International Grammar School

Year 12
Mathematics
Trial HSC Examination
2015

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page of this question paper
- In Questions 11 - 16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

90 marks

- Attempt Questions 11-16
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 2 hours 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

BLANK PAGE

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 The period and amplitude of $y = 3\cos 2x$ is:

(A) Amplitude = 2, Period = $\frac{2\pi}{3}$

(B) Amplitude = 3, Period = π

(C) Amplitude = π , Period = 3

(D) Amplitude = $\frac{2\pi}{3}$, Period = 2

2 What is the solution to the equation $2\cos^2 x - 1 = 0$ in the domain $0 \le x \le 2\pi$?

(A) $x = \frac{\pi}{6}, \frac{11\pi}{6}$

(B) $x = \frac{\pi}{4}, \frac{7\pi}{4}$

(C) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(D) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

3 An infinite geometric series has a first term of 3 and a limiting sum of 1.8. What is the common ratio?

(A) $-0.\dot{3}$

(B) $-0.\dot{6}$

(C) -1.5

(D) -3.75

4 Evaluate the $\int_2^7 \frac{5}{x}\,dx$.

(A) $5(\ln 7 - \ln 2)$

(B) $\frac{1}{5}(\ln 7 - \ln 2)$

(C) $\frac{5}{49} - \frac{5}{4}$

(D) 0

5 Let α and β be the solution of $2x^2 - 5x - 9 = 0$. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$

(A) $-\frac{9}{2}$

(B) $-\frac{9}{5}$

(C) $-\frac{5}{9}$

(D) $\frac{5}{2}$

6 The equation of the normal to the curve $x^2 = 4y$ at the point where $x = 2$ is:

(A) $y = 1$

(B) $x - y - 1 = 0$

(C) $y = -1$

(D) $y + x - 3 = 0$

7 The acceleration of a particle moving in a straight line is given by the formula $a = 12t + 6$ with an initial velocity of –36 m/s. When is the particle at rest?

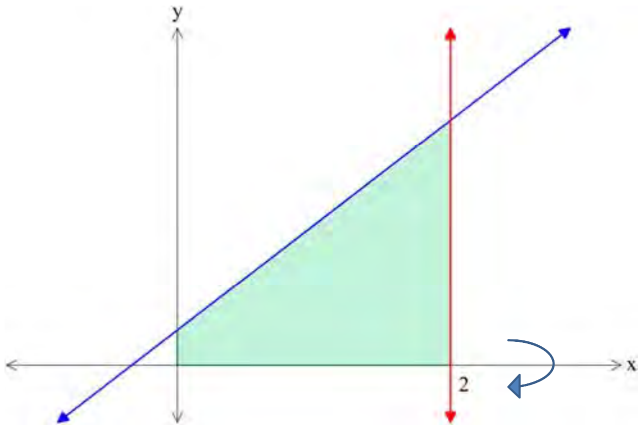
(A) $t = 0$

(B) $t = 1$

(C) $t = 2$

(D) $t = 3$

8 A region in the first quadrant is bounded by the line $y = 3x + 1$, the x-axis, the y-axis, and the line $x = 2$.



What is the volume of the solid of revolution formed when this region is rotated about the x-axis?

(A) 8 units^3

(B) 38 units^3

(C) $8\pi \text{ units}^3$

(D) $38\pi \text{ units}^3$

9 What are the coordinates of the focus of the parabola $x^2 - 2x = 6y + 11$

(A) $\left(-\frac{3}{2}, 1\right)$

(B) $\left(-\frac{1}{2}, 1\right)$

(C) $\left(1, -\frac{3}{2}\right)$

(D) $\left(1, -\frac{1}{2}\right)$

10 What is an expression for $[f(x)]^2$ if $f(x) = 4 + 2^{-x}$?

(A) $16 + 2^{3-x} + 2^{-2x}$

(B) $16 + 2^{2-x} + 2^{-2x}$

(C) $17 + 2^{3-x}$

(D) $17 + 2^{2-x}$

END OF SECTION I

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Simplify $\frac{y}{y^2-4}-\frac{2}{y-2}$ 2

(b) Rationalise the denominator $\frac{\sqrt{2}}{\sqrt{7}+3}$ 2

(c) Evaluate $\int_0^{\frac{\pi}{6}}(x^2+\sin 2x)dx$ 2

(d) For the arithmetic sequence 4, 9, 14, 19, ….

(i) Write the rule to describe the nth term. 1

(ii) What is the 100^{th} term? 1

(iii) Find the sum of the first 100 terms. 1

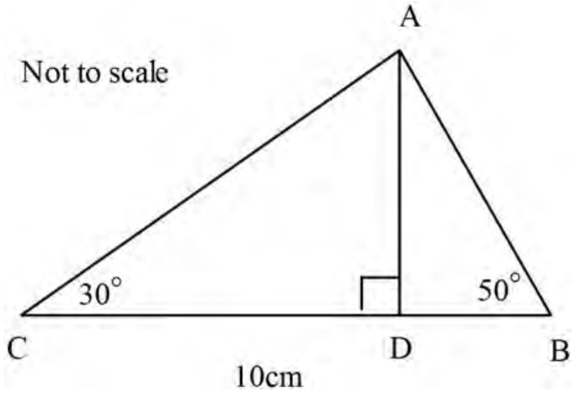
(e) Differentiate

(i) $\tan 5x$ 1

(ii) $\frac{\log_e x}{x}$ 1

(iii) $x\cos x$ 1

(f) In the triangle ABC, $\angle ACB = 30^0$, $\angle ABC = 50^0$ and BC = 10 cm.
The foot of the perpendicular from A to BC is D.



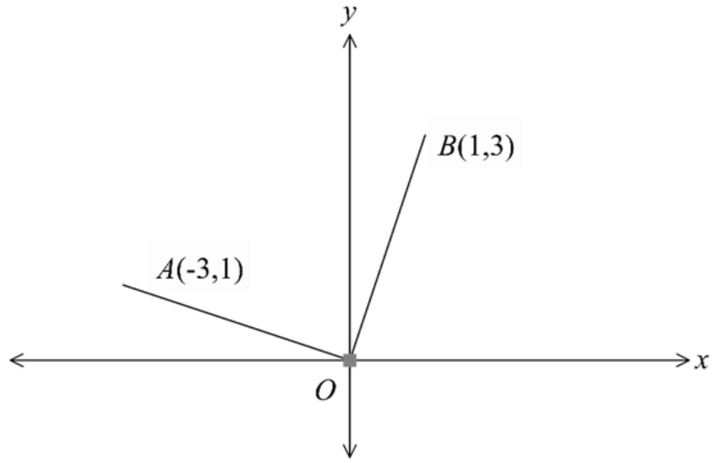
(i) Use the Sine Rule to find the length of AB correct to 2 decimal places. 2

(ii) Hence or otherwise, find the length of AD. 1
Answer correct to two decimal places.

End of Question 11

Question 12 (15marks) **Marks**

(a) Points $A(-3,1)$ and $B(1,3)$ are on a number plane.



(i) Find the gradient of line OA. 1

(ii) Show that OA is perpendicular to OB. 1

(iii) $OACB$ is a quadrilateral in which BC is parallel to OA. 2
Show that the equation of BC is $x+3y-10=0$.

(iv) The point C lies on the line $x=-2$. 1
What are the coordinates of point C?

(v) Show that the length of the line BC is $\sqrt{10}$. 1

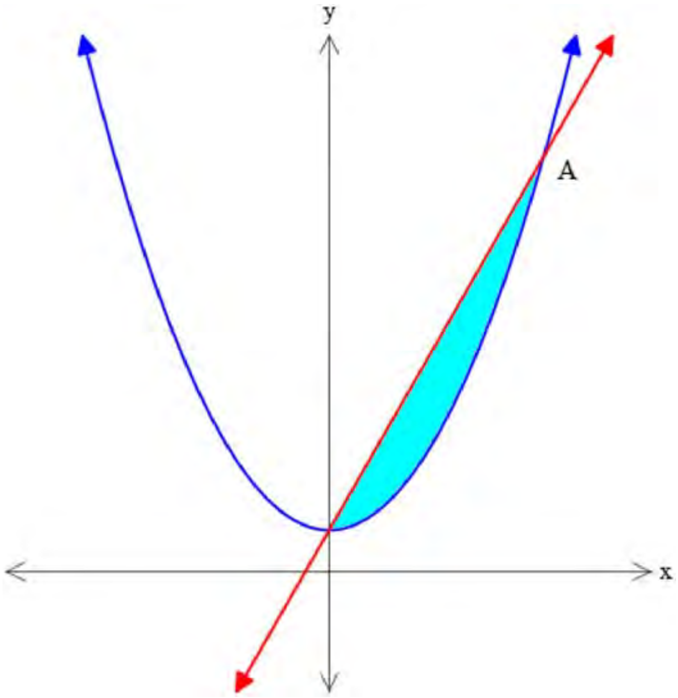
(vi) Find the area of $OACB$. 1

(b) The table shows the values of a function $f(x)$ for five values of x. 2

x	1	1.5	2	2.5	3
$f(x)$	4	1.5	–2	2.5	8

Use Simpson's rule with these five vales to estimate $\int_1^3 f(x)dx$.

(c) The parabola $y = x^2 + 1$ and the line $y = 3x + 1$ intersect at the point A



(i) Find the x-coordinate of point A 1

(ii) What is the area enclosed between the curves 2

(d) Evaluate $\log_6 9 + \log_6 24$ 1

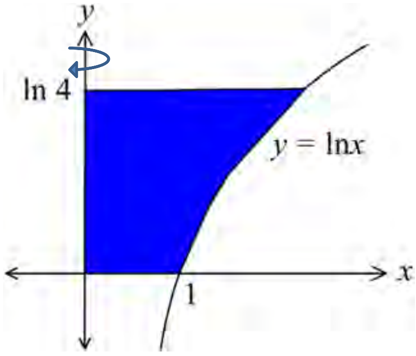
(e) Solve $4^{x+1} = 9$ correct to 2 decimal places 2

End of Question 12

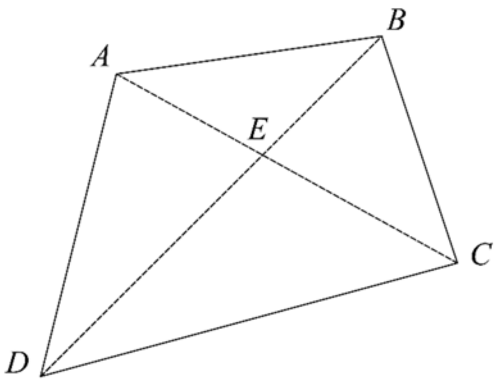
Question 13 (15 marks) Marks

(a) In the diagram, the shaded region bounded by the curve $y = \ln x$, the coordinate axes and the line $y = \ln 4$, is rotated about the y-axis. 3

Find the exact volume of the solid of revolution.



(b) In quadrilateral $ABCD$ the diagonals AC and BD intersect at E.
$AE = 3$, $CE = 6$, $BE = 4$, $ED = 8$.



Copy or trace the diagram into your booklet.

(i) Show that $\Delta ABE \parallel\!| \Delta CDE$ 3

(ii) What type of quadrilateral is $ABCD$? Justify your answer. 2

(c) The parabola $y = ax^2 + bx + c$ has a vertex at (3, 1) and passes through (0, 0).

(i) Find the other x-intercept of the parabola. 1

(ii) Find a, b and c. 3

(d) John plays computer games competitively. Everytime he plays, John has a 0.8 chance of winning a game of *Beastie* and a 0.6 chance of winning a game of *Dragonfire*. In one afternoon of competition he plays one game of *Beastie* and one of *Dragonfire*.

(i) What is the probability that he will win both games? 1

(ii) What is the probability that he will win exactly one game? 1

(iii) What is the probability that he will win at least one game? 1

End of Question 13

Question 14 (15 marks) **Marks**

(a) A function $f(x)$ is defined by $f(x)=x^2(3-x)$.

(i) Find the stationary points for the curve $y=f(x)$ and determine their nature. 3

(ii) Sketch the graph of $y=f(x)$ showing the stationary points and x-intercepts. 2

(b) The displacement of an object at time (t) seconds is given by: 3

$$x=3e^{2t}-4e^{t}-10t$$

Find the time the object comes to rest.

(c) The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term? 2

(d) At the start of the year John puts \$10 into his personal safe which earns no interest. At the start of each month, John puts \$5 more then the previous month (ie. \$15 in the second month, \$20 in the third month...)

At the same time, Henry invests *M* dollars into a new saving account and deposits *M* dollars at the start of each following month. The money in the account earns interest at the rate of 0.4% per month compounding monthly.

(i) Show that the John has saved \$3510 after 3 years. 2

(ii) Write an expression involving *M* showing how much Henry saved after 2 months 1

(iii) If the total money saved by Henry and John is \$18035 after 3 years, find the value of *M* to the nearest dollar. 2

End of Question 14

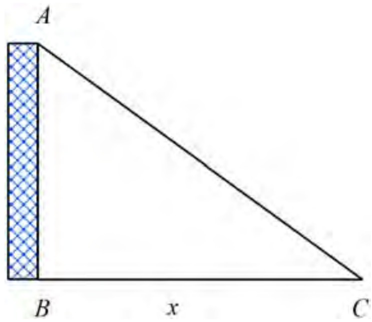
Question 15 (15 marks) **Marks**

(a) (i) On the same number plane graph: 2
$y = \sin x$ and $y = \cos 2x$ for $0 \le x \le \pi$

(ii) Show that the curves intersect at $A = \frac{\pi}{6}$ *and* $B = \frac{5\pi}{6}$. 1

(iii) Hence find the area bounded by 2
$y = \sin x$ and $y = \cos 2x$ for $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$.

(b)



3 metres of fencing is being used to form a triangular garden against an existing wall. Let the length of the base BC be x metres.

(i) Show that the area of the triangle ABC is $0.5x\sqrt{9-6x}$. 2

(ii) What value of x gives the maximum possible area of the triangle? 3

(c) The radiation in a rock after a nuclear accident was 8,000 becquerel (bq). One year later, the radiation in the rock was 7,000 bq. It is known that the radiation in the rock is given by the formula:

$$R = R_0 e^{-kt}.$$

where R_0 and k are constants and t is the time measured in years.

(i) Evaluate the constants R_0 and k. 2

(ii) What is the radiation of the rock after 10 years? 1
Answer correct to the nearest whole number.

(iii) The region will become safe when the radiation of the rock reaches 50 bq. After how many years will the region become safe? 2

End of question 15

Question 16 (15 marks) **Marks**

(a) An object is moving in a straight line and its velocity is given by;

$$v = 1 - 2\sin 2t \text{ for } t \geq 0$$

where v is measured in metres per second and t in seconds.
Initially the object is at the origin.

(i) Find the displacement x, as a function of t. 2

(ii) What is the position of the object when $t = \frac{\pi}{3}$? 1

(iii) Find the acceleration a, as a function of t. 1

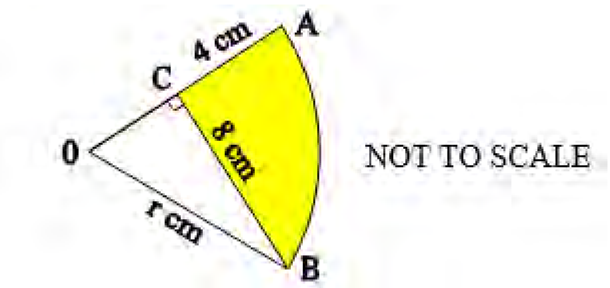
(iv) Sketch the graph of a, as a function of t, for $0 \leq t \leq \pi$. 1

(v) What is the maximum acceleration of the object? 1

(b) (i) Prove that $\frac{1}{1+\sin 3\theta} + \frac{1}{1-\sin 3\theta} = 2\sec^2 3\theta$. 2

(ii) Hence find $\int \left(\frac{4}{1+\sin 2\theta} + \frac{4}{1-\sin 2\theta} \right) d\theta$ 2

(c) The diagram shows the sector *OAB* with centre 0 and radius r.
The perpendicular from B meets the radius *OA* at *C*, *BC* = 8 cm and *AC* = 4 cm.



(i) Show that the radius r of the circle is 10 cm. 2

(ii) Find the area of the shaded region in this sector. 3

End of examination

BLANK PAGE

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \;, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

2015 Year 12 Trial HSC Examination **Student Name: . . . .**

Mathematics

Section I Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

Trial HSC 2015

Multiple choice

1. $y = 3\cos 2x$.

amplitude $= 3$

period $\frac{2\pi}{2} = \pi$

B

2. $2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

D

3. $a = 3$

$S = 1.8$

$S = \frac{a}{1-r}$

$1 - r = \frac{a}{S}$

$r = 1 - \frac{a}{S}$

$= 1 - \frac{3}{1.8}$

$= -0.\dot{6}$

B

4. $\int_{2}^{7} \frac{5}{x}\, dx$

$= 5 \ln x \Big|_{2}^{7}$

$= 5(\ln 7 - \ln 2)$

A

5. $2x^2 - 5x - 9 = 0$

$\alpha + \beta = -\frac{b}{a} = \frac{5}{2}$

$\alpha\beta = \frac{c}{a} = \frac{-9}{2}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{5}{2} \div -\frac{9}{2}$

$= \frac{5}{2} \times -\frac{2}{9}$

$= -\frac{5}{9}$

C

6. $x^2 = 4y.$

$y = \frac{x^2}{4}$

$y' = \frac{2x}{4} = \frac{x}{2}$

$x = 2 \rightarrow y = \frac{4}{4} = 1$

$y' = \frac{2}{2} = 1$

normal $m_2 = -1$

$y - 1 = -1(x - 2)$

$y - 1 = -x + 2$

$y + x - 3 = 0$

D

7. $a = 12t + 6.$

$v = 6t^2 + 6t + c.$

$t = 0 \rightarrow v = -36$

$-36 = c.$

$v = 6t^2 + 6t - 36$

$= 6(t^2 + t - 6)$

$= 6(t+3)(t-2)$

$v = 0 \rightarrow t = -3 < 0$

$t = 2.$

C

8. $y = 3x + 1$

$V = \pi \int_0^2 (3x+1)^2 \, dx$

$= \pi \int_0^2 (9x^2 + 6x + 1) \, dx$

$= \pi \left| 3x^3 + 3x^2 + x \right|_0^2$

$= \pi \left| 3 \times 2^3 + 3 \times 2^2 + 2 \right|$

$= 38\pi$ unit^3

D

9. $x^2 - 2x = 6y + 11.$

$(x-1)^2 = 6y + 12$

$(x-1)^2 = 6(y+2)$

$(x-1)^2 = 4 \times \frac{3}{2}(y+2)$

Vertex $(1, -2)$

Focal length : $\frac{3}{2}$

Focus $(1, -\frac{1}{2})$

D

10. $[f(x)]^2 = [4+2^{-x}]^2$

$= 16 + 8 \times 2^{-x} + 2^{-2x}.$

$= 16 + 2^3 \times 2^{x} + 2^{-2x}$

$= 16 + 2^{3-x} + 2^{-2x}.$

A

Question 11

11.

a.

$$\frac{y}{y^2-4} - \frac{2}{y-2}$$

$$= \frac{y}{(y-2)(y+2)} - \frac{2}{y-2}$$ ✓

1 mark finds a common denominator

$$= \frac{y-2(y+2)}{(y-2)(y+2)}$$

$$= \frac{y-2y-4}{y^2-4}$$

$$= \frac{-y-4}{y^2-4}.$$ ✓

1 mark – correct answer

Question 11

b $\dfrac{\sqrt{2}}{\sqrt{7}+3} \times \dfrac{\sqrt{7}-3}{\sqrt{7}-3}$ ✓

$= \dfrac{\sqrt{14} - 3\sqrt{2}}{7-9}$

$= \dfrac{\sqrt{14} - 3\sqrt{2}}{-2}$ ✓

1 mark.

1 mark - correct answer

Questioon 11

c. $\displaystyle\int_0^{\pi/6} (x^2 + \sin 2x\, dx.$

$= \dfrac{x^3}{3} - \dfrac{1}{2}\cos 2x \Big|_0^{\pi/6}$ ✓ | 1 mark - Find the primitive function

$= \left(\dfrac{(\pi/6)^3}{3} - \dfrac{1}{2}\cos 2\dfrac{\pi}{6}\right) - \left(0 - \dfrac{1}{2}\cos 0\right)$

$= \dfrac{\pi^3}{648} - \dfrac{1}{4} + \dfrac{1}{2}$

$= \dfrac{\pi^3}{648} + \dfrac{1}{4}$

$= 0.2978$ ✓ | 1 mark - correct answer

Question 11

d. 4, 9, 14, 19.

i.

$a = 4$

$d = 5$

$T_n = a + (n-1)d$

$= 4 + nd - d$

$= 4 + 5n - 5$

$= 5n - 1$ ✓ | 1 mark _ correct answer.

ii. $T = 5 \times 100 - 1$

$= 499$ ✓ | 1 mark

iii. $S_{100} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{100}{2}[2 \times 5 + (100-1) \times 5]$

$= 25150$ ✓ | 1 mark

Question 11

e.

i. $y = \tan 5x$

$y' = 5\sec^2 5x$

1 mark - correct answer

ii. $y = \dfrac{\log_e x}{x}$

$u = \log_e x \quad , \quad u' = \dfrac{1}{x}$

$v = x \quad , \quad v' = 1$

$y' = \dfrac{u'v - v'u}{v^2}$

$= \dfrac{\frac{1}{x} \times x - \log_e x}{x^2}$

$= \dfrac{1 - \log_e x}{x^2}$

1 mark

iii. $y = x \cos x$

$u = x \quad , \quad u' = 1$

$v = \cos x \quad , \quad v' = -\sin x$

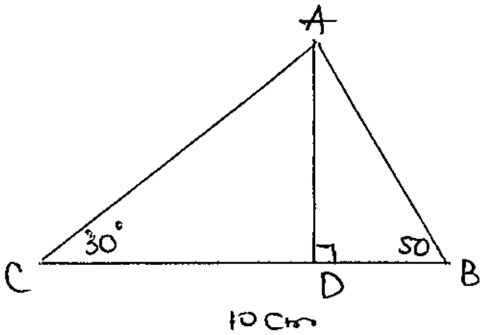
$y' = u'v + v'u$

$= \cos x - x \sin x$

1 mark.

Question 11

f.



$\angle BAC = 180 - 30 - 50 = 100°$ ✓

i. $\frac{AB}{\sin 30} = \frac{BC}{\sin 100}$

$AB = \frac{BC \times \sin 30}{\sin 100}$

$= \frac{10 \times \sin 30}{\sin 100}$ ✓

ii. $\sin 50 = \frac{AD}{AB}$

$AD = AB \times \sin 50$

$= \frac{10 \times \sin 30}{\sin 100} \times \sin 50$ ✓

$\simeq 3.89$ cm

– 1 mark to find angle BAC or using sine rule with correct values

– 1 mark – correct answer

1 mark – correct answer

Question 12

a.

i. $A(-3,1)$, $B(1,3)$, $O(0,0)$

$$m_{OA} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-1}{0+3} = -\frac{1}{3} \checkmark$$

1 mark – correct answer

ii.

$$m_{OB} = \frac{0-3}{0-1} = 3.$$

$$m_{OA} \times m_{OB} = -\frac{1}{3} \times 3 = -1$$

$\therefore OA \perp OB.$ ✓

1 mark

iii. $BC \parallel OA$

$$m_{BC} = m_{OA} = -\frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{3}(x-1) \checkmark$$

$$3y - 9 = -x + 1$$

$$x + 3y - 10 = 0 \checkmark$$

1 mark – uses the gradient intercept form

1 mark – correct answer

iv. $x = -2.$

Sub $x=-2$ into $x + 3y - 10 = 0$

$$-2 + 3y - 10 = 0$$

$$3y = 12$$

$$y = 4.$$

$\therefore C(-2,4)$ ✓

1 mark – correct answer

v.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$BC = \sqrt{(-2-1)^2 + (4-3)^2} = \sqrt{10} \checkmark$$

1 mark

vi. $OA = OB = \sqrt{10} = BC.$

$OA \perp OB.$

$\therefore OACB$ is a square.

$A = S \times S = \sqrt{10} \times \sqrt{10} = 10$ square unit. ✓

1 mark.

Question 12

b. $$\int_1^3 f(x)\,dx = \frac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2y_2\right]$$

$$= \frac{0.5}{3}\left[4 + 8 + 4(1.5 + 2.5) + 2\times(-2)\right]$$

$$= 4.$$

1 mark – uses Simpson's rule

1 mark – correct answer.

Question 12

e. i. $y = x^2 + 1$

$y = 3x + 1$

points of intersection

$x^2 + 1 = 3x + 1$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$

$x = 3.$

$\therefore$ The x coordinate of A is 3 ✓

1 mark - find x coordinate of A

ii. $\int_0^3 (3x+1) - (x^2+1)\, dx.$

$= \int_0^3 (3x - x^2)\, dx$

$= \frac{3x^2}{2} - \frac{x^3}{3} \Big|_0^3$ ✓

1 mark - Find the primitive function

$= \frac{3 \times 3^2}{3} - \frac{3^3}{3}$

$= \frac{9}{2}$ Square units ✓

1 mark - correct answer

Question 12

d.

$\log_6 9 + \log_6 24$

$= \log_6 (9 \times 24)$

$= \log_6 (216)$

$= \log_6 6^3$ ✓

$= 3$

1 mark – correct answer

e. $4^{x+1} = 9.$

$\log 4^{x+1} = \log 9$ ✓

1 mark – using log.

$(x+1)\log 4 = \log 9$

$x+1 = \dfrac{\log 9}{\log 4}$

$x+1 = 1.58$

$x = 0.58$ ✓

1 mark – correct answer

Question 13

a.

$y = \ln x$

$e^y = \ln e^x$

$e^y = x \ln e$

$x = e^y$

$V = \pi \int_0^{\ln 4} (e^y)^2 \, dy$ ✓

$= \pi \int_0^{\ln 4} e^{2y} \, dy$

$= \pi \left[\frac{1}{2} e^{2y} \right]_0^{\ln 4}$ ✓

$= \frac{\pi}{2} \left(e^{2\ln 4} - e^0 \right)$

$= \frac{15\pi}{2}$ cubic units ✓

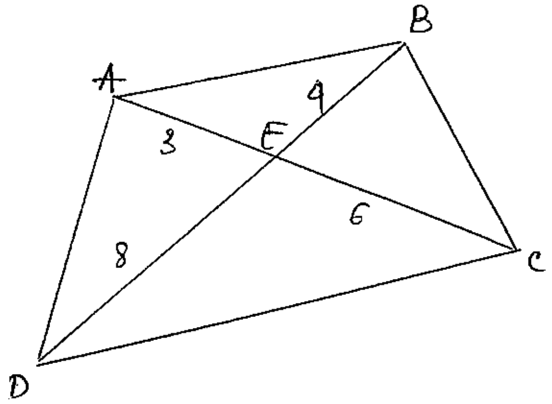
1 mark – uses volume formula

1 mark – find primitive function

1 mark – correct answer

Question 13

b.



i. In ΔABE and ΔCDE

$\frac{AE}{EC} = \frac{3}{6} = \frac{1}{2}$ ✓

$\frac{BE}{DE} = \frac{4}{8} = \frac{1}{2}$ — 1 mark – one relevant statement

$\angle AEB = \angle DEC$ (vertically opposite angles are equal) ✓ — 1 mark.

$\therefore \Delta ABC \ ||| \ \Delta CDE$

(Two pairs of corresponding sides are in proportion and the include angles are equal) ✓ — 1 mark.

ii. $\angle BAE = \angle DCE$ (matching angles in similar triangles) ✓ — 1 mark – relevent statement or show some understanding.

Therefore $\angle BAE$ and $\angle DCE$ are alternate angles and equal

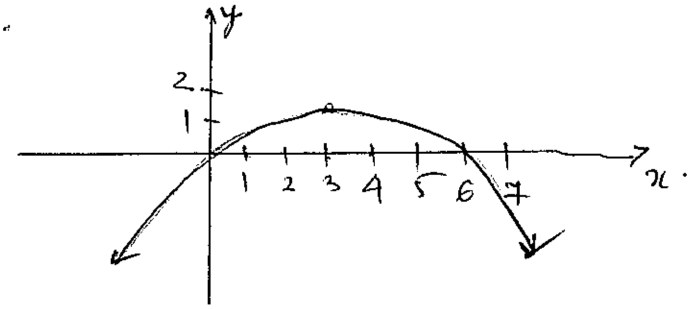
$\therefore AB \parallel CD$

Therefore ABCD is a trapezium. ✓ — 1 mark

Question 13.

c.

i.



- The parabola is symmetrical about the vertex of (3,1)
- if the parabola passes through the origin it is concave down and the other x intercept is

(6,0)

1 mark - correct answers.

ii. For (0,0)

$y = ax^2 + bx + c$

$0 = c.$

For (6,0)

$y = ax^2 + bx + c$

$0 = 36a + 6b.$

For (3,1)

$y = ax^2 + bx + c.$

$1 = 9a + 3b$

$\begin{cases} 36a + 6b = 0 \\ 9a + 3b = 1 \quad \times(2) \end{cases}$

$\begin{cases} 36a + 6b = 0 \quad (1) \\ 18a + 6b = 2 \quad (2) \end{cases}$

(1) - (2)

$18a = -2.$

$a = -\frac{1}{9}$ ✓

sub $a = -\frac{1}{9}$ into (2)

$18 \times -\frac{1}{9} + 6b = 2$ ✓

$b = \frac{2}{3}$

$\therefore a = -\frac{1}{9},\ b = \frac{2}{3},\ c = 0$ ✓

1 mark - finds the value of a or shows some understanding

1 mark - finds value of b

1 mark. finds value of c.

Question 13

d.

i. $P(WW) = 0.8 \times 0.6$

$= 0.48$ ✓ | 1 mark – correct answer

ii. $P(\text{exactly 1 game}) = 0.8 \times 0.4 + 0.2 \times 0.6$

$= 0.44$ ✓ | 1 mark

iii $P(\text{atleast 1 game}) = 1 - P(LL)$

$= 1 - 0.2 \times 0.4$

$= 0.92$ ✓ | 1 mark

Question 14.

a. i. $f(x) = x^2(3-x)$

$= 3x^2 - x^3$

For stationary points

$f'(x) = 0$.

$f'(x) = 6x - 3x^2$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

- $x = 0 \rightarrow y = 0$
- $x = 2 \rightarrow y = 4$.

$\therefore (0,0), (2,4)$

At $(0,0)$, $f''(0) = 6 - 6x$

$= 6 > 0$, Min

At $(2,4)$, $f''(2) = 6 - 12$

$= -6 < 0$, Max.

1 mark – stationary points

1 mark – max

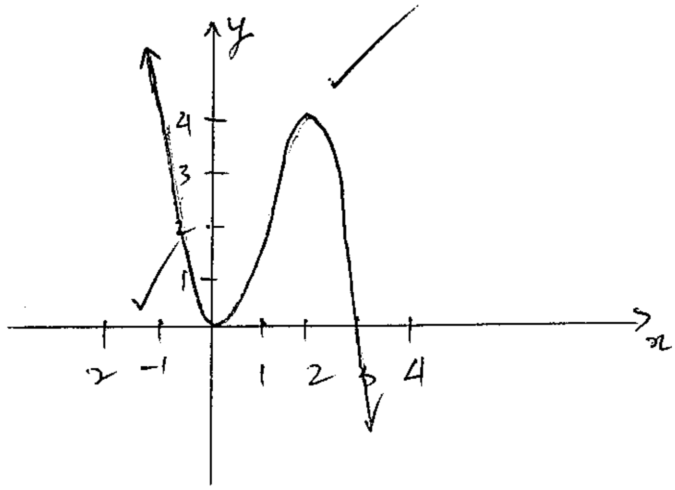
1 mark – min

ii. x intercept

$y = 0 \rightarrow 3x^2 - x^3 = 0$

$x^2(3-x) = 0$

$x = 0$ and $x = 3$



1 mark – shows max or min

1 mark – correct shape.

Question 14

b.

$x = 3e^{2t} - 4e^{t} - 10t$

$v = 6e^{2t} - 4e^{t} - 10$ ✓

$= 2(3e^{2t} - 2e^{t} - 5)$

The object comes to rest when $v = 0$

$2(3e^{2t} - 2e^{t} - 5) = 0$

Let $e^{t} = m$

$2(3m^2 - 2m - 5) = 0$

$2(m+1)(3m-5) = 0$ ✓

- $m + 1 = 0$

  $m = -1$

  $e^{t} = -1$ (no solutions)

- $3m - 5 = 0$

  $m = \frac{5}{3}$

  $e^{t} = \frac{5}{3}$

  $t = \ln\frac{5}{3}$ s

  $t = 0.51$ s ✓

1 mark – find derivative

1 mark – factorising as quadratic.

1 mark – correct answer having excluded $e^{t} = -1$

Question 14

c.

$T_3 = ar^2 = 1.25 \quad (1)$

$T_7 = ar^6 = 20 \quad (2)$

$(2) \div (1)$

$$\frac{ar^6}{ar^2} = \frac{20}{1.25}$$

$r^4 = 16$

$r = \pm 2$ ✓

1 mark – Finds r or two equations using n terms of a GP

$T_3 = ar^2$

$= a \times (\pm 2)^2 = 1.25$

$4a = 1.25$

$a = \frac{1.25}{4}$

$= \frac{5}{16}$ or 0.3125 ✓

1 mark. correct answer

Question 14

d.

i. John's Saving

$\$10 + \$15 + \$20 + \cdots$

AP: $a = 10$

$d = 5$

$n = 3 \times 12 = 36$ ✓

1 mark – finds a, d, n

$S_{36} = \frac{36}{2}(2 \times 10 + 35 \times 5)$

$= \$3510$ ✓

1 mark – correct answer

ii. Henry saved after 1 month

$A_1 = M \times 1.004$

Henry saved after 2 months

$A_2 = M \times 1.004$

So total saved is

$M \times 1.004^2 + M \times 1.004$

$= M(1.004^2 + 1.004)$ ✓

1 mark for sum

iii. After 3 years Henry and John save together \$18035

Henry must save $\$18035 - \$3510 = \$14525$.

Henry's total saving

$M(1.004 + 1.004^2 + 1.004^3 + \cdots + 1.004^{36})$

GP: $a = 1.004$

$r = 1.004$

$n = 36$

So $\$14525 = M \times 1.004 \dfrac{(1.004^{36} - 1)}{1.004 - 1}$ ✓

1 mark – sum using GP correctly.

$= M \times 38.79266$

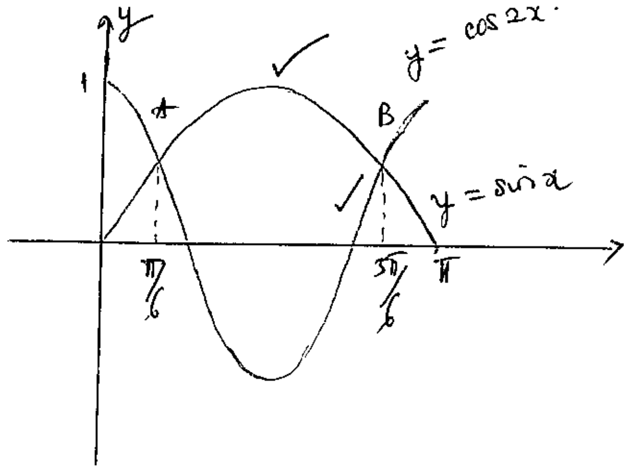
$M = \$374.43$ ✓

1 mark

Question 15

a.

i.



1 mark - y = sin x

1 mark - y = cos 2x.

ii.

$\sin x = \cos 2x.$

- When $x = \frac{\pi}{6}$. $\sin \frac{\pi}{6} = \frac{1}{2}$

$\cos 2\frac{\pi}{6} = \frac{1}{2}$

- When $x = \frac{5\pi}{6}$. $\sin \frac{5\pi}{6} = \frac{1}{2}$

$\cos 2 \times \frac{5\pi}{6} = \frac{1}{2}$

1 mark.

iii. $\int_{\pi/6}^{5\pi/6} (\sin x - \cos 2x)\, dx$

$= -\cos x - \frac{1}{2}\sin 2x \Big|_{\pi/6}^{5\pi/6}$

1 mark

$= \left(-\cos 5\pi/6 - \frac{1}{2}\sin 2 \times \frac{5\pi}{6}\right) - \left(-\cos \pi/6 - \frac{1}{2}\sin 2 \times \frac{\pi}{6}\right)$

$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}\frac{\sqrt{3}}{2}\right)$

$= \frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{4}$

1 mark - correct answer.

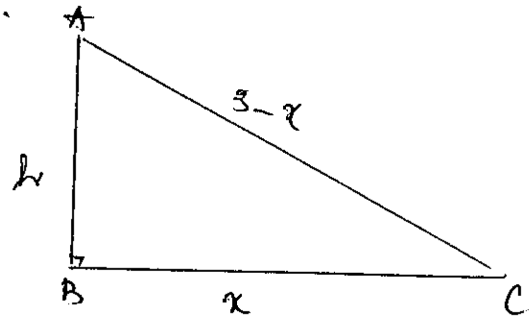
$= \frac{6\sqrt{3}}{4}$ or 2.59808 unit2

$= \frac{3\sqrt{3}}{2}$

Question 15

b.

i.



$h^2 = (3-x)^2 - x^2$

$= 9 - 6x + x^2 - x^2$

$= 9 - 6x$

$h = \sqrt{9-6x}$

1 mark - Find the height of the triangle

$A = \frac{1}{2} b \times h$

$= \frac{1}{2} \times x \times \sqrt{9-6x}$

$= 0.5x \times \sqrt{9-6x}$ m^2

1 mark - correct answer

Question 15

b.

ii. $A = 0.5x\sqrt{9-6x}$

Max occurs when $A' = 0$.

$u = \frac{1}{2}x \rightarrow u' = \frac{1}{2}$

$v = (9-6x)^{1/2} \rightarrow v' = \frac{1 \times -6}{2\sqrt{9-6x}}$

$A' = \frac{\sqrt{9-6x}}{2} + \frac{1}{2}x \times \frac{-6}{2\sqrt{9-6x}}$ ✓ — 1 mark - find the first derivative

$= \frac{\sqrt{9-6x}}{2} - \frac{6x}{4\sqrt{9-6x}}$

$= \frac{2(9-6x)-6x}{4\sqrt{9-6x}}$

$= \frac{18-18x}{4\sqrt{9-6x}}$

$= \frac{9(1-x)}{2\sqrt{9-6x}} = 0$.

✓ — 1 mark.

$\Rightarrow 9(1-x) = 0$

$x = 1$

Test.

$x = 0.9 \rightarrow A' = \frac{9(1-0.9)}{2\sqrt{9-6\times 0.9}} > 0$

$x = 1.1 \rightarrow A' = \frac{9(1-1.1)}{2\sqrt{9-6\times 1.1}} < 0$.

✓

$\therefore$ Max occurs when $x = 1$ — 1 mark - correct answer

Question 15

c.

i. Initially $t=0$, $R=8000$.

$$R=R_0 e^{-kt}$$

$$8000 = R_0 e^{-k0}$$

$$R_0 = 8000.$$ ✓ — 1 mark

. When $t=1$ and $R=7000$.

$$7000 = 8000\, e^{-k}$$

$$e^{-k} = \frac{7000}{8000}$$

$$-k = \log_e \frac{7}{8}$$

$$k = -\log_e \frac{7}{8}$$

$$= 0.1335313 9$$ ✓ — 1 mark

ii. When $t=10$

$$R = 8000\, e^{(-\log_e \frac{7}{8})\times 10}$$

$$= 2104.604609$$

$$\approx 2105 \text{ bq}$$ ✓ — 1 mark

iii When $R=50$.

$$50 = 8000\, e^{-kt}$$

$$e^{-kt} = \frac{50}{8000} = \frac{1}{160}$$

$$-kt = \log_e \frac{1}{160}$$

$$t = -\frac{1}{k} \log_e \frac{1}{160}$$ ✓ — 1 mark - shows some understanding

$$= \log_e \frac{1}{160} \div \log_e \frac{7}{8}$$

$$\approx 38.0073485$$

$$\approx 38 \text{ years.}$$ ✓ — 1 mark - correct answer

Question 16

a.

i. $x = \int (1 - 2\sin 2t)\,dt$

$= t + \cos 2t + C$. ✓ | 1 mark - finds primitive function

initially $t = 0$ and $x = 0$

$0 = 0 + \cos 0 + C$.

$C = -1 \quad (\cos 0 = 1)$

$\therefore \quad x = t + \cos 2t - 1$. ✓ | 1 mark - correct answer

ii. when $t = \frac{\pi}{3}$ then

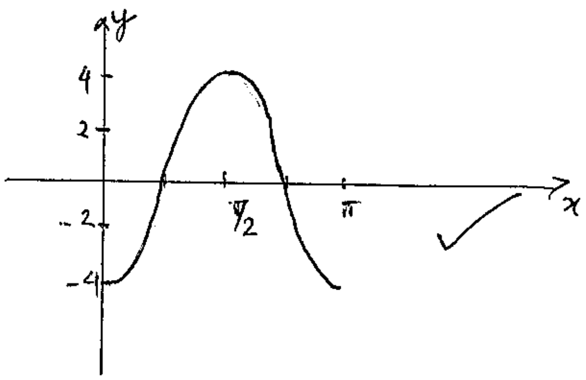
$x = \frac{\pi}{3} + \cos 2 \times \frac{\pi}{3} - 1$

$= \frac{\pi}{3} - \frac{1}{2} - 1$

$= \frac{\pi}{3} - \frac{3}{2}$ ✓ OR -0.4528 | 1 mark

iii. $a = -4\cos 2t$ ✓ | 1 mark.

iv.



✓ | 1 mark

v. $-4 \leq -4\cos 2t \leq 4$

Max acceleration is $4\,ms^{-2}$ ✓ | 1 mark

Question 16

b.

i. $\dfrac{1}{1+\sin 3\theta} + \dfrac{1}{1-\sin 3\theta}$

$= \dfrac{1-\sin 3\theta + 1 + \sin 3\theta}{(1+\sin 3\theta)(1-3\sin 3\theta)}$ ✓ 1 mark

$= \dfrac{2}{1-\sin^2 3\theta}$

$= \dfrac{2}{\cos^2 3\theta}$ 1 mark

$= 2\sec^2\theta$ ✓

ii. $\displaystyle\int\left(\frac{4}{1+\sin 2\theta} + \frac{4}{1-\sin 2\theta}\right) d\theta$

$= 4\displaystyle\int\left(\frac{1}{1+\sin 2\theta} + \frac{1}{1-\sin 2\theta}\right) d\theta$ ✓ 1 mark

$= 4\displaystyle\int 2\sec^2 2\theta \, d\theta$

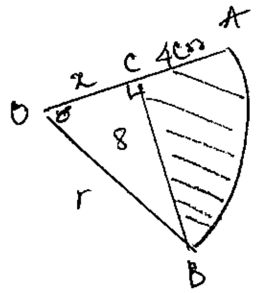
$= 4\tan 2\theta + C$ ✓ 1 mark

Question 16

c.

i. Let $OC = x$

so $r = x + 4$



In ΔOCB

$x^2 + 8^2 = (x+4)^2$ ✓ — 1 mark - Use Pythagoras' theorem

$x^2 + 64 = x^2 + 8x + 16$

$48 = 8x$

$x = 6$

$\therefore \quad r = 6 + 4$

$= 10$ cm ✓ — 1 mark - correct answer

ii.

$\sin\theta = \frac{8}{10}$

$\theta = 0.927295$

- Area of sector $= \frac{1}{2} \times 10^2 \times 0.927295$ ✓ — 1 mark

$= 46.3647 \text{ cm}^2$

- Area of triangle $= \frac{1}{2} \times 8 \times 6$ ✓ — 1 mark

$= 24 \text{ cm}^2$

- Shaded region $= 46.3647 - 24$ ✓ — 1 mark

$= 22.3647 \text{ cm}^2$

End of Examination.