Question 1 (Start a new page)

(a) Factor fully : $x^3y - xy^3$.

(b) Simplify : $\frac{1}{x} - \frac{1-x}{2x}$.

(c) Solve: (i) $4 - x = 5$,

(ii) $2x^2 - 7x - 4 = 0$,

(iii) $2 - (1 - x) \leq 3 + 2x$,

(iv) $2\log_4 8 = x$.

(d) Find the centre and radius of the circle with equation $x^2 + y^2 - 6x + 4y - 12 = 0$.

(e) Find the value of x if 2^x is half of 2^{400}.

Question 2 (Start a new page)

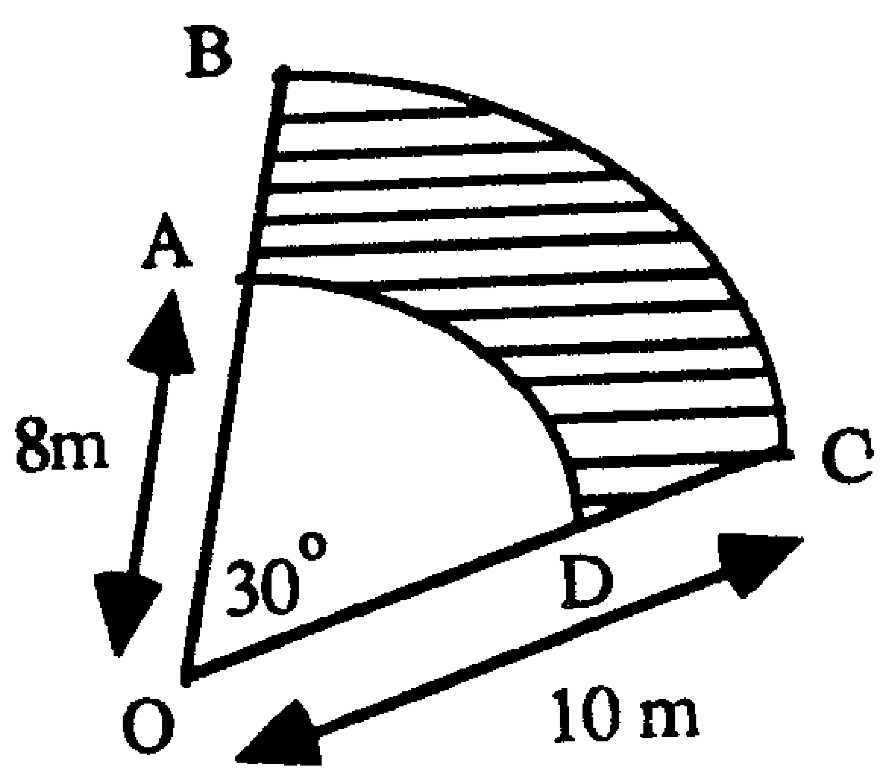
(a) Differentiate: (i) $f(x) = x^4 + \sqrt{\pi}$,

(ii) $f(x) = x^2e^x$,

(iii) $f(x) = \frac{\sin x}{1 + \sin x}$.

(b) Find the equation of the tangent to the curve $y = \ln(x^3 - 7)$ at the point where $x = 2$.

(c) AD and BC are arcs of concentric circles with O as their centre.
Find the perimeter of the shaded region if OA = 8 metres and OC = 10 metres.



Question 3 (Start a new page)

(a) Solve $2\cos x + 1 = 0$ for $0^\circ \le x \le 360^\circ$.

(b) Evaluate $\int_0^{\frac{\pi}{4}} \left(1 + \tan^2 x\right) dx$.

(c) In ΔABC, $AB = 5$, $BC = 8$ and $A\hat{C}B = 30^\circ$. Use the sine rule to find $C\hat{A}B$ correct to the nearest degree.

(d) A hiker sets up camp at point P. From camp P she then walks 8 km in the direction 038°T to a point A. From point A she then walks in the direction 134°T for 5 km till she reaches point B.

(i) Draw a neat sketch showing all the given information.

(ii) Find, to the nearest kilometre, the distance from P to B.

Question 4 (Start a new page)

(a) Find (i) $\int e^{4x+1}\, dx$,

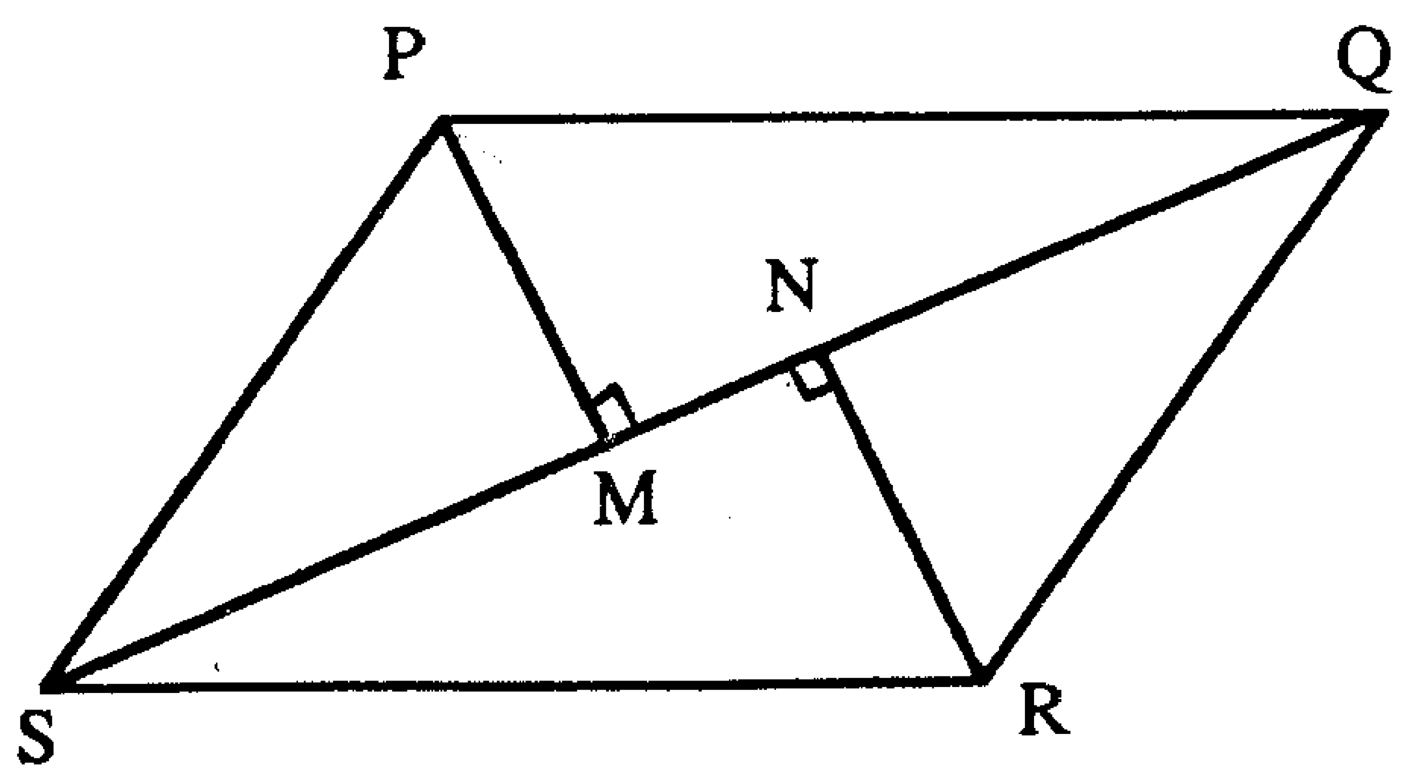
(ii) $\int_0^{\frac{\pi}{2}} (1 + \cos x)\, dx$,

(iii) $\int_1^2 \frac{1}{x^2}\, dx$.

(b) PQRS is a parallelogram. PM and RN are perpendicular to QS.

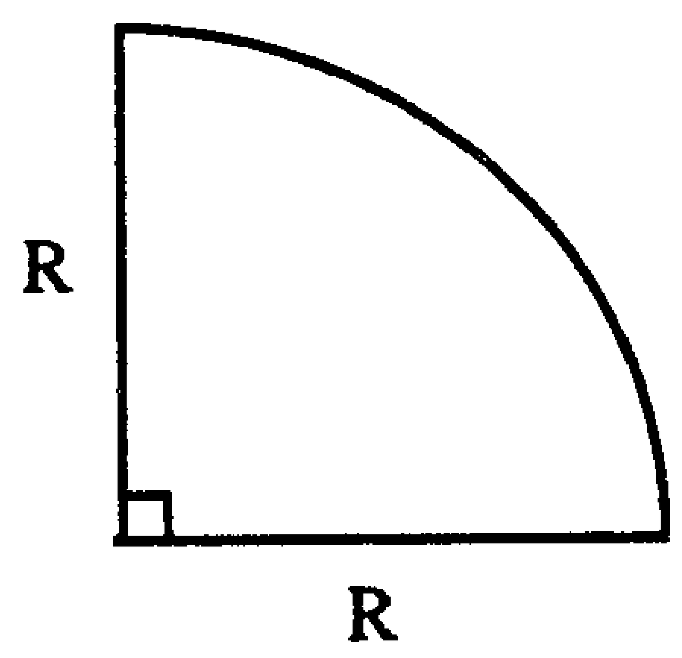
(i) Copy the diagram onto your answer sheet.

(ii) Prove that PNRM is a parallelogram.



Question 5 (Start a new page)

(a) A quadrant of a circle radius R metres has a perimeter of 1 metre.



(i) Show that its area is given by:

$$A = \frac{\pi}{(4+\pi)^2},$$

(ii) Find its area correct to 2 decimal places.

(b) (i) Show that the line l_1 with equation $3x + 2y - 12 = 0$ passes through the point A(2,3).

(ii) Find the equation of the line l_2 through point A and perpendicular to line l_1.

(iii) Find the co-ordinates of the points B and C where the lines l_1 and l_2 pass through the y-axis.

(iv) Find the area of ΔABC.

(v) Find all values of k for which the line $x + y = k$ passes through ΔABC.

Question 6 (Start a new page)

(a) If α and β are roots of the equation $2x^2 - 3x - 7 = 0$, find:

(i) $\alpha + \beta$,

(ii) $\alpha\beta$,

(iii) $(\alpha + 1)(\beta + 1)$.

(b) Find the value(s) of k if $x^2 - kx + 4 = 0$ has equal roots.

(c) A parabola has equation $8y = x^2 - 4x + 12$. Find:

(i) the co-ordinates of its vertex,

(ii) the equation of its directrix,

(iii) the co-ordinates of its focus.

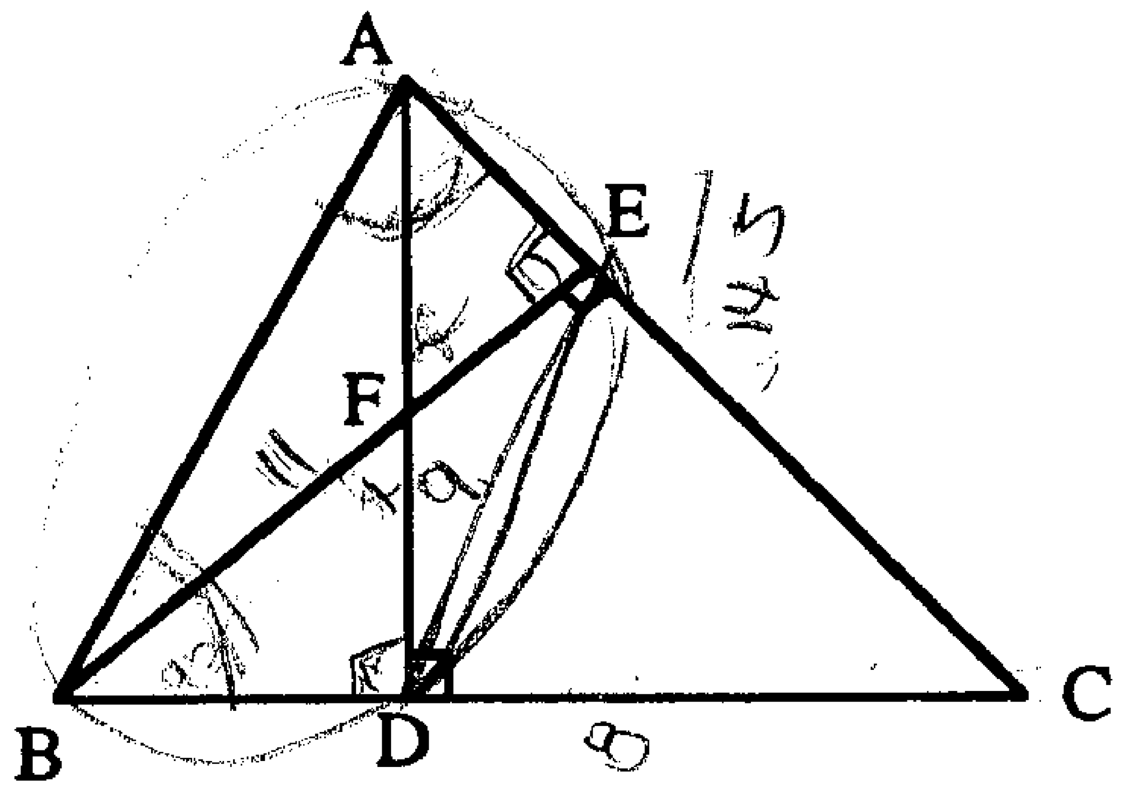
(d) In a raffle, 100 red tickets and 100 blue tickets are sold and three prizes are to be won. One ticket is drawn for first prize, one for second prize and one for third prize. Each winning ticket is destroyed after each draw. What is the probability that:

(i) all three prizes are won by red tickets?

(ii) at least one red ticket wins a prize?

Question 7 (Start a new page)

(a) In the diagram, AD⊥ BC and BE ⊥ AC. If BE = 16, AD = 9 and CD = 8, find the length of CE, giving all reasons.



(b) (i) Using calculus, show that the curve $y = x^3 - 3x^2$ has two turning points and one point of inflexion.

(ii) Draw a neat sketch of the curve $y = x^3 - 3x^2$ for $-1 \le x \le 4$, clearly showing all maxima, minima and points of inflexion.

(c) The position of a particle, x metres from the origin O, after t seconds is given by $x = t^3 - 3t^2$. Briefly describe its motion. (In your description, include initial conditions and points where the particle changes direction)

Question 8 (Start a new page)

(a) (i) Find the points where the line $y = 2x - 3$ intersects the parabola $y = x^2 - 2x - 3$.

(ii) Find the exact area bounded by the line and the parabola.

(b) The amount M grams of Thorium is given by $M = M_0e^{-kt}$, where M_0 and k are positive constants and the time t is measured in years.

(i) Show that M satisfies the equation $\frac{dM}{dt} = -kM$,

(ii) Find the value of k if 200 gms of Thorium decomposes to 150 gms at the end of two years,

(iii) Find the half-life of Thorium. (Give your answer correct to the nearest year)

(iv) Find the amount of Thorium which has decomposed by the end of 10 years. (Give your answer correct to the nearest gram)

Question 9 (Start a new page)

(a) The gradient function of a curve is given by $\frac{dy}{dx} = \frac{2}{x-1}$. If the curve passes through the point (4,0) find the equation of the curve.

(b) The area bounded by the curve $y = \sqrt{1 + \ln x}$, the x-axis and the lines $x = 1$ and $x = 3$ is rotated once about the x-axis to form a solid. Use Simpson's rule with 3 function values to find the volume generated. (Give your answer correct to one decimal place)

(c) A tank containing 18 000 litres of water is to be drained. After t minutes, the rate at which the volume of water is decreasing is given by: $\frac{dV}{dt} = -40(30 - t)$.

(i) Derive a formula for the volume of water remaining in the tank after t minutes.

(ii) How much water will be left in the tank after 10 minutes?

(iii) How long will it take the tank to empty?

Question 10 (Start a new page)

(a) An open box is to be made of thin sheet metal with a square base having edges x metres long and vertical sides y metres long. It is to have a volume of 2 m^3.

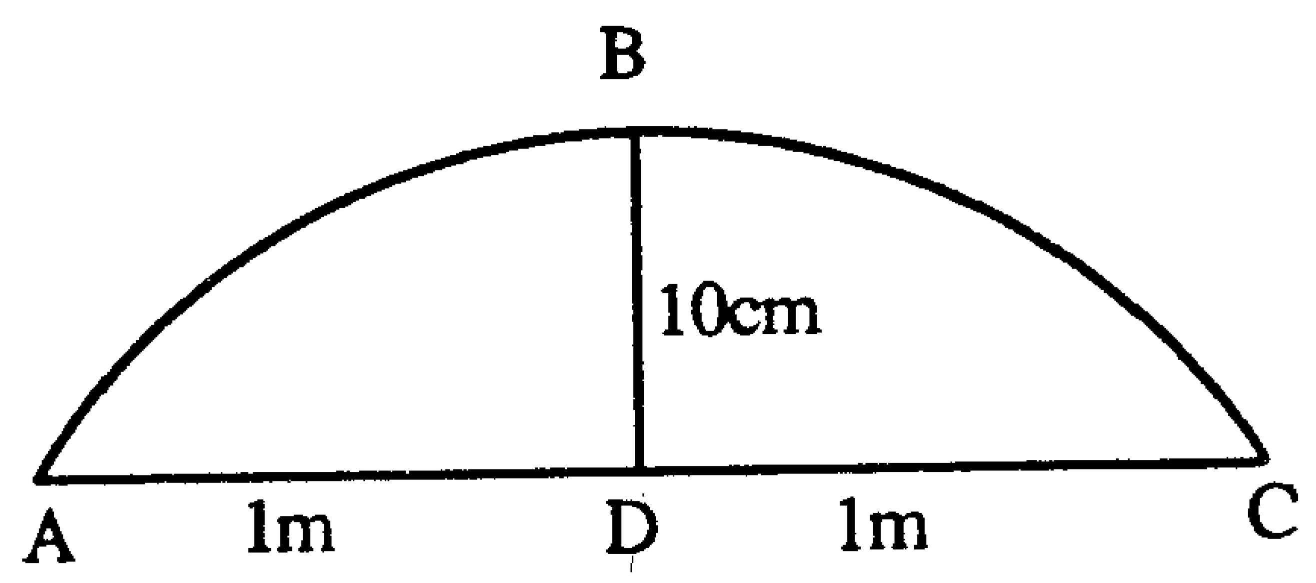
(i) Show that the surface area of the box (assuming negligible thickness) is given by $A = \left(\frac{8}{x} + x^2\right)$.

(ii) Prove that the cost of insulating the inside of the box will be a minimum if $x = 2y$.

(b) An arch ABC is part of a circle of radius R. D bisects the chord AC which is 2 metres long. A strip BD of length 10 cm is used to strengthen the arch.

(i) Prove that the radius R is 505 cm.

(ii) Find the area of the segment ABCD correct to the nearest square centimetre.



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J.R.A.H.S TRIAL H.S.C.
2/3 Unit 1994

1 (a) $xy(x-y)(x+y)$

(b) $\frac{1+x}{2x}$

(c)(i) $x=-1$

(ii) $x=-\frac{1}{2}, 4$

(iii) $x \geq -2$

(iv) $x=3$

(d) Centre $(3,-2)$ radius 5

(e) $x=399$

2 (a)(i) $f'(x)=4x^3$

(ii) $f'(x)=xe^x(x+2)$

(iii) $f'(x)=\frac{\cos x}{(1+\sin x)^2}$

(b) $12x-y-24=0$

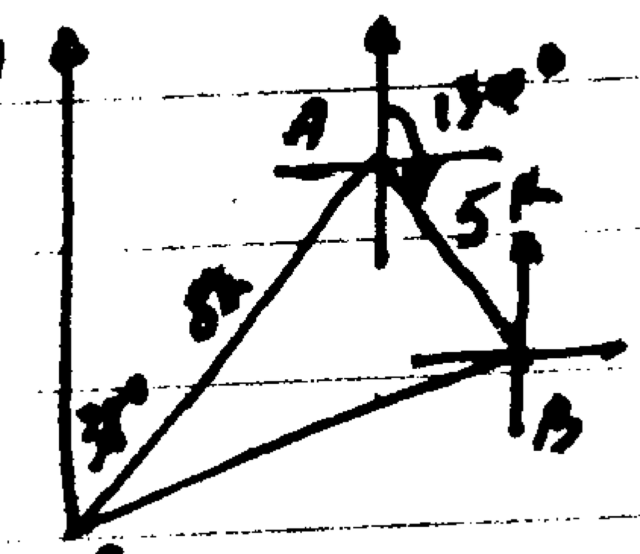
(c) $P=(4+3\pi)$ m

3 (a) $x=120°, 240°$

(b) 1

(c) $C\hat{A}B = 53°, 127°$

(d)(i)



(ii) Distance from P to B is 9 km

4 (a)(i) $\frac{1}{4}e^{4x+1}+c$

(ii) $\left(\frac{\pi}{2}+1\right)$

(iii) $\frac{1}{2}$

(b) –

5 (a)(i) $R=\frac{2}{4+\pi}$

(ii) $A=0.06$

(b)(i) –

(ii) $2x-3y+5=0$

(iii) $B(0,6)$ and $C(0,\frac{5}{3})$

(iv) $A=4\frac{1}{3}\ u^2$

(v) $\frac{5}{3} \leq k \leq 6$

6 (a)(i) $\frac{3}{2}$

(ii) $-\frac{7}{2}$

(iii) -1

(b) $k=\pm 4$

(c)(i) vertex $(2,1)$

(ii) Directrix $y+1=0$

(iii) Focus $(2,3)$

(d)(i) $\frac{49}{398}$ (ii) $\frac{349}{398}$

7 (a) $CE = 9\frac{7}{9}$

(b) –

(c) –

8 (a)(i) $(0,-3), (4,5)$

$A = 10\frac{2}{3}\ u^2$

(b)(i) –

(ii) $k=0.144$

(iii) $t \doteqdot 5$ years.

(iv) 153 gms.

9 (a) $y = 2\ln\left(\frac{x-1}{3}\right)$

(b) $V \doteqdot 10.3\ u^3$

(c)(i) $t=30$

(ii) $V = 8{,}000$ L

(iii) $t=30$ minutes.

10 (a)(i)

$A = x^2 + \frac{8}{x}$

(ii) MIN. at $x=2$

(b)(i) $R = 505$ cm

(ii) $A = 1{,}339\ m^2$