1997 JRAHS 2U TRIAL.

QUESTION 1: **(START A NEW PAGE)**

2 (a) Write down the exact value of $\tan\left(\frac{11\pi}{6}\right)$.

2 (b) Find the value of $\log_3 7$ correct to 2 significant figures.

2 (c) If $F(h) = \frac{e^h - 1}{h}$, find $F(2)$ correct to 2 decimal places.

2 (d) Find the probability of throwing a sum of 8 with two regular six sided dice.

2 (e) Find the gradient of the tangent to the curve $y = \sin 2x$ at the point $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$.

2 (f) The gradient function of a curve is given by $\frac{dy}{dx} = e^{4x}$. Find the equation of the curve if it passes through the point (0,2).

QUESTION 2: **(START A NEW PAGE)**

(a) Differentiate with respect to x:

2 (i) $\tan 3x$,

2 (ii) $\log_e(4x - 5)$,

2 (iii) xe^{2x} .

(b) Find:

2 (i) $\int x^2 + \cos 6x \ dx$,

2 (ii) $\int (3x - 4)^5 \ dx$,

2 (iii) $\int_0^3 \frac{x}{x^2 + 1} \ dx$.

QUESTION 3: **(START A NEW PAGE)**

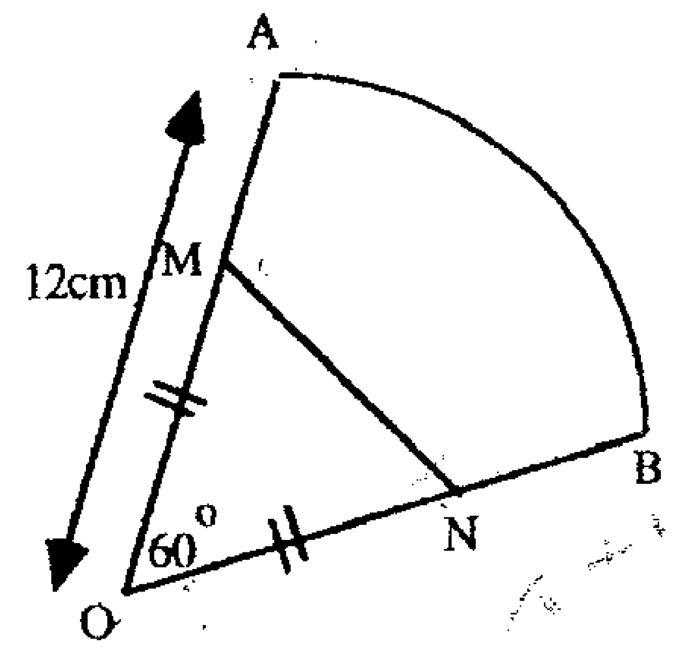
3 (a) A pensioner deposits $1200 in an account. If the interest rate is 8% per annum and is compounded every 6 months, find the value of the deposit at the end of 5 years. Give your answer correct to the nearest dollar.

(b) The lines $y = x - 2$ and $2x + y - 10 = 0$ meet at the point A.

2 (i) Find the co-ordinates of the point A.

2 (ii) Show that the interval OA (where O is the origin) is perpendicular to one of the above lines.

5 (c) OAB is a sector with OA = OB = 12cm and $A\hat{O}B = 60^\circ$. Points M and N are chosen so that OM = ON (see diagram).
If the area of ΔMON is half the area of sector AOB find the length of OM giving your answer correct to 2 decimal places.



QUESTION 4: **(START A NEW PAGE)**

3 (a) Find the area bounded by the curve $y = 6x - x^2$ and the x-axis.

(b) The letters of the word JAMES are placed into a cup and the letters of the word RUSE are placed into another cup. A letter is chosen at random from each cup.

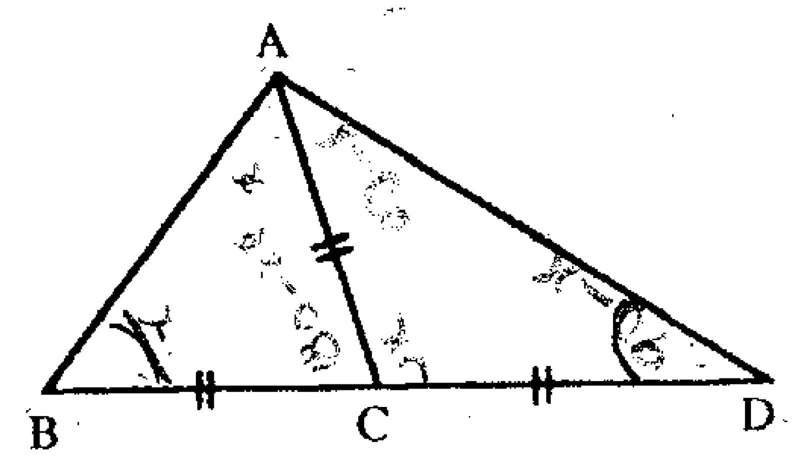
1 (i) Draw a dot diagram to represent the above sample space.

Hence find the probability that the two letters chosen are

1 (ii) the same,

2 (iii) both vowels given that at least one vowel has been chosen.

5 (c) ΔABC is isosceles with AC = CB.
BC is extended to D so that BC = CD (see diagram).
Prove that $A\hat{B}C$ and $A\hat{D}C$ are complementary.



QUESTION 5: (START A NEW PAGE)

3 (a) Find the volume of the solid formed when the area bounded by $y = \sqrt{4 - x}$ and the co-ordinate axes is rotated one revolution about the x-axis.

2 (b) (i) Find the points of intersection of the curves $y = \frac{1}{2 - x}$ and $y = 2 - x$.

3 (ii) On the same set of axes sketch the above curves showing all asymptotes and intercepts with the co-ordinate axes.

4 (iii) Find the area bounded by these curves and both co-ordinate axes.

QUESTION 6: (START A NEW PAGE)

(a) The rate of flow (R litres/minute) of water through a filter is given by $R = 8t - \frac{t^2}{2}$.

2 (i) At what times is no water flowing through the filter?

3 (ii) Find the volume of water that flows through the filter in the first 5 minutes.

(b) ABCD is a quadrilateral. The diagonal AC bisects both $B\hat{A}D$ and $B\hat{C}D$.

3 (i) Prove that AB = AD.

4 (ii) Hence prove that AC $\perp$ BD.

QUESTION 7: (START A NEW PAGE)

(a) In a chemical reaction the amount (M kilograms) of undissolved solid after t hours is given by $M = Ae^{-kt}$. If a chemical reaction starts with 20 kg of solid and 5 kg remain after 24 hours, find

2 (i) the values of A and k,

1 (ii) the amount of undissolved solid after 3 hours (give your answer to the nearest gram),

2 (iii) the time taken to dissolve 19 kg of solid (give your answer to the nearest hour).

2 (b) (i) Sketch the area bounded by $y = x^3$, the x-axis and the line x=2.

5 (ii) Find the volume of the solid formed when this area is rotated one revolution about the y-axis

QUESTION 8: (START A NEW PAGE)

(a) (i) For the function $f(x) = \frac{\sin x}{x}$, copy and complete the following table giving your answers correct to 4 decimal places.

2

x	0	0.5	1
f(x)			

2 (ii) Use the above table and Simpsons rule to find an approximation for $\int_0^1 \frac{\sin x}{x}\,dx$. Give your answer correct to 2 decimal places.

3 (b) On the same set of axes sketch the graphs of $y = \frac{1}{4}x$ and $y = \sin\frac{1}{2}x$ for $0 \le x \le 4\pi$.

(c) A particle is moving in a straight line has velocity (v m/s) at time t seconds given by:

$$v = \frac{1}{4}t + \sin\frac{1}{2}t$$

1 (i) Find the initial velocity.

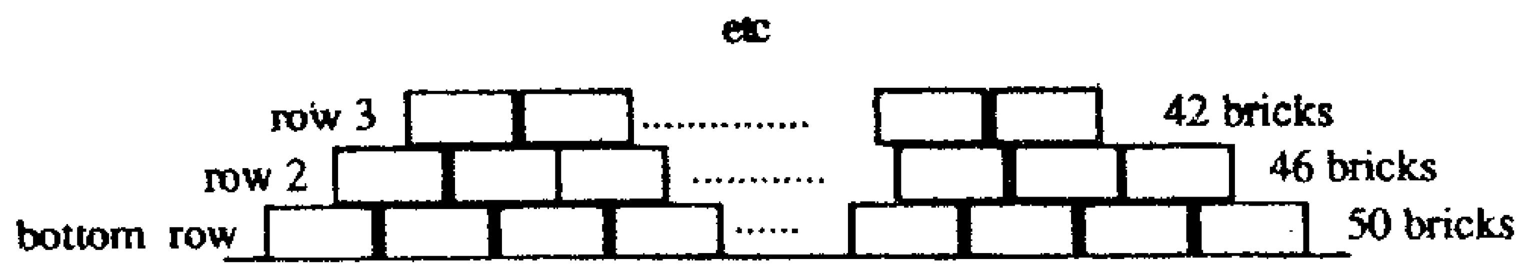
1 (ii) With the aid of the graphs in part (b) explain why the particle is always moving to the right.

(iii) Find the distance travelled by the particle during the first 2π seconds.

3

QUESTION 9: (START A NEW PAGE)

(a) Bricks are stacked in a neat pile at a building site. There are 50 bricks in the bottom row, 46 in the 2nd row, 42 on the 3rd row and so on until the total number of bricks in the pile is 320. (see diagram)



2 (i) Show that the number of bricks (T_n) in the n^{th} row is given by $T_n = 54 - 4n$ and hence prove that there are less than 14 rows of bricks.

2 (ii) How many rows of bricks are in the pile?

2 (iii) How many bricks are in the top row?

(b) James Ruse pays \$80 into a superannuation fund at the start of each month. The fund pays 6% p.a. interest compounded on the balance in the fund at the end of each month.

2 (i) What is the final value of the first deposit at the end of 5 years?

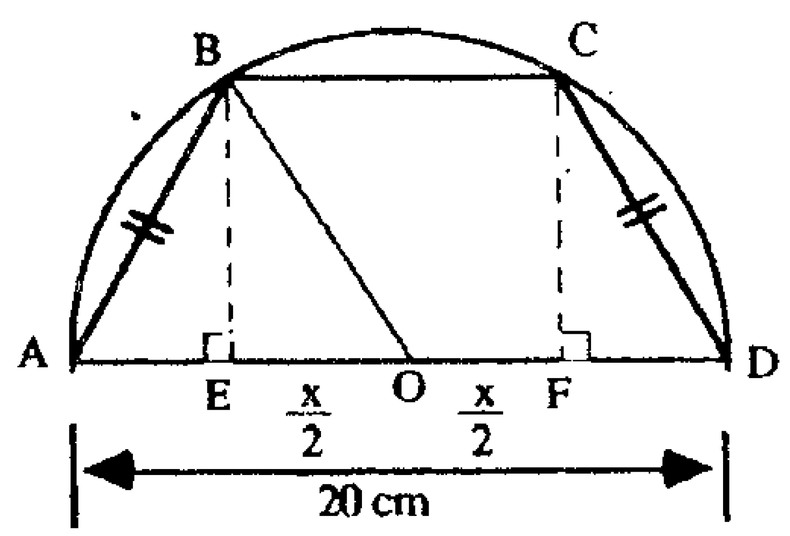
2 (ii) Write an expression for the total value of the fund at the end of n months.

2 (iii) For approximately how many months must he contribute to the fund so that it will be worth \$10000?

QUESTION 10: (START A NEW PAGE)

4 (a) An urn contains W white and B black marbles. If the probability of selecting 2 white marbles at random is $\frac{1}{3}$ while the probability of selecting 3 white marbles at random is $\frac{1}{6}$ find the number of white marbles in the urn.

(b) An isosceles trapezium ABCD is drawn with its vertices on a simicircle centre O and diameter 20 cm (see diagram).



2 (i) If $EO = OF = \frac{x}{2}$, show that:

$$BE = \frac{1}{2}\sqrt{400 - x^2}.$$

2 (ii) Show that the area (A cm^2) of the trapezium ABCD is given by:

$$A = \frac{1}{4}(x + 20)\sqrt{400 - x^2}.$$

4 (iii) Hence find the length of BC so that the area of trapezium ABCD is a maximum.

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JRAHS 2UNIT 1997 TRIAL HSC.

QUESTION 1

(a) $-1/\sqrt{3}$

(b) 1.8

(c) 3.19

(d) 5/36

(e) -1

(f) $y=\frac{1}{4}e^{4x}+1\frac{3}{4}$

QUESTION 2

(a)(i) $y'=3\sec^2 3x$

(ii) $y'=\dfrac{4}{4x-5}$

(iii) $y'=e^{2x}(1+2x)$

(b)(i) $\frac{1}{3}x^2+\frac{1}{6}\sin 6x+c$

(ii) $\frac{1}{18}(3x-4)^6+c$

(iii) $\frac{1}{2}\ln 10$

QUESTION 3

(a) \$1776

(b)(i) (4,2)

(ii) $\frac{1}{2}\times -2=-1$

$\therefore \perp$

(c) 9.33

QUESTION 4

(a) 36 u^2

(b)(i) —

(ii) 1/10

(iii) 2/7

(c) —

QUESTION 5

(a) $8\pi\ u^3$

(b)(i) (1,1), (3,−1)

(ii) —

(iii) $\ln 2+\frac{1}{2}\ u^2$

QUESTION 6

(a)(i) $t=0$ or 16

(ii) $79\frac{1}{6}$ l.

(b)(i) —

(ii) —

QUESTION 7

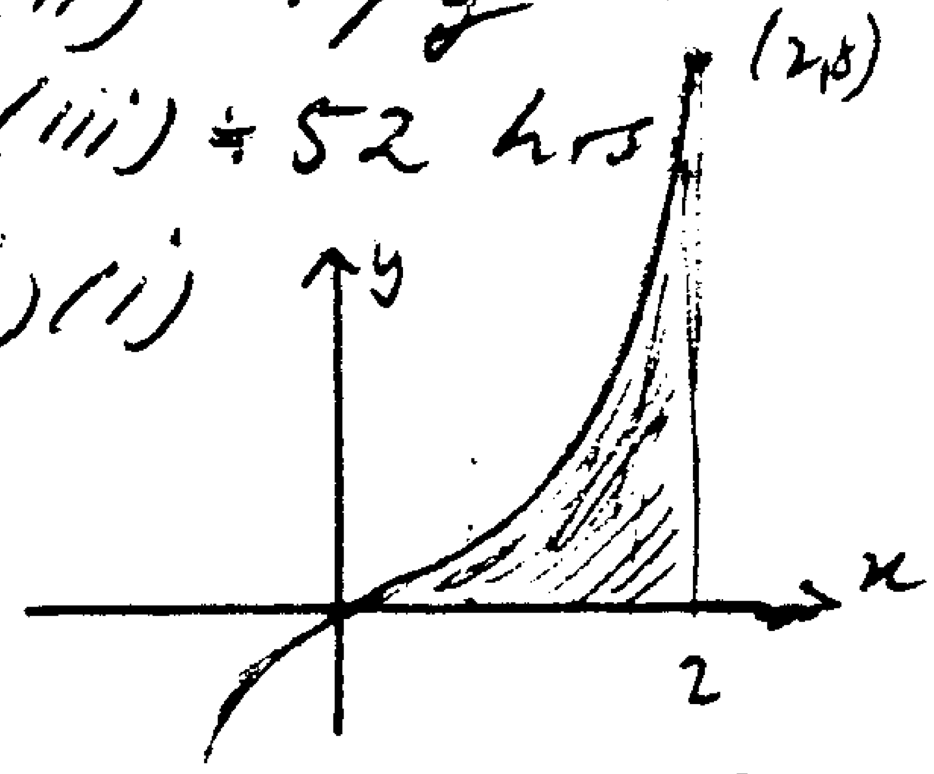
(a)(i) $A=20$

$k=-\frac{1}{24}\ln\frac{1}{4}$

(ii) 17 g

(iii) ≑ 52 hrs

(b)(i)



(ii) $\dfrac{64\pi}{5}\ u^3$

QUESTION 8

(a)(i)

x	0	$\frac{1}{2}$	1
$f(x)$	1.0000	0.9589	0.8415

(ii) ≑ 0.95

(b) —

(c)(i) $v=0$

(ii) $v>0$ for $t>0$

(iii) $4+\dfrac{\pi^2}{2}$ m.

QUESTION 9

(a)(i) $a=50$, $d=-4$

$T_n>0 \Rightarrow n<13.5$

(ii) 10 rows

(iii) $T_{10}=14$.

(b)(i) \$107.91

(ii) —

(iii) ≑ 97 month.

QUESTION 10

(a) 6 white

(b)(i) —

(ii) —

(iii) BC=10