Section I Multiple Choice (10 marks)

Attempt Question 1 – 10 (1 mark each)
Allow approximately 15 minutes for this section.

Question 1

Factorise $8x^6 - 27$

A) $(2x^2 - 3)(4x^4 - 6x^2 + 9)$

B) $(2x^2 + 3)(4x^4 - 6x^2 + 9)$

C) $(2x^2 - 3)(4x^4 + 6x^2 + 9)$

D) $(2x^2 + 3)(4x^4 - 6x^2 + 9)$

Question 2

What is the greatest value taken by the function $f(x) = 3 - 2\cos x$?

A) 1 B) 2 C) 3 D) 5

Question 3

What is the value of $\int_2^6 \frac{1}{x+2}dx$?

A) ln 2 B) ln 4 C) ln 6 D) ln 8

Question 4

The table below shows the values of a function *f(x)* for five values of *x*.

x	2	2.25	2.5	2.75	3
f(x)	3	4	-1	3	7

What value is an estimate for $\int_2^3 f(x)dx$ using Simpson's Rule with these five values ?

A) 3 B) 4 C) 5 D) 6

Question 5

The perpendicular distance from the point $(-3, 2)$ to the line $x - 4y = 1$ is closet to:

A) 1.1 B) 2.8 C) 2.9 D) 3.3

Question 6

Thirty tickets are sold in a raffle. There are two prizes (first and second). Lewis buys 4 tickets. What is the probability of him winning the second prize but not the first?

A) $\frac{2}{15}$ B) $\frac{26}{225}$ C) $\frac{52}{435}$ D) $\frac{26}{435}$

Question 7

What is the equation of the **normal** to the curve $y = x^2 - 4x$ at $(1, -3)$?

A) $x + 2y - 7 = 0$ B) $x - 2y - 7 = 0$

C) $2x - y + 1 = 0$ D) $2x + y + 1 = 0$

Question 8

The concave up parabola with focus $S(3, 2)$, focal length 1 and axis of symmetry perpendicular to the $x - axis$ is:

A) $(x-3)^2 = 4(y-2)$ B) $(x-3)^2 = 4(y-1)$

C) $(y-2)^2 = 4(x-3)$ D) $(x-3)^2 = 4(y-3)$

Question 9

What is the derivative of $\frac{x}{cosx}$?

A) $\frac{cosx+xsinx}{cos^2x}$ B) $\frac{cosx-xsinx}{cos^2x}$ C) $\frac{xsinx-cosx}{cos^2x}$ D) $\frac{-xsinx-cosx}{cos^2x}$

Question 10

What are the solutions to the equation $25^x - 5^{x+1} + 6 = 0$?

A) $x = 2$ or $x = 3$ B) $x = \frac{ln2}{ln5}$ or $x = \frac{\ln 3}{ln5}$

C) $x = ln\frac{2}{5}$ or $x = ln\frac{3}{5}$ D) $No\ Solutions$

End of Section I

Section II Total Marks is 90

Attempt Questions 11 – 16.
Allow approximately 2 hours & 45 minutes for this section.

Answer all questions, starting each new question on a new sheet of paper with your **student ID number** in the top right hand corner and the question number on the left hand side of your paper.

All necessary working must be shown in each and every question.

Question 11 (15 Marks) Start a new piece of paper Marks

a) For the series $2 - 1 + \frac{1}{2} - \frac{1}{4} + \cdots$

i. Find the 8th term of the series. **1**

ii. Find the limiting sum of the series. **1**

b) Differentiate with respect to x:

i. $sin\left(\frac{x-1}{x^2}\right)$ **2**

ii. $x^2 \mathrm{e}^{-x}$ **2**

c) Find:

i. $\int \sqrt{3x+1}\,dx$ **1**

ii. $\int \frac{2x+1}{x^2}\,dx$ **2**

d)

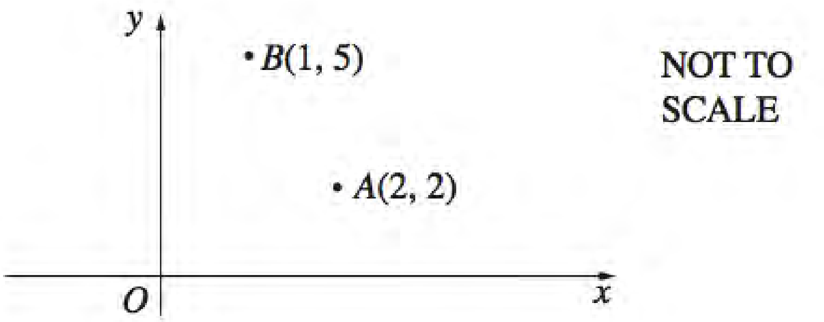
i. Prove that $\frac{cos^2x}{1-sinx} = 1 + sinx.$ **2**

ii. Hence or otherwise, find the range of $y = \frac{cos^2x}{1-sinx}$. **1**

iii. Sketch $y = \frac{cos^2x}{1-sinx}$ $\;-\pi \le x \le \pi$ showing all important features. **3**

Question 12 (15 Marks) Start a new piece of paper Marks

a)



The diagram shows two points $A(2\,,2)$ and $B(1\,,5)$ on the number plane.

Copy the diagram into your writing paper.

i. Find the coordinates of M, the midpoint of AB. 1

ii. Show that the equation of the perpendicular bisector of AB is $x - 3y + 9 = 0$. 2

iii. Find the coordinates of the point C that lies on the y axis and is equidistant from A and B. 1

iv. The point D lies on the intersection of the line $y = 5$ and the perpendicular bisector $x - 3y + 9 = 0$. Find the coordinates of D. 1

v. Find the area of ΔABD. 2

b) A particle travels in a straight line. Its velocity $\dot{x}$ at time t is given by

$$\dot{x} = (3t^2 - 9t + 6)\ ms^{-1}$$

i. Find the expression for the particle's displacement x in terms of t, if after 1 second the particle is at $x = 3$. 2

ii. Find the expression for the particle's acceleration in terms of t. 1

iii. At what time(s) is the particle at rest? 1

iv. Draw a velocity-time graph representing the motion of this particle showing all important features. 2

v. Find the total distance travelled by the particle after 2 seconds. 2

Question 13 (15 Marks) Start a new piece of paper Marks

a) Let $f(x) = (x + 2)(x^2 + 4)$.

i. Show that $y = f(x)$ has no stationary points. 2

ii. Find the values of x for which $y = f(x)$ is concave up and concave down respectively. 2

iii. Sketch the graph $y = f(x)$, indicating all intercepts and any points of inflexion. 2

b) On the 1st of January 2001, Mary decides to put \$5000 into a savings account that earns her interest at a rate of 4% p.a. compounded at the end of each month.

i. Show that (to the nearest dollar) Mary will have \$5636 in her account on the 1st of January 2004. 2

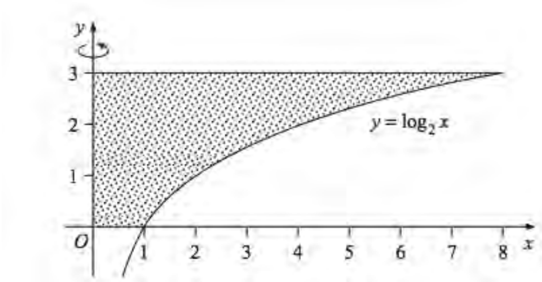
ii. At this rate, at the end of which year and month will Mary's account exceed \$10 000? 2

iii. Mary has started working and is able to deposit \$500 on the 2nd of every month starting from January 2004. How much will Mary have in her account on the 1st of January 2007? (Use \$5636 as the starting amount in 2004). 3

c) Given that $y = (3x + 1)^2$. Show that $y - y' + 4 = x(y' - 12) - y$. 2

Question 14 (15 Marks) **Start a new piece of paper** **Marks**

a)



The diagram shows the graph $y = log_2x$ between $x = 1$ and $x = 8$. The shaded region, bounded by $y = log_2x$, the line $y = 3$, and the x and y axes, is rotated about the y axis to form a solid.

i. Show that the volume of the solid is given by **2**

$$V = \pi \int_0^3 e^{y ln 4} dy$$

ii. Hence find the volume of the solid. **2**

b) The death rate of an endangered species on an island is given by

$$\frac{dP}{dt} = -kP,$$

where P is the population of the species after t days and k is a constant.

i. Show that $P = Ae^{-kt}$ is a solution to the equation, where A is a constant. **1**

ii. Initially there were 2000 of the species on the island, after 300 days only 1000 were left. What is the population (to the nearest whole number) after 400 days? **3**

iii. After how many days will the population of the species drop below 400? **2**

c) Use the Trapezoidal Rule with 5 function values to obtain an estimate for: **2**

$$\int_{-2}^{6} \log_e \sqrt{x+3}\, dx$$

Simplify your answer as much as possible.

d)

i. Rationalise the denominator in the expression **1**

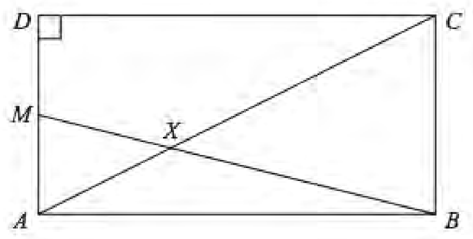
$\frac{1}{\sqrt{n}+\sqrt{n+1}}$, where n is a positive integer

ii. Using your result from part i or otherwise, find the value of the sum **2**

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{99}+\sqrt{100}}$$

Question 15 (15 Marks) **Start a new piece of paper** **Marks**

a)



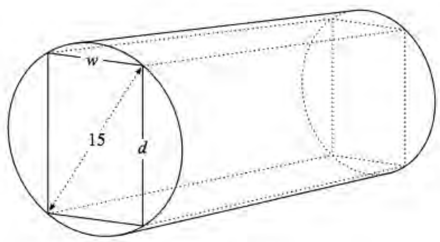
In the diagram, ABCD is a rectangle and $AB = 2AD$. The point M is the midpoint of AD. The line BM meets AC at X.

i. Show that the triangles AXM and BXC are similar. 2

ii. Show that 3CX = 2AC 2

iii. Show that $9(CX)^2 = 5(AB)^2$ 2

b)



A rectangular beam of width w cm and depth d cm is cut from a cylindrical pine log as shown.

The diameter of the cross-section of the log (and hence the diagonal of the cross-section of the beam is 15 cm.

The strength S of the beam is propotional to the product of its width and the square of its depth, so that

$$S = kd^2w.$$

i. Show that $S = k(225w - w^3)$. 1

ii. Find numerically the dimensions that will maximise the strength of the beam. Justify your answer. 3

iii. Find the strength S of the beam if its cross sectional area is a square with diagonal 15 cm. 1

iv. Express as a percentage, how much stronger will the beam of maximum strength be in comparison to the square beam in part iii to the nearest %. 2

c) 65 blue marbles are placed into a bag with red marbles in it. Then a sample of 32 marbles were drawn out of the bag. It was noted that of the 32 drawn out, 27 of them were red. What is the expected number of red marbles in the bag to begin with? 2

Please Turn Over

Question 16 (15 Marks) **Start a new piece of paper** **Marks**

a)

i. Show that the tangent to $y = x^3$ at the point where $x = 2$ is $y = 12x - 16$. 3

ii. Show that the tangent meets the curve again at $(-4, -64)$. 1

iii. Find the area of the region enclosed between the curve and the tangent. 2

b) Joe tosses an unbiased coin continuously, until, for the first time, the same result is repeated in two consecutive throws (that is, 2 heads in a row or 2 tails in a row).

i. What is the probability that the game ends within the first 3 throws? 1

ii. Joe decides that he wins if the game ends after an odd number of throws, and he loses if the game ends after an even number of throws. Find the probability that Joe wins the game. 2

c) Two ships P and Q are observed to be at a bearing of $315°T$ and $045°T$ respectively from a port A. From a second port B, which is 1 km due east of A, the ships P and Q are observed to be at a bearing of $293°T$ and $023°T$ respectively. Let the point of intersection of QA and PB be T.

i. Draw a diagram illustrating the information given above. 1

ii. Show that BT is given by $\frac{sin45°}{sin112°}$ and AT is given by $\frac{sin23°}{sin112°}$. 2

iii. Hence, or otherwise, find the distance between the two ships P and Q correct to 2 decimal places, giving reasons. 3

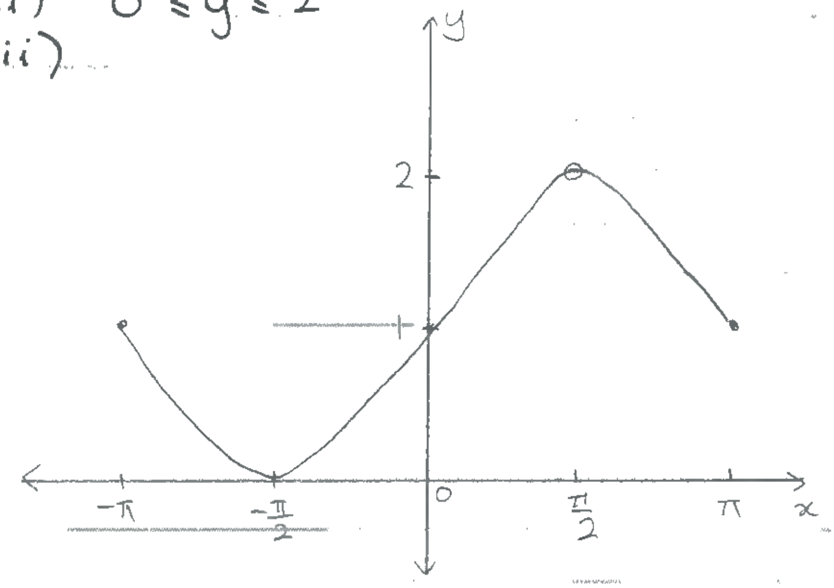
End of Exam.

MATHEMATICS: Question multiple choice		
Suggested Solutions	**Marks**	**Marker's Comments**
1. $(2x^2)^3 - 3^3 = (2x^2-3)(4x^4+6x^2+9)$ (C) 2. greatest value at $x = 180$ $f(x) = 3 - 2\cos(180)$ $= 5$ (D) 3. $\int_2^6 \frac{1}{x+2}\,dx = [\ln(x+2)]_2^6$ $= \ln 8 - \ln 4$ $= \ln 2$ (A) 4. $\int_2^3 f(x)dx \approx \frac{0.25}{3}(3 + 4\times4 + 2(-1) + 4(3) + 7)$ $= 3$ (A) 5. $d = \frac{\lvert 1(-3) - 4(2) - 1 \rvert}{\sqrt{1^2 + (-4)^2}}$ $= 2.910\ldots$ (C) 6. $\frac{26}{30} \times \frac{4}{29} = \frac{52}{435}$ (C) 7. $y = x^2 - 4x$ $y' = 2x - 4$ at $(1,-3)$ $y' = 2(1) - 4$ $= -2$ $m_\perp = \frac{1}{2}$ $\therefore$ equation is: $y + 3 = \frac{1}{2}(x-1)$ $x - 2y - 7 = 0$ (B) 8. vertex: $(3,1)$ $a = 1$ $(x-3)^2 = 4(y-1)$ (B) 9. $\frac{d}{dx}\left(\frac{x}{\cos x}\right) = \frac{\cos x + x\sin x}{\cos^2 x}$ (A) 10. $25^x - 5^{x+1} + 6 = 0$ $5^{2x} - 5\times5^x + 6 = 0$ $(5^x - 2)(5^x - 3) = 0$ $\therefore 5^x = 2$, $5^x = 3$ $x\ln 5 = \ln 2$, $x \ln 5 = \ln 3$ $x = \frac{\ln 2}{\ln 5}$, $x = \frac{\ln 3}{\ln 5}$ (B)		

MATHEMATICS: Question 11

Suggested Solutions	Marks	Marker's Comments
a) i) $a = 2$, $r = -\frac{1}{2}$		
$T_8 = ar^{n-1}$		
$= 2(-\frac{1}{2})^7$		
$= -\frac{1}{64}$	1	
ii) $S_\infty = \frac{a}{1-r}$		
$= \frac{2}{1+\frac{1}{2}}$		
$= \frac{4}{3}$	1	
b) i) $\frac{d}{dx}\left(\sin\left(\frac{x-1}{x^2}\right)\right)$		
$= \cos\left(\frac{x-1}{x^2}\right) \times \left(\frac{x^2-(x-1)2x}{x^4}\right)$	2	
$= \frac{2-x}{x^3} \cos\left(\frac{x-1}{x^2}\right)$		
ii) $\frac{d}{dx}(x^2e^{-x}) = 2xe^{-x} - x^2e^{-x}$	2	
$= xe^{-x}(2-x)$		
c) i) $\int \sqrt{3x+1}\, dx = \frac{1}{3} \times \frac{2}{3}(3x+1)^{3/2} + c$	1	
$= \frac{2}{9}(3x+1)^{3/2} + c$		
ii) $\int \frac{2x+1}{x^2}\, dx = \int\left(\frac{2}{x} + \frac{1}{x^2}\right) dx$	1	
$= 2\ln x - \frac{1}{x} + c$	1	
d) i) $\frac{\cos^2 x}{1-\sin x} = 1+\sin x$		
$\text{LHS} = \frac{\cos^2 x}{1-\sin x}$		
$= \frac{1-\sin^2 x}{1-\sin x}$	1	

MATHEMATICS: Question 11

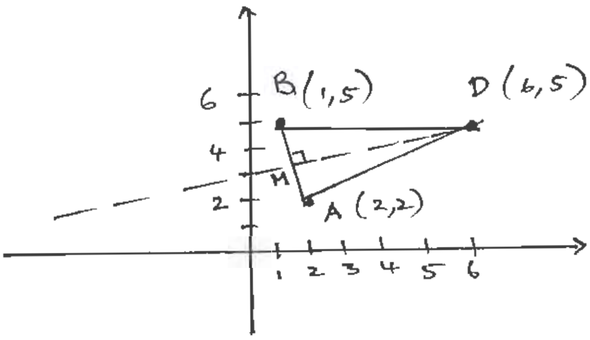
Suggested Solutions	Marks	Marker's Comments
$= \frac{(1-\sin x)(1+\sin x)}{(1-\sin x)}$	1	
$= 1+\sin x$		
ii) $0 \leq y \leq 2$	1	
iii) 		1 open circle at $x=\frac{\pi}{2}$ 1 shape 1 correct end points & scale

MATHEMATICS: Question 12

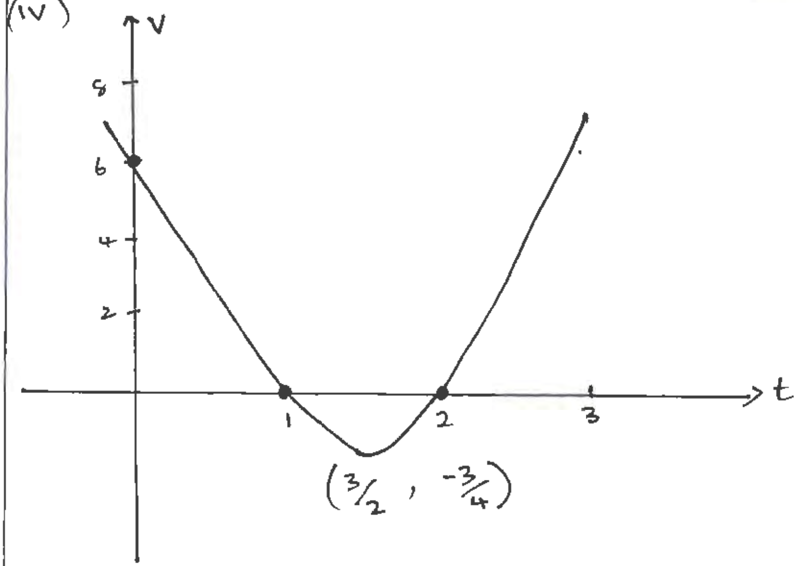
Suggested Solutions	Marks	Marker's Comments
a) (i) midpoint $\left(\frac{1+2}{2}, \frac{5+2}{2}\right)$ $= \left(\frac{3}{2}, \frac{7}{2}\right)$ ✓	1	both x and y have to correct to get 1 mark
(ii) Equation of perpendicular bisector of AB $M_{AB} = \frac{5-2}{1-2} = -3$ ∴ Perpendicular bisector has gradient $\frac{1}{3}$ ✓ Equation is: $y - \frac{7}{2} = \frac{1}{3}\left(x - \frac{3}{2}\right)$ ✓ $3y - \frac{21}{2} = x - \frac{3}{2}$ $6y - 21 = 2x - 3$ $2x - 6y + 18 = 0$ $x - 3y + 9 = 0$	2	1 mark for getting the gradient $\left(\frac{1}{3}\right)$ for perpendicular bisector 1 mark for subing in $x = \frac{3}{2}$ and $y = \frac{7}{2}$ and gradient $\frac{1}{3}$ into equation
(iii) Coordinates of C since C lies on the y axis the coordinate would be $(0, c)$ $AC = BC$ $\sqrt{(2-0)^2 + (2-c)^2} = \sqrt{(1-0)^2 + (5-c)^2}$ $4 + (4 - 4c + c^2) = 1 + (25 - 10c + c^2)$ $c^2 - 4c + 8 = c^2 - 10c + 26$ $6c = 18$ $c = \frac{18}{6}$ $c = 3$ ✓ ∴ coordinate of C is $(0, 3)$	1	Answer had to be written as a coordinate

J:\Maths\marking templates\Suggested Mk solns template_V2_no Ls.doc

MATHEMATICS: Question 12

Suggested Solutions	Marks	Marker's Comments
IV) Coordinates of D when $y=5$ $x-3(5)+9=0$ $x-15+9=0$ $x-6=0$ $x=6$ $\therefore$ D is $(6,5)$ ✓	1	1 mark for coordinate $(6,5)$
(v) Area of triangle ABD.  $d_{AB}=\sqrt{(2-1)^2+(2-5)^2}$ $=\sqrt{1+9}$ $=\sqrt{10}$ $d_{MD}=\sqrt{\left(6-\frac{3}{2}\right)^2+\left(5-\frac{7}{2}\right)^2}$ $=\sqrt{\frac{81}{4}+\frac{9}{4}}$ $=\sqrt{\frac{90}{4}}$ $=\frac{3\sqrt{10}}{2}$ Area $=\frac{1}{2}bh$ $=\frac{1}{2}\times\sqrt{10}\times\frac{3\sqrt{10}}{2}$ $=\frac{30}{4}$ $=\frac{15}{2}$ Area is $\frac{15}{2}\ u^2$.	2	Alternatively From diagram BD is 5 units perpendicular height is 3 units Area $=\frac{1}{2}bh$ $=\frac{1}{2}\times5\times3$ $=\frac{15}{2}$ Area is $\frac{15}{2}\ u^2$ ✓1 mark for getting distances and ✓1 mark for area calculation

MATHEMATICS: Question 12

Suggested Solutions	Marks	Marker's Comments
b (i) $\dot{x} = (3t^2 - 9t + 6)\ ms^{-1}$ Displacement $x = \int 3t^2 - 9t + 6\ dt$ $x = t^3 - \frac{9t^2}{2} + 6t + c$ ✓ when $t = 1$, $x = 3$ $3 = 1 - \frac{9}{2} + 6 + c$ $c = \frac{1}{2}$ ✓ $\therefore x = t^3 - \frac{9t^2}{2} + 6t + \frac{1}{2}$	2.	1 mark for calculating $c = \frac{1}{2}$ 1 mark for getting integration $t^3 - \frac{9t^2}{2} + 6t$ correctly.
(ii) $\ddot{x} = 6t - 9$ ✓	1	
(iii) time when particle is at rest $\dot{x} = 0$ $\therefore 3t^2 - 9t + 6 = 0$ $t^2 - 3t + 2 = 0$ $(t-1)(t-2) = 0$ $t = 1$ or $t = 2$ ✓	1	1 mark for getting both $t = 1$ or 2 sec.
(iv) 	2	✓ 1 mark for getting x intercepts as $x=1$ and $x=2$ and $y=6$ ✓ 1 mark for labelling the TP $(\frac{3}{2}, -\frac{3}{4})$

MATHEMATICS: Question 12

Suggested Solutions	Marks	Marker's Comments

(v) Distance $= \int_0^1 (3t^2 - 9t + 6)\,dt + \left| \int_1^2 (3t^2 - 9t + 6)\,dt \right|$

$= \left[t^3 - \frac{9t}{2} + 6t \right]_0^1 + \left| \left[t^3 - \frac{9}{2}t + 6t \right]_1^2 \right|$

$= (3 - \frac{1}{2}) + \left| (\frac{5}{2} - 3) \right|$

$= 2\frac{1}{2} + \left| \frac{-1}{2} \right|$

$= 2\frac{1}{2} + \frac{1}{2}$

$= 3$

$\therefore$ distance $= 3$ units

Alternatively.

Use distance $x = t^3 - \frac{9}{2}t^2 + 6t + \frac{1}{2}$

t	0	1	2
x	$\frac{1}{2}$	3	$\frac{5}{2}$

$x = 2\frac{1}{2}$.

$\frac{1}{2}$

Total distance = 3 units

Need absolute value for area between $x = 1$ and $x = 2$ (below x-axis) to get 1 mark

total distance equals 3 units

Second method needed proper working out to get full 2 marks

QUESTION 13   2 units Trial 2016.

a) i) show $f(x)$ has no stationary points.

$f(x) = (x+2)(x^2+4)$

$f'(x) = (x+2)\cdot 2x + (x^2+4)\cdot 1$

$= 2x^2 + 4x + x^2 + 4$

$= 3x^2 + 4x + 4$ ——— 1

$\Delta = b^2 - 4ac$

$= 16 - 4(3)(4)$

$= -32$

$f'(x) \neq 0$ since $\Delta < 0$ (no real root)

$\therefore$ no stationary points. ——— 1

ii) $f''(x) = 6x + 4$

$f''(x) > 0$ concave up

$\therefore 6x + 4 > 0$

$6x > -4$

$x > -\frac{2}{3}$

1

$f''(x) < 0$ concave down

$6x + 4 < 0$

$6x < -4$

$x < -\frac{2}{3}$

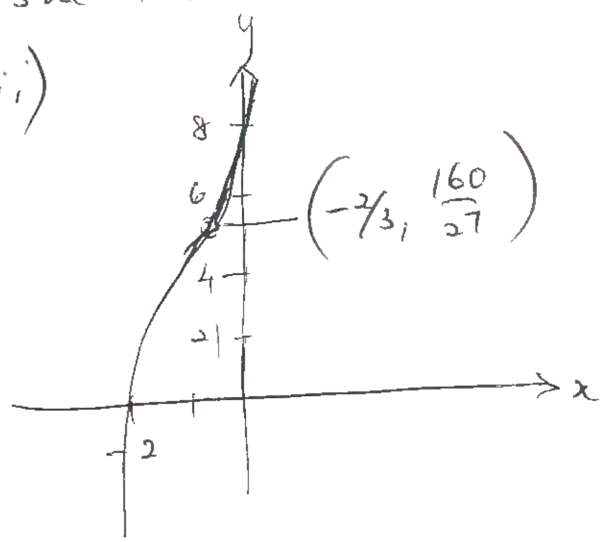
1

or

	ie -0.7		ie 0.6
x	$< -\frac{2}{3}$	$-\frac{2}{3}$	$> -\frac{2}{3}$
$f''(x)$	-0.2	0	0.4
	$\cap$		$\cup$
	concave down		concave up

Q13 a cont ...

a iii)



to get 2 marks.
① show y and x intercept & shape
② pt of inflexion

b) 1s Jan 2001, $P = 5000$, $r = 4\%$ p.a compounded at end of each mth.

i) 1st Jan 2004 $\therefore$ $n = 3 \times 12 = 36$ $r = \frac{4\%}{12} = \frac{0.04}{12}$.

① for correct n and r

$A = P(1+r)^n$
$= 5000\left(1+\frac{0.04}{12}\right)^{36}$
$= 5636.36$
$= \$5636$ (showed)

① for subbing correctly

ii) $A > 10000$

$\therefore$ $5000\left(1+\frac{0.04}{12}\right)^n = 10000$

$\left(1+\frac{0.04}{12}\right)^n = 2$

$n = \frac{\lg_e 2}{\lg_e\left(1+\frac{0.04}{12}\right)}$

①

$= 208.29.$ $\div 12$ to get years & mths.

for $A > 10000$ $\therefore$ 17 yrs 4 ... mths

$\therefore$ 2001 + 17 years = 2018 Jan + 4 .... mths May $\therefore$ end of May 2018

(1

Q 13 b cont ...

iii) deposit \$500 on 2nd of every mth.
starting from Jan 2004.
? much in Jan on 2007?

$\therefore\ P = 5636 \qquad I = 1 + \frac{0.04}{12} = \frac{301}{300}$

$M = 500$

$A_1 = 5636\left(\frac{301}{300}\right) + 500\left(\frac{301}{300}\right)$

$A_2 = 5636\left(\frac{301}{300}\right)^2 + 500\left(\frac{301}{300} + \frac{301}{300}^2\right)$ 1

$A_n = 5636\left(\frac{301}{300}\right)^n + 500\left(\frac{301}{300} + \cdots + \frac{301}{300}^n\right)$

$GP = 500 \times \dfrac{\left(\frac{301}{300}\left(\frac{301}{300}^{36} - 1\right)\right)}{\frac{301}{300} - 1}$ 1

$A_{36} = (5636 + 500) \times \frac{301}{300}^{36} + 500\left(\frac{301}{300}^{35} + \cdots + \frac{301}{300}\right)$

$= 25507.72$ 1

OR.

Let $P = 5636 \quad M = 500 \qquad R = \frac{301}{300}$

$A_1 = (P + M)R = RP + RM$

$A_2 = (RP + RM + M)R = R^2P + R^2M + RM$

$A_3 = (R^2P + R^2M + RM + M)R = R^3P + R^3M + R^2M + RM$

$\therefore\ A_n = R^nP + R^nM + \cdots + R^2M + RM.$ easier to work with.

$= R^nP + RM\dfrac{(R^n - 1)}{R - 1}$

then sub the values

Q 13c cont.

c) $y = (3x+1)^2$

show $y - y' + 4 = x(y' - 12) - y$ (1)

$y' = 2(3x+1) \cdot 3$

$= 6(3x+1)$ or $18x + 6$.

LHS

$(3x+1)^2 - (18x+6) + 4$

$= 9x^2 + 6x + 1 - 18x - 6 + 4$

$= 9x^2 - 12x - 1$

RHS

$x(18x + 6 - 12) - (3x+1)^2$

$= 18x^2 - 6x - (9x^2 + 6x + 1)$

$= 18x^2 - 6x - 9x^2 - 6x - 1$

$= 9x^2 - 12x - 1$

LHS = RHS (1)

QUESTION 14 2units trial 2016. (1)

ai) $V = \pi \int_0^3 e^{y\ln 4}\, dy$. (show)

$y = \log_2 x$ (log is an exponent)

$2^y = x$ (square) — (1)

$\cancel{(2^y)^2 = x^2}$

$2^{2y} = x^2$ we know $a^x = e^{x\ln a}$

$x^2 = e^{2y\ln 2}$

$= e^{y\ln 2^2}$ } — (1)

$= e^{y\ln 4}$ (shown)

OR. $y = \frac{\ln x}{\ln 2}$

$\ln 2\, y = \ln x$

$\ln x = \ln 2\, y$

$x = e^{y\ln 2}$

$x^2 = e^{y\ln 2^2}$

$= e^{y\ln 4}$

ii) Find the volume $V = \pi \int_0^3 x^2\, dy$.

$= \pi \int_0^3 e^{y\ln 4}\, dy$

$= \pi \left[\frac{e^{y\ln 4}}{\ln 4}\right]_0^3$ — (1)

$= \frac{\pi}{\ln 4}\left(e^{3\ln 4} - e^0\right)$

$= \frac{\pi}{\ln 4}\left(e^{3\ln 4} - 1\right)$

or $\frac{63\pi}{\ln 4}$, 142.77 } (1)

Q 14 cont...

(2)

b) i)

$$P = Ae^{-kt}$$

$$\frac{dP}{dt} = -kAe^{-kt}$$

$$= -kP$$

(1)

ii) $t = 0$, $P = 2000$

$t = 300$ $P = 1000$

$t = 400$ $P = ?$

$\therefore$ when $t = 0$

$$P = Ae^{-kt}$$

$$2000 = Ae^{-0}$$

$$A = 2000$$ (1)

when $t = 300$ $P = 1000$

$$\therefore 1000 = 2000(e^{-k(300)})$$

$$1000 = 2000e^{-300k}$$

$$e^{-300k} = 1/2$$

$$-300k = \frac{\ln 1/2}{-300} \text{ or } \frac{\ln 2}{300} \text{ or } 0.00231$$ (1)

when $t = 400$ find P.

$$\therefore P = 2000e^{-k(400)}$$

$$= 793.7$$

after 400 days $= 794$ (1)

$P = 794$ (nearest whole number)

14 biii) $P < 400$

$\therefore\ 400 = 2000\, e^{-kt}$

$\frac{1}{5} = e^{-kt}$

$t = \dfrac{\ln \frac{1}{5}}{-k}$ (1)

$= 696 \cdot 6 \ldots$ (1)

after 697 days $P < 400$

14 c) Trapezoidal Rule 5 func.

$\int_{-2}^{6} \log_e \sqrt{x+3}\ dx$

x	$f(x)$	w	$w \times f(x)$
-2	0	1	0
0	$\ln\sqrt{3}$	2	$2\ln\sqrt{3}$
2	$\ln\sqrt{5}$	2	$2\ln\sqrt{5}$
4	$\ln\sqrt{7}$	2	$2\ln\sqrt{7}$
6	$\ln\sqrt{9}$	1	$\ln 3$

$\Sigma w = 8$

$\therefore\ A \simeq \dfrac{b-a}{\Sigma w}\left(\Sigma\, w f(x)\right)$

$\simeq \left[\dfrac{6 - -2}{8}\left(0 + 2\left(\ln\sqrt{3} + \ln\sqrt{5} + \ln\sqrt{7}\right) + \ln 3\right)\right]$ (1)

$\simeq (\ln 3 + \ln 5 + \ln 7 + \ln 3)$

$\simeq (2\ln 3 + \ln 5 + \ln 7)$

$\simeq \ln(9 \times 5 \times 7)$

(1) $\simeq \ln 315\ u^2$ (simplified as much as possible)

$*\ (5.75)$

(4)

14 d i) $\dfrac{1}{\sqrt{n}+\sqrt{n+1}} \times \dfrac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}}$ $\longrightarrow$ multiply with $a^2-b^2=(a+b)(a-b)$

$= \dfrac{\sqrt{n}-\sqrt{n+1})}{n-(n+1)}$

$= \dfrac{\sqrt{n}-\sqrt{n+1}}{-1}$ —— common mistake

$= \sqrt{n+1}-\sqrt{n}$ —— 1

ii) Find the sum of

$\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \dots + \dfrac{1}{\sqrt{99}+\sqrt{100}}$

using the result from i) etc —— 1

$(\cancel{\sqrt{2}}-\sqrt{1})+(\cancel{\sqrt{3}}-\cancel{\sqrt{2}})+(\sqrt{4}-\cancel{\sqrt{3}})+\dots+(\cancel{\sqrt{99}}-\sqrt{98})+(\sqrt{100}-\cancel{\sqrt{99}})$

* everything in the middle cancels off.

$\therefore (-\sqrt{1}+\sqrt{100})$ $(n>0)$ from question

$= -1+10$

$= 9$ —— 1

MATHEMATICS: Question 15		
Suggested Solutions	**Marks**	**Marker's Comments**
a) i) $AD \parallel BC$ (opposite sides of a rectangle are parallel) In ΔAXM & ΔCXB $\angle MAX = \angle BCX$ (alternate angles, $AD \parallel BC$)	1	
$\angle MXA = \angle BXC$ (vertically opposite angles are equal) $\therefore \Delta AXM \,\vert\vert\vert\, \Delta CXB$ (equiangular)	1	
ii) $AM = \frac{1}{2}AD$ (M is the midpoint) $AD = BC$ (opposite sides of a rectangle are equal) $\therefore AM = \frac{1}{2}BC$ $\frac{AM}{BC} = \frac{1}{2}$ $\frac{AX}{CX} = \frac{AM}{BC} = \frac{1}{2}$ (corresponding sides in similar triangles are in proportion)	1	
$\therefore CX = 2AX$ $AX = \frac{1}{2}CX$ $AC = CX + AX$ $= CX + \frac{1}{2}CX$ $AC = \frac{3}{2}CX$ $\therefore 2AC = 3CX$	1	
iii) $AB = 2AD$ (given) $AD = BC$ (proven above) $\therefore AB = 2BC$ $BC = \frac{1}{2}AB$ $AB^2 + BC^2 = AC^2$ (Pythagoras' Theorem)	1	
$AB^2 + (\frac{1}{2}AB)^2 = AC^2$ $\frac{5}{4}AB^2 = AC^2$		
from (ii) $\Rightarrow$ $2AC = 3CX$ $4AC^2 = 9CX^2$ $AC^2 = \frac{9}{4}CX^2$ $\therefore \frac{5}{4}AB^2 = \frac{9}{4}CX^2$ $\quad \therefore 5AB^2 = 9CX^2$	1	

MATHEMATICS: Question 15

Suggested Solutions	Marks	Marker's Comments
b) i) $d^2 = 15^2 - w^2$ (pythagoras theorem) $= 225 - w^2$ $S = kwd^2$ $\therefore S = kw(225 - w^2)$ $= k(225w - w^3)$	1	
ii) $\frac{dS}{dw} = k(225 - 3w^2)$ for st. pt. $\frac{dS}{dw} = 0$ $k(225 - 3w^2) = 0$ $3w^2 = 225$ $w^2 = 75$ $w = 5\sqrt{3}$, $(w > 0)$	1	
$\frac{d^2S}{dw^2} = -6kw$ when $w = 5\sqrt{3}$, $\frac{d^2S}{dw^2} = -30\sqrt{3}\,k$ < 0 $\therefore$ curve is concave down $\therefore$ S is a maximum	1	can use table with first derivative.
when $w = 5\sqrt{3}$, $d^2 = 225 - 75$ $= 150$ $d = \sqrt{150}$, $d > 0$ $= 5\sqrt{6}$ $\therefore$ dimensions are $5\sqrt{3}$ by $5\sqrt{6}$	1	
iii) $d = w$ $\therefore w^2 = 15^2 - w^2$ $2w^2 = 225$ $w^2 = \frac{225}{2}$ $w = \frac{15}{\sqrt{2}}$ $S = kw^3$ $= k\left(\frac{15}{\sqrt{2}}\right)^3$ $= \frac{3375k}{2\sqrt{2}}$	1	
iv) max strength: $S = k(5\sqrt{3})(5\sqrt{6})^2$ $= 750\sqrt{3}\,k$	1	

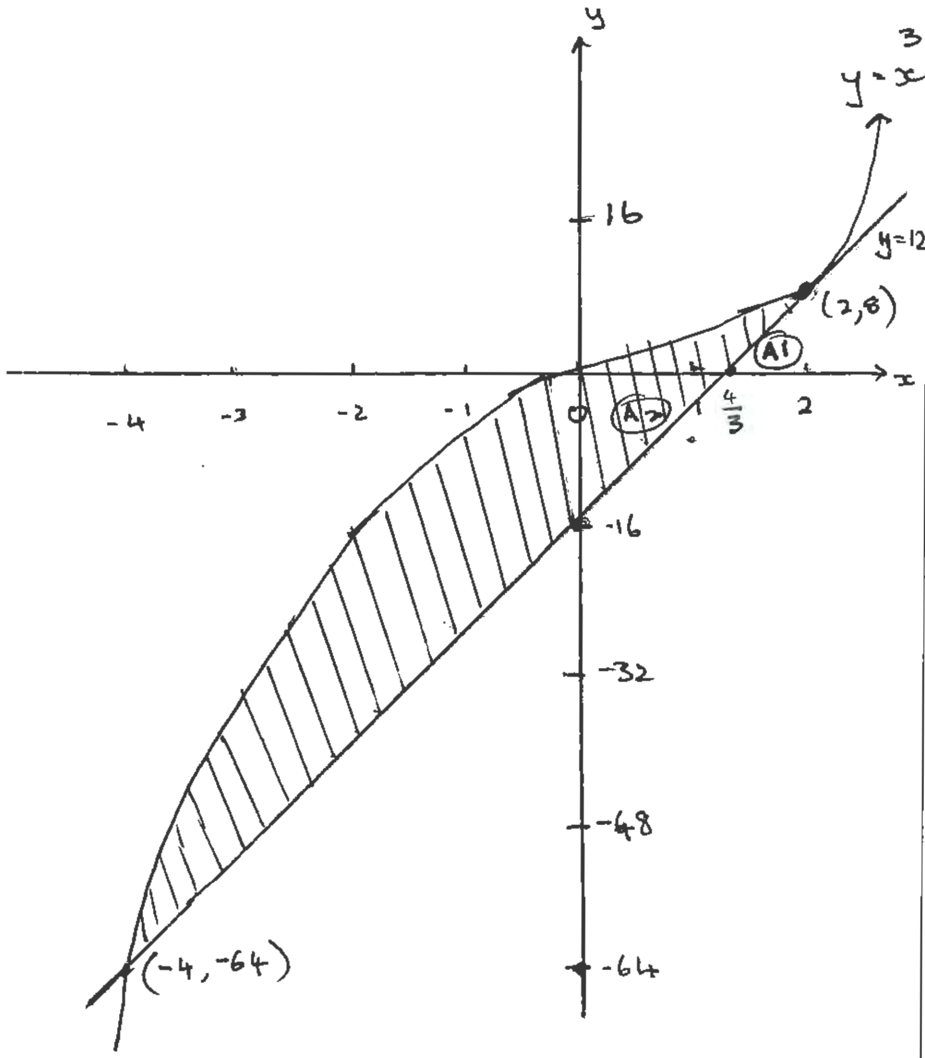
MATHEMATICS: Question 15

Suggested Solutions	Marks	Marker's Comments
$\therefore$ percentage is: $\dfrac{750\sqrt{3}}{\frac{3375}{2\sqrt{2}}} \times 100$		
$= 108.866...$		
$= 109\%$ (nearest %)	1	
$\therefore$ it is 9% stronger		
c) let the number of red marbles be x.		
$\dfrac{x}{65+x} = \dfrac{27}{32}$	1	
$32x = 1755 + 27x$		
$5x = 1755$		
$x = 351$		
$\therefore$ There are 351 red marbles to begin with	1	

MATHEMATICS: Question 16			
	Suggested Solutions	**Marks**	**Marker's Comments**
a	(i) $y = x^3$ $\frac{dy}{dx} = 3x^2$ when $x = 2$ $y = 8$ then $\frac{dy}{dx} = 3(2^2) = 12$ $\therefore$ equation of tangent is $y - 8 = 12(x-2)$ $y - 8 = 12x - 24$ $y - 12x + 16 = 0$ or $y = 12x - 16$	3	1 mark - finding $\frac{dy}{dx}$ 1 mark - getting gradient of 12 1 mark - sub in $y - y_1 = m(x - x_1)$
	(ii). $y = x^3$ Test $(-4, -64)$ LHS $= -64$ RHS $= (-4)^3 = -64$ $y = 12x - 16$ LHS $= -64$ RHS $= 12(-4) - 16$ $= -48 - 16$ $= -64$ Since LHS = RHS for both equations we can say tangent and curve meets again at $(-4, -64)$ Alternatively: $y = x^3$ and $y = 12x - 16$ $\therefore x^3 - 12x + 16 = 0$ Sub in $x = -4$ to get true statement then sub in to find y in the two equations	1	To get 1 mark you needed to test both the tangent and curve - Some found $f(2) = 0$ $\therefore x - 2$ is a factor Then did long division $x - 2 \overline{)x^3 - 12x + 16}$ ended up with $(x-2)(x^2 + 2x - 8)$ $(x-2)(x-2)(x+4)$ $x = 2$ or $x = -4$ then sub in x value to get y as -64.

MATHEMATICS: Question 16

Suggested Solutions	Marks	Marker's Comments
(iii) Area enclosed between curve and tangent.	2	Alternatively



$$A = \int_{-4}^{2} x^3 - 12x + 16 \, dx$$

$$= \left[\frac{x^4}{4} - 6x^2 + 16x \right]_{-4}^{2}$$

$$= (4 - 24 + 32) - (64 - 96 - 64)$$

$$= (12 + 96)$$

$$= 108$$

Area is $108\,u^2$

Marker's Comments

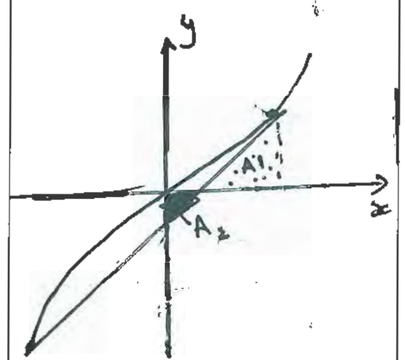
Alternatively

$$\int_0^2 (x^3 - 12x + 16)\,dx + \left| \int_{-4}^{0} (x^3 - 12x + 16)\,dx \right|$$

OR

$$\int_0^2 x^3 - \underbrace{\left(\frac{1}{2} \times \frac{2}{3} \times 8\right)}_{A_1} + \underbrace{\left(\frac{1}{2} \times 16 \times \frac{4}{3}\right)}_{A_2}$$

$$+ \int_{-4}^{0} x^3 - 12x + 16$$



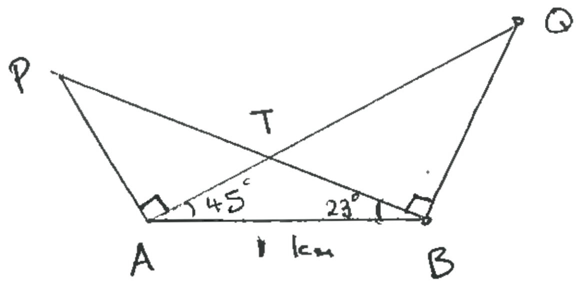
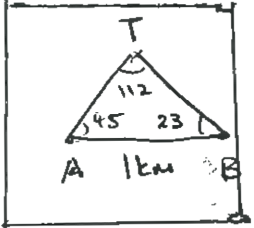
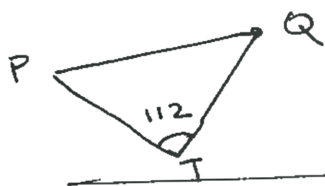
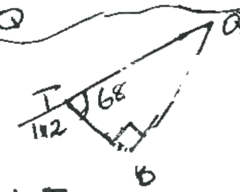
Note $\frac{4}{3}$ is the x-intercept

Note
If you got 12 units as your answer you ignored the other point of intersection (-4,-64)
✓ 1 mark for limits
✓ 1 mark for correct region to get 108

MATHEMATICS: Question..16.....

Suggested Solutions	Marks Awarded	Marker's Comments
(b) i) $P_{(\leq 3)} = P(1) + P(2) + P(3)$ $= 0 + \left(1 \times \frac{1}{2}\right) + \left(1 \times \frac{1}{2} \times \frac{1}{2}\right)$ $= 0 + \frac{1}{2} + \frac{1}{4}$ $= \frac{3}{4}$	1 mark	✓ 1 mark for calculating to $\frac{3}{4}$.
(ii) $P\begin{pmatrix}\text{odd}\\ \text{no}\\ \text{of}\\ \text{throws}\end{pmatrix} = P(3) + P(5) + P(7) \ldots\ldots$ $= \left(1 \times \frac{1}{2} \times \frac{1}{2}\right) + \left(1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \ldots\ldots$ $= \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \ldots\ldots$ S_∞ where $a = \frac{1}{4}$ and $r = \frac{1}{4}$ $\therefore S_\infty = \dfrac{\frac{1}{4}}{1 - \frac{1}{4}}$ $= \frac{1}{4} \div \frac{3}{4}$ $= \frac{1}{3}$.	2 marks	wins if game ends after an odd number of throws $P(3) + P(5) \ldots\ldots$ loses if game ends after an ~~even~~ even number of throws $P(4) + P(6) \ldots\ldots$ ✓ 1 mark for recognising it is a Geometric sequence ie $S_\infty = \frac{a}{1-r}$ ✓ 1 mark for obtaining $\frac{1}{3}$

MATHEMATICS: Question........

	Suggested Solutions	Marks Awarded	Marker's Comments
C	(i) 	1 mark	✓ 1 mark for all information given
	(ii) In $\triangle$ ABT $\angle ATB = 180 - 45 - 23$ (angle sum of $\triangle$ ATB) $= 112$  Using Sine Rule * $\dfrac{\sin 112}{1} = \dfrac{\sin 23}{AT}$ $\therefore AT = \dfrac{\sin 23}{\sin 112}$ * similarily $\dfrac{\sin 112}{1} = \dfrac{\sin 45}{BT}$ $\therefore BT = \dfrac{\sin 45}{\sin 112}$	2 marks	✓ 1 mark Using correct angle of 112 and Sine Rule to get AT ✓ 1 mark get BT
	(iii) Method 1 . Using Cosine Rule  $\boxed{PQ^2 = PT^2 + QT^2 - 2(PT)(QT)\cos 112^\circ}$ Calculate QT $\angle BTQ = 180 - 112$ (angle sum of a straight line ATQ) $= 68$ $\cos 68 = \dfrac{A}{H} = \dfrac{BT}{QT} \Rightarrow \therefore QT = \dfrac{BT}{\cos 68}$	3 marks	✓ Giving reasons and calculating PT. ✓ Giving reasons to calculate QT ✓ Using cosine Rule to calculate PQ  $= \dfrac{\sin 45}{(\sin 112)(\cos 68)}$

$= 2.035840382$

MATHEMATICS: Question……..

Suggested Solutions	Marks Awarded	Marker's Comments

Similarily calculate PT :

$\angle ATP = 68^{\circ}$ (Vertically opposite angles)

$$\cos 68 = \frac{AT}{PT}$$

$$PT = \frac{\sin 23}{(\sin 112)(\cos 68)}$$

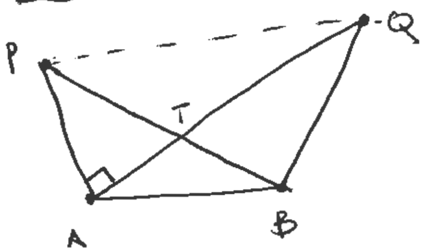
$$= 1.124959103$$

<u>Using cosine rule</u>

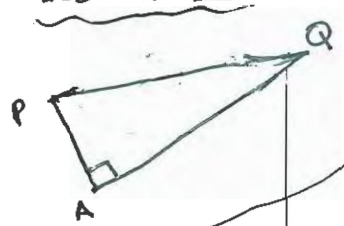
$$PQ^2 = PT^2 + QT^2 - 2(PT)(QT)\cos 112$$

$$PQ = 2.67 \text{ km } (2 \text{ dp})$$

<u>Method 2</u>



Look at $\triangle PAQ$



Using pythagoras $PA^2 + QA^2 = PQ^2$

<u>To get QA</u>

$$QA = AT + QT \text{ (given)}$$

$$= \frac{\sin 23}{\sin 112} + \frac{\sin 45}{(\sin 112)(\cos 68)}$$

$$= 2.457$$

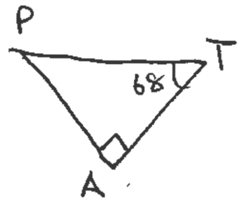
<u>To get PA</u>

$$\frac{PA}{\sin 68} = \frac{AT}{\sin APT} = \frac{\sin 23}{(\sin 112)(\cos 68)}$$

$$= 1.043$$

then use pythagoras theorem.

Marker's Comments:



Some found $\angle P = 22^{\circ}$ and used $\sin 22$ instead of $\cos 68$

✓ 1 mark showing $\angle PAQ$ is a right angle and finding $\angle PTA = 68$ and use pythagoras

✓ Calculate QA as AT + QT

✓ Calculate PA using sine rule