JRAHS 2U 1993

Question 1. (*Start a new page.*)

a) Find, correct to one decimal place, the value of : $\dfrac{1.09^2 + 0.89}{1.09^2 - 0.89}$.

b) Express $\dfrac{2}{3}$ of $\dfrac{1}{4}$ as an exact percentage.

c) Simplify $a - 3(a - 2)$.

d) Find the coordinates of the vertex of the parabola
$$y = 2x^2 - 20x + 53.$$

e) Solve $x^2 - 3x - 28 = 0$.

f) Evaluate $\log_5 45 + \log_5 40 - \log_5 72$.

Question 2. (*Start a new page.*)

a) Two lines l: $x + y = 12$ and m: $2x - y = -3$ intersect at the point M.

(i) Find the coordinates of M.

(ii) Prove that the point A(6,6) lies on the line l.

(iii) The line through A parallel to the y-axis intersects the line m at B. Find the coordinates of B.

(iv) If lines l and m intersect the y-axis at C(0,12) and D(0,3) respectively, show that the quadrilateral ABCD is a parallelogram. **Give reasons.**

(v) Hence find the area of the triangle AMD.

b) Find the limiting sum of the geometric series $\dfrac{1}{3} + \dfrac{1}{6} + \dfrac{1}{12} + \dfrac{1}{24} + \ldots$

Question 3. *(Start a new page.)*

a) Differentiate :

(i) $x^2 - \frac{3}{7}$.

(ii) $\frac{\sin x}{x}$.

(iii) $(3-4x)^5$.

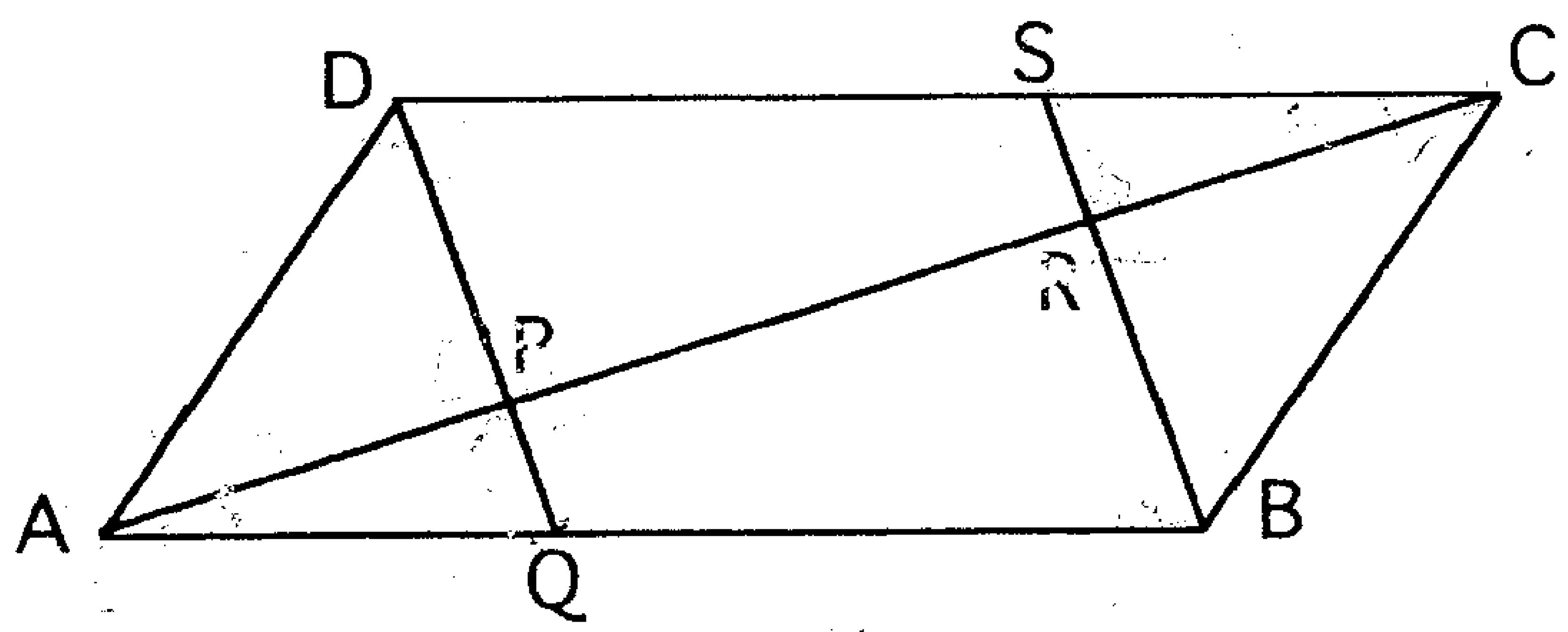
b) Find the equation of the tangent to the curve $y = x \ln x$ at the point $(1,0)$.

c) Find $\int \sec^2 3x \, dx$.

d) Evaluate $\int_1^3 \frac{2x}{x^2+3} dx$

Question 4. *(Start a new page.)*

a) In the parallelogram ABCD, the straight lines DPQ and BRS are perpendicular to the diagonal AC.



Copy the figure on your answer sheet and prove, giving reasons :

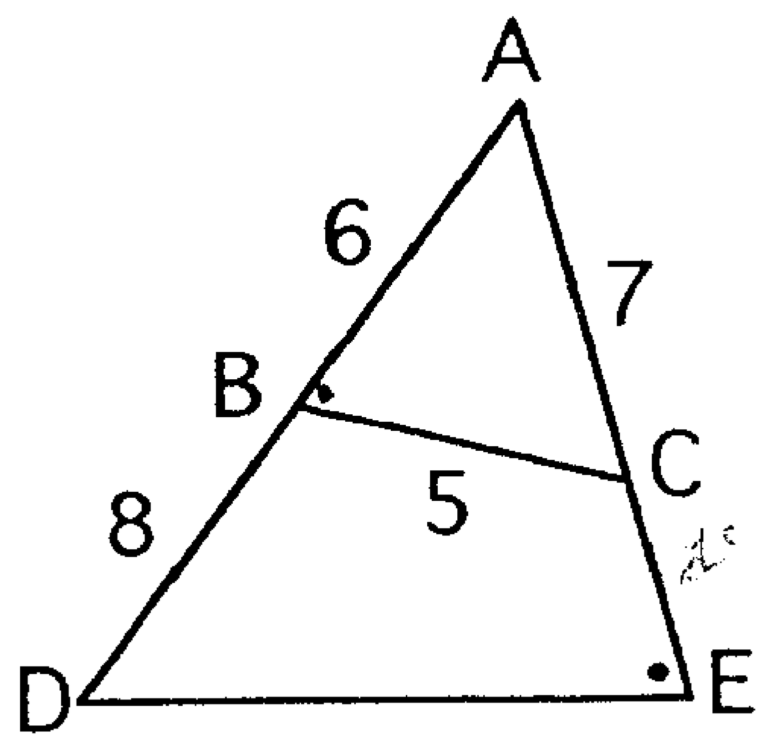
(i) DQ || SB.

(ii) Quadrilateral QBSD is a parallelogram.

(iii) SC = AQ.

b) In the figure above, if the area of the triangle AQD is 20 cm^2 and AQ : QB = 2 : 3 find the area of the parallelogram ABCD.

c) In the figure below, not drawn to scale, $\angle ABC = \angle CED$, $AB = 6\,cm$, $BD = 8\,cm$, $AC = 7\,cm$ and $BC = 5\,cm$.



(i) Prove that the triangles ABC and ADE are similar.

(ii) Find the length of CE. Give reasons.

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Question 5. *(Start a new page.)*

a) Two 6 sided, unbiased dice, one red and the other green, are tossed. What is the probability that :

(i) The red one is an odd number ?

(ii) Their sum is 8 ?

(iii) Their sum is 8 given that the red one shows an odd number ?

b) A particle initially at ~~rest~~ origin, moves in a straight line. The displacement x metres from the origin at time t seconds is given by $x = t^3 - 6t^2 + 9t,\ t \geq 0$.

(i) Find the velocity of the particle at time t seconds.

(ii) Hence prove that the particle changes direction twice in the first 4 seconds, and find the positions of the particle at those times.

(iii) Sketch the displacement-time graph showing all information obtained so far, for $t \leq 4$.

(iv) Calculate the average speed over the first 3 seconds.

Question 6. *(Start a new page.)*

a) The number of bacteria, N, in a culture increased from 600 to 1800 in two hours. Assuming that the rate of increase is directly proportional to the number of the bacteria present, $\frac{dN}{dt} = kN$,

(i) Show that $N = Ae^{kt}$, where A is a constant, satisfies the equation $\frac{dN}{dt} = kN$.

(ii) Find the constant A and then the constant k .

(iii) How many hours does it take for the number of bacteria to increase from 1800 to 18000 ?

b) (i) Sketch the graph of $y = \sqrt{4 - x^2}$.

(ii) Hence, find $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$.

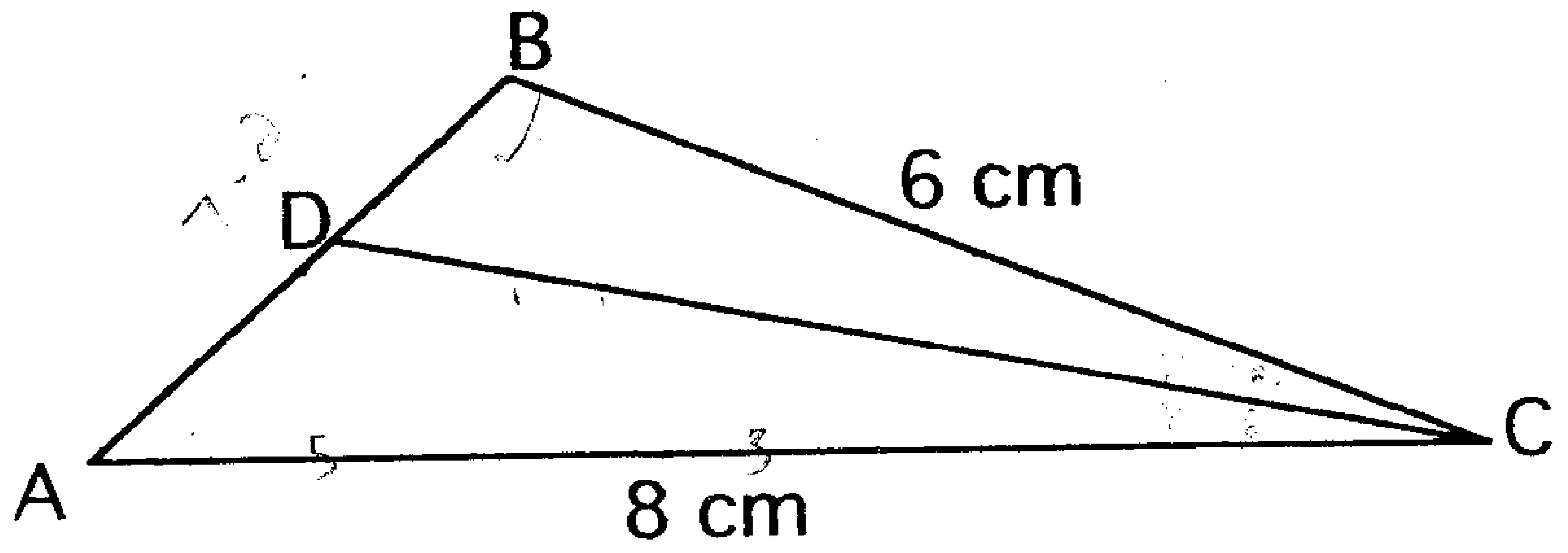
Question 7. *(Start a new page.)*

a) A curve $y = f(x)$ has derivative $f'(x) = kx^2 + 2$ and a stationary point at $(-1,3)$.

(i) Find k.

(ii) Find the function.

b) In ΔABC, not drawn to scale, CD bisects the angle BCA and D lies on side AB. $\angle C = 60^\circ$, AC=8 cm and BC=6 cm.



(i) Calculate the length of AB, giving your answer correct to one decimal place.

(ii) Calculate the exact area of ΔABC.

(iii) Hence, calculate the exact length of CD.

Question 8. *(Start a new page.)*

a) Calculate the area bounded by $y = x^2 - 5x + 6$ and $y = 2x$.

b) Every year, starting on Peter's first birthday, his grandparents gave him \$100 which was deposited in the same bank account at the rate of 8% per annum. On his 21st birthday, instead of the \$100, they gave him a lump sum of \$5,000, deposited in the same account. After this they stopped giving Peter any more money.

(i) How much money did Peter have in this account before his 21st birthday ?

(ii) Peter left all this money in the bank, at the usual rate of 8%, until his 28th birthday when he decided to use it for a new car. How much money did he collect from this account ?

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Question 9. *(Start a new page.)*

a) The area in the first quadrant, under the curve $y = xe^x$, the line $x = 2$ and the x-axis, is rotated about the x-axis.

(i) Write down a definite integral which gives the volume of the generated solid.

(ii) Calculate an estimate of the volume of this solid of revolution using Simpson's rule with three function values.

b) Towns A, B and C are located 6 km west, 6 km east and 10 km south, respectively, of a point D. A road is to run north from C to a point P and from P a branch road is to run to A and another branch road to B. (Assume all roads to be straight lines.)

(i) Show that, the total length L of road PA + PB + PC is given by $L = x + 2\sqrt{x^2 - 20x + 136}$, where x is the length of the road from C to P.

(ii) Show that the minimum total length for the three roads, PA + PB + PC, is $(10 + 6\sqrt{3})\,km$.

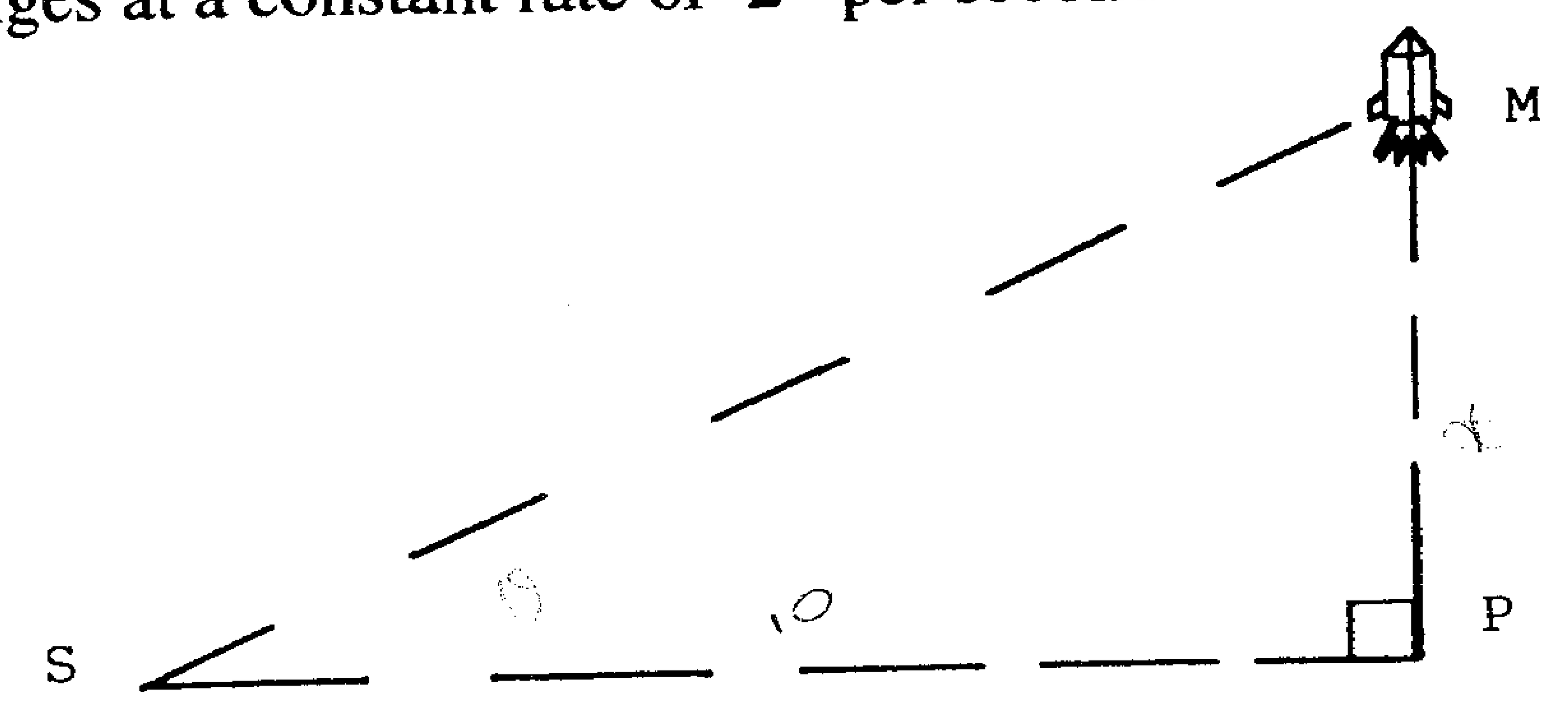
Question 10. *(Start a new page.)*

a) Given the quadratic equation $x^2 - 3x + k = 0$,

(i) Find the value of k for which the equation has two equal roots.

(ii) Find the value of k for which one of the roots exceeds the other root by two.

b) A missile, M, is fired vertically from a point P which is 10 km from a tracking station S at the same elevation as P, as shown below. For the first 20 seconds of flight its angle of elevation θ changes at a constant rate of 2° per second.



(i) Find θ as a function of t (in the first 20 seconds).

(ii) Hence express the distance x travelled by the missile, as a function of t.

(iii) Find the velocity of the missile when the angle of elevation is 30°, giving your answer in km/hr.

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Q1 a) 7.0

b) $16\frac{2}{3}\%$

c) $-2a+6$

d) $(5,3)$

e) $x=7$ or -4

f) 2

Q2 a) i) $M(3,9)$

ii) L.H.S. $= 6+6=12=$ R.H.S.

iii) $B(6,15)$

iv) ABCD is a parm.
(one pair of opposite sides equal and parallel)

v) $13\frac{1}{3}$ units^2

b) $S_\infty = \frac{2}{3}$

Q3 a) i) $2x$

ii) $\dfrac{x\cos x - \sin x}{x^2}$

iii) $-20(3-4x)^4$

b) $m=1$, $y=x-1$

c) $\frac{1}{3}\tan 3x + c$

d) $\ln 3$

Q4. a) i) $DQ \parallel SB$ (one pair of alternate angles are equal).

ii) QBSD is a parm (2 pairs of opposite sides parallel).

iii) $SC = AQ$

b) $100\,cm^2$

c) i) equiangular test

ii) $CE = 5\,cm$

Q5 i) $\frac{1}{2}$

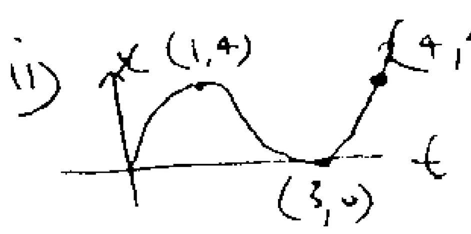
ii) $\frac{5}{36}$

iii) $\frac{2}{18} = \frac{1}{9}$

b) $x = t^3 - 6t^2 + 9t$
$= t(t-3)^2$

i) $v = 3t^2 - 12t + 9$

ii) $t=1$, $t=3$

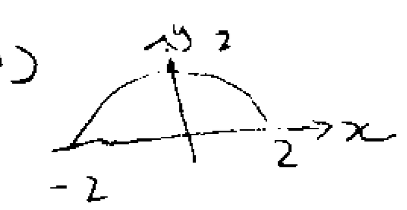
iii) 

iv $\frac{4+4}{3} = \frac{8}{3}$ m/s

Q6. a) i) $\frac{dN}{dt} = Ake^{kt}$
$= kN$

ii) $A = 600$, $k = \frac{\ln 3}{2}$

iii) $t = 6.2$ hours $- 2$
$=$ 4hrs 10mins

b) i)  $y \geq 0$

ii) 2π units^2

Q7. a) i) $k = -2$

ii) $c = 4\frac{1}{3}$
$f(x) = -\frac{2}{3}x^3 + 2x + 4\frac{1}{3}$

b) i) $AB = 7.2\,cm$

ii) Area $= 12\sqrt{3}\,cm^2$

iii) $DC = \dfrac{24\sqrt{3}}{7}$

Q8 i) a) $x=1$, $x=6$

$A = 20\frac{5}{6}$ units^2

b) \$4942

ii) \$9,942.29
worth $=$ \$17039

Q9 a) i) $V = \int_0^2 \pi x^2 e^{2x}\,dx$

ii) 236 units^3

b) i) $PA = x + 2\sqrt{x^2 - 20x + 136}$

ii) min. TP. at $x = 10 - 2\sqrt{3}$

abs. min $= 10 - 2\sqrt{3} + 4\sqrt{3} + 4\sqrt{3}$

Q10. a) i) $k = \frac{9}{4}$

ii) $P = \frac{1}{2}\left(\frac{1}{2} + 2\right)$
$= \frac{5}{4}$

b) i) $\theta = (2t)^\circ$

ii) $x = 10\tan(2t)$
$x = 10\tan\left(\frac{\pi}{90}t\right)$

iii) $V = 10 \times \frac{\pi}{90} \times \sec^2\left(\frac{\pi}{90}t\right)$

$= \frac{4\pi}{27}$ km/s

$= \frac{1600\pi}{3}$ km/h