Class:	

JAMES RUSE A H S

GESTA NON VERBA

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2011

MATHEMATICS

Time Allowed – 3 Hours
(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators and templates are permitted
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate *stapled* bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

Question 1 - Start a new sheet of paper **Marks**

a) Solve $x^2 - 3x - 18 = 0$ **1**

b) Convert $\dfrac{7\pi}{9}$ radians to degrees **1**

c) Write down a primitive of $\sec^2 3x$ **1**

d) Solve $|2x-1| = 7$ **2**

e) Write $\dfrac{2}{\sqrt{17}-3}$ with a rational denominator **1**

f) Given $f(x) = \begin{cases} \dfrac{1}{3^x} & x < 3 \\ (x-3)^2 + 1 & x \geq 3 \end{cases}$,

evaluate $2f(4) - f(-1)$ **2**

g) Find the non-zero values of x for which the geometric series

$2 - 6x + 18x^2 - .....$ has a limiting sum **2**

h) i) Write down the equation of the locus of a point P that is 3 units from the point $A(2,-4)$ **1**

ii) How many times does this locus cut the x axis ? **1**

Question 2 - Start a new sheet of paper **Marks**

a) Differentiate the following with respect to x :

i) $\ln\sqrt{x^2+3}$ **2**

ii) $(2x^2+7)^3$ **1**

iii) $x^2 \cos x$ **2**

b) The equation of a parabola is given by $8y = x^2 + 4x + 12$

i) Find the coordinates of the vertex **2**

ii) Write down the focal length of the parabola **1**

iii) Sketch the parabola, clearly showing the focus and directrix **2**

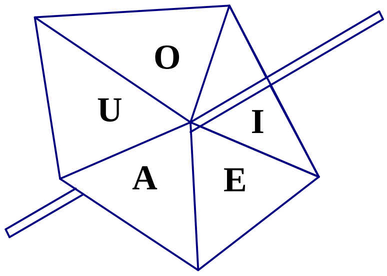
c) Solve the equation $\sec^2 x = 4$ for $0 \leq x \leq 2\pi$ **2**

Question 3 - Start a new sheet of paper **Marks**

a) Given that $\log_k 5 = 1.838$ and $\log_k 2 = 0.792$, find the values of

i) $\log_k 50$ ii) $\log_k (0 \cdot 4)$ iii) $\log_{10} k$ **4**

b) An unbiased spinner, as shown, is equally likely to stop at any one of the letters ***A***, ***E***, ***I***, ***O*** or ***U***



i) If the spinner is spun twice, find the probability that it stops on the same letter both times. **1**

ii) How many times must the spinner be spun for it to be 99% certain that the spinner will have stopped on ***E*** at least once? **2**

c) i) Sketch the graph of $y = 3\sin 2x$ for $0 \le x \le 2\pi$ (Your diagram should take up at least one quarter of a page.) **2**

ii) Draw the line $y = \dfrac{x}{2}$ on the same diagram, noting particularly the point on the line where $x = 2\pi$ **1**

iii) Hence determine the **total** number of solutions there are to the equation

$$6\sin 2x - x = 0 \qquad \textbf{for all } \boldsymbol{x}$$ **2**

(**NB** You are **not** required to solve the equation but you should include a short explanation of your answer)

Question 4 - Start a new sheet of paper **Marks**

a) The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 4$

i) Find $\frac{d^2y}{dx^2}$ **1**

ii) Find those values of x for which the curve is both increasing and concave downwards. Show your reasoning. **2**

iii) If the curve passes through the point (1 , -2) , find the equation of the curve. **2**

b) The coordinates of *A*, *B* and *C* are (4 , -6), (-18 , 0) and (0 , 6) respectively. *D* is the midpoint of *AB*.

i) Draw these points on a clear coordinate diagram and show that *D* has coordinates (-7 , -3). **1**

ii) Show that the equation of the line *AC* is $3x + y - 6 = 0$ **2**

iii) Show that the line *BC* is perpendicular to the line *AC* **1**

iv) Therefore *AB* is the diameter of a circle which passes through *A*, *B* and *C*. Find the equation of this circle. **2**

v) Find the circumference of the circle , accurate to 1 decimal place. **1**

Question 5 - Start a new sheet of paper **Marks**

a) A ship sails from *O* on a bearing of 130°T to the point *P*. *P* lies on a bearing of 030°T from *M* where *M* is 120 kms due south of O.

i) Show this information on a clear diagram and confirm that $\angle OPM = 100°$ **1**

ii) How far, to the nearest kilometre, is *P* from *M* ? **2**

iii) A lighthouse is located at *L* which is 40 kms due south of *O*. What was the closest that the ship came to *L* as it made its way from *O* to *P*. **2**

b) A substance decomposes such that the mass M (kgs) remaining at any time t (hrs) is given by $M = M_0 e^{-kt}$. If $\frac{3}{4}$ of the substance has decomposed in 4 hours, find:

i) the exact value of k **2**

ii) the value of M_0, correct to 2 decimal places, given that 4kg of the substance remains after the first 90 minutes. **2**

c) Simplify $\frac{1+\cot\theta}{\text{cosec}\theta} - \frac{\sec\theta}{\tan\theta + \cot\theta}$ **3**

Question 6 - Start a new sheet of paper **Marks**

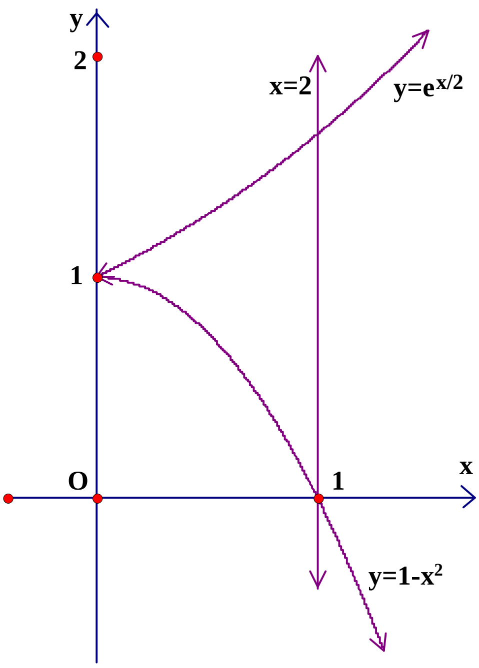
a) i) Find $\int x\sin(x^2)dx$ **1**

ii) Evaluate $\int_1^{16}\sqrt{x}dx$ **2**

b) The quadratic equation $5x^2-(21+a)x+a=0$ has two roots, with one of the roots being the reciprocal of the other. Find the value of a and hence find the two roots. **2**

c) A polygon has 60 sides. The lengths of the sides, starting with the smallest, form an arithmetic sequence. The perimeter of the polygon is 1770 cms and the length of the longest side is three times the length of the shortest side. Find the lengths of the two shortest sides. **4**

d)



The diagram shows the region enclosed between the two curves $y=e^{x/2}$ and $y=1-x^2$ and the line $x=1$. Find the area of this enclosed region. **3**

Question 7 - Start a new sheet of paper **Marks**

a) *PQRS* is a rhombus, angle *P* being acute. *X* is the point on *PS* such that *QX* is perpendicular to *PS*. *QX* intersects the diagonal *PR* at *Y*.

i) Show this information on a clear diagram and explain why $\angle SRP=\angle QRP$ **1**

ii) Prove that triangle *SYR* is congruent to triangle *QYR*. **3**

iii) Show that $\angle RQY$ is 90° and hence find the size of $\angle YSR$ **2**

Question 7 is continued on the next page…..

Question 7 (continued) **Marks**

b) The velocity v (kms/min) of a train travelling from Epping to Eastwood, is given by $v = t^2(3-t)/3$, where t is the time in minutes since leaving Epping.

i) If the first stop is at Eastwood, how long does the journey take? **1**

ii) Find an expression for the distance (x km) travelled from Epping after time t (where $0 \le t \le 3$) **2**

iii) Hence find the distance from Epping to Eastwood. **1**

iv) Where and when, between the two stations, was the train travelling the fastest? **2**

Question 8 - Start a new sheet of paper **Marks**

a) Stella sets up a prize fund with a single investment of $1000 to provide her school with an annual prize of $72. The fund accrues interest at a rate of 6% per annum, compounded annually. The first prize is awarded one year after the investment is set up.

i) Calculate the balance in the fund at the beginning of the second year, after the first prize has been awarded. **1**

ii) Let $\$B_n$ be the balance in the fund at the end of n years (after the nth prize has been awarded and while funds are still available).
Show that $B_n = 1200 - 200 \times (1.06)^n$ **2**

iii) At the end of the tenth year (after that prize has been awarded), it is decided that the prize will henceforward be increased to $90. For how many more years can the fund be used to award the full prize? **3**

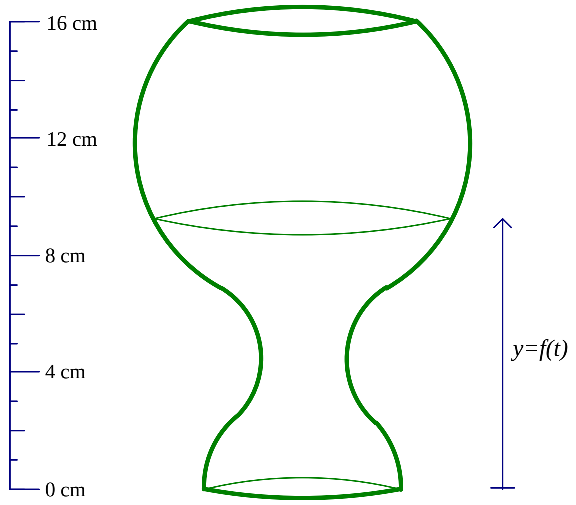
b) i) Sketch the graph of $y = e^x$ for $0 \le x \le 2$ **1**

ii) The portion of the curve $y = e^x$ between $y = 3$ and $y = 5$ is rotated about the y-axis to give a solid. Show that the volume of the solid formed is given by $\pi\int_3^5 (\ln y)^2\,dy$ cubic units. **2**

iii) Use Simpson's Rule and 5 function values to find an approximation to this volume (answer to 1 decimal place). **3**

Question 9 - Start a new sheet of paper **Marks**

a) The diagram shows a 16 cm high wine glass that is being filled with water at a constant rate (by volume). Let $y = f(t)$ be the depth of the liquid in the glass as a function of time.

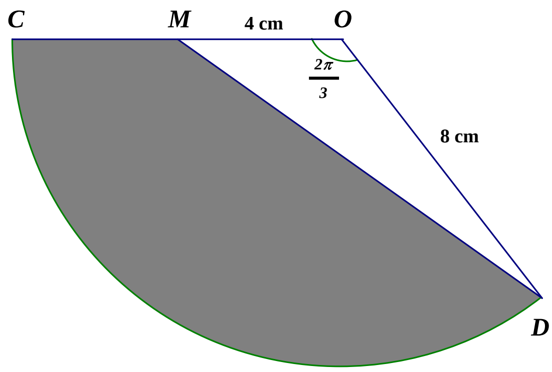


i) Write down the approximate depth y_1 at which $\frac{dy}{dt}$ is a minimum. **1**

ii) Write down the approximate depth y_2 at which $\frac{dy}{dt}$ is a maximum. **1**

iii) If the glass takes 8 seconds to fill, graph $y = f(t)$ and identify any points on your graph where the concavity changes. **2**

b) In the diagram, *CD* is an arc of a circle with radius 8cm and centre *O*. *M* is the midpoint of *OC* and angle *COD* is $\frac{2\pi}{3}$ radians.



Find the perimeter of the shaded area *CDM* in exact form. **3**

c) Find the exact value of *k* (where *k*>2) such that $\int_2^k \frac{2t}{3t^2 - 1}\,dt = \frac{1}{3}\log_e 13$ **3**

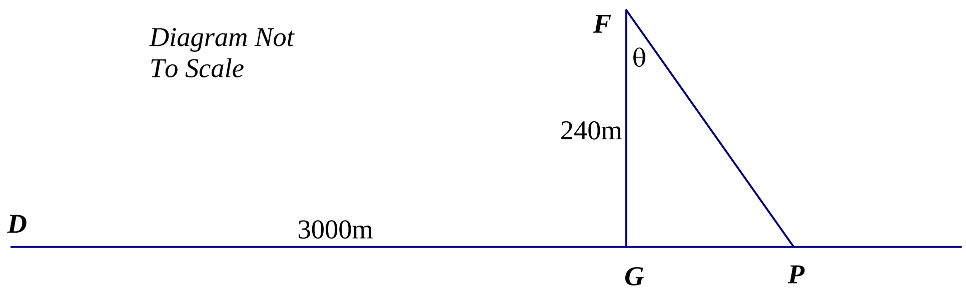
d) If *a*, *b* and *c* are consecutive positive terms of a geometric sequence, show that $\log a$, $\log b$ and $\log c$ are consecutive terms in an arithmetic sequence. **2**

Question 10 - Start a new sheet of paper **Marks**

a) State the Domain and Range of the function $y = 2\sqrt{25 - x^2}$ **2**

b) Evaluate $\displaystyle\sum_{r=1}^{63} \frac{1}{\sqrt{r+1} + \sqrt{r}}$ **2**

c)



The diagram shows a farmhouse F that is located 240m from a straight section of road, at the end of which is the bus depot D. The front gate G of the farmhouse is 3000m from the bus depot. The school bus leaves the depot at 8am and travels along the road at 15 ms^{-1}. Peter lives in the farmhouse and can run across the open paddock at a speed of 4 ms$^{-1.}$ The bus will stop for Peter anywhere on the road but will not wait.

Assume that Peter catches the bus at the point P where $\angle GFP = \theta$.

i) Show that the time, in seconds, taken for the bus to go from D to P is given by

$$200 + 16\tan\theta$$ **2**

ii) Find an expression, in terms of θ, for the time taken by Peter to run from F to P. **1**

iii) If Peter leaves home T seconds after 8am and he and the bus arrive at P at the same time, show that $T = 200 + 16\tan\theta - 60\sec\theta$ **1**

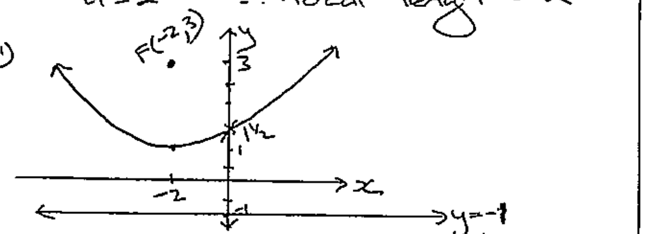
iv) What is the latest time, to the nearest second, that Peter can leave home and still catch the bus? **4**

******** THIS IS THE END OF THE EXAM ********

2011 TRIAL HSC 2U MATHEMATICS: Question 1

Suggested Solutions	Marks	Marker's Comments
(a) $x^2-3x-18=0$ $(x-6)(x+3)=0$ $\therefore x=-3$ or 6	1	$\frac{1}{2}$ mark for each value
(b) $\frac{7\pi}{9} = \frac{7\pi}{9}\times\frac{180}{\pi}$ $=140^\circ$	1	
(c) $\int \sec^2 3x\,dx = \frac{1}{3}\tan 3x + C$	1	+C not penalized as a primitive of $\sec^2 3x$ is $\frac{1}{3}\tan 3x$
(d) $\lvert 2x-1\rvert = 7$ $2x-1=7$ or $-(2x-1)=7$ $2x=8$ $-2x+1=7$ $x=4$ or $x=-3$	2	1 mark for each answer
(e) $\frac{2}{\sqrt{17}-3}\times\frac{\sqrt{17}+3}{\sqrt{17}+3} = \frac{2(\sqrt{17}+3)}{8}$ $=\frac{\sqrt{17}+3}{4}$	1	no marks deducted for not simplifying
(f) $f(4)=(4-3)^2+1=2$ $f(-1)=\frac{1}{3^{-1}}=3$	1	
$2f(4)-f(-1)=4-3$ $=1$	1	
(g) $r=\frac{T_2}{T_1}=-3x$	1	$x\neq 0$ $\frac{1}{2}$ mark
For limiting sum $\lvert r\rvert<1$ $\therefore \lvert 3x\rvert<1$ $-\frac{1}{3}<x<\frac{1}{3}$ $x\neq 0$	1	
(h) (i) $(x-2)^2+(y+4)^2=9$	1	
(ii) 0 times (the centre is 4 units below the axis and the radius is only 3)	1	

MATHEMATICS: Question 2   2011

Suggested Solutions	Marks	Marker's Comments
(a) (i) $y=\ln\sqrt{x^2+3}$ $\therefore y=\frac{1}{2}\ln(x^2+3)$	½	
$\frac{dy}{dx}=\frac{1}{2}\cdot\frac{2x}{x^2+3}$	1	
$=\frac{x}{x^2+3}$	½	
(ii) $y=(2x^2+7)^3$ $\frac{dy}{dx}=3(2x^2+7)^2\cdot 4x$	½	
$=12x(2x^2+7)^2$	½	
(iii) $y=x^2\cos x$ $\frac{dy}{dx}=2x\cos x - x^2\sin x$ (Product Rule) $=x(2\cos x - x\sin x)$	1	for each part
(b) (i) $8y=x^2+4x+12$ $8y=x^2+4x+4+8$ $8y=(x+2)^2+8$	½	If they just write down $(-2,1)$ they get the marks
$8y-8=(x+2)^2$ $8(y-1)=(x+2)^2$	½	
$\therefore$ vertex is $(-2,1)$	1	
(ii) $4a=8$ $a=2$ $\therefore$ focal length is 2	1	
(iii) 	2	½ for y-intercept ½ for focus ½ for directrix ½ for shape
(c) $\sec^2 x=4$ $\cos^2 x=\frac{1}{4}$ $\rightarrow$ $\cos x=\pm\frac{1}{2}$ $x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$	2.	* If they only had 2 answers — 1 mark * If they had $\cos x=\frac{1}{2}$ and then 2 answers =

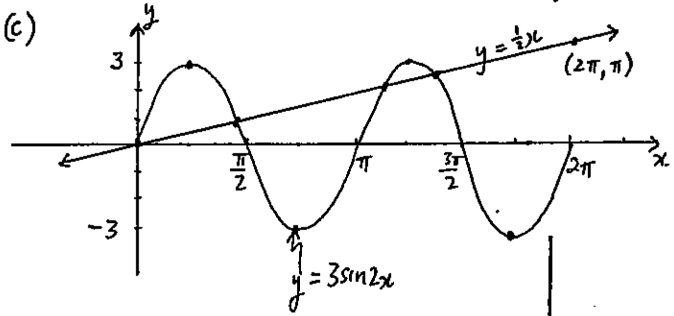
JRAHS : TRIALS MATHEMATICS 2UNIT AUG 2011

QUESTION 3

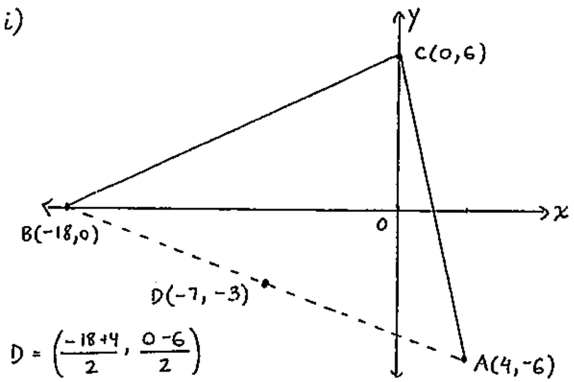
EXPECTED ANSWERS	MARKS	COMMENTS
) (i) $\log_k 50$		Fairly well done
$= \log 5^2 \times 2$		
$= 2\log 5 + \log 2$	$\frac{1}{2}$ ... sum	
$= 2 \times 1.838 + 0.792$		
$= 4.468$	$\frac{1}{2}$	
(ii) $\log_k (0.4)$		
$= \log_k 2/5$		
$= \log 2 - \log 5$	$\frac{1}{2}$ ... difference	
$= 0.792 - 1.838$		
$= -1.046$	$\frac{1}{2}$	
i) $\log_{10} k$		• Generally not well done
$= \dfrac{1}{\log_k 10}$	$\frac{1}{2}$ ... quotient	• Many provided lengthy solutions
$= \dfrac{1}{\log_k 5 + \log_k 2}$	$\frac{1}{2}$ ... sum	• Some "creative" students found k numerically and then resubstituted!! ($k = 2.4$)
$= \dfrac{1}{1.838 + 0.792}$		• max $1\frac{1}{2}$ if answer was left as $\frac{1}{2.63}$
$= \dfrac{1}{2.63}$	$\frac{1}{2}$	
$= \dfrac{100}{263}$		
$= 0.380$ (3dp)	$\frac{1}{2}$	

JRAHS TRIALS : MATHEMATICS 2U : AUG 2011

QUESTION 3

EXPECTED ANSWERS	MARKS	COMMENTS
(b) (i) P(same letter, both times)		• No marks if $\frac{1}{25}$ only.
$= \left(\frac{1}{5}\right)^2 \times 5$		
$= \frac{1}{5}$	1	
(ii) $P(E) = \frac{1}{5} \Rightarrow P(\bar{E}) = \frac{4}{5}$	$\frac{1}{2}$	• max $\frac{1}{2}$ if $n = 2.75$
Let n = number of spins		• generally poorly done
Then $\left(\frac{4}{5}\right)^n < 0.01$	$\frac{1}{2}$	
$n \log(4/5) < \log \frac{1}{100}$		
$n > \log \frac{1}{100} / \log(4/5)$		
$n > 20.6$ (1dp)	$\frac{1}{2}$	
$\therefore$ Number of spins is 21	$\frac{1}{2}$ for correct conclusion	
(c) 		(i) $\frac{1}{2}$ amplitude $\frac{1}{2}$ period $\frac{1}{2}$ scale $\frac{1}{2}$ shape (ii) $\frac{1}{2}$ for $(2\pi, \pi)$ $\frac{1}{2}$ for labelled line thro' the origin
(iii) There are 4 solutions for $x \geq 0$ as can be seen on the graph	$\frac{1}{2}$	(iii) $(\frac{1}{2})$ for recognising that $6\sin 2x - x = 0$ transforms to $3\sin 2x = \frac{1}{2}x$
and also due to $\pi > 3$, there would be no points of intersection beyond $x > 2\pi$. For $x < 0$ and $x > -2\pi$, there would be a further 3 PoI's.	$\frac{1}{2}$	$(\frac{1}{2})$ for 4 solutions; $x \geq 0$
Hence total number of solutions is 7	1	(1) for the 3 solutions; $x < 0$ and total of 7

MATHEMATICS: Question 4

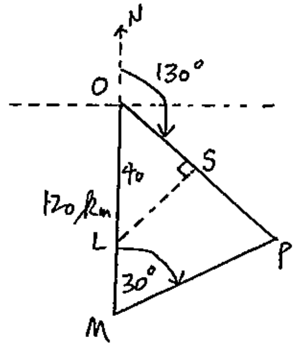
Suggested Solutions	Marks	Marker's Comments
a) i) $\frac{d^2y}{dx^2} = 6x$	1	Correctly differentiates
ii) For curve to be increasing, $y' > 0$. i.e. $3x^2 - 4 > 0$, $x^2 > \frac{4}{3}$		
$\therefore x > \frac{2}{\sqrt{3}}$ or $x < \frac{-2}{\sqrt{3}}$	$\frac{1}{2}$	Solves inequality for y'
For curve to be concave down, $y'' < 0$. i.e. $6x < 0$, $x < 0$	$\frac{1}{2}$	Solves inequality for y''
$\therefore x < \frac{-2}{\sqrt{3}}$ only.	1	Takes intersection
iii) $y = \int (3x^2 - 4)\,dx$, $y = x^3 - 4x + c$	1	Indefinite integral
Since curve passes through $(1,-2)$, $-2 = 1 - 4 + c$, $c = 1$	1	Correctly evaluates constant
$\therefore y = x^3 - 4x + 1$		
b) i) 	$\frac{1}{2}$	Accurate diagram
$D = \left(\frac{-18+4}{2}, \frac{0-6}{2}\right) = (-7,-3)$	$\frac{1}{2}$	Calculate midpoint of AB

MATHEMATICS: Question 4

Suggested Solutions	Marks	Marker's Comments
b) Continued.		
ii) $m_{AC} = \frac{6-(-6)}{0-4} = -3$ $\therefore$ Equation of AC is $y - 6 = -3(x-0)$, $y = -3x + 6$, $3x + y - 6 = 0$.	2	Correct use of point-gradient or two-point form
iii) $m_{AC} = -3$ (shown above)		
$m_{BC} = \frac{6-0}{0-(-18)} = \frac{1}{3}$	$\frac{1}{2}$	Calculate both gradients
$\therefore m_{AC} \times m_{BC} = -3 \times \frac{1}{3} = -1$ $\therefore AC \perp BC$.	$\frac{1}{2}$	Relate gradients
iv) If AB is diameter, $D(-7,-3)$ is centre.	$\frac{1}{2}$	Centre co-ordinates
Radius $AD = \sqrt{(-7-4)^2 + (-3+6)^2} = \sqrt{121+9} = \sqrt{130}$ units	$\frac{1}{2}$	Radius
$\therefore$ Circle equation: $(x+7)^2 + (y+3)^2 = 130$.	1	Equation
v) Circumference $= 2\pi r = 2\pi \times \sqrt{130} = 71.639334\ldots$	$\frac{1}{2}$	Exact value
≈ 71.6 units (1dp)	$\frac{1}{2}$	Correctly approximates

JRAHS TRIAL : MATHEMATICS 2U : Aug 2011

Q 5

EXPECTED ANSWERS	MARKS	COMMENTS
a) (i) 	$\frac{1}{2}$ for correct diagram	
$\angle POM = 50°$ .... (adjacent supplem on a straight line add up to 180°)		Many students just assumed $\angle POM = 50°$
$\therefore \angle OPM = 180° - (50+30)°$ ... (angles of ΔPOM add to 180°) $= 100°$	$\frac{1}{2}$ for confirmation	
(ii) Using sine rule on ΔPOM: $\frac{PM}{\sin \angle POM} = \frac{OM}{\sin \angle OPM}$	$\frac{1}{2}$ for sine rule	• many students omitted the calculator display
$\Rightarrow PM = \frac{120 \times \sin 50}{\sin 100°}$	$\frac{1}{2}$ for correct substitution	• Some students omitted final conclusion, to answer Question
$= 93.344...$ km $= 93$ km (nearest km)	$\frac{1}{2}$ for nearest km	
$\therefore$ P is 93 km away from M	$\frac{1}{2}$ for final conclusion	
(iii) Shortest distance is LS ($\perp$ OP) $\therefore$ In Δ LOS : $\sin 50° = \frac{LS}{40}$	1 mark	• Some made SL $\perp$ OM
$\therefore LS = 40 \sin 50$ $= 30.641...$ closest the ship is from L is 31 km (nearest km)	1 mark	

EXPECTED ANSWERS	MARKS	COMMENTS
b) (i) $M = M_0 e^{-kt}$ = amount remaining		• A large number of students used $M = \frac{3}{4} M_0$, maximum then was 1 mark.
When $t = 4$, $M = \frac{1}{4} M_0$ $\therefore \frac{1}{4} = e^{-4k}$	1 for recognising $M = \frac{1}{4} M_0$	
$4k = \ln 4$ $k = \frac{1}{2} \ln 2$	1 for final step	In this case the exact value for k was $\frac{1}{4} \ln(4/3)$
(ii) When $t = 90$ minutes $= 1\frac{1}{2}$ hrs, then $M = 4$ kg		• Many students are still not showing how the initial equations are obtained
$\therefore 4 = M_0 e^{-3/2 k}$ $\therefore M_0 = 4e^{3/2 k}$	1 for setting up correct equation	
$= 4e^{3/2 \times \frac{1}{2} \ln 2}$ $= 4e^{3/4 \ln 2}$		• Again, the calculator display not written down
$= 6.727...$ kg	$\frac{1}{2}$ for display	
$= 6.73$ kg (2dp)	$\frac{1}{2}$ for ans to 2dp	• no marks docked if concluding statement omitted
$\therefore M_0$ is 6.73 kg		

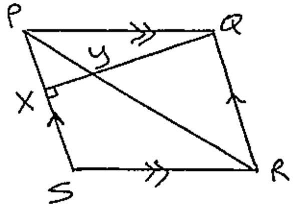
QUESTION 5

EXPECTED ANSWERS	MARKS	COMMENTS
(c) $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} - \dfrac{\sec\theta}{\tan\theta+\cot\theta}$		• Students are still skipping many parts of their working
$= \dfrac{1+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} - \dfrac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$	$\frac{1}{2}$ for expressing everything in terms of $\sin\theta$ and/or $\cos\theta$	• $-\frac{1}{2}$ if $\sin^2\theta+\cos^2\theta=1$ was not stated or expressed accordingly in the expression
$= \sin\theta\left(1+\dfrac{\cos\theta}{\sin\theta}\right) - \dfrac{1}{\cos\theta\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)}$	$\frac{1}{2}$ for simplifying 2nd term	
$= \sin\theta+\cos\theta - \dfrac{1}{\cos\theta\left(\frac{1}{\cos\theta\sin\theta}\right)}$		
$= \sin\theta+\cos\theta - 1/\tfrac{1}{\sin\theta}$	$\frac{1}{2}$ for $\sin^2\theta+\cos^2\theta=1$	
$= \sin\theta+\cos\theta-\sin\theta$	$1\frac{1}{2}$ for general simplification	
$= \cos\theta$		

2011 TRIAL HSC 2U MATHEMATICS: Question 6

Suggested Solutions	Marks	Marker's Comments
Qu 6 (i) $\int x\sin(x^2)\,dx = -\frac{1}{2}\cos(x^2)+k$	1	$\cos(x^2)+k$ without the minus loses $\frac{1}{2}$ mark
(ii) $\int_1^{16} x^{\frac{1}{2}}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^{16}$	1	
$= \frac{2}{3}(64-1)$		
$= 42$	1	
(b) $5x^2-(21+a)x+a=0$ Let the roots be α and $\frac{1}{\alpha}$		1 for each root
Product of roots $= \alpha\times\frac{1}{\alpha}$ $= 1$		
$\therefore \frac{c}{a} = 1$ where $c=a$, $a=5$		
$\therefore \frac{a}{5} = 1$		
$a = 5$	1	
$5x^2-26x+5 = 0$ $(5x-1)(x-5) = 0$ Roots are 5 and $\frac{1}{5}$	1	
(c) Let a be the first term and d the common difference.		
$S_{60} = \frac{60}{2}[2a+59d] = 1770$		
$6a+177d = 177$ .....(1)	$\frac{1}{2}$	
Also $a+59d = 3a$ $2a = 59d$ $\therefore 6a = 177d$ .....(2)	$\frac{1}{2}$	
Sub (2) in (1)		
$177d+177d = 177 \quad \therefore d=\frac{1}{2}$	1	
$\therefore$ Lengths of shortest sides are $14\frac{3}{4}$, $15\frac{1}{4}$ cms	2	1 mark for each length
(d) Area $= \int_0^1 e^{x/2} - (1-x^2)\,dx$		
$= \left[2e^{x/2} - x + \frac{x^3}{3}\right]_0^1$	1	
$= 2e^{\frac{1}{2}} - 1 + \frac{1}{3} - 2$	1	
$\therefore$ Area $= \left(2\sqrt{e}-\frac{8}{3}\right)$ units^2 $= 0.6307758\ldots$	1	

MATHEMATICS: Question 7 2011

Suggested Solutions	Marks	Marker's Comments
a) 	$\frac{1}{2}$	Some students didn't use a ruler!!
(i) $\angle SRP = \angle QRP$ (diagonals of a rhombus bisect the angles at the vertex).	$\frac{1}{2}$	
(ii) In Δs YSR, YQR	$\frac{1}{2}$	
$\angle SRP = \angle QRP$ (proven above)	$\frac{1}{2}$	
$QR = SR$ (sides of a rhombus are all equal).	$\frac{1}{2}$	
YR is common	$\frac{1}{2}$	
$\Delta YSR \equiv \Delta YQR$ (SAS)	1	
(iii) $QR \parallel PS$ (opposite sides of a rhombus are parallel).	$\frac{1}{2}$	
$\angle RQY = \angle QXP$ (alternate angles are equal, $QR \parallel PS$).	$\frac{1}{2}$	
but $Q\hat{X}P = 90^\circ$ $\therefore R\hat{Q}Y = 90^\circ$		
$\therefore \angle YSR = R\hat{Q}Y$ (corresponding angles in congruent triangles are equal).	$\frac{1}{2}$	
$= 90^\circ$	$\frac{1}{2}$	

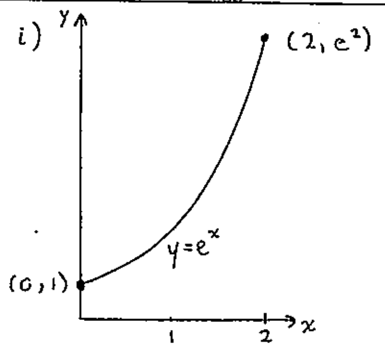
MATHEMATICS: Question 7 2011

Suggested Solutions	Marks	Marker's Comments
b)(i) stops when $v=0$ $\frac{t^2(t-3)}{3} = 0$ $\therefore$ stops when $t=0$ or $t=3$ $\therefore$ Journey takes 3 minutes		1 right or wrong only
(ii) $\frac{dx}{dt} = \frac{t^2(3-t)}{3} = \frac{3t^2}{3} - \frac{t^3}{3} = t^2 - \frac{t^3}{3}$ $\therefore x = \frac{1}{3}t^3 - \frac{t^4}{12} + c$	1	lost one mark if they forgot the "+c".
when $t=0$, $x=0$ $\therefore c=0$ $\therefore x = \frac{1}{3}t^3 - \frac{1}{12}t^4$	1	
(iii) when $t=3$, $x = \frac{1}{3}\times 27 - \frac{1}{12}\times 3^4$ $= 2\frac{1}{4}$	$\frac{1}{2}$	
$\therefore$ It is $2\frac{1}{4}$km from Epping to Eastwood	$\frac{1}{2}$	
(iv) $v = t^2 - \frac{t^3}{3}$ $\frac{dv}{dt} = 2t - t^2$ $\frac{dv}{dt} = 0$ at $t=0$ or $t=2$	$\frac{1}{2}$	
$\frac{d^2v}{dt^2} = 2 - 2t$ when $t=2$, $\frac{d^2v}{dt^2} = 2-4 = -2 < 0$ $\therefore$ concave down $\therefore$ relative maximum	$\frac{1}{2}$	If they didn't discuss why it was the fastest or test the nature at $t=2$, they lost $\frac{1}{2}$ a mark!!
when $t=2$, $x = \frac{1}{3}(2)^3 - \frac{1}{12}(2^4)$ $= \frac{4}{3}$	$\frac{1}{2}$	
$\therefore$ Maximum speed at $x = \frac{4}{3}$km after 2 minutes.	$\frac{1}{2}$	

MATHEMATICS: Question 8

Suggested Solutions	Marks	Marker's Comments
a) i) $B_1 = 1000 \times 1.06 - 72$	$\frac{1}{2}$	Interprets question
$= 988$		
$\therefore$ Balance is \$988.	$\frac{1}{2}$	Evaluates
ii) $B_2 = B_1 \times 1.06 - 72$		
$= 1000(1.06)^2 - 72(1.06) - 72$		
$= 1000(1.06)^2 - 72(1.06 + 1)$		
$B_3 = 1000(1.06)^3 - 72(1.06^2 + 1.06 + 1)$	1	Establish pattern
$\therefore B_n = 1000(1.06)^n - 72(1.06^{n-1} + \cdots + 1.06 + 1)$		
$B_n = 1000(1.06)^n - 72\left[\frac{1.06^n - 1}{1.06 - 1}\right]$ — EQUATION ONE	$\frac{1}{2}$	Identify GP
$= 1000(1.06)^n - 1200(1.06^n - 1)$		
$= 1000(1.06)^n - 1200(1.06)^n + 1200$	$\frac{1}{2}$	Simplify
$B_n = 1200 - 200(1.06)^n$ as required.		
iii) $B_{10} = 1200 - 200(1.06)^{10}$		
$= 841.83046\cdots$	$\frac{1}{2}$	Evaluating B_{10}
Let C_n be the balance at the end of n years after the prize increases.		
$\therefore C_n = 841.83(1.06)^n - 90\left[\frac{1.06^n - 1}{1.06 - 1}\right]$ from EQUATION ONE		
$= 841.83(1.06)^n - 1500(1.06^n - 1)$		
$C_n = 1500 - 658.17(1.06)^n$	$\frac{1}{2}$	Equation for new balance
The fund will run out when $C_n = 0$,		
i.e. $1500 - 658.17(1.06)^n = 0$		
$(1.06)^n = \frac{1500}{658.17}$		
$n = \log_{1.06}\left[\frac{1500}{658.17}\right]$		
$= \frac{\ln 1500 - \ln 658.17}{\ln 1.06}$		
$n = 14.14\cdots$	1	Make n the subject
$\therefore$ Full prize can be given for a further $\underline{14}$ years.	1	Final answer

MATHEMATICS: Question 8

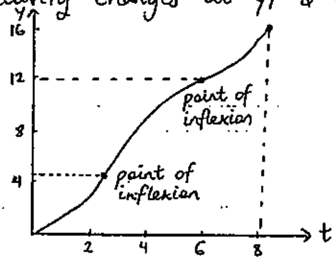
Suggested Solutions	Marks	Marker's Comments
b) i) 	$\frac{1}{2}$	Accurate shape
	$\frac{1}{2}$	Correct domain
ii) $y = e^x$		
$x = \ln y$	1	Make x the subject
$x^2 = (\ln y)^2$		
$V = \pi\int_a^b x^2\,dy$ where $a = 3$, $b = 5$	1	Form integral
$\therefore V = \pi\int_3^5 (\ln y)^2\,dy$		

iii)

y	3	3.5	4	4.5	5
x	1.099	1.253	1.386	1.504	1.609
x^2	1.207	1.569	1.922	2.262	2.590

Suggested Solutions	Marks	Marker's Comments
(table above)	1	Function values
$V = \pi\int_3^5 (\ln y)^2\,dy$		
$\approx \pi \times \frac{h}{3}(f_0 + 4f_1 + 2f_2 + 4f_3 + f_4)$		
$\approx \frac{\pi}{6}\left[1.207 + 4(1.569) + 2(1.922) + 4(2.262) + 2.590\right]$	1	Correct use of Simpson's formula
$\approx 12.02576\cdots$		
≈ 12.0 units^3 (1dp)	1	Approximated answer

MATHEMATICS: Question 9

Suggested Solutions	Marks	Marker's Comments
a) i) $\frac{dy}{dt}$ is minimum when glass is widest, i.e. $y_1 = 12$.	1	Accepted $\pm\frac{1}{2}$
ii) $\frac{dy}{dt}$ is maximum when glass is thinnest, i.e. $y_2 = 4\frac{1}{2}$.	1	Accepted $\pm\frac{1}{2}$
iii) Concavity changes at y_1 & y_2.	1	Points of inflexion
	$\frac{1}{2}$	Endpoint (8, 16)
	$\frac{1}{2}$	Shape (i.e. correct concavity)
b) As M bisects CO, $CM = 4$cm.	$\frac{1}{2}$	Radius
Arc $CD = r\theta$ $= 8 \times \frac{2\pi}{3}$ $= \frac{16\pi}{3}$ or $5\frac{1}{3}\pi$ cm.	1	Arc length
In ΔDOM, $MD^2 = 4^2 + 8^2 - 2(4\times 8)\cos\left(\frac{2\pi}{3}\right)$ $= 16 + 64 - (-32)$	1	Cosine rule
$MD^2 = 112$ $MD = \sqrt{112}$ (as $MD > 0$) $\doteqdot 4\sqrt{7}$ cm	$\frac{1}{2}$	Simplify MD
$\therefore$ Perimeter is $\left[4 + 4\sqrt{7} + \frac{16\pi}{3}\right]$cm.		

MATHEMATICS: Question 9

Suggested Solutions	Marks	Marker's Comments
c) $\int_2^k \frac{2t}{3t^2-1}\,dt = \frac{1}{3}\int_2^k \frac{6t}{3t^2-1}\,dt$		
$= \frac{1}{3}\left[\ln(3t^2-1)\right]_2^k$	1	Integrate
$\frac{1}{3}\ln 13 = \frac{1}{3}\left[\ln(3k^2-1) - \ln(12-1)\right]$ $\ln 13 = \ln\left[\frac{3k^2-1}{11}\right]$		
$\frac{3k^2-1}{11} = 13$	1	Eliminate log terms
$3k^2 - 1 = 143$ $3k^2 = 144$	$\frac{1}{2}$	Simplify
$k^2 = 48$ $k = \pm 4\sqrt{3}$ but $k > 2$	$\frac{1}{2}$	Consider restriction
$\therefore k = 4\sqrt{3}$ only.		
d) Since a, b, c is a GP, $\frac{c}{b} = \frac{b}{a}$. $\therefore \log\left(\frac{c}{b}\right) = \log\left(\frac{b}{a}\right)$	1	Take logs of both sides
$\log c - \log b = \log b - \log a$ $\therefore \log a$, $\log b$, $\log c$ is an AP.	1	Manipulate with log laws
<u>ALTERNATIVE METHOD</u> Let r be the common ratio of the GP.		
$\therefore b = ar$ $\log b = \log(ar)$ $= \log a + \log r$	1	
$c = ar^2$ $\log c = \log(ar^2)$ $= \log a + \log(r^2)$ $= \log a + 2\log r$	1	
$\therefore \log a$, $\log b$, $\log c$ is an AP where the common difference is $\log r$.		

2011 TRIAL HSC **MATHEMATICS: Question** 10

Suggested Solutions	Marks	Marker's Comments
Q (a) $25-x^2 \geq 10$ $x^2 \leq 25$ ∴ Domain: $-5 \leq x \leq 5$	1	$\frac{1}{2}$ mark for each correct limit of domain or range
Min value 0 at ends of range ∴ Range: $0 \leq y \leq 10$	1	
(b) $\displaystyle\sum_{r=1}^{63} \frac{1}{\sqrt{r+1}+\sqrt{r}} = \sum_{r=1}^{63} \frac{1}{\sqrt{r+1}+\sqrt{r}} \times \frac{\sqrt{r+1}-\sqrt{r}}{\sqrt{r+1}-\sqrt{r}}$	$\frac{1}{2}$	
$= \displaystyle\sum_{r=1}^{63} \sqrt{r+1}-\sqrt{r}$ $= (\sqrt{1+1}-\sqrt{1}) + (\sqrt{2+1}-\sqrt{2}) + \dots + (\sqrt{63+1}-\sqrt{63})$	1	
$= -\sqrt{1} + \sqrt{63+1}$ $= -1 + 8$ $= 7$	$\frac{1}{2}$	
(c) (i) $GP = 240\operatorname{Tan}\theta$	$\frac{1}{2}$	
∴ $DP = 3000 + 240\operatorname{Tan}\theta$	$\frac{1}{2}$	
∴ Time $= \dfrac{DP}{\text{speed}} = \dfrac{3000+240\operatorname{Tan}\theta}{15}$ $= 200 + 16\operatorname{Tan}\theta$	1	
(ii) $EP = 240\operatorname{Sec}\theta$ ∴ Time $= \dfrac{240\operatorname{Sec}\theta}{4}$ $= 60\operatorname{Sec}\theta$	1	$\dfrac{60}{\cos\theta}$, $\sqrt{3600(1+\tan^2\theta)}$ also received full marks
(iii) Time taken for Peter to reach P $= T + 60\operatorname{Sec}\theta$ If Peter and Bus arrive at P at the same time $T + 60\operatorname{Sec}\theta = 200 + 16\operatorname{Tan}\theta$ ∴ $T = 200 + 16\operatorname{Tan}\theta - 60\operatorname{Sec}\theta$	1	
(iv) $T = 200 + 16\operatorname{Tan}\theta - 60\operatorname{Sec}\theta$ $\dfrac{dT}{d\theta} = 16\operatorname{Sec}^2\theta - 60\operatorname{Sec}\theta\operatorname{Tan}\theta$ $\dfrac{dT}{d\theta} = 4\operatorname{Sec}\theta(4\operatorname{Sec}\theta - 15\operatorname{Tan}\theta)$	1	
$\dfrac{dT}{d\theta} = 0$ for stationary pts as $4\operatorname{Sec}\theta \neq 0$		

2011 TRIAL HSC **MATHEMATICS: Question** 10

Suggested Solutions	Marks	Marker's Comments
(iv) cont. ∴ $4\operatorname{Sec}\theta - 15\operatorname{Tan}\theta = 0$ $4\operatorname{Sec}\theta = 15\operatorname{Tan}\theta$ $\dfrac{4}{\cos\theta} = \dfrac{15\sin\theta}{\cos\theta}$ ∴ $\sin\theta = \dfrac{4}{15}$	1	
Test nature		
<table><tr><td>θ</td><td>0.26</td><td>$\sin^{-1}(\frac{4}{15})$</td><td>0.27</td></tr><tr><td>$\frac{dT}{d\theta}$</td><td>0.6159</td><td>0</td><td>-0.0042</td></tr><tr><td>slope</td><td>/</td><td>–</td><td>\</td></tr></table>	1	
There is a change in gradient from positive to negative hence a local max. occurs at $\theta = \sin^{-1}(\frac{4}{15})$ As there is only one turning pt for $0.26 \leq \theta \leq 0.27$ then the local max. is also the absolute max.		
∴ Maximum Time $= 200 + 16\operatorname{Tan}(\sin^{-1}(\frac{4}{15})) - 60\operatorname{Sec}(\sin^{-1}(\frac{4}{15}))$ $= 142.17$ (2dp) $\doteqdot 142$ seconds (nearest second)	$\frac{1}{2}$	or 2 minutes 22 seconds
<u>OR</u> Test nature $\dfrac{d^2T}{d\theta^2} = 4\operatorname{Sec}\theta(4\operatorname{Tan}\theta\operatorname{Sec}\theta - 15\operatorname{Sec}^2\theta)$ $+ (4\operatorname{Sec}\theta - 15\operatorname{Tan}\theta) \times 4\operatorname{Tan}\theta\operatorname{Sec}\theta$ $= 16\operatorname{Tan}\theta\operatorname{Sec}^2\theta - 60\operatorname{Sec}^3\theta + 16\operatorname{Tan}\theta\operatorname{Sec}^2\theta$ $- 60\operatorname{Tan}^2\theta\operatorname{Sec}\theta$ $= 32\operatorname{Tan}\theta\operatorname{Sec}^2\theta - 60\operatorname{Sec}^3\theta - 60\operatorname{Tan}^2\theta\operatorname{Sec}\theta$ $= \dfrac{1}{\cos^3\theta}\left[32\sin\theta - 60 - 60\sin^2\theta\right] \doteqdot -62.254$ Now as $60 > 32\sin\theta$ $\dfrac{d^2T}{d\theta^2} < 0$ Hence the curve is concave down and a max exists when $\sin\theta = \dfrac{4}{15}$		
∴ Latest time that Peter can leave home and still catch the bus is <u>8:02:22 am</u>	$\frac{1}{2}$	