QUESTION 1

(a) Find the exact value of $\sin\left(\frac{\pi}{4}\right)$. 1

(b) 1500 identical sheets of paper are laid on top of each other to form a pile of sheets 12cm high. Find the thickness of an individual sheet of paper. 2
Give your answer in millimeters.

(c) Factorise $2p^2 + p - 6$. 2

(d) Solve $2\cos\theta = 1$ for $0^\circ \le \theta \le 360^\circ$. 2

(e) Simplify $\frac{3}{x+1} - \frac{2}{x^2-1}$. 2

(f) Evaluate $\int_1^4 y\sqrt{y}\, dy$ 3

QUESTION 2 (START A NEW PAGE)

(a) Differentiate with respect to x:

(i) $x^4 - \frac{3}{x}$, 2

(ii) $\log_e(7-3x)$. 1

(b) Find the equation of the tangent to $y = \sqrt{x+3}$ at the point (1,2). 3

(c) (i) The lines $y = 2x$ and $x + 2y = 20$ meet at point A. Find the coordinates of A. 2

(ii) The line $x + 2y = 20$ meets the *x-axis* at the point B and M is the midpoint of AB. Find the coordinates of the points B and M. 2

(iii) Given that O is the origin, show that ΔOAM is isosceles. 2

QUESTION 3 (START A NEW PAGE)

(a) The gradient of a curve $y = f(x)$ is given by $f'(x) = \dfrac{2x^2+1}{x}$. Find the equation of the curve if it passes through the point (1,3). 3

(b) An arc PQ has length 12cm and is drawn on the circumference of a circle with radius 8cm. Find

(i) the size of the angle subtended at the center of the circle by the arc PQ, 1

(ii) the length of the chord PQ, correct to 2 decimal places. 2

(c) (i) Sketch the parabola $y = x^2 - 4x - 12$, clearly showing its intercepts with the coordinate axes and the coordinates of its vertex. 4

(ii) Hence, or otherwise, solve $x^2 - 4x - 12 \geq 0$. 2

QUESTION 4 (START A NEW PAGE)

(a) Given the parabola $y = \frac{1}{2}x^2 - 3x + 1$,

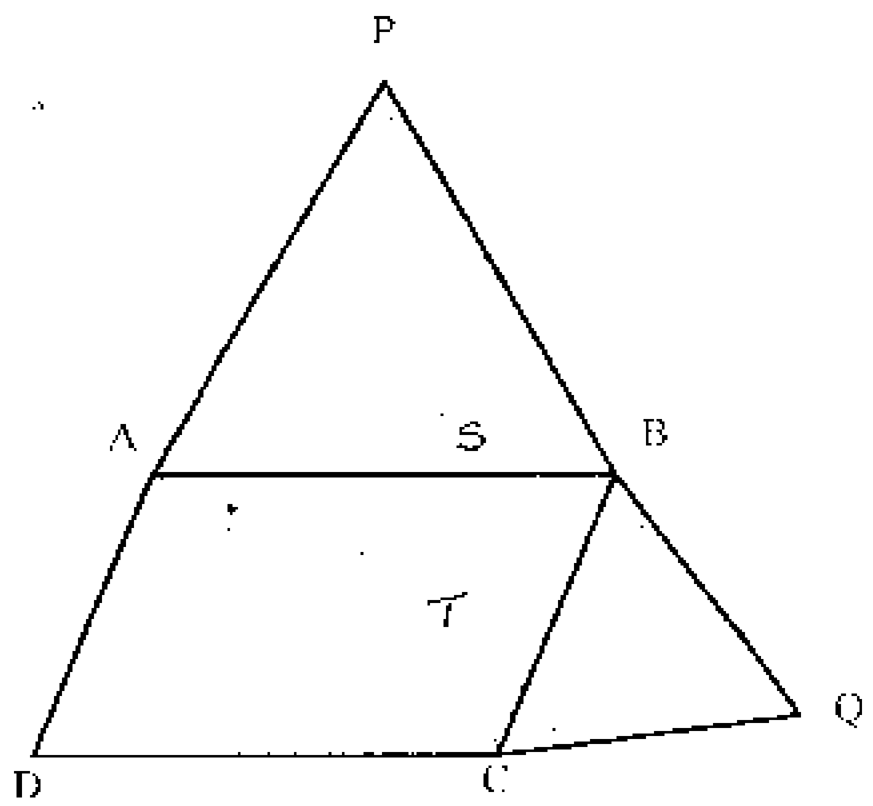
(i) Express the equation in the form $(x-h)^2 = 4a(y-k)$, where a, h and k are constants. 3

(ii) Write down the coordinates of the focus of this parabola. 2

(b) $ABCD$ is a parallelogram. ΔAPB and ΔBQC are equilateral. (see diagram)

(i) Prove that $\Delta ABQ \equiv \Delta PBC$. 4

(ii) Find the size of the acute angle between AQ and PC. (Give reasons) 3



QUESTION 5 (START A NEW PAGE)

(a) The 4^{th} term of an Arithmetic Progression is 30 and its 10^{th} term is 54. Find

(i) the common difference and the first term, 4

(ii) the sum of the first 20 terms. 2

(b) An object, initially at rest at the origin, moves in a straight line with velocity $v\ ms^{-1}$ so that $v = 4t(5-t)$ where t is the time elapsed in seconds. Find

(i) the acceleration of the object at the end of the third second, 2

(ii) an expression for the displacement x metres of the particle in terms of t, 2

(iii) the position of the particle when it again comes to rest. 2

QUESTION 6 (START A NEW PAGE)

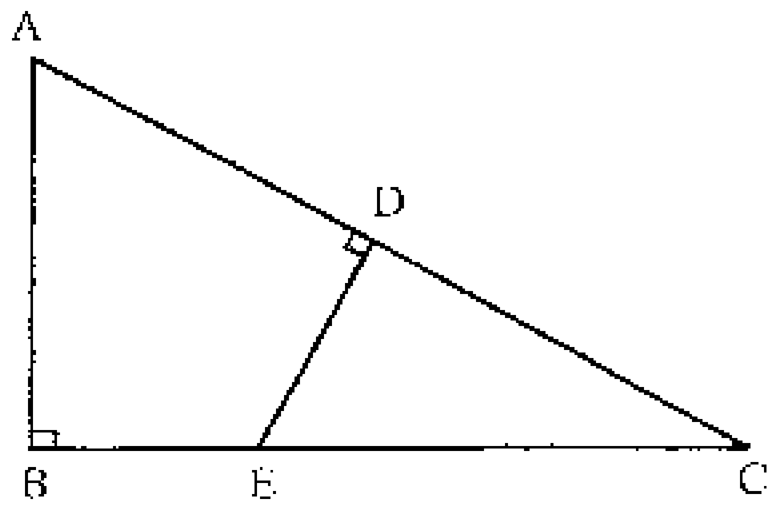
(a) ΔABC is right-angled at B and DE is perpendicular to AC (see diagram)

(i) Prove that ΔABC and ΔCDE are similar. 2

(ii) Prove that $BC \times CE = AC \times CD$ 2

(iii) Prove that: 2

$$DE^2 = AD \times DC - BE \times EC$$



(b) (i) Sketch the parabola $y = x^2 - 4x$ and the line $y = 2x$. 3
Clearly show their points of intersection.

(ii) Find the area bounded by the above curves. 3

QUESTION 7 (START A NEW PAGE)

(a) Water flows into and out of a tank at a rate (in litres/hour) given by $R = 2\pi \sin \pi t$. If the tank is initially empty at 10am, find:

(i) The first time (after 10am) when the tank is filling at its greatest rate. 2

(ii) An expression for the volume (V litres) of water in the tank after t hours. 2

(iii) The maximum volume of water in the tank. 1

(b) Given the curve $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$, $x > 0$.

(i) Find the coordinates of any stationary points and determine their nature. 3

(ii) Find the coordinates of any points of inflexion. 2

(iii) Sketch the curve showing all stationary points and inflexion points. 2

QUESTION 8 (START A NEW PAGE)

(a) (i) Prove that the line with equation $y = px + (1 - 2p^2)$ is a tangent to the parabola $x^2 = 8(y - 1)$ for all values of p. 3

(ii) Find the angle between the tangents drawn to $x^2 = 8(y - 1)$ from the point $(0, -7)$. 3

(b) (i) If $f(x) = \sin^2 x$, find $f'(x)$. 1

(ii) The area bounded by the curve $y = \sin x + \cos x$ and the *x-axis* for $0 \le x \le 2\pi$ is rotated one revolution about the *x-axis*. Find the volume of the solid formed. 5

QUESTION 9 **(START A NEW PAGE)**

(a) In a hat are six red and four green discs. Two discs are chosen at random from the hat one disc before the other without replacing the first disc that was chosen.

(i) Draw a probability tree diagram for the above information. 2

Find the probability that:

(ii) two red discs are chosen, 1

(iii) the second disc chosen is green, 1

(iv) at least one green is chosen. 1

(b) Figure 1 shows the end view of a small rectangular aquarium filled with water to a depth of 1 metre. The end of the tank has dimensions 2m by 1.5m. Figure 2 shows the same aquarium with the base tilted 30^0 to the horizontal.

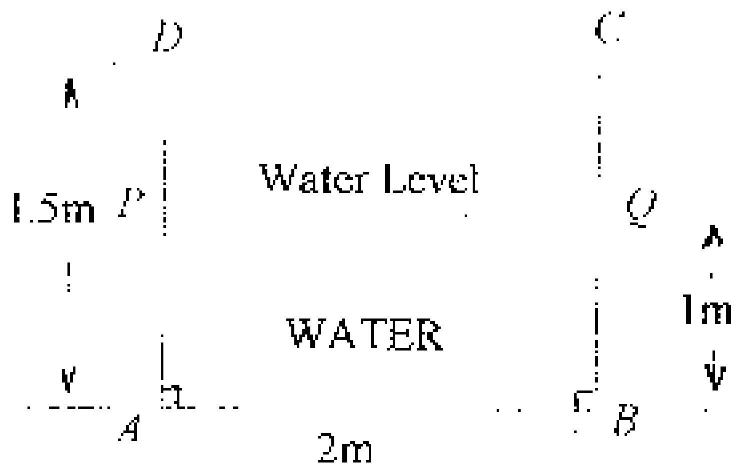


Figure 1

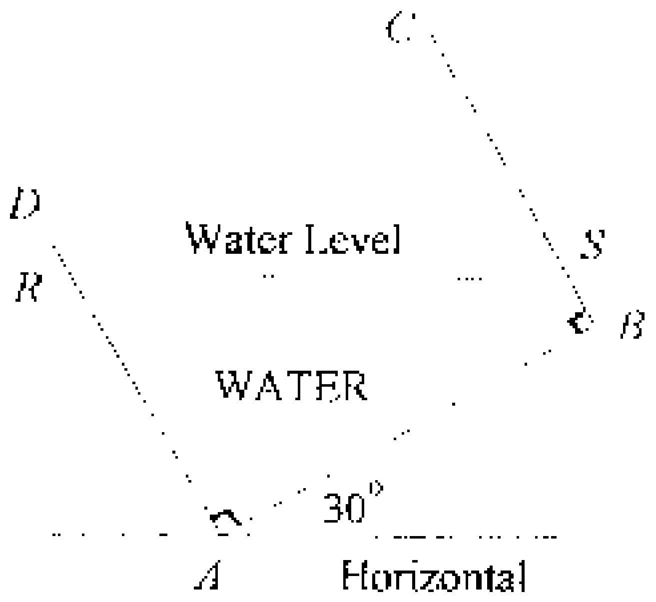


Figure 2

(i) Show that the length $SB = \frac{\sqrt{3}-1}{\sqrt{3}}$. 3

(ii) Find the height of R above the horizontal. 4

QUESTION 10 **(START A NEW PAGE)**

(a) The number (N) of bacteria in a colony after t minutes is given by the formula $N = 200e^{kt}$. If the population grows to 5000 in 40 minutes, find

(i) the exact value of k, 2

(ii) the number of bacteria in the colony at the end of 1 hour. 2

(b) The velocity of a train increases from 0 to V at a constant rate a. The velocity then remains constant at V for a certain time. After this time the velocity decreases to 0 at a constant rate b. Given that the total distance travelled by the train is s and the time for the journey is T,

(i) Draw a velocity-time graph for the above information, 2

(ii) Show that the time (T) for the journey is given by $T = \frac{s}{V} + \frac{1}{2}V\left(\frac{1}{a} + \frac{1}{b}\right)$. 3

(iii) When a, b and s are fixed, find the speed that will minimize the time for the journey. 3

THIS IS THE END OF THE EXAMINATION PAPER

Question 1

(a) $1/\sqrt{2}$

(b) thickness $= \frac{120}{1500}$ mm

$= 0.08$ mm

(c) $(2p-3)(p+2)$

(d) $\cos\theta = \frac{1}{2}$

$\theta = 60°, 300°$

(e) $\frac{3(x-1)-2}{(x-1)(x+1)} = \frac{3x-5}{x^2-1}$

(f) $\int_1^4 y^{3/2}\,dy = \left[\frac{2}{5}y^{5/2}\right]_1^4$

$= \frac{2}{5}(4^{5/2}-1^{5/2})$

$= \frac{2}{5}(32-1)$

$= 62/5$

Question 2

(a)(i) $y = x^4 - 3x^{-1}$

$y' = 4x^3 + 3x^{-2}$

$= 4x^3 + \frac{3}{x^2}$

(ii) $y' = \frac{-3}{7-3x}$

(b) $y = (x+3)^{1/2}$

$y' = \frac{1}{2}(x+3)^{-1/2}\cdot 1$

$= \frac{1}{2\sqrt{x+3}}$

when $x = 1$, $y' = \frac{1}{2\sqrt{4}}$

$= \frac{1}{4}$

Tangent: $y - 2 = \frac{1}{4}(x-1)$

$4y - 8 = x - 1$

$x - 4y + 7 = 0$

(c) (i) $y = 2x$ —①

$x + 2y = 20$ —②

sub. ① into ②

$5x = 20$

$x = 4$

$\therefore y = 8$

A is $(4, 8)$

(ii) at B, $y = 0$ $\therefore$ $x = 20$

B is $(20, 0)$

$M\left(\frac{20+4}{2}, \frac{0+8}{2}\right) = M(12, 4)$

(iii) $OA = \sqrt{4^2+8^2}$

$= \sqrt{80}$

$AM = \sqrt{(-8)^2+4^2}$

$= \sqrt{80}$

$\therefore OA = AM$ $\therefore \Delta$ is isosceles (two equal sides).

Question 3

(a) $f'(x) = 2x + 1/x$

$f(x) = x^2 + \ln x + c$

at $(1, 3)$ $3 = 1 + \ln 1 + c$

$c = 2$

$f(x) = x^2 + \ln x + 2$

(b) (i) $l = r\theta$

$12 = 8\theta$

$\theta = 1\frac{1}{2}$

(ii) $PQ^2 = 8^2 + 8^2 - 2(8)(8)\cos 1.5$

$= 128 - 128\cos 1.5$

$PQ = 10.91$ (to 2 dp)

(c) $y = (x-6)(x+2)$

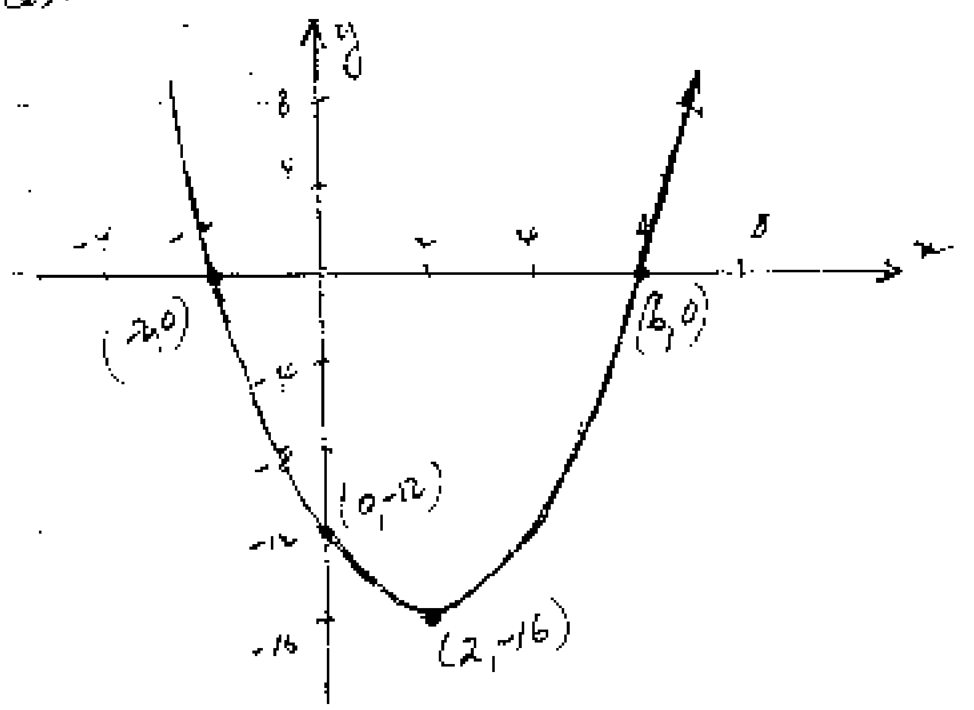
x-int: $(-2, 0)$, $(6, 0)$

y-int: $(0, -6)$

axis $x = 2$

vertex $(2, -16)$

Q3 (cont)



(ii) $x \le -2$ or $x \ge 6$

QUESTION 4

(a) (i) $x^2 - 6x = 2y - 2$

$x^2 - 6x + 9 = 2y + 7$

$(x-3)^2 = 2(y + 3\tfrac{1}{2})$

$(x-3)^2 = 4(\tfrac{1}{2})(y + 3\tfrac{1}{2})$

(ii) ~~Focus (3, 4)~~ $(3, -3)$

(b) (i) In ΔABQ & ΔPBC

$AB = PB$ (equal sides of equilateral ΔAPB)

$BQ = BC$ (equal sides of equilateral ΔBQC)

$A\hat{B}Q = P\hat{B}C$ (both $60° + A\hat{B}C$, all angles of equilateral triangle are $60°$)

$\therefore \Delta ABQ \equiv \Delta PBC$ (SAS)

(ii) Let $B\hat{P}C = \theta°$

$\therefore B\hat{A}Q = \theta°$ (corresponding angles in congruent triangles)

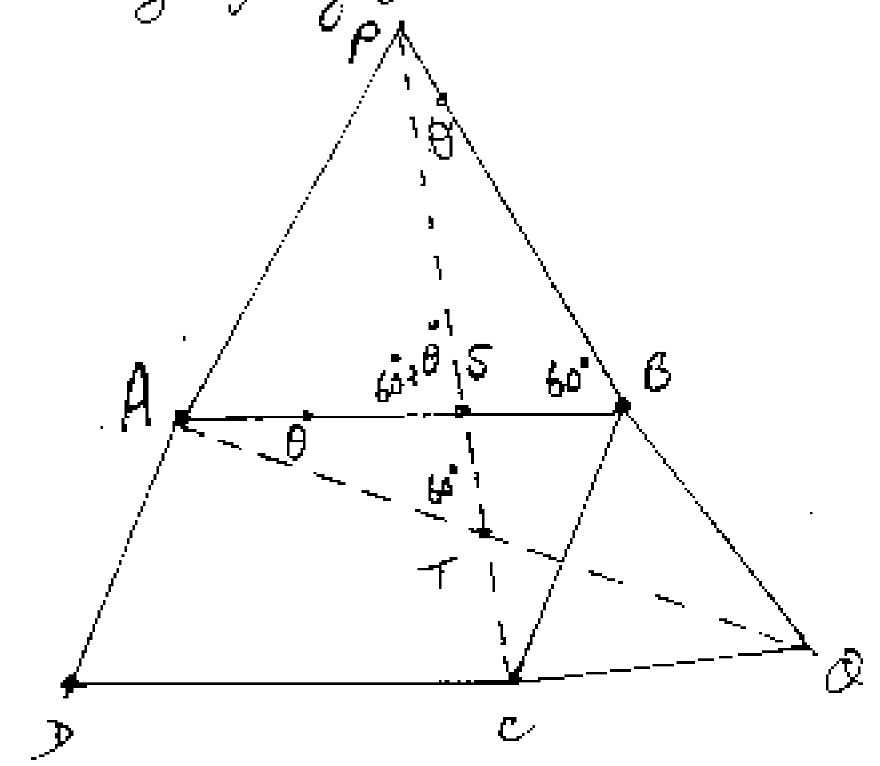
Let AB & PC meet at S.

$P\hat{S}A = \theta° + 60°$ (exterior angle of ΔPSB equals sum of opposite interior angles, $P\hat{B}S = 60°$)

Let AQ & PC meet at T

$\therefore P\hat{T}A = 60°$ (exterior angle of ΔAST equals sum of opposite interior angles)

$\therefore$ size of angle $= 60°$



QUESTION 5

(a) (i) $T_4 = a + 3d \Rightarrow a + 3d = 30$ — ①

$T_{10} = a + 9d \Rightarrow a + 9d = 54$ — ②

② − ①: $6d = 24$

$d = 4$

sub into ① $a = 30 - 3d$

$= 30 - 12$

$= 18$

$\therefore$ common diff $= 4$ first term $= 18$

(ii) $S_{20} = \frac{n}{2}(2a + (n-1)d)$

$= 10(36 + 19(4))$

$= 1120$

(b)(i) $v = 20t - 4t^2$

$a = 20 - 8t$

when $t = 3$, $a = 20 - 24$

$= -4 \text{ ms}^{-2}$

Q5(b) cont

(ii) $x = 10t^2 - \frac{4}{3}t^3 + c$

when $t=0$, $x=0$ $\therefore c=0$

$x = 10t^2 - \frac{4}{3}t^3$

(iii) when $v=0$ $\therefore$ $4t(5-t) = 0$

$t = 0, 5$

when $t=5$, $x = 10(5)^2 - \frac{4}{3}(5)^3$

$= 83\frac{1}{3}$ m.

QUESTION 6

(a)(i) In ΔABC & ΔEDC

$A\hat{B}C = E\hat{D}C$ (both 90°)

$A\hat{C}B = E\hat{C}D$ (common)

$\therefore \Delta ABC ||| \Delta EDC$ (equiangular)

(ii) $\frac{BC}{DC} = \frac{AC}{EC}$ (ratio of corresponding sides)

$BC \times EC = AC \times DC$

(iii) $AC = AD + DC$

$BC = BE + EC$

$(BE+EC)EC = (AD+DC)DC$

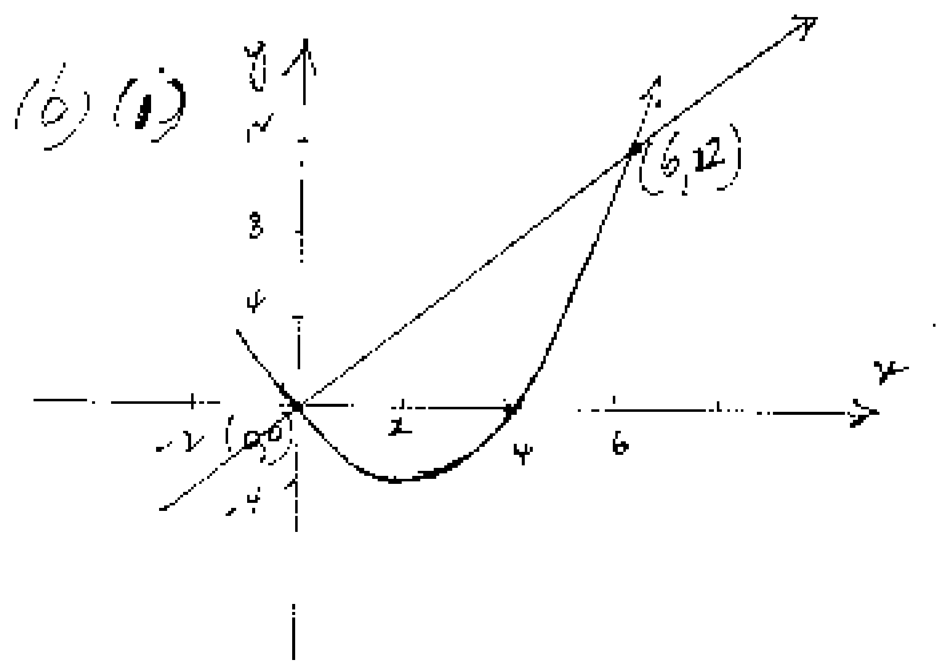
$BE.EC + EC^2 = AD.DC + DC^2$

$EC^2 - DC^2 = AD.DC - BE.EC$

$ED^2 = AD.DC - BE.EC$

(since $EC^2 - DC^2 = ED^2$ by Pythag. Theorem)

(b) (i)



$y = 2x$, $y = x^2 - 4x$

$x^2 - 4x = 2x$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0, 6$

$x=0$, $y=0$ (0,0)

$x=6$, $y=12$ (6,12)

(ii) $A = \int_0^6 2x - (x^2 - 4x)\, dx$

$= \int_0^6 6x - x^2\, dx$

$= \left[3x^2 - \frac{1}{3}x^3\right]_0^6$

$= 3(6)^2 - \frac{1}{3}(6)^3 - 0$

$= 36\ u^2$

QUESTION 7

(a)(i) $R = 2\pi \sin \pi t$

max R when $\pi t = \pi/2$

$t = \frac{1}{2}$

$\therefore$ time $= 10{\cdot}30$ am

(ii) $V = \int 2\pi \sin \pi t\, dt$

$= -2\cos \pi t + c$

$t=0$, $V=0 \Rightarrow 0 = -2 + c$

$c = 2$

$\therefore V = 2 - 2\cos \pi t$ l

(iii) max vol. $= 4$ l (when $\cos \pi t = -1$)

(b)(i) $y = x^{1/2} + x^{-1/2}$

$y' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$

$= \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$

for stat pt $y' = 0$

$\frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}} = 0$

Q7 (cont)

$2x\sqrt{x} = 2\sqrt{x}$

$2\sqrt{x}(x-1) = 0$

$x = 1 \quad (x>0)$

when $x=1$, $y=2$ stat pt $(1,2)$

$y'' = -\frac{1}{4}x^{-1\frac{1}{2}} + \frac{3}{4}x^{-2\frac{1}{2}}$

when $x=1$, $y'' = -\frac{1}{4}+\frac{3}{4}$

> 0

∴ concave up ∴ local min tp.

(ii) $y'' = 0$

$-\frac{1}{4x\sqrt{x}} + \frac{3}{4x^2\sqrt{x}} = 0$

$x^2\sqrt{x} - 3x\sqrt{x} = 0$

$x\sqrt{x}(x-3) = 0$

$x = 3 \quad (x>0)$

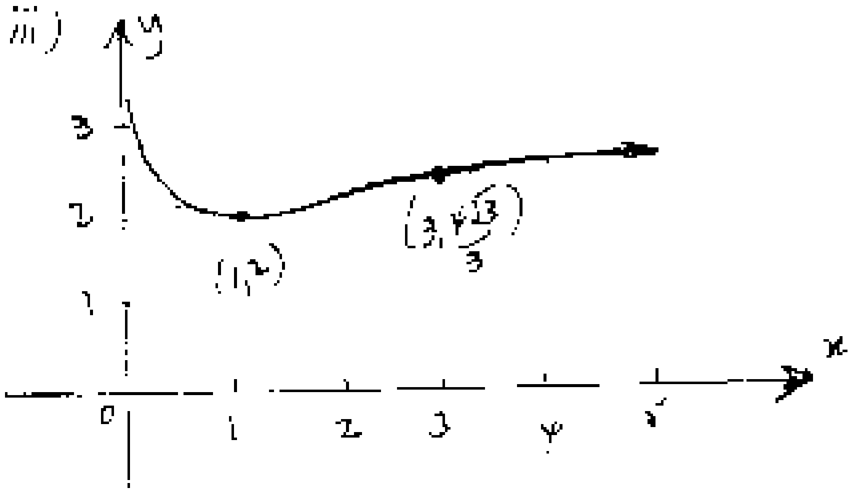
x	1	3	4
y''	$\frac{1}{2}$ >0	0	$-\frac{1}{128}$ <0

Since curve is cts for $1<x<4$ & concavity changes then there is an inflexion pt when $x=3$

$x=3$, $y = \sqrt{3} + \frac{1}{\sqrt{3}}$

$= \frac{4}{3}\sqrt{3}$

(iii)



QUESTION 8

(a)(i) $x^2 = 8y - 8$ —①

$y = px + (1-2p^2)$ —②

sub ② into ①

$x^2 = 8px + 8(1-2p^2) - 8$

$= 8px - 16p^2$

$x^2 - 8px + 16p^2 = 0$

quadratic has only one solution if $\Delta = 0$ & ∴ line will be a tangent to parabola

$\Delta = (-8p)^2 - 4(16p^2)$

$= 64p^2 - 64p^2$

$= 0$ for all p

∴ line is tangent to parabola

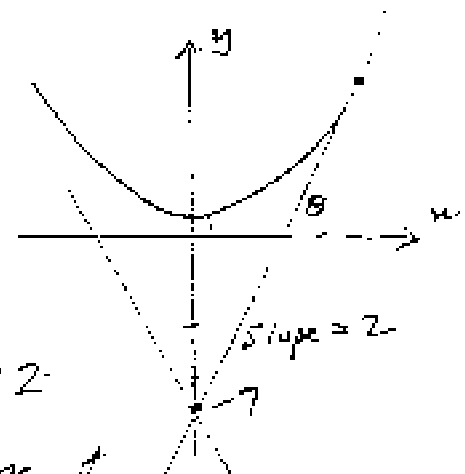
(ii) if tangents pass thru' $(0,-7)$

$-7 = 1 - 2p^2$

$2p^2 = 8$

$p^2 = 4$

$p = \pm 2$



∴ slope of tangents are ± 2

∴ angle (θ) between tangent an x-axis is given by $\tan\theta = 2$

∴ $\theta = 63°26'$

∴ angle between y-axis & tangent $= 90 - \theta$

$= 26°34'$

∴ angle between tangents $= 53°08'$

(b) (i) $f'(x) = 2\sin x\cos x$

(ii) $V = \pi\int_0^{2\pi} (\sin x + \cos x)^2\,dx$

$= \pi\int_0^{2\pi} \sin^2 x + \cos^2 x + 2\sin x\cos x\,dx$

8 (cont.)

$$= \pi \int_0^{2\pi} 1 + 2\sin x \cos x \, dx$$

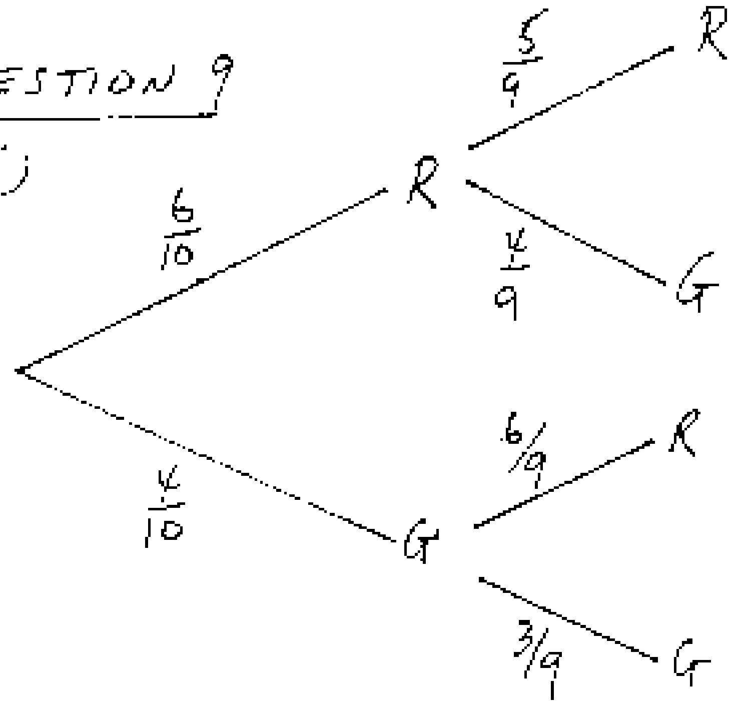
$$= \pi \left[x + \sin^2 x \right]_0^{2\pi}$$

$$= \pi \left\{ (2\pi + 0) - (0 + 0) \right\}$$

$$= 2\pi^2 \ u^3$$

QUESTION 9

(a)(i)



(ii) $P(RR) = \frac{6}{10} \times \frac{5}{9}$

$= 1/3$

(iii) $P(\text{2nd } G) = P(RG) + P(GG)$

$= \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{3}{9}$

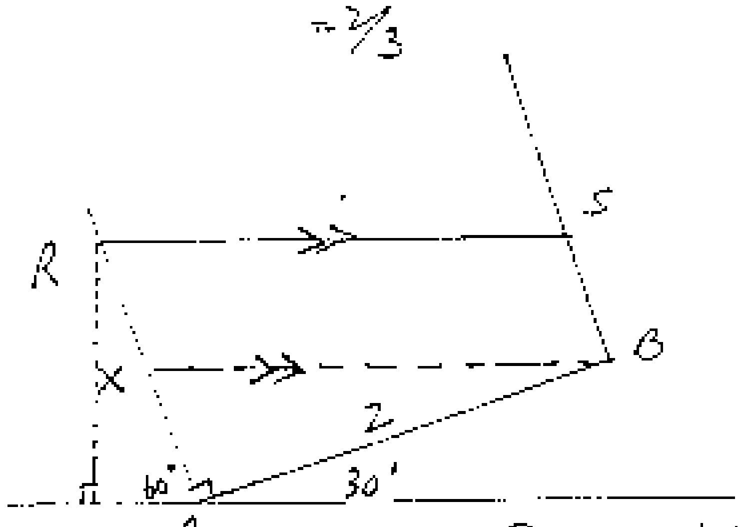
$= 2/5$

(iv) $P(\text{at least } 1G) = 1 - P(RR)$

$= 1 - 1/3$

$= 2/3$

(b)



Draw XB // RS

$\frac{XA}{2} = \tan 30°$

$AX = 2/\sqrt{3}$

Also area ARSB = area APQB = 2

Let SB = x (= RX since XRSB is a parallelogram)

$\therefore \frac{1}{2}(2)\left(x + 2/\sqrt{3} + x\right) = 2$

$2x + 2/\sqrt{3} = 2$

$x = 1 - 1/\sqrt{3}$

$x = \frac{\sqrt{3} - 1}{\sqrt{3}} = SB$

(ii) $\frac{\text{height}}{RA} = \sin 60°$

$RA = 1 - \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}} = 1 + \frac{1}{\sqrt{3}}$

$h = \frac{\sqrt{3} + 1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$

$\text{height} = \frac{\sqrt{3} + 1}{2}$

QUESTION 10

(a)(i) $5000 = 200e^{40k}$

$25 = e^{40k}$

$40k = \ln 25$

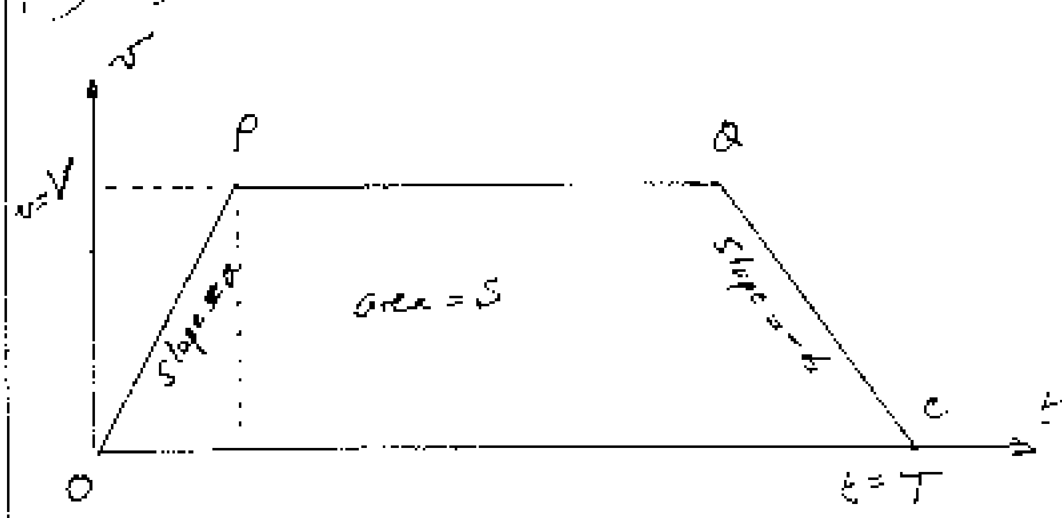
$k = \ln 25 / 40$

(ii) $t = 60$

$N = 200e^{60 \times \frac{\ln 25}{40}}$

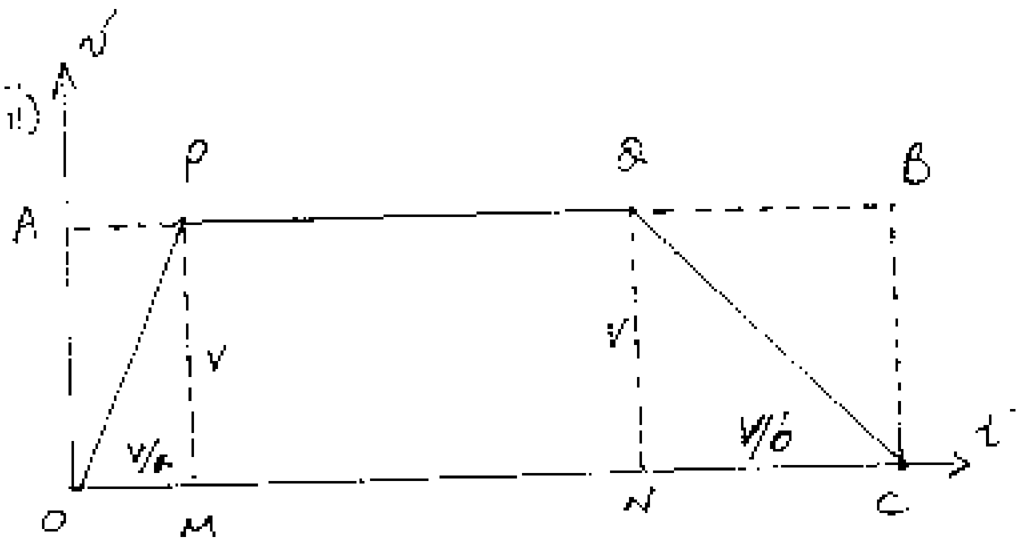
$= 250\,000$

(b) (i)



Q10 (cont)

ii)



$OA = PM = V$

$\therefore \frac{PM}{OM} = a \quad OM = V/a \ (= AP)$

$QN = BC = V$

$\frac{QN}{NC} = b \quad NC = V/b \ (= QB)$

Area of OABC = VT

Area of OABC = area ΔOPM + trapezium OPQC + area ΔNQC

$= \frac{1}{2}V^2/a + S + \frac{1}{2}V^2/b$.

$\therefore VT = \frac{V^2}{2a} + \frac{V^2}{2b} + S$

$T = \frac{S}{V} + \frac{V}{2a} + \frac{V}{2b}$

$T = \frac{S}{V} + \frac{V}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$

(iii) $\frac{dT}{dV} = \frac{-S}{V^2} + \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$

for max/min $\frac{dV}{dT} = 0$

$\frac{S}{V^2} = \frac{b+a}{2ab}$

$\frac{2abS}{a+b} = V^2$

$V = \sqrt{\frac{2abS}{a+b}} \quad$ (speed $V > 0$)

$\frac{d^2T}{dV^2} = \frac{2S}{V^3}$

> 0 since $V > 0$

∴ concave up ∴ local min tp.

& Since the function for T is cts for V > 0 & there is only one tp. which is a local min tp then it is the absolute min tp.

∴ speed = $\sqrt{\frac{2abS}{a+b}}$.