1998 2U JRAHS TRIAL

QUESTION 1. [Start a *new* page]

(a) Calculate, correct to one decimal place

$$\frac{7.04}{8.04-\sqrt{27.04}}$$

(b) Mary bought a dress for \$75 during the sales. How much did she save if the sales discount on the dress was 40% ?

(c) If $f(x)=\dfrac{1}{1+x^3}$.

(i) Evaluate $f(1)$ and $f(2)$.

(ii) Using the trapezoidal rule with three function values, find the approximate area under the curve between $x=0$ and $x=2$.

(d) Evaluate $\log_4\left(\dfrac{1}{\sqrt{2}}\right)$.

(e) Factorise $a^2-b^2+2a+2b$.

QUESTION 2. [Start a *new* page]

(a) Solve the equation $\frac{1}{2}(x+1)-\frac{1}{5}(x-2)=3$.

(b) Solve:

(i) $x^2+6x-16=0$.

(ii) $x^2+6x-16\geq 0$.

(c) A box contains 4 red and 3 green apples. Peter took two apples at random.

(i) Find the probability that they are:

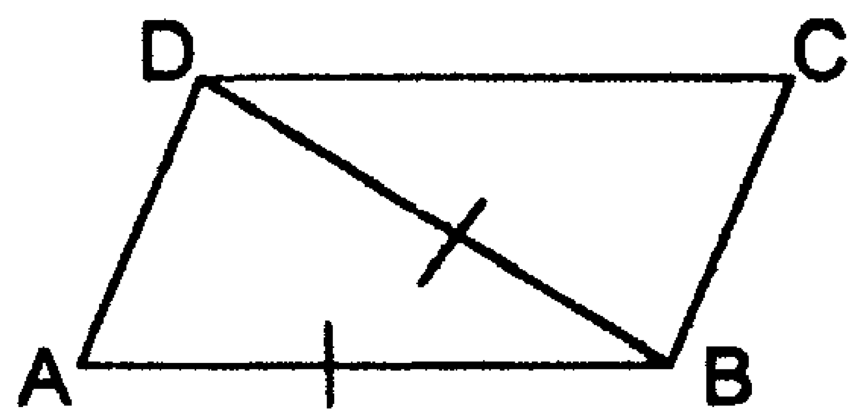
(1) both red.

(2) one of each colour.

(ii) If it is known that at least one of the apples is red, find the probability that they are both red.

QUESTION 3. [Start a *new* page]

(a)



ABCD is a parallelogram. Angle ADC equals 120°. Diagonal DB equals side AB.

(i) Find $\angle DAB$ (give reasons).

(ii) Show that triangle ABD is equilateral (give reasons).

(iii) If smaller diagonal DB = 8 cm, find the area of the parallelogram.

(b) Consider the parabola with equation $y=x^2+2x-5$.

(i) Express the equation in the form $a(y-k)=(x-h)^2$, where a, h, and k are constants.

(ii) Write down the coordinates of the vertex and the coordinates of the focus.

(c) The roots of the equation $x^2-4x+7=0$ are α and β. Evaluate $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$.

QUESTION 4. [Start a *new* page]

A particle is moving in a straight line such that its displacement x from a fixed point O is given by:

$x = 3t^2 - t^3$, x in metres, t (in seconds) ≥ 0.

(i) Find the initial velocity and acceleration of the particle.

(ii) Find the stationary points on the displacement-time curve and state their nature.

(iii) Evaluate time at which the acceleration of the particle is zero.

(iv) Graph the function $x(t)$, $t \geq 0$.

(v) Evaluate the total distance travelled by the particle in the first 4 seconds.

(vi) Describe the motion, making use of the information obtained above.

QUESTION 5. [Start a *new* page]

(a) The first term of an arithmetic sequence is 1 and the fifteenth is 29.

(i) Find the value of the common difference.

(ii) Find the sum of the first 150 terms.

(b) The equation of a parabola is $y = \frac{1}{4}x^2$ and the points P(8, p) and Q(q, 1) are on the curve.

(i) If P and Q are on opposite sides of the axis of symmetry of the parabola, find p and q.

(ii) Show that the mid point M of segment PQ is $(3, 8\frac{1}{2})$.

(iii) The equation of the tangent to the parabola at Q is $y = -x - 1$. Show that the equation of the tangent at P is $y = 4x - 16$.

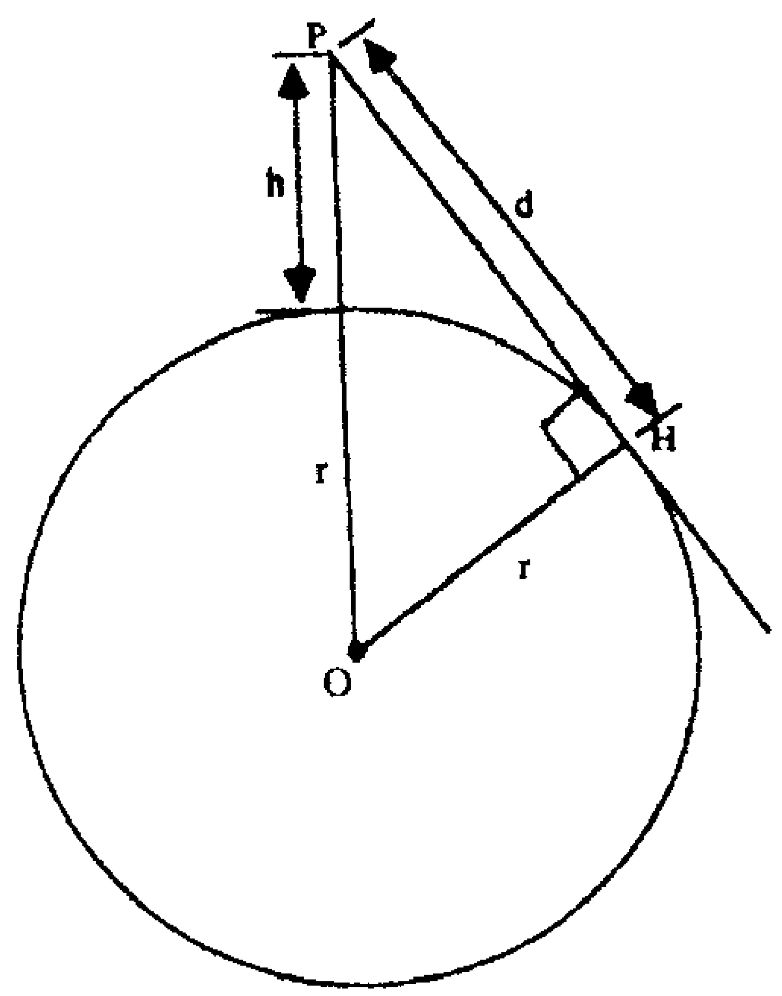
(iv) Find the coordinates of the point T, the intersection of the two tangents.

(v) Show that the parabola bisects the segment TM.

QUESTION 6. [Start a *new* page]

(a) Find the area of a sector of 20° cut from a circle of radius 5 cm.

(b) The diagram (not to scale) represents the planet Mars (spherical shape assumed) and an astronaut at P, h metres above the surface. PH is the distance d, in metres, to the horizon, and r is the radius of Mars, also in metres.



(i) Show that d and h are connected by the relation $h^2 + 2hr - d^2 = 0$.

(ii) Hence show that $h = r\sqrt{1 + \frac{d^2}{r^2}} - r$.

(iii) When k is very small, say $k < 10^{-4}$, then $\sqrt{1+k}$ can be approximated by $1 + \frac{1}{2}k$.

Show that in such a case, $h = \frac{d^2}{2r}$ approximately.

(iv) If the observer at P has his horizon 20 km distant and the radius of Mars is 3398 km,

(1) Show that the formula in (iii), $h = \frac{d^2}{2r}$, is valid to calculate h.

(2) How many metres is the astronaut above the surface of Mars ?

QUESTION 7. [Start a *new* page]

(a) Triangle ABC has area equal to 5 cm ^{2}. If AC = 4 cm and BC = 5 cm, find $\angle$ ACB.

(b) A mining company started three mining towns A, B, and C with a population of 500 each and it was planned that they should grow by roughly 50 inhabitants each, each year for the first 10 years. Only town A grew as planned!

(i) Write down an expression for the intended population P of the town A, t years after its opening ($t \leq 10$).

(ii) For various reasons, towns B and C did not grow as planned. Their populations are better modelled by:

Town B: $\dfrac{dP}{dt} = -0.3\,P$

Town C: $P = 100\left(5 + t - \dfrac{1}{4}t^2\right)$

(1) Prove that the population for town B is given by $P = 500e^{-0.3t}$ after t years.

(2) Calculate the population of town B after 7 years.

(3) Find the rate of change of the population of town C after 1, 2, and 3 years.

(4) What was the maximum population of town C? And in which year was it reached?

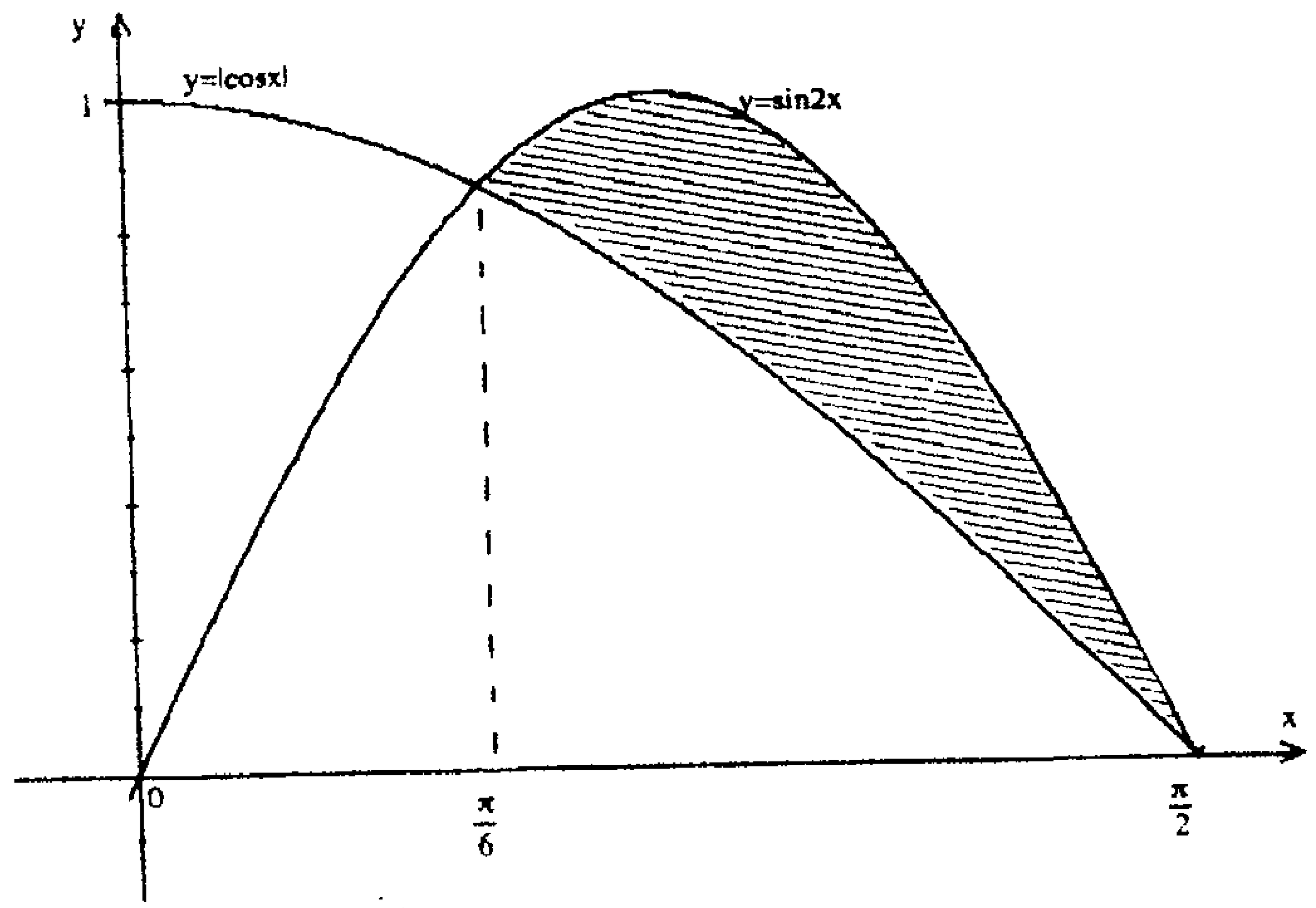
(5) The mining company considers the mines unprofitable and will close the towns if the population gets to below 50. Show that town C will be abandoned before town B.

QUESTION 8. [Start a *new* page]

(a) Graph the solution of $2 - x \geq 0$ on the number line.

(b) Differentiate $f(x) = \dfrac{x^2}{\ln x}$ and find $f'(e^2)$.

(c) The diagram below shows the graphs of $y = |\cos x|$ and $y = \sin 2x$ between 0 and $\dfrac{\pi}{2}$. The area completely enclosed by the two curves is shaded. Show that the shaded area is equal to $\dfrac{1}{4}$ square units.



(d) (i) Copy the diagram of (c) and extend it to $x = 2\pi$, that is, graph $y = |\cos x|$ and $y = \sin 2x$ between 0 and 2π.

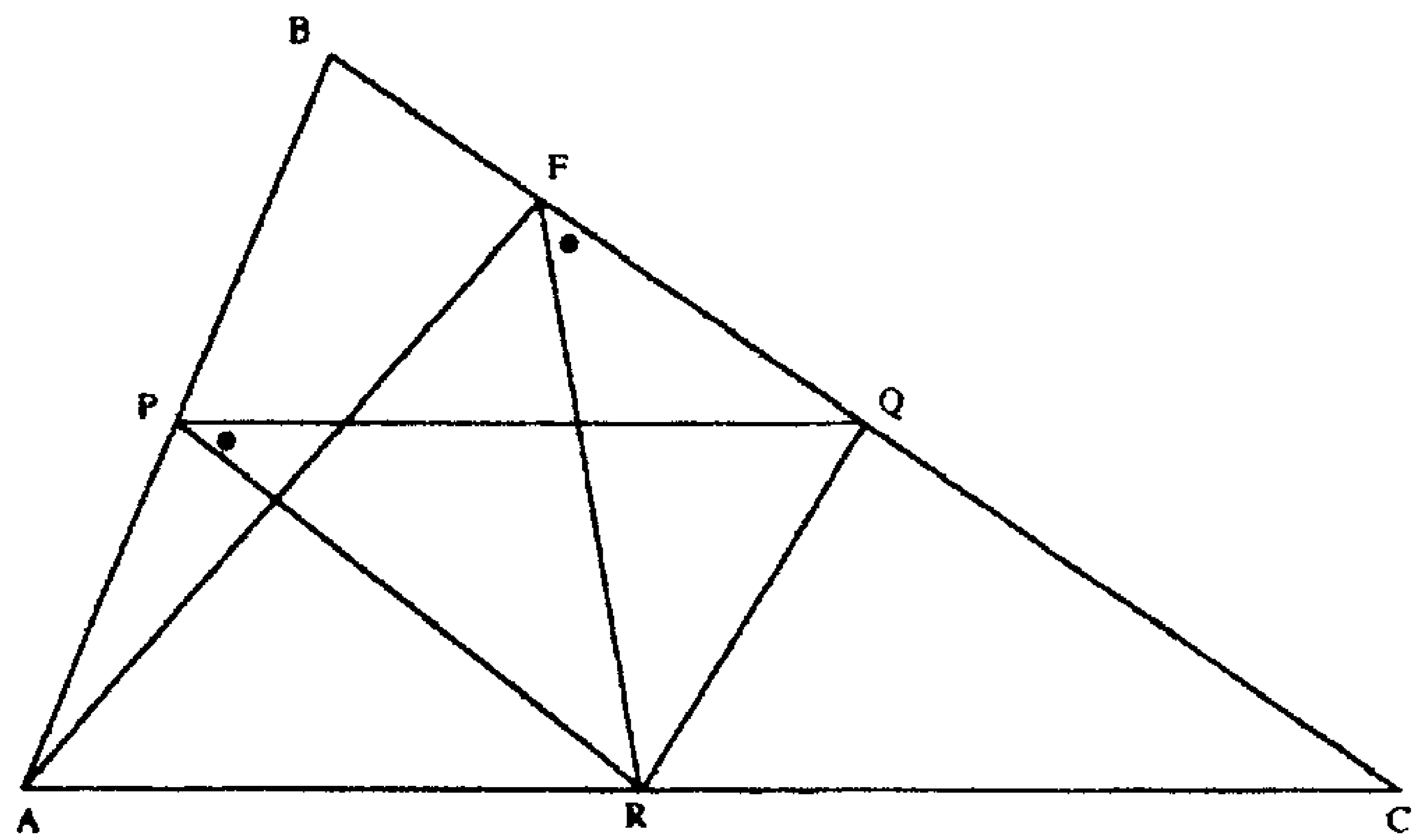
(ii) Find the area of the 3 regions now enclosed by the two curves for $\dfrac{\pi}{6} \leq x \leq \dfrac{3\pi}{2}$.

QUESTION 9. [Start a *new* page]

(a) (i) Graph $y = \dfrac{2x}{x^2+1}$.

(ii) Find the area under the curve, the x axis and the lines $x = -1$ and $x = 2$.

(b) In ΔABC, P, Q, and R are mid-points of sides AB, BC, and CA, respectively. F is a point in side BC, such that $\angle RPQ = \angle RFQ$.



Show that:

(i) AC = 2PQ (give reasons)

(ii) ΔABC is similar to ΔQRP (give reasons)

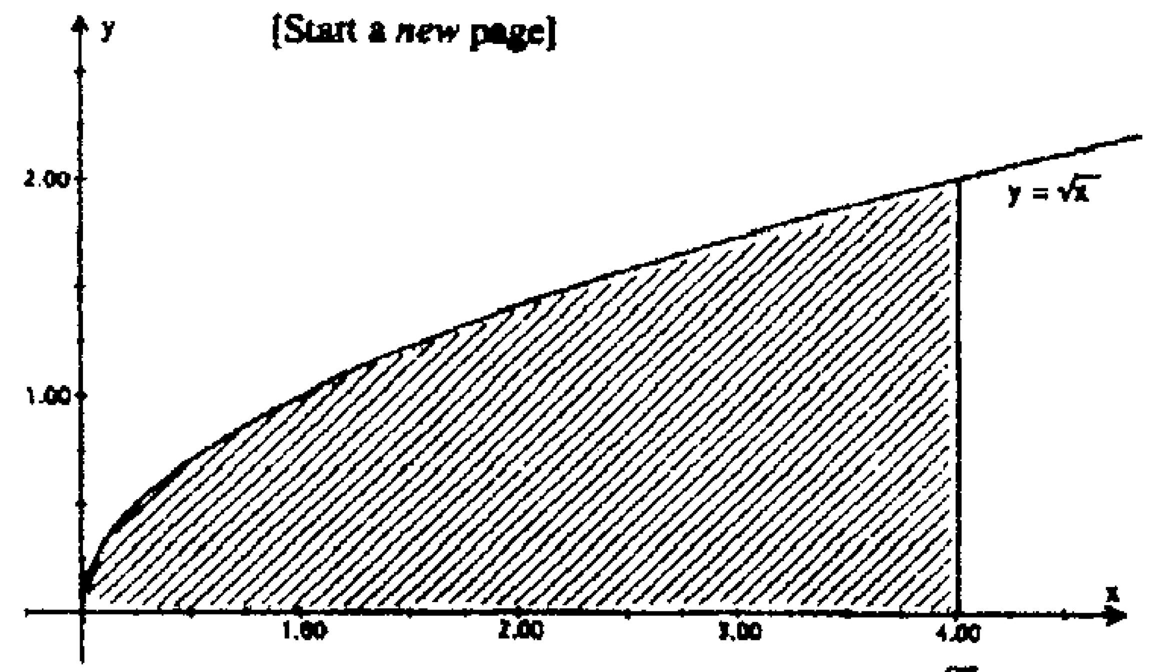
(iii) $\angle ACB = \angle RFQ$ (give reasons)

(iv) $\angle RFA = \angle FAR$ (give reasons)

(v) line AF is perpendicular to line FC (give reasons).

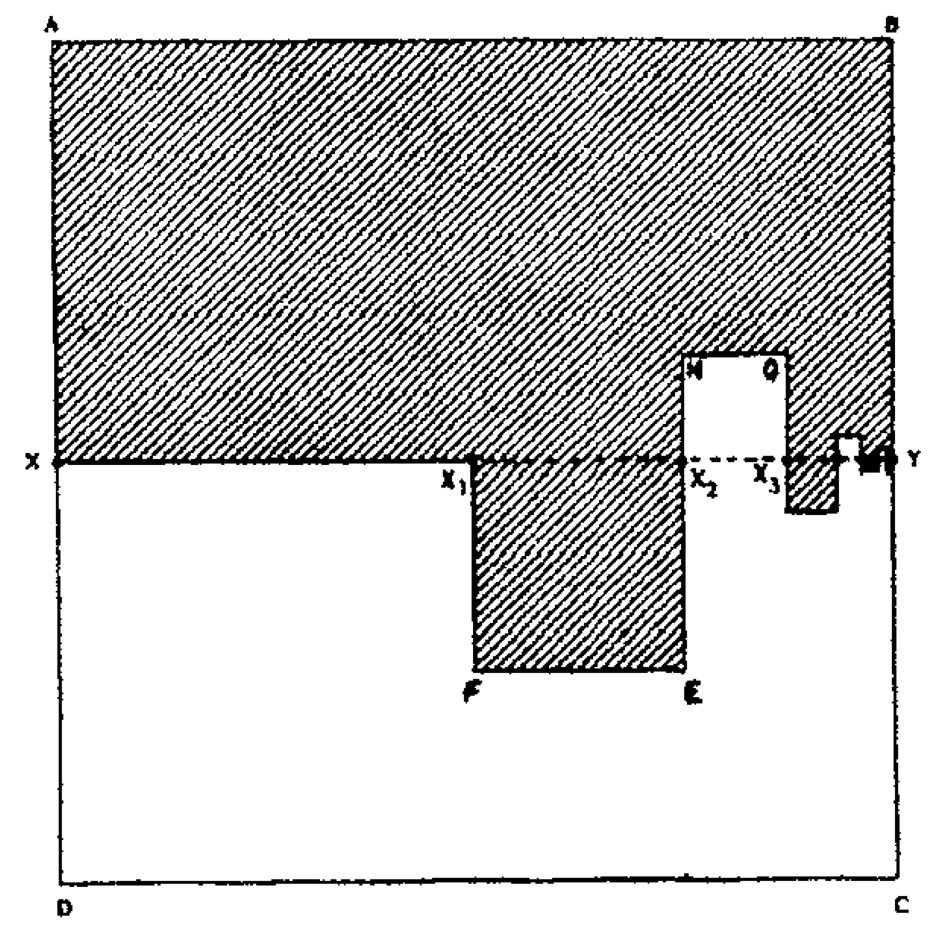
QUESTION 10. [Start a *new* page]

(a)



The shaded region is bounded by the lines $y = 0$, $x = 4$, and $y = \sqrt{x}$.
Find the volume of the solid of revolution when the shaded region is rotated about the y-axis.

(b)



$[ABCD]$ is a square of side a. X and Y are mid points of AD and BC respectively.
X_1 is the mid point of XY and X_1X_2EF is a square.
X_2 is the mid point of X_1Y and X_2X_3GH is also a square.
X_3 is the mid point of X_2Y ....

This process is continued indefinitely, with squares smaller and smaller alternately above and below the line XY.

(i) Express the area of square X_1X_2EF as a function of a, the side of the square ABCD.

(ii) Calculate the shaded area as a fraction of the area of the square ABCD.

(c) Given the relation $\dfrac{y}{x} + \dfrac{x+2}{y+1} = 2$, $x \neq 0$, $y \neq -1$

(i) Show that $y^2 + y(1-2x) + x^2 = 0$.

(ii) Find the greatest integer x for which y is real and rational.

JRAHS 2U 1998 TRIAL

QUESTION 1

(a) 3.8

(b) \$50

(c) $f(1) = \frac{1}{2}$

$f(2) = \frac{1}{9}$

(ii) $1\frac{1}{18}$

(d) $-\frac{1}{4}$

(e) $(a+b)(a-b+2)$

QUESTION 2

(a) $x = 7$

(b)(i) $x = -8, 2$

(ii) $x \leq -8,\ x \geq 2$

(c)(i)(1) $2/7$

(2) $4/7$

(ii) $6/17$

QUESTION 3

(a)(i) $60°$

(ii) —

(iii) $32\sqrt{3}$ cm²

(b)(i) $(x+1)^2 = 4(\frac{1}{4})(y+6)$

(ii) $V(-1, -6)$

$S(-1, -5\frac{3}{4})$

(iii) $4/7$

QUESTION 4

(i) $v = 0$ m/s

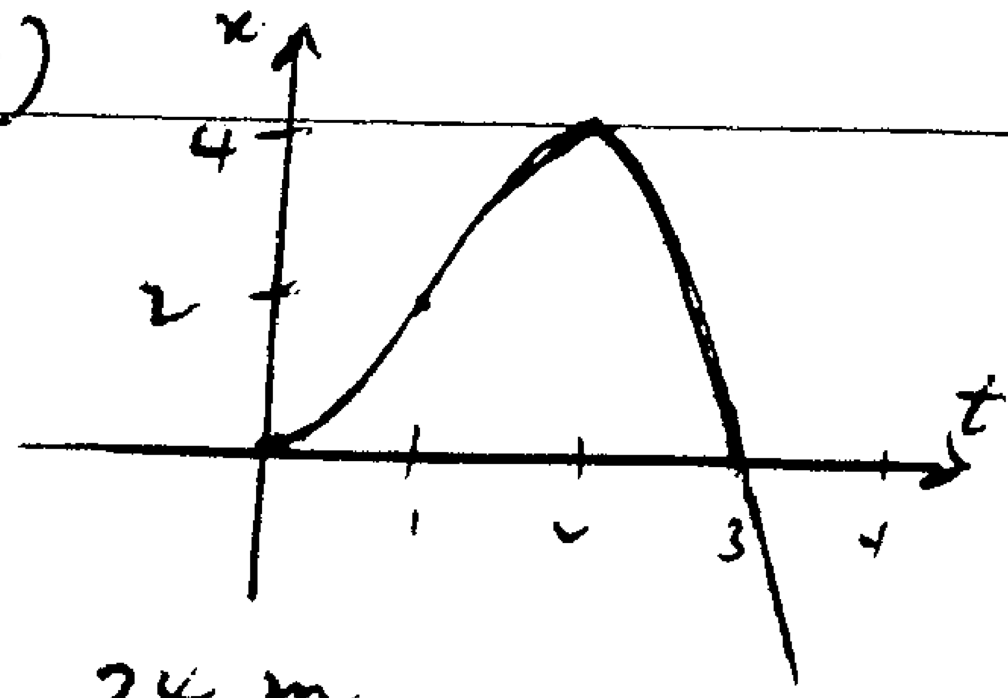
$a = 6$ m/s²

(ii) min tp $(0,0)$

max tp $(2,4)$

(iii) $t = 1$ sec.

(iv)



(v) 24 m

(vi) —

QUESTION 5

(a)(i) $d = 2$

(ii) $S_{150} = 22500$

(b)(i) $p = 16,\ q = -2$

(ii) —

(iii) $y = 4x - 16$

(iv) $T(3, -4)$

(v) midpt of TM $(3, 2\frac{1}{4})$ lies on parabola.

QUESTION 6

(a) $\frac{25\pi}{8}$ cm²

(b)(i) —

(ii) —

(iii) —

(iv)(1) —

(2) $\doteqdot 59$ metres

QUESTION 7

(a) $30°$ or $150°$

(b)(i) $P = 500 + 50t$

(ii)(1) —

(2) $\doteqdot 61$ people

(3) 50, 0, −50

(4) 600

(5) when $t = 7$,

population of B $\doteqdot 61$

population of C < 0

∴ C closes before B

QUESTION 8

(a)

(b) $f'(x) = \frac{2x\ln x - x}{(\ln x)^2}$

$f'(e^2) = \frac{3e^2}{4}$

(c) $\frac{1}{4}$

(d)(i) —

(ii) $2\frac{3}{4}$

QUESTION 9

(a)(i) —

(ii) $\ln 10$

(b) —

QUESTION 10

(a) $128\pi/5$

(b)(i) $\frac{1}{6}a^2$

(ii) $11/20$

(c)(i) —

(ii) $x = -2$