# Year 12 Mathematics - Trial HSC 2004

## QUESTION 1

MARKS

(a) Find the value of $e^{-2.5}$ correct to 3 significant figures. **2**

(b) Factorise fully: $16x^2 - 36y^2$. **2**

(c) Solve for $t$: $\frac{4}{2t-3} = \frac{5}{t}$. **3**

(d) If $\frac{12}{2+\sqrt{10}}$ is written in the form $m + n\sqrt{10}$, where $m$ and $n$ are rational numbers, find the values of $m$ and $n$. **3**

(f) A customer is given a 6% discount on the purchase of a radio. If the customer paid \$42.30, find the price of the radio before the discount. **2**

## QUESTION 2: (START A NEW PAGE)

(a) Differentiate the following with respect to $x$, leaving your answer in simplest form.

(i) $(3-4x)^7$. **2**

(ii) $\frac{2x}{3x+1}$. **2**

(b) (i) Find: $\int \frac{6}{1-2x}\,dx$. **2**

(ii) Evaluate: $\int_0^{\frac{\pi}{4}} \sec^2 3x\,dx$. **3**

(c) Find the equation of the curve $y = f(x)$, if $f'(x) = \frac{\sqrt{x}-4}{x}$ and the curve passes through the point $(1, 5)$. **3**



## QUESTION 3: (START A NEW PAGE)

MARKS

(a) (i) Sketch the graph of $y = 3\cos 2\theta$ for $0 \le \theta \le \pi$. **2**

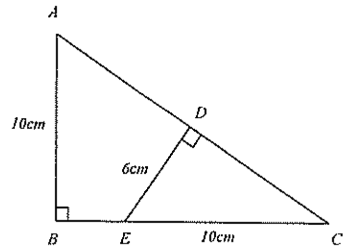
(ii) Solve $3\cos 2\theta = 1$ for $0 \le \theta \le \pi$. Give your answer correct to 2 decimal places. **2**

(b) (i) On the same set of coordinate axes, sketch the functions $y = 6x - x^2$ and $y = 2x$, clearly showing the coordinates of their intersection points. **4**

(ii) Find the area bounded by the above curves and the $x$-axis. **4**

## QUESTION 4: (START A NEW PAGE)

(a) Triangles $ABC$ and $CDE$ are right angled at $B$ and $D$ respectively (as shown in the diagram).




(i) Copy the diagram onto your examination answer sheet and prove that $\Delta ABC$ and $\Delta CDE$ are similar. **2**

(ii) If $AB = EC = 10$cm and $DE = 6$cm, find the length of $AC$. **2**

(b) $A(5, 20)$, $B(30, 15)$, $C(20, -10)$ and $D$ are the vertices of a quadrilateral $ABCD$.

(i) Given that the diagonals $AC$ and $BD$ are perpendicular, prove that the point $D$ lies on the line $y = \frac{1}{2}x$. **2**

(ii) If also $AB = AD$, prove that the coordinates of $D$ are $(-6, -3)$. **3**

(iii) Prove that $AC$ bisects $BD$. **3**

QUESTION 5: (START A NEW PAGE)

MARKS

(a) A balloon drifts 100km from point $A$ to point $B$ on a bearing of 028°T. At point $B$ the balloon changes direction and drifts 160km to point $C$ on a bearing of 114°T.

(i) Draw a neat diagram showing the above information. 1

(ii) Find the distance from point $A$ to point $C$. Give your answer correct to the nearest kilometre. 2

(iii) Find the true bearing of point $C$ from point $A$. Give your answer correct to the nearest degree. 3

(b) Water flows into then out of a container at a rate ($R$ litres/minute) given by $R = t(10-t)$.

(i) Find the maximum flow rate. 2

(ii) Find an expression for the volume, $V$ litres, of water in the container at time $t$ minutes assuming that the container is initially empty. 2

(iii) Find the total time for the container to fill and then empty. 2

QUESTION 6: (START A NEW PAGE)

(a) (i) Sketch the region bounded by the curve $y = \sqrt[3]{x}$, the $y$-axis and the line $y = 2$. 1

(ii) Find the *exact* volume of the solid formed when the area in part (i) is rotated one revolution about the $x$-axis. 4

(b) The velocity $v$ m/s of an object at time $t$ seconds is given by $v = 3t^2 - 14t + 8$. The object is initially 30m to the right of the origin.

(i) Find the initial acceleration of the object. 1

(ii) Find when the object is at rest. 2

(iii) Find the minimum distance between the origin and the object during its motion. 4



QUESTION 7: (START A NEW PAGE)

MARKS

(a) On an interval $x_1 \le x \le x_2$, a curve $y = f(x)$ has the following three properties:
$f(x_1) < 0$ , $f'(x) > 0$ and $f''(x) < 0$. 3
Draw a section of the curve $y = f(x)$ that illustrates all of above information.

(b) The mass $M$ grams of a radioactive isotope of Carbon (called Carbon 14 and written as $C_{14}$) found in a rock sample at time $t$ years is given by the formula $M = Ae^{-kt}$, where $A$ and $k$ are constants.

(i) Prove that the rate of decay of the mass of $C_{14}$ is proportional to the mass present at any time $t$. 2

(ii) If there is initially 100 grams of $C_{14}$ and this mass decays to 75 grams in 2500 years, find the values of the $A$ and $k$. Give your value of $k$ correct to three significant figures. 3

(iii) Find the amount of $C_{14}$ present at the end of 4000 years. Give your answer correct to the nearest gram. 2

(iv) Find the time required for the mass of $C_{14}$ to decay to 5 grams. Give your answer correct to the nearest 100 years. 2

QUESTION 8: (START A NEW PAGE)

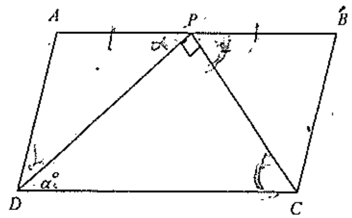
(a) As wire is unwound from a cylinder, the mass of wire remaining on the cylinder decreases. It is given that the mass, $M$ kg, of wire remaining after $t$ minutes can be calculated by the formula $M = 240 - 40\sqrt{t+1}$.

(i) Find the initial mass of wire on the cylinder. 1

(ii) Find the time taken to remove all the wire from the cylinder. 2

(iii) Find the rate at which the wire is being removed from the cylinder when half the wire has been removed. 3

(b) $ABCD$ is a parallelogram. $P$ is a point chosen on side $AB$ so that $PD$ bisects $\angle ADC$ and $\angle DPC = 90°$. (as shown in the diagram)



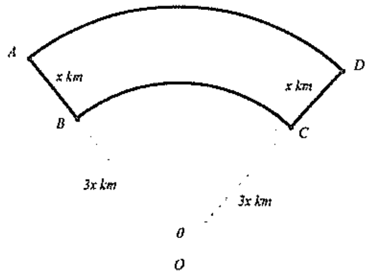

(i) If $\angle PDC = \alpha°$, prove that $\angle BPC = (90-\alpha)°$. 3

(ii) Prove that $\Delta BPC$ is isosceles. 3

## QUESTION 9: (START A NEW PAGE)

MARKS

(a) Four towns $A$, $B$, $C$ and $D$ are joined by roads that are either straight or arcs of concentric circles with centre at $O$. Towns $B$ and $C$ are distance $3x$ km from $O$ and towns $A$ and $D$ are both distance $x$ km from $B$ and $C$ respectively and $\angle AOD = \theta$ radians. (see diagram)



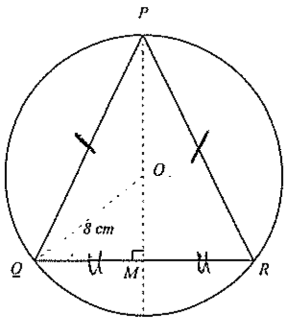

(i) Write an expression, in terms of $x$ and $\theta$, for the length of the journey from town $A$ to town $D$ along the arc $AD$. **1**

(ii) A salesperson wants to travel from town $A$ to town $D$ but must visit towns $B$ and $C$ on the way. Write an expression, in terms of $x$ and $\theta$, for the length of this journey from town $A$ to town $D$. **1**

(iii) Find the value of $\theta$ for which the journeys described in parts (i) and (ii) are the same distance. **2**

(b) An isosceles triangle $PQR$ with $PQ = PR$ is inscribed in a circle of radius 8cm (as shown in the diagram).

Given that $O$ is the centre of the circle and $M$ is the midpoint of the base $QR$ of the triangle, you may assume that $P$, $O$ and $M$ are collinear and $PM$ is perpendicular to $QR$.




(i) If the height, $PM$ cm, of $\Delta PQR$ is $h$ cm, prove that its area, $A$ cm$^2$, is given by $A = h\sqrt{16h - h^2}$. **3**

(ii) Write down the restriction on the values for $h$. **1**

(iii) Find the maximum area of $\Delta PQR$. **4**



## QUESTION 10: (START A NEW PAGE)

MARKS

A fund is established to provide prizes for a basketball team's annual Awards night. $10 000 is placed in the fund one year before the first Awards night. It is decided that $450 will be withdrawn from the fund each year to purchase the annual prizes. The money in the fund is invested at 3% p.a. compounded annually with the interest paid into the fund before each annual Awards night.

(i) Show that the fund contains $9695.50 after the second Awards night. **2**

(ii) If $A_n$ is the amount in dollars remaining in the fund after the n$^{th}$ Awards night, prove that $A_n = 5000(3 - 1.03^n)$. **3**

(iii) Find the amount of money in the fund after the 25$^{th}$ Awards night. Give your answer correct to the nearest dollar. **1**

(iv) Find the maximum number of Awards nights that can be financed using this fund. **2**

(v) For the fund described above it is decided to increase the amount of money withdrawn for each Awards night by 2% each year.

(α) Show that the amount remaining in the fund after the 2$^{nd}$ Awards night is $9686.50. **2**

(β) Find the amount remaining in the fund after the 25$^{th}$ Awards night. Give your answer correct to the nearest dollar. **2**

⌘ ☙ THE END ❧ ⌘

## Year 12 Mathematics - Trial HSC 2004

### QUESTION 1

(a) $e^{-2.5} = 0.0821$ (to 3 significant figures) 2

(b) $16x^2 - 36y^2 = 4(4x^2 - 9y^2)$ 2

$= 4(2x - 3y)(2x + 3y)$

(c) $4t = 5(2t - 3)$ 3

$= 10t - 15$

$6t = 15$

$t = 2\frac{1}{2}$

(d) $\dfrac{12}{2+\sqrt{10}} = \dfrac{12(2-\sqrt{10})}{(2+\sqrt{10})(2-\sqrt{10})}$ 3

$= \dfrac{12(2-\sqrt{10})}{4-10}$

$= \dfrac{12(2-\sqrt{10})}{-6}$

$= -2(2-\sqrt{10})$

$= -4 + 2\sqrt{10}$

$\therefore\ m = -4,\ n = 2$

(f) 94% of radio price = \$42.30 2

1% of radio price = $\dfrac{\$42.30}{94}$

100% of radio price = $\dfrac{\$42.30}{94} \times 100$

= \$45.00

### QUESTION 2: (STAR A NEW PAGE)

(a) (i) Let $f(x) = (3-4x)^7$ 2

$f'(x) = 7(3-4x)^6 \times (-4)$

$= -28(3-4x)^6$

(ii) Let $f(x) = \dfrac{2x}{3x+1}$ 2

$f'(x) = \dfrac{(3x+1)(2) - (2x)(3)}{(3x+1)^2}$

$= \dfrac{6x + 2 - 6x}{(3x+1)^2}$

$= \dfrac{2}{(3x+1)^2}$

(b) (i) $\int \dfrac{6}{1-2x}\,dx = -3\ln(1-2x) + c$ 2

(ii) $\int_0^{\frac{\pi}{4}} \sec^2 3x\,dx = \frac{1}{3}[\tan 3x]_0^{\frac{\pi}{4}}$ 3

$= \frac{1}{3}\{\tan\frac{3\pi}{4} - \tan 0\}$

$= -\frac{1}{3}$

(c) $\dfrac{dy}{dx} = \dfrac{\sqrt{x} - 4}{x}$ 3

$= x^{-\frac{1}{2}} - \dfrac{4}{x}$

$y = 2x^{\frac{1}{2}} - 4\ln x + c$

at point $(1,5)$

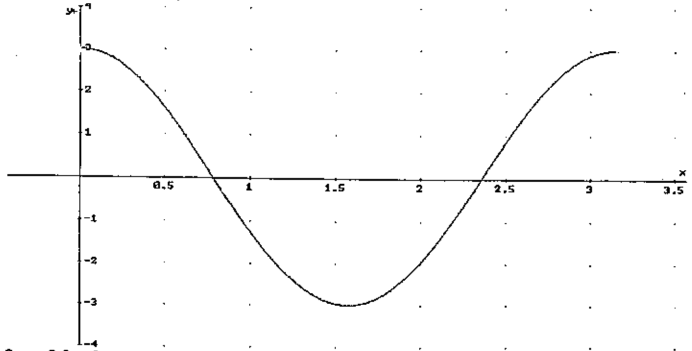
$5 = 2\sqrt{1} - 4\ln 1 + c$

$\therefore c = 3$

$y = 2\sqrt{x} - 4\ln x + 3$

**QUESTION 3: (STAR A NEW PAGE)**

(a) (i) Sketch the graph of $y = 3\cos 2\theta$ for $0 \le \theta \le \pi$. 2



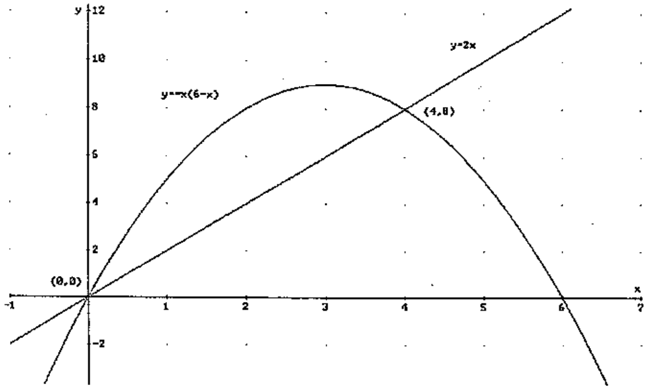
(ii) $3\cos 2\theta = 1$ 2

$\cos 2\theta = \frac{1}{3}$

$2\theta = 1.230959$ or $5.052226$

$\theta = 0.62$ or $2.53$ (to 2 decimal places)

(b) (i) 4




(ii) 4

$$A = \int_0^4 2x\,dx + \int_4^6 \left(6x - x^2\right)dx$$

$$= \left[x^2\right]_0^4 + \left[3x^2 - \tfrac{1}{3}x^3\right]_4^6$$

$$= \left\{4^2 - 0\right\} + \left\{\left(3\times 6^2 - \tfrac{1}{3}\times 6^3\right) - \left(3\times 4^2 - \tfrac{1}{3}\times 4^3\right)\right\}$$

$$= 25\tfrac{1}{3}$$

Area $= 25\frac{1}{3}\ \text{u}^2$

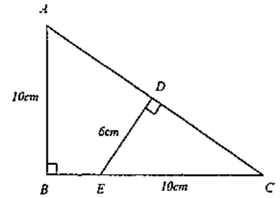
**QUESTION 4: (STAR A NEW PAGE)**

(a) (i) In $\Delta ACB$ and $\Delta CDE$ 2

$A\hat{C}B = D\hat{C}E$ (common)

$A\hat{B}C = C\hat{D}E$ (both $90^\circ$)

$\Delta ABC \equiv \Delta EDC$ (equiangular)




(ii) $\dfrac{AC}{10} = \dfrac{10}{6}$ (ratio of corresponding sides in similar triangles) 2

$AC = 16\frac{2}{3}$

length of $AC = 16\frac{2}{3}$cm

(b) (i) 2

slope $AC = \dfrac{20+10}{5-20}$

$= -2$

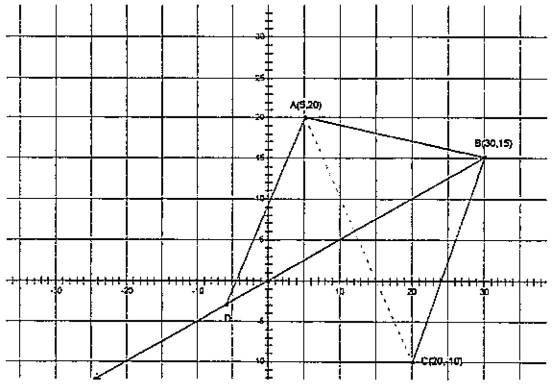
$\therefore$ slope $DB = \frac{1}{2}$

equation $DB$ is

$y - 15 = \frac{1}{2}(x - 30)$

$y - 15 = \frac{1}{2}x - 15$

$y = \frac{1}{2}x$




(ii) Let $D$ be the point $(2a, a)$ 3

$AD^2 = AB^2$

$(2a-5)^2 + (a-20)^2 = (5-30)^2 + (20-15)^2$

$5a^2 - 60a - 225 = 0$

$a^2 - 12a - 45 = 0$

$(a+3)(a-15) = 0$

$a = -3$ or $15$

at point $D$, $a = -3$

$\therefore D$ is $(2a, a) = (-6, -3)$

(iii) Using coordinate geometry 3

Midpoint of $BD$ is $P(12,6)$

Equation of line $AC$ is

$y-20=-2(x-5)$

$y=-2x+30$

at point $P(12,6)$

$LHS = y$

$= 6$

$RHS = -2x+30$

$= -2\times(-12)+30$

$= 6$

$\therefore LHS = RHS$

$\therefore$ midpoint $P$ lies on line $AC$

i.e. line $AC$ bisects $DB$

or

Using congruent triangles

Let $AC$ meet $BD$ at $P$

In $\Delta ADP$ and $\Delta ABP$

$AD = AB$ (given)

$AP = AP$ (common)

$A\hat{P}D = A\hat{P}B$ (both $90°$, $AC \perp BD$)

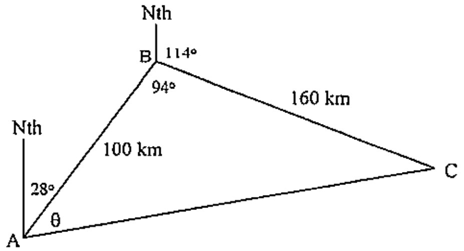
$\therefore \Delta ADP \equiv \Delta ABP$ (RHS)

$\therefore DP = BP$ (corresponding sides in congruent triangles)

$\therefore$ line $AC$ bisects $DB$

**QUESTION 5: (STAR A NEW PAGE)**

(a) (i) 1




(ii) $AC^2 = 100^2 + 160^2 - 2(100)(160)\cos 94°$ 2

$AC = 195\text{km}$ (to nearest km)

(iii) $\cos\theta = \dfrac{100^2 + AC^2 - 160^2}{2(100)(AC)}$ 3

$\approx 0.5715$

$\theta = 55°8'$

bearing $= (28° + 55°)\text{T}$

$= 083°\text{T}$ (to nearest degree)

(b) (i) when $t = 5$, $R = 5(10-5)$ 2

$= 25$

max. flow rate = 25 L/min

(ii) $V = \int (10t - t^2)\,dt$ 2

$= 5t^2 - \frac{1}{3}t^3 + c$

when $t = 0, V = 0 \Rightarrow c = 0$

$V = 5t^2 - \frac{1}{3}t^3$

(iii) when $V = 0$ 2

$5t^2 - \frac{1}{3}t^3 = 0$

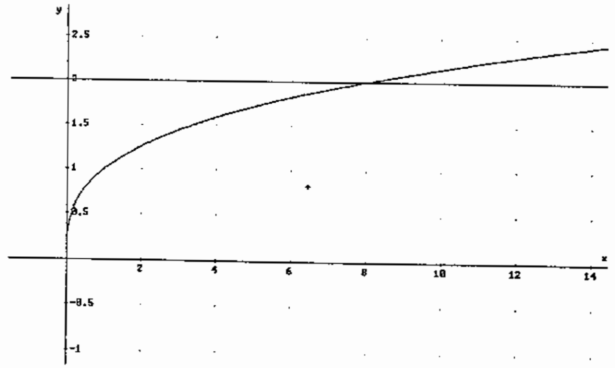
$\frac{1}{3}t^2(15 - t) = 0$

$t = 0$ or $15$

$\therefore$ time taken = 15 minutes

**QUESTION 6: (STAR A NEW PAGE)**

(a) (i) 1




(ii) $V = \pi\int_0^8 \left(2^2 - x^{\frac{2}{3}}\right)dx$ 4

$= \pi\left[4x - \frac{3}{5}x^{\frac{5}{3}}\right]_0^8$

$= \pi\left\{\left(4\times 8 - \frac{3}{5}\times 8^{\frac{5}{3}}\right) - (0)\right\}$

$= \frac{64\pi}{5}$

$\therefore$ volume $= \frac{64\pi}{5}$ $u^3$

(b) (i) $v = 3t^2 - 14t + 8$ 1

$a = 6t - 14$

when $t = 0$, $a = -14$

initial acceleration $= -14\text{ms}^{-2}$

(ii) at rest when $v = 0$ 2

$3t^2 - 14t + 8 = 0$

$(3t-2)(t-4) = 0$

$t = \frac{2}{3}$ or 4

at rest after $\frac{2}{3}$ seconds or 4 seconds

(iii) $x = t^3 - 7t^2 + 8t + c$ 4

$t = 0, x = 30 \Rightarrow c = 30$

$\therefore x = t^3 - 7t^2 + 8t + 30$

when $t = 0$, $x = 30$

when $t = \frac{2}{3}$, $\ddot{x} = -10 < 0$, $\therefore$ concave down $\Rightarrow$ local max.tp

when $t = 4$, $\ddot{x} = 10 > 0$, $\therefore$ concave up $\Rightarrow$ local min.tp.

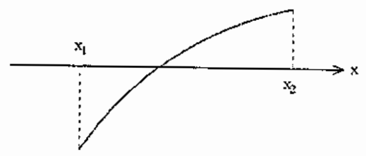
$x = 4^3 - 7\times 4^2 + 8\times 4 + 30$

$= 14$

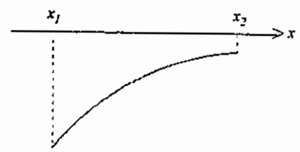
$\therefore$ minimum distance = 14km

**QUESTION 7: (STAR A NEW PAGE)**

(a) 3




or




(b) (i) $\frac{dM}{dt} = -kAe^{-kt}$ 2

$= -kM$ since $M = Ae^{-kt}$

$\therefore \frac{dM}{dt} \propto M$

(ii) when $t = 0$, $M = 100$ 3

$100 = Ae^0$

$A = 100$

when $t = 2500$, $M = 75$

$75 = 100e^{-2500k}$

$e^{-2500k} = 0.75$

$-2500k = \ln 0.75$

$k = \frac{\ln 0.75}{-2500}$

$= 1.15\times 10^{-4}$ (to 3 significant figures)

(iii) when $t = 4000$ 2

$M = 100e^{-4000\times 1.15\times 10^{-4}}$

$\approx 63$

mass $\approx$ 63 grams

(iv) when $M = 5$ 2

$5 = 100e^{-kt}$

$e^{-kt} = 0.05$

$-kt = \ln 0.05$

$t = \frac{\ln 0.05}{-k}$

$\approx 26049$

time = 26000years (to nearest 100 years)

## QUESTION 8: (STAR A NEW PAGE)

(a) (i) when $t = 0$ 1

$M = 240 - 40\sqrt{1}$

$= 200$

$\therefore$ initial mass = 200kg

(ii) when $M = 0$ 2

$0 = 240 - 40\sqrt{t+1}$

$40\sqrt{t+1} = 240$

$\sqrt{t+1} = 6$

$t + 1 = 36$

$t = 35$

$\therefore$ time = 35 minutes

(iii) $\frac{dM}{dt} = 0 - 40 \times \frac{1}{2}(t+1)^{-\frac{1}{2}}$ 3

$= \frac{-20}{\sqrt{t+1}}$

when $M = 100$

$100 = 240 - 40\sqrt{t+1}$

$40\sqrt{t+1} = 140$

$\sqrt{t+1} = 3.5$

$\therefore \frac{dM}{dt} = \frac{-20}{\sqrt{t+1}}$

$= -\frac{20}{3.5}$

$= -\frac{40}{7}$

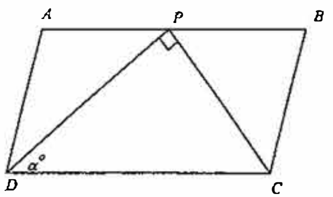
$\therefore$ rate $= -\frac{40}{7}$ kg/min

(b) (i) $AB \parallel DC$ (opposite sides of parallelogram are parallel) 3

$\angle APD = \alpha°$ ($AB \parallel DC$, alternate angles are equal)

$\angle BPC + \alpha° + 90° = 180°$ (straight angle $\angle APB = 180°$)

$\angle BPC = (90 - \alpha)°$




(ii) $\angle ADC = 2\alpha°$ ($PD$ bisects $\angle ADC$) 3

$\angle ABC = 2\alpha°$ (opposite angles of parallelogram are equal)

$\angle BCP + 2\alpha° + (90 - \alpha)° = 180°$ (angle sum of $\Delta BPC = 180°$)

$\angle BCP = (90 - \alpha)°$

$\therefore \Delta BPC$ is isosceles ($\angle BPC = \angle BCP = (90 - \alpha)°$)

## QUESTION 9: (STAR A NEW PAGE)

(a) (i) $AD = 4x\theta$ 1

(ii) $ABCD = 2x + 3x\theta$ 1

(iii) $AD = ABCD$ 2

$4x\theta = 2x + 3x\theta$

$x\theta - 2x = 0$

$x(\theta - 2) = 0$

$\theta = 2$ $(x \neq 0)$

$\therefore$ angle = 2 radians

(b) (i) $Area = \frac{1}{2}QR.PM$ 3

$QR = 2QM$

Case (1) if $h \geq 8$

$QM^2 = 8^2 - (h-8)^2$ (Pythagoras' Theorem)

Case 2 if $h < 8$

$QM^2 = 8^2 - (8-h)^2$ (Pythagoras' Theorem)

$\therefore QM^2 = 16h - h^2$

$QM = \sqrt{16h - h^2}$ $(QM > 0)$

$QR = 2\sqrt{16h - h^2}$

$Area: A = \frac{1}{2} \times 2\sqrt{16h - h^2} \times h$

$\therefore A = h\sqrt{16h - h^2}$

$= 16h - h^2$

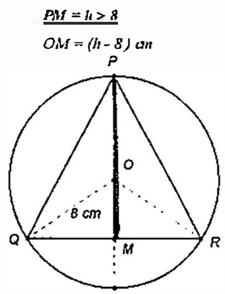
$QM = \sqrt{16h - h^2}$ $(QM > 0)$

$QR = 2\sqrt{16h - h^2}$

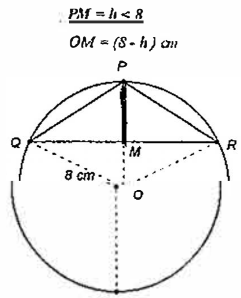
$Area: A = \frac{1}{2} \times 2\sqrt{16h - h^2} \times h$

$\therefore A = h\sqrt{16h - h^2}$

Case (1)




Case (2)




(ii) $0 \leq h \leq 16$ 1

(iii) $A = h(16h - h^2)^{\frac{1}{2}}$ 4

$\frac{dA}{dh} = (1)(16h - h^2)^{\frac{1}{2}} + (h) \times \frac{1}{2}(16h - h^2)^{-\frac{1}{2}} \times (16 - 2h)$

$= \sqrt{16h - h^2} + \frac{8h - h^2}{\sqrt{16h - h^2}}$

$= \frac{2h(12 - h)}{\sqrt{16h - h^2}}$

for stat. pt. $\frac{dA}{dh} = 0$

$\frac{2h(12-h)}{\sqrt{16h-h^2}} = 0$

$2h(12-h) = 0$

$h = 0 \text{ or } 12$

test stat. points

when $h = 0,\ A = 0,\ \therefore$ min. area

when $h = 12$

| $h$ | <12 (=11) | 12 | >12 (=13) |
|---|---|---|---|
| $\frac{dA}{dh}$ | $\frac{2(11)(1)}{\sqrt{16\times 11-11^2}}$ $>0$ | 0 | $\frac{2(13)(-1)}{\sqrt{16\times 13-13^2}}$ $<0$ |

Change in gradient (+ , 0 , -) and curve is continuous for $11 \le h \le 13$ $\therefore$ stat. pt. is a local max. tp.
Since the area function is continuous for $0 \le h \le 16$ and there is only one max. tp. for $0 < h < 16$ then the local max. tp. is the absolute max.

maximum $A = 12\sqrt{12(16-12)}$

$= 48\sqrt{3}$

$\therefore$ maximum area $= 48\sqrt{3}\ \text{cm}^2$

**QUESTION 10: (STAR A NEW PAGE)**

(i) Let $\$A_n$ = amount in the fund after the $n^{\text{th}}$ awards night 2

$A_1 = 10000\times 1.03 - 450$

$A_2 = A_1 \times 1.03 - 450$

$= (10000\times 1.03 - 450)\times 1.03 - 450$

$= 10000\times 1.03^2 - (1.03+1)\times 450$

$= 9695.50$

$\therefore$ amount in fund = \$9695.50

(ii) $A_1 = 10000\times 1.03 - 450$ 3

$A_2 = A_1 \times 1.03 - 450$

$= (10000\times 1.03 - 450)\times 1.03 - 450$

$= 10000\times 1.03^2 - (1.03+1)\times 450$

$\vdots$

$A_n = 10000\times 1.03^n - (1.03^{n-1} + 1.03^{n-2} + \cdots + 1.03 + 1)\times 450$

$= 10000\times 1.03^n - 1\times\left(\frac{1.03^n - 1}{1.03-1}\right)\times 450$

$= 10000\times 1.03^n - \left(\frac{1.03^n - 1}{0.03}\right)\times 450$

$= 10000\times 1.03^n - (1.03^n - 1)\times 15000$

$= 10000\times 1.03^n - 1.03^n \times 15000 + 15000$

$= 15000 - 5000\times 1.03^n$

$A_n = 5000(3 - 1.03^n)$

(iii) when $n = 25$ 1

$A_{25} = 5000(3 - 1.03^{25})$

$= 4531.11$

$\therefore$ amount = \$4351 (to nearest dollar)

(iv) $A_n \ge 0$ 2

$5000(3 - 1.03^n) \ge 0$

$3 - 1.03^n \ge 0$

$1.03^n \le 3$

$n\ln(1.03) \le \ln 3$

$n \le \frac{\ln(1.03)}{\ln 3}$

$n \le 37.16$

$\therefore$ max. number of awards nights = 37

(v) Let $\$B_n$ = amount in the fund after the $n^{\text{th}}$ awards night 2

(α) $B_1 = 10000\times 1.03 - 450$

$B_2 = B_1 \times 1.03 - 450\times 1.02$

$= (10000\times 1.03 - 450)\times 1.03 - 450\times 1.02$

$= 10000\times 1.03^2 - (1.03 + 1.02)\times 450$

$= 9686.5$

$\therefore$ amount in fund = \$9686.50

(β) Let $\$B_n$ = amount in the fund after the $n^{\text{th}}$ awards night 2

$B_1 = 10000\times 1.03 - 450$

$B_2 = B_1 \times 1.03 - 450\times 1.02$

$= (10000\times 1.03 - 450)\times 1.03 - 450\times 1.02$

$= 10000\times 1.03^2 - (1.03 + 1.02)\times 450$

$\vdots$

$B_n = 10000\times 1.03^n - 450\times \left\{1.03^{n-1} + 1.03^{n-2}(1.02) + 1.03^{n-3}(1.02^2) + \cdots + 1.03(1.02^{n-2}) + 1.02^{n-1}\right\}$

$B_n$ series is a GP with $a = 1.02^{n-1}$ and $r = \dfrac{1.03}{1.02}$

$$B_n = 10000\times 1.03^n - 450\times 1.02^{n-1}\times\left\{\frac{\left(\frac{1.03}{1.02}\right)^n - 1}{\left(\frac{1.03}{1.02}\right) - 1}\right\}$$

$$= 10000\times 1.03^n - \frac{450\times 1.02^{n-1}\left\{\frac{1.03^n - 1.02^n}{1.02^n}\right\}}{\left(\frac{1.03-1.02}{1.02}\right)}$$

$$= 10000\times 1.03^n - \frac{450\times(1.03^n - 1.02^n)}{0.01}$$

$= 10000\times 1.03^n - 45000\times(1.03^n - 1.02^n)$

$= 10000\times 1.03^n - 45000\times 1.03^n + 45000\times 1.02^n$

$B_n = 45000\times 1.02^n - 35000\times 1.03^n$

$B_{25} = 45000\times 1.02^{25} - 35000\times 1.03^{25}$

$\therefore$ amount in fund = \$545

**THE END**