x^2+1}$ $y = 2\int \frac{2x}{x^2+1} dx$ $= 2\ln\left[x^2+1\right] + c$ $when \;\; x = 0 \;\; y = e$ $e = 2\ln\left[0+1\right] + c$ $\therefore c = e$ $\therefore y = 2\ln\left[x^2+1\right] + e$	2.	 1 1
		/12	

Question 6	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	Let $\angle ADB = x°$ ΔADB is isosceles ($AB = AD$) $\angle ADB = \angle ABD$(*base angles of* $\Delta ADB =$) *then* $2x + 100 = 180°$(*angle sum* Δ) *so* $x = 40°$ $\therefore \angle ADB = 40°$ *then* $\angle DBC = 40°$(*alt* $\angle$'s = AD⎕ BC) $\because \Delta DBC$ is isosceles (DB=DC) $\therefore \angle DBC = 40°$(*base* $\angle$'*s* ΔDBC) $\therefore \angle BDC = 180° - 40° - 40°$(*angle* sum ΔDBC) $=100°$ $\therefore \angle BDC = 100°$	3	1 1 1
b)	$f(x) = x^3 - 6x^2$	2	
	i) $x^3 - 6x^2 = 0$ $x^2(x-6) = 0$. $x = 0, 6$. $\therefore$ crosses axes $(0,0)$ $(6,0)$		2.
	ii) $f'(x) = 3x^2 - 12x$. SP where $f'(x) = 0$ $3x(x-4) = 0$ $x = 0,\ 4$ $y = 0, -32$. Nature: $f''(x) = 6x - 12$ at $(0,4)$ $f''(x) < 0$ $\therefore$ max $(0,4)$ $(4,-32)$ $f''(x) > 0$ $\therefore$ min. $(4,-32)$ so max TP at $(0,4)$ min TP at $(4,-32)$	2	must use a test for nature 2 (1 each)
	iii) POI when $f''(x) = 0$ and concavity changes. $f''(x) = 0$ $6x - 12 = 0$ $x = 2$ $y = -16$. at $x = 1$, $f''(x) < 0$ } $\therefore$ concavity $x = 3$, $f''(x) > 0$ } changes. $\therefore$ POI at $(2, -16)$	1	1 mark but must test for concavity.

Question 6	**Trial HSC Examination - Mathematics**		**2010**
Part	**Solution**	**Marks**	**Comment**
iv)		3	1 shape 2 all pts marked (intercepts, TP and POI) [1 mark if intercepts and TP].
v)	$y = f(x)$ increasing for $x < 0$, $x > 4$.	1	1
		/12	

<table>
<tr><td colspan="2">Question 7</td><td colspan="2">Trial HSC Examination - Mathematics</td><td>2010</td></tr>
<tr><td>Part</td><td colspan="2">Solution</td><td>Marks</td><td>Comment</td></tr>
<tr><td>a)</td><td colspan="2">
<table>
<tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr>
<tr><td>$f(x)$</td><td>2</td><td>1.936</td><td>1.732</td><td>1.323</td><td>0</td></tr>
</table>
$\int_a^b f(x)dx \approx \frac{h}{3}\left[(y_0+y_n)+(4\times odds)+(2\times evens)\right]$
$\approx \frac{0.5}{3}\left[(2+0)+4(1.936+1.323)+2(1.732)\right]$
≈ 3.083 or $3\frac{1}{12}$
</td><td>1

2</td><td>1

1

1</td></tr>
<tr><td>b)</td><td colspan="2">
i)
$l = r\theta$
$40 = 50\theta$
$\theta = \frac{40}{50}$
$\theta = 0.8 radians$

ii)
$a^2 = b^2 + c^2 - 2bc\cos A$
$a^2 = 50^2 + 50^2 - 2\times 50\times 50\times \cos\frac{4}{5}$
$a = 38.94 cm$
</td><td>1

2</td><td>1

1
1</td></tr>
<tr><td>c)</td><td colspan="2">
i)
$N = 10000e^{0.05t}$
$N = 10000e^{0.05\times 10}$
$N = 16487$

ii)
$100000 = 10000e^{0.05t}$
$10 = e^{0.05t}$
$\ln 10 = \ln(e^{0.05t})$
$\ln 10 = 0.05t(\ln e)$
$t = \frac{\ln 10}{0.05}$
$= 46 hours$

iii)
$N' = 500e^{0.05t}$
$when\ t = 24 hours$
$N' = 500e^{0.05\times 24}$
$= 1660$ bacteria/hour
</td><td>1

2

1</td><td>1

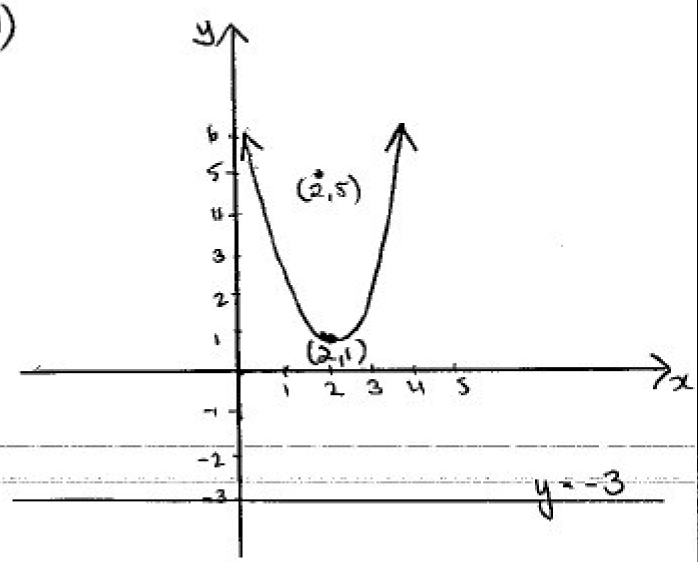
1

1

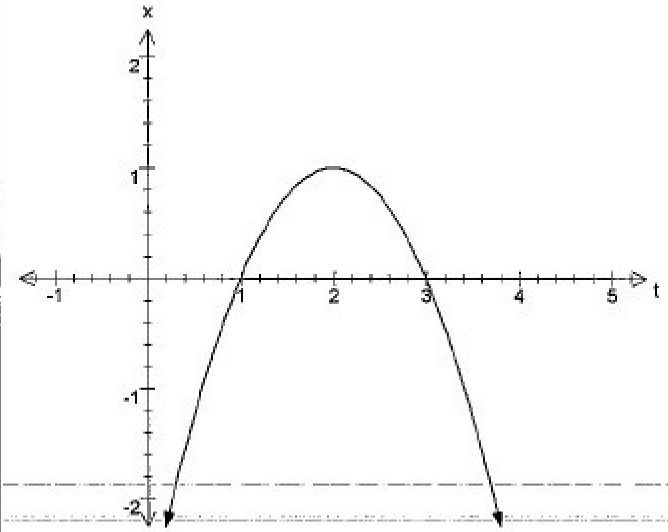
1</td></tr>
</table>

Question 7	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
d)	$y^2 = 3 - 2x - x^2$ $V = \pi\int_a^b y^2 dx$ $V = \pi\int_{-3}^{1} 3 - 2x - x^2 dx$ $= \pi\left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^{1}$ $= \pi\left[\left(3 - 1 - \frac{1}{3}\right) - (-9 - 9 + 9)\right]$ $= \frac{32\pi}{3}$	2	1 1
		/12	

Question 8	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	i)	2	
	$r = 9 \div 100 \div 12$		
	$r = 0.0075$		
	$A = P(1+r)^n$		
	$A_1 = 500000(1.0075)^1 - R$		1
	$A_2 = A_1(1.0075)^1 - R$		
	$A_2 = 500000(1.0075)^2 - R(1.0075) - R$		
	$A_3 = A_2(1.0075)^1 - R$		
	$A_3 = 500000\left[(1.0075)^2 - R(1.0075) - R\right](1.0075) - R$		
	$= 500000(1.0075)^3 - R(1.0075)^2 - R(1.0075) - R$		
	$= 500000(1.0075)^3 - R\left[1 + 1.0075 + 1.0075^2\right]$		1
	as required		
	ii)		
	$A_n = 0$ as all money is repaid	2	
	$\therefore 0 = 500000(1.0075)^{300} -$		
	$R\left[1 + 1.0075 + 1.0075^2 + ... + 1.0075^{n-1}\right]$		1
	$R = \dfrac{500000(1.0075)^{300}}{\left[1 + 1.0075 + 1.0075^2 + ... + 1.0075^{n-1}\right]}$		
	geometric series with $a = 1, r = 1.0075, n = 300$		
	$S_n = \dfrac{a\left(r^n - 1\right)}{r - 1}$		
	$S_{300} = \dfrac{1\left(1.0075^{300} - 1\right)}{1.0075 - 1}$		
	$S_{300} \approx 1121.121937$		
	$R = \dfrac{500000(1.0075)^{300}}{S_{300}}$		
	$R = \$4195.98$		1
	iii)		
	Total repaid $= \$4195.98 \times 300$	1	
	$= \$1258794.00$		
	$Interest = \$1258794 - 500000$		
	$= \$758794.00$		1

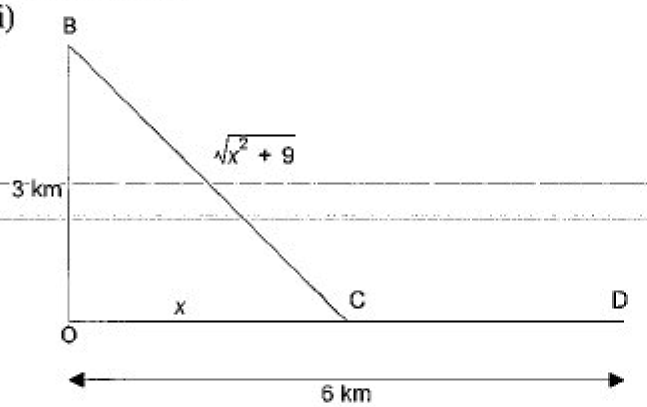
Question 8	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	iv) $SI = \Pr n$ $758794 = 500000 \times r \times 25$ $r = 6.07\%$	1	1
b)	1)  ii) $(x-2)^2 = 4.4(y-1)$ $= 16(y-1)$	2. 1 1	1 correct shape 2 correct vertex
c)	i) $f(x) = 4 - 2^{-x}$ $f(x^2) = 4 - 2^{-x^2}$ ii) $\left[f(x)\right]^2 = \left[4 - 2^{-x}\right] \times \left[4 - 2^{-x}\right]$ $= 16 - 2^3 \times 2^{-x} + (2^{-x})^2$ $= 16 - 2^{3-x} + 2^{-2x}$ iii) $f(-x) = 4 - 2^{-(-x)}$ $= 4 - 2^x$ $\neq f(x) or - f(x)$ $\therefore$ the function is neither odd nor even	1 1 1	1 1 1
		/12	

Question 9	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	i)	2	
	$a=-2 \quad x=-3 \quad v=4ms^{-1}$		
	$a=-2$		
	$v=\int -2dt$		
	$=-2t+c$		
	$when\ t=0 \quad v=4$		
	$4=-2\times 0+c$		
	$c=4$		
	$\therefore v=-2t+4$		1
	$x=\int -2t+4dt$		
	$=-t^2+4t+c$		
	$when\ t=0 \quad x=-3$		
	$-3=0+0=c$		
	$c=-3$		
	$\therefore x=-t^2+4t-3$		1
	ii)		
	Particle at rest when v=0	2	1
	$v=-2t+4$		
	$0=-2t+4$		
	$2t=4$		
	$t=2\sec onds$		1
	$\therefore$ *particle* at rest when $t=2\sec onds$		
	iii)		
	Particle at the origin when $x=0$	2	
	$x=-t^2+4t-3$		
	$0=-t^2+4t-3$		
	$0=-(t^2-4t+3)$		1
	$0=-(t-3)(t-1)$		
	$\therefore t=1\ or\ \ 3\sec onds$		1
	particle first at the origin when $t=1\sec ond$		

Question 9	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	iv) 	2	1 for correct shape 1 for correct intercepts
b)	i) $y=3^{4x-2}$ $4x-2=\log_3 y$ $4x-2=\dfrac{\ln y}{\ln 3}$ $x=\dfrac{\frac{\ln y}{\ln 3}+2}{4}$ $x=\dfrac{\ln y}{4\ln 3}+\dfrac{1}{2}$ $\dfrac{dx}{dy}=\dfrac{1}{4\ln 3}\times\dfrac{1}{y}$ $=\dfrac{1}{4\ln 3y}$ $\dfrac{dy}{dx}=4\ln 3y$ $=4\ln 3(3^{4x-2})$	3	1 1 1
		/12	

Question 9	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
	ii) $\int 3^{4x-2}dx$ $=\dfrac{1}{4\ln 3}\int 4\ln 3(3^{4x-2})$ $=\dfrac{1}{4\ln 3}\times(3^{4x-2})+c$ $=\dfrac{(3^{4x-2})}{4\ln 3}+c$	1	1
		/12	

Question 10	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	i) $Q(t)=2000(25-t)^2, t \geq 0$ $Q'(t)=-4000(25-t)$ $\therefore$ it is emptying at a rate of $4000(25-t)$ litres/minute	1	 1
	ii) Pool full at $t=0$ $Q(t)=2000(25-0)^2$ $=1250000 litres$ $\therefore$ half full = 625000 litres $625000=2000(25-t)^2$ $312.5=625-50t+t^2$ $t^2-50t+312.5=0$ $2t^2-100t+625=0$ $t=\dfrac{--100\pm\sqrt{100^2-4\times2\times625}}{2\times2}$ $t=\dfrac{100\pm\sqrt{5000}}{4}$ $t=\dfrac{100\pm50\sqrt{2}}{4}$ $t=\dfrac{2(50\pm25\sqrt{2})}{4}$ $t=\dfrac{50\pm25\sqrt{2}}{2}$ $t=7.322$ or 42.68 $\therefore t=7$ minutes $\therefore$ it will take $\approx$ 7 minutes to half empty the pool	2	 1 1
	iii) $20kL=20000L/\min$ $20000=-4000(25-t)$ $20000=-100000+4000t$ $4000t=120000$ $t=30\min s$ $\therefore$ the flow rate will be 20kL after 30 minutes	2	 1 1

Question 10	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
a)	iv) when $t=10$ $Q(t)=2000(25-10)^2$ $=2000\times 225$ $=450000L$ left in the pool when $t=0$ $Q(t)=1250000L$ $Average=\dfrac{(1250000-450000)}{10}$ $=80000L/\min$	2	 1 1
b)	i)  ii) Using Pythagoras and $S=\dfrac{D}{T}$ $\therefore$ he paddles a distance of $\sqrt{x^2+9}$ at 4km/h $\therefore Paddles$ - $\dfrac{\sqrt{x^2+9}}{4}hours$ $\therefore$ he walks a distance of $6-x$ at 5km/h $\therefore Walks$ - $\dfrac{6-x}{5}hours$ The total time $T(x)=\dfrac{\sqrt{x^2+9}}{4}+\dfrac{6-x}{5}$	1 2	1 1 1

Question 10	Trial HSC Examination - Mathematics		2010
Part	**Solution**	**Marks**	**Comment**
b)	iii)		
	$T(x)=\dfrac{\sqrt{x^2+9}}{4}+\dfrac{6-x}{5}$	2	
	$T'(x)=\dfrac{x}{4\sqrt{x^2+9}}-\dfrac{1}{5}$		1
	$=\dfrac{5x-4\sqrt{x^2+9}}{20\sqrt{x^2+9}}$		
	Min when $T'(x)=0$		
	$0=\dfrac{5x-4\sqrt{x^2+9}}{20\sqrt{x^2+9}}$		
	$0=5x-4\sqrt{x^2+9}$		
	$5x=4\sqrt{x^2+9}$ (square both sides)		
	$25x^2=16x^2+144$		
	$9x^2=144$		
	$x^2=16$		
	$x=\pm 4\ (x\neq -4)$		
	$\therefore x=4$		
	check min*imum*		
	when $x<4, T'(x)<0$		
	when $x>4, T'(x)>0$		
	$\therefore$ min*imum* at $x=4$		
	$\therefore$ Adam paddles to C - 4 kilometres from O		
	$T(x)=\dfrac{\sqrt{x^2+9}}{4}+\dfrac{6-x}{5}$		
	$T(4)=\dfrac{\sqrt{4^2+9}}{4}+\dfrac{6-4}{5}$		
	$=1.65\,hours$		
	$=1\,hour\ \&\ 39\min s$		1
		/12	