Name:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2019

MATHEMATICS

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Reference Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 100

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new sheet for each new question.
- Allow about 2 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

Section I Multiple Choice (10 marks)

Attempt Question 1 – 10 (1 mark each)
Allow approximately 15 minutes for this section.

1. The graph of $y = e^{-x}$ is:

A. Monotonically increasing

B. Monotonically decreasing

C. Neither monotonically increasing nor monotonically decreasing

D. Cannot be determined

2. What is the 6th term of the series $10 - 15 + 22.5 - 33.75 \ldots$

A. -75.9375

B. 75.9375

C. -113.90625

D. 113.90625

3. Evaluate $\int_{-2}^{2} 4x^3 - 4x\, dx$:

A. 0

B. 8

C. 16

D. 24

4. $\frac{3+\sqrt{2}}{3-\sqrt{2}} - 5$ is equivalent to:

A. $\frac{-21(2+\sqrt{2})}{7}$

B. $\frac{2(23+3\sqrt{2})}{7}$

C. $\frac{6(5\sqrt{2}-8)}{7}$

D. $\frac{6(\sqrt{2}-4)}{7}$

5. The axis of symmetry for the parabola $y = -x^2 + 8x + 3$ is:

A. $y - axis$

B. $x = -4$

C. $x = 2$

D. $x = 4$

6. John has 10 marbles in a bag, 3 of which are green and the rest are yellow. He randomly draws 1 out of the bag, notes its colour and then puts it back before drawing another out of the bag. What is the probability that both marbles drawn are the same colour?

A. 0.42 B. 0.58 C. 0.7 D. 3

7. The parabola with vertex $(-\frac{3}{2}, 0)$, focal length $\frac{1}{2}$, and exists only on the negative side of the y-axis is given by:

A. $y^2 = 2(x + \frac{3}{2})$

B. $y^2 = -2(x + \frac{3}{2})$

C. $x^2 = 2(y + \frac{3}{2})$

D. $x^2 = -2(y + \frac{3}{2})$

8. The quadrilateral formed by $(1, 2), (3, 3), (1, 7), (-1, 3)$ is: (Give the most specific answer)

A. Parallelogram

B. Rhombus

C. Square

D. Kite

9. For what values of m will the geometric series $1 + 2m^2 + 4m^4 + \cdots$ have a limiting sum?

A. $m < \frac{1}{2}$ B. $-\frac{1}{2} < m < \frac{1}{2}$ C. $-\frac{1}{4} < m < \frac{1}{4}$ D. $-\frac{1}{\sqrt{2}} < m < \frac{1}{\sqrt{2}}$

10. Suppose that the point $P\ (a, f(a))$ lies on the curve $y = f(x)$. If $f'(a) = 0$ and $f''(a) > 0$, which of the following statements describes the point P on the graph of $y = f(x)$?

A. P is a maximum turning point

B. P is a horizontal point of inflexion

C. P is a minimum turning point

D. P is a point of inflexion

End of Section I

Section II   Total Marks is 90

Attempt Questions 11 – 16.
Allow approximately 2 hours & 45 minutes for this section.

Answer all questions, starting each new question on a new sheet of paper with your **student ID number** in the top right hand corner and the question number on the left hand side of your paper.

All necessary working must be shown in each and every question.

Question 11 (15 marks)

a) Find the derivative of:

i. $(x^2 + 9x - 3)^3$ 1

ii. $\frac{x}{x^2+1}$ (Simplify your answer) 1

iii. $x^2\sqrt{x+1}$ (Simplify your answer fully over one denominator) 2

b) If $f(x) = \sqrt{x}$, show that $f'(x) = \frac{1}{2\sqrt{x}}$ using first principles. 2

c) A remote-controlled toy car travels in a straight line from an origin according to the equation $x = (t-7)(t-2)$ where x is in metres and t is in seconds.

i. Find the displacement of the car after 5 seconds. 1

ii. Find the velocity of the car in terms of t. 1

iii. According to the velocity expression, when will the car first change direction? 1

iv. The car unfortunately will malfunction if it reaches a speed of exactly 5 ms^{-1}, when will this happen first? 2

d) Let α and β be the roots of the quadratic equation $2x^2 + 4x - 9 = 0$. Find value of:

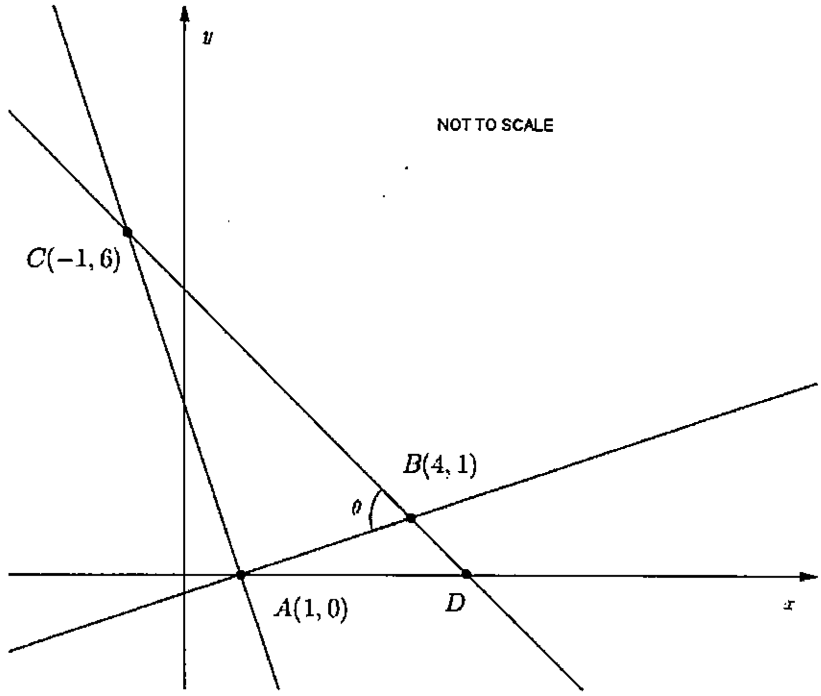
i. $\alpha + \beta$ 1

ii. $\alpha\beta$ 1

iii. $\alpha^3 + \beta^3$ 2

Question 12 (15 marks) Start a new sheet of paper

a)



The diagram shows points $A(1,0)$ and $B(4,1)$ and $C(-1,6)$ in the Cartesian plane. $\angle ABC = \theta$ and D is where CB cuts the x-axis.

Copy or trace this diagram into your answer sheet.

i.	Show that the line AC is given by $3x + y - 3 = 0$.	2
ii.	Hence, find the angle AC makes with the positive x-axis to the nearest degree.	1
iii.	Show that of $AB \perp AC$.	2
iv.	Find the length of AB and AC.	2
v.	Hence, by considering the right angle ΔABC, find the value of θ to the nearest degree.	1
vi.	Find the coordinates of E, the midpoint of AC.	1
vii.	Find the area of ΔCEB.	2
viii.	Suppose F lies on BC such that $EF \parallel AB$. Prove that $\Delta CEF \,|||\, \Delta CAB$.	2
ix.	Hence, find the length of EF. Give reasons.	2

Question 13 (15 marks) Start a new sheet of paper

a) Find $\frac{d}{dx}(\sin^2 x)$ 1

b) Solve $\sqrt{3}\sin^2\theta = sin\theta cos\theta$ for $0 \le \theta \le 2\pi$ (Give your answer in radians) 3

c) Solve $\log_2 x + \log_2(x-3) = 2$ 3

d) In a group of 10 birds, 5 are red and 5 are green. If three birds are selected at random, Find the probability that:

i. They are all red 1

ii. At least one is green 1

e) A coin is tossed continually, until for the first time successive tosses give the same result. Find the probability that the experiment finishes before the 4^{th} throw. 2

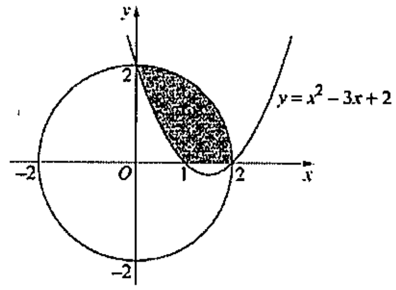
f) The sum to n^{th} term of a series is given by $S_n = 3n^2 - 2n + 1$.

i. Use the formula $T_n = S_n - S_{n-1}$ to show that the n^{th} term of the series is given by $T_n = 6n - 5$ 2

ii. It can be shown that the sum of this series is actually given by $S_n = C + T_1 + T_2 + \cdots T_n$ where C is a constant (DO NOT PROVE THIS FORMULA). Find the value of C. 2

Question 14 (15 marks) Start a new sheet of paper

a)

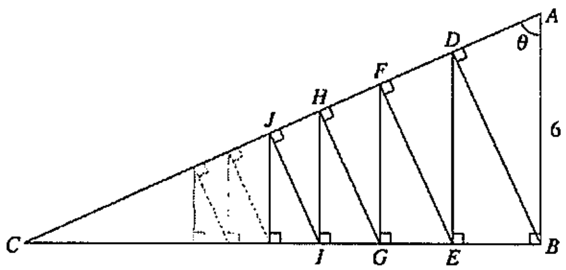


The shaded region in the diagram is bounded by the circle of radius 2 centred at the origin, the parabola $y = x^2 - 3x + 2$, and the x-axis.

Find the area of the shaded region. 3

b) Use Simpsons Rule with 5 function values to approximate $\int_0^{\pi} x\sin x \, dx$ correct to 2 decimal places. 3

c)



The triangle ABC has a right angle at B, $\angle BAC = \theta$ and $AB = 6$. The line BD is drawn perpendicular to AC. The line DE is then drawn perpendicular to BC. This process continues indefinitely as shown in the diagram. You may assume other angles are also θ without proof.

i. Find the length of the interval BD, and hence show that the length of the interval EF is $6\sin^3\theta$. 2

ii. Show that the limiting sum $BD + EF + GH + \cdots$ is given by $6\sec\theta\tan\theta$ 3

d) A restaurant offers choices of 3 vegetarian options, 3 meat options and 2 carb options. I wish to select **2 different** dishes.

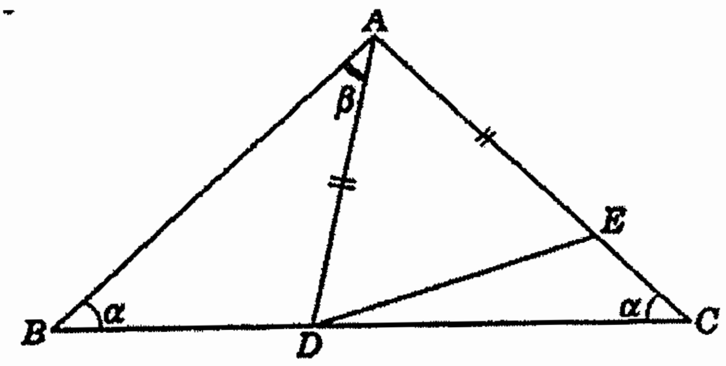
i. Draw a grid diagram (or dot diagram) to illustrate the sample space 2

ii. Hence or otherwise, find the probability that both my dishes belong to the same category. 1

iii. Amy wishes to also order 2 different dishes at random. Find the probability that we will have ordered exactly the same dishes. 1

Question 15 (15 marks) Start a new sheet of paper

a)



In the isosceles triangle ABC, $\angle ABC = \angle ACB = \alpha$. The points D and E lie on BC and AC respectively, so that $AD = AE$, as shown in the diagram. Let $\angle BAD = \beta$.

Copy the diagram onto your answer sheet.

i. Find $\angle DAC$ in terms of α and β. Give reasons. **1**

ii. Hence or otherwise, find $\angle EDC$ in terms of β. Give reasons. **3**

b) The flow rate of water first into and then out of a tank is given by

$$R = 3\pi t(12 - t) \text{ litres/second.}$$

i. When does the water stop flowing into the tank? **1**

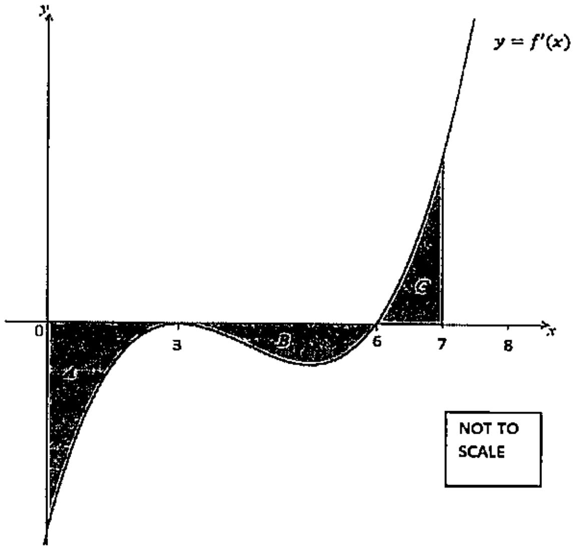
ii. The tank was initially empty. Show that the volume of water in the tank is given by **2**

$$V = \pi t^2(18 - t)$$

iii. Find the rate at which the water is flowing out of the tank when the tank is empty again. **1**

Question 15 continues on the next page

c)

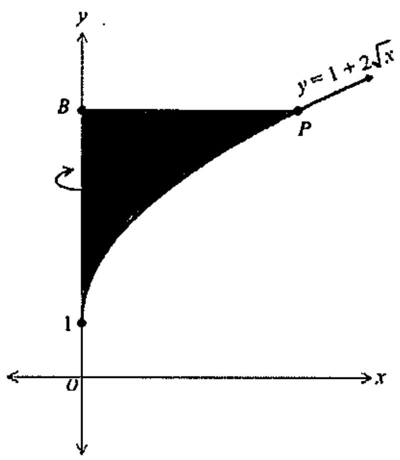


The regions A, B and C are enclosed by the curve $y = f'(x)$, the x-axis, the y-axis, and the line $y = 7$. The area of region A is $10u^2$, while the area of regions B and C are both $5u^2$.

i. Given that $f(0) = 12$, find $f(7)$. 2

ii. Let $g(x) = \left(f(x)\right)^2$. Given that $f'(7) = 6$, find the equation of the tangent to the graph $y = g(x)$ at $x = 7$. 2

d)



The shaded region in the diagram is bounded by the curve $y = 1 + 2\sqrt{x}$, the y-axis and the horizontal interval BP. The x-coordinate of P is 4.

Calculate the exact volume of the solid of revolution formed when the shaded region is rotated about the y-axis. 3

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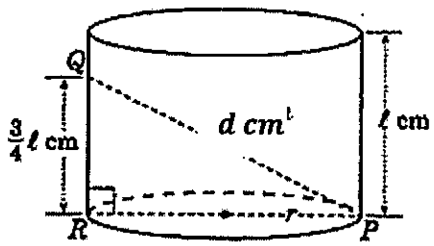
Question 16 (15 marks) Start a new sheet of paper

a)



The diagram shows a hollow cylindrical tube of length l cm and radius r cm. P is a point on the circumference at one edge of the tube, at the very bottom of the rim of the tube. Q is a point on the opposite side of the tube, three-quarters of the way up the tube. RP is the diameter with R directly below Q. The length of PQ is d cm.

i. Show that the volume of the tube is given by $V = \frac{\pi l}{4}(d^2 - \frac{9l^2}{16})$ **2**

ii. If d remains constant, find l in terms of d that will give the maximum volume for the tube. **3**

b) A certain insect plague is following the law of natural growth. The insect population P satisfies the equation $P = P_0 e^{kt}$. Time t is measured in months and P_0 and k are constants.

Initially there were 10 000 insects in the plague and after 8 months there were 40 000.

i. Show that $P_0 = 10\,000$ and $k = \frac{1}{4} ln2$ **2**

ii. After how many whole months would the population exceed 1 million? **2**

c) \$30 000 is borrowed at 9 % per annum reducible interest, calculated monthly. The loan is repaid in equal monthly instalments of $\$M$ over 5 years. Interest is charged before repayments are made every month.

Let A_n be the amount owing after n monthly repayments.

i. Show that $A_n = 30\,000 \times R^n - M(\frac{R^n-1}{R-1})$ where $R = \frac{403}{400}$ **3**

ii. Show that the monthly repayment is \$622.75 **1**

iii. What is the balance owing after the 24^{th} payment? **2**

End of Paper

2U Trials

MATHEMATICS: Question 11		3
Suggested Solutions	**Marks**	**Marker's Comments**
a) i) $y = (x^2 + 9x - 3)$ $y' = 3(x^2 + 9x - 3) \cdot (2x + 9)$ — (1)		
ii) $y = \dfrac{x}{x^2 + 1}$ $y' = \dfrac{x^2 + 1 \cdot 1 - x(2x)}{(x^2 + 1)^2}$ $= \dfrac{1 - x^2}{(x^2 + 1)^2}$ — (1)		simplify your answer.
iii. $y = x^2 \sqrt{x + 1}$ $y' = (x+1)^{1/2} \cdot 2x + x^2 \cdot \frac{1}{2}(x+1)^{-1/2}$ — (1) $= \dfrac{2 \cdot 2x(x+1) + x^2}{2(\sqrt{x+1})}$ $= \dfrac{4x^2 + 4x + x^2}{2(\sqrt{x+1})} = \dfrac{5x^2 + 4x}{2(\sqrt{x+1})}$ — (1) OR. $y = x^2 (x+1)^{\frac{1}{2}}$ $y = (x^5 + x^4)^{1/2}$ $y' = \frac{1}{2}(x^5 + x^4)^{-1/2} \times 5x^4 + 4x^3$ — (1) $= \dfrac{\cancel{x^2}(5x^2 + 4x)}{2\cancel{x}(x+1)^{1/2}}$ $= \dfrac{5x^2 + 4x}{2(\sqrt{x+1})}$ — (1)		

MATHEMATICS: Question........ 11 $\qquad$ 2/3

Suggested Solutions	Marks	Marker's Comments

b) $f(x) = \sqrt{x}$

using 1st principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \qquad 1$$

$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}} \qquad 1$$

$$= \frac{1}{2\sqrt{x}}$$

c) i) $x = ?$ when $t = 5$

$x = (-2)(3)$

$= -6\text{m}$ from origin $\qquad$ 1

ii) $\cancel{x} = t^2 - 9t + 14$ $\qquad$ 1

$v = 2t - 9$

iii) change direction when $v = 0$

$\therefore\ 2t = 9$ $\qquad$ 1

$t = 4.5\text{s}$

iv) $|v| = 5\ \therefore$ $\qquad$ 1

$5 = 2t - 9 \qquad -5 = 2t - 9$

$t = 7 \qquad t = 2$

$\therefore$ happens first when $t = 2\text{s}$. $\qquad$ 1

MATHEMATICS: Question 11		3/3
Suggested Solutions	**Marks**	**Marker's Comments**
d) $2x^2+4x-9=0$ i) $\alpha+\beta = -\frac{4}{2} = -2$ ii) $\alpha\beta = \frac{-9}{2}$ iii) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $= (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ $= (\alpha+\beta)((\alpha+\beta)^2-2\alpha\beta-\alpha\beta)$ $= -2 \times ((-2)^2-(-\frac{9}{2}))$ $= -35$ — 1		— 1

<table>
<tr><th colspan="3">MATHEMATICS: Question 12 - page 1</th></tr>
<tr><th>Suggested Solutions</th><th>Marks</th><th>Marker's Comments</th></tr>
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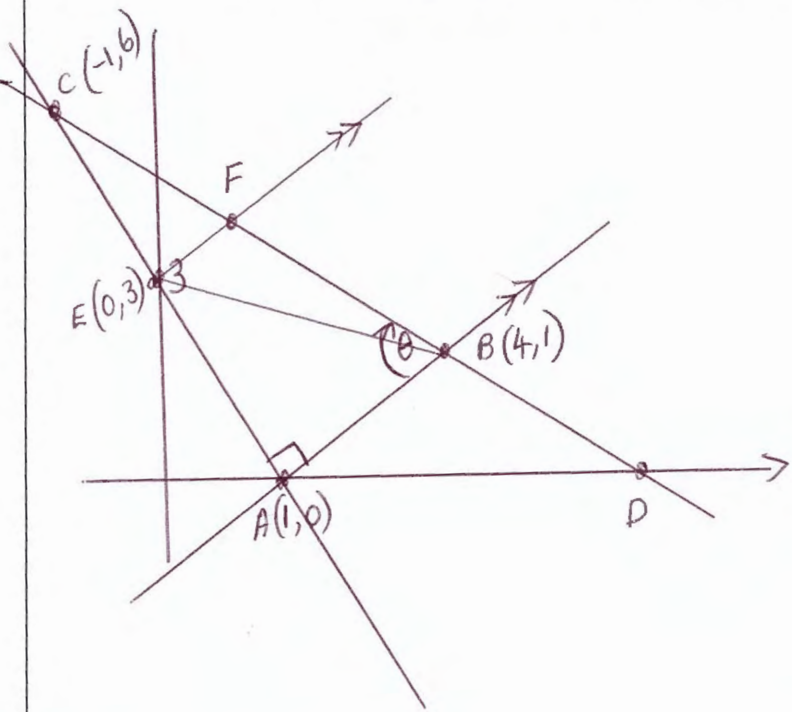

</td><td></td><td>Draw a diagram at least $\frac{1}{3}$ page in size.</td></tr>
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(i) $m_{AC} = \frac{-3}{1}$ & y intercept = 3

∴ eqn of AC: $y = -3x + 3$

$3x + y - 3 = 0$

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(ii) $\tan \alpha = -\frac{3}{1}$ ∴ $\alpha = 108^\circ$.

α=Angle DAC

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(iii) $m\,AB = \frac{1-0}{4-1} = \frac{1}{3}$ $\quad m\,AC = -\frac{3}{1}$

$m_{AB} \cdot m_{AC} = \frac{1}{3} \times -\frac{3}{1} = -1$

∴ $AB \perp AC$.

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MATHEMATICS: Question 2 – [illegible]

Suggested Solutions	Marks	Marker's Comments
(iv) $d_{AB} = \sqrt{(4-1)^2 + (1-0)^2}$		
$= \sqrt{9+1}$	1	
$= \sqrt{10}$		
$d_{AC} = \sqrt{(0-6)^2 + (1+1)^2}$		2
$= \sqrt{36+4}$		
$= \sqrt{40}$	1	
$= 2\sqrt{10}$		
(v) In ΔABC $\tan\theta = \frac{AC}{AB}$		
$= \frac{2\sqrt{10}}{\sqrt{10}}$		
$\theta = 63°$	1	
(vi) $E = \left(\frac{1+-1}{2}, \frac{0+6}{2}\right)$	1	
$E = (0, 3)$		
(vii) Area $\Delta CEB = \frac{1}{2} \times EC \times AB$		
$= \frac{1}{2} \times \sqrt{(-1-0)^2 + (6-3)^2} \times \ldots$	1	
$= \frac{1}{2} \times \sqrt{10} \times \sqrt{10}$		2
$= \frac{10}{2}$		
$= 5u^2$	1	
Note: You could also use $A = \frac{1}{2} \times EC \times EB \times \sin\angle BEC$.		

MATHEMATICS: Question

Suggested Solutions	Marks	Marker's Comments
(viii) In ΔCEF and ΔCAB		
$\angle C$ is common	1	finding equal angles
$\angle CEF = \angle CAB$ (corresponding angles, $EF \parallel AB$)		(2)
$\therefore \Delta CEF \parallel\!\mid \Delta CAB$ (equiangular).	1	identifying triangles & giving reason for similarity.
(ix) $\dfrac{EF}{AB} = \dfrac{CE}{CA}$ (corresponding sides of similar triangles are in proportion (or same ratio).	1	
$\dfrac{EF}{\sqrt{10}} = \dfrac{\sqrt{10}}{2\sqrt{10}}$		(2)
$EF = \dfrac{10}{2\sqrt{10}}$	1	
$= \dfrac{\sqrt{10}}{2}$.		

(Alternatively, you could use $CE = \frac{1}{2}AC$ (a line drawn from one side of a triangle to another, parallel to the 3rd side, divides the other 2 sides in same ratio)

MATHEMATICS: Question 13

Suggested Solutions	Marks	Marker's Comments
a) $\frac{d}{dx}(\sin^2 x) = 2\sin x\cos x$ $= \sin 2x$	1	accepted both answers
b) Solve $\sqrt{3}\sin^2\theta = \sin\theta\cos\theta$ for $0 \le \theta \le 2\pi$ $\sqrt{3}\sin^2\theta - \sin\theta\cos\theta = 0$ $\sin\theta(\sqrt{3}\sin\theta - \cos\theta) = 0$ ✓ $\sin\theta = 0$ or $\sqrt{3}\sin\theta - \cos\theta = 0$ $\theta = 0, \pi, 2\pi$ or $\sqrt{3}\sin\theta = \cos\theta$ $\tan\theta = \frac{1}{\sqrt{3}}$ $\theta = \frac{\pi}{6}$ or $\frac{7\pi}{6}$ $\therefore \theta = 0^\circ, \frac{\pi}{6}, \pi, \frac{7\pi}{6}$ and 2π Note: If students did only $\sqrt{3}\sin\theta = \cos\theta$ got only 1 mark out of 3	3	question says answer in radians 1 mark for factorising 1 mark for solving $\sqrt{3}\sin\theta - \cos\theta = 0$ 1 mark for final 5 answers accepted 0.523599 and 3.665182
c) Solve $\log_2 x + \log_2(x-3) = 2$ $\log_2(x^2-3x) = \log_2 4$ $x^2 - 3x = 4$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $x = 4$ or -1 but $x > 3$ $\therefore x = 4$ only	3	1 mark for simplifying logarithmic law 1 mark for factorising correctly 1 mark for accepting 4 only

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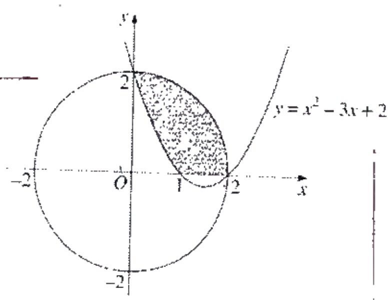
MATHEMATICS: Question 13

Suggested Solutions	Marks	Marker's Comments
d) 10 birds < 5 red, 5 green. i) $p(\text{all red}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$ ii) $p(\text{at least one is green}) = 1 - p(\text{all red})$ $1 - \frac{1}{12}$ $= \frac{11}{12}$	1 1	Getting $\frac{1}{12}$ Getting $\frac{11}{12}$
e) Find probability that expt finishes before the 4^{th} row **Method 1** $p(HH) = \frac{1}{4}$ $p(TT) = \frac{1}{4}$ $P(HTT) = \frac{1}{8}$ $P(THH) = \frac{1}{8}$ add $\frac{3}{4}$ **Method 2** $1 - p(\text{not finishes})$ $1 - \left(\frac{1}{8} + \frac{1}{8}\right)$ $= \frac{3}{4}$ **Method 3** $P_{(\text{even})} = P_2 + P_3$ probability for second throw + probability for 3rd throw $= \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$ $= \frac{3}{4}$	2	1 mark for correct working of probabilities 1 mark for getting the answer $\frac{3}{4}$.
f) (i) $S(n) = 3n^2 - 2n + 1$ and use $T_n = S_n - S_{n-1}$ to find $T_n = 6n - 5$ $S(n-1) = 3(n-1)^2 + 2(n-1) + 1$ $= 3(n^2 - 2n + 1) - 2(n-1) + 1$ $= 3n^2 - 6n + 3 - 2n + 2 + 1$ $= 3n^2 - 8n + 6$ $\therefore$ $S_n - S_{n-1}$ gives	2	✓ 1 mark for finding S_{n-1} and ✓ mark for working $T_n = S_n - S_{n-1}$ to give $6n-1$

MATHEMATICS: Question 13 Page 3

Suggested Solutions	Marks	Marker's Comments
$\therefore T_n = (3n^2-2n+1) - (3n^2-8n+6)$ $= 6n-5$ $\therefore T_n = 6n-5$		
(ii) $S_n = c + T_1 + T_2 + \dots \dots T_n$. Show it is an AP. $T_1 = 1$ $T_2 = 7$ $T_3 = 13$ $T_4 = 19 \dots\dots$		1 mark. show it is an AP
Method 1 $S_2 = 3(2^2)-2(2)+1$ $= 9$. From formula $S_2 = c + T_1 + T_2$ $9 = c+1+7$ $\therefore c = 1$ Method 2 $S_n = \frac{n}{2}[2a+(n-1)d]$ $= \frac{n}{2}[2+(n-1)6]$ $= \frac{n}{2}[6n-4]$ $= 3n^2-2n$ Compare with $S_n = 3n^2-2n+1$ $3n^2-2n+c = 3n^2-2n+1$ $\therefore c = 1$		working getting value of c.



MATHEMATICS: Question 14

Suggested Solutions	Marks	
a) Area of the shaded Region.	3	
$A =$ Area of first quadrant of a circle $- \int_0^1 (x^2-3x+2)\,dx$		· 1 mark for finding area of quater of circle as π
$= \frac{\pi(2^2)}{4} - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^1$		correct limits and integrating x^2-3x+2
$= \pi - \left(\frac{1}{3} - \frac{3}{2} + 2\right)$		
$= \pi - \frac{5}{6}$		· Correct working to get answer $\pi - \frac{5}{6}$
Area is $\pi - \frac{5}{6}$ unit^2.		
b). Simpson's rule with 5 function values. $\int_0^{\pi} x\sin x \, dx$	3	
		1 mark to get $\frac{\pi}{12}, \left(\frac{1}{4} \pm 3\right)$
$I \simeq \frac{\pi}{12}\left[(0+0) + 4\left(\frac{\pi}{4\sqrt{2}} + \frac{3\pi}{4\sqrt{2}}\right) + 2\left(\frac{\pi}{2}\right)\right]$		correct Use of formula.
$\simeq \frac{\pi}{12}\left[\frac{4\pi}{\sqrt{2}} + \pi\right]$		
$= \frac{\pi}{12}\left[\frac{4\pi + \sqrt{2}\pi}{\sqrt{2}}\right]$		
$= \frac{\pi^2}{12\sqrt{2}}(4+\sqrt{2})$		· Correct manipulations to get 3.15
$= \frac{\pi^2(2\sqrt{2}+1)}{12}$		
$\simeq 3.15$		

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	$\frac{\pi}{4\sqrt{2}}$	$\frac{\pi}{2}$	$\frac{3\pi}{4\sqrt{2}}$	0

MATHEMATICS: Question 14

Suggested Solutions	Marks	Marker's Comments
c) Find the length of BD and show interval EF is $6\sin^3\theta$ i) $\sin\theta = \frac{BD}{6}$ $\therefore BD = 6\sin\theta$ $\angle EBD = \theta$ also $\sin\theta = \frac{DE}{BD} = \frac{DE}{6\sin\theta}$ $\therefore DE = 6\sin^2\theta$ $\angle FDE = \theta$ $\sin\theta = \frac{EF}{DE} = \frac{EF}{6\sin^2\theta}$ $\therefore EF = 6\sin^3\theta$	2.	> Show BD is $6\sin\theta$ Working to get to $6\sin^3\theta$
(ii) show that the limiting sum $BD + EF + GH + \cdots$ is given by $6\sec\theta\tan\theta$ Series is $6\sin\theta + 6\sin^3\theta + 6\sin^5\theta + \cdots$ $a = 6\sin\theta$ $r = \sin^2\theta$ Sum $= \frac{a}{1-r}$ $= \frac{6\sin\theta}{1-\sin^2\theta}$ $= \frac{6\sin\theta}{\cos^2\theta}$ $= 6\frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}$ $= 6\tan\theta\sec\theta$	3	1 mark to getting the series and values of a and r 1 mark for correct formula and substitution Showing trig identities and leading to $6\tan\theta\sec\theta$ Note if students claim $\frac{6\sin\theta}{\cos^2\theta} = 6\tan\theta\sec\theta$ lost a mark

MATHEMATICS: Question 14

Suggested Solutions	Marks	Marker's Comments
d). (i) 3 VG, 3 meat and 2 carb.	2.	To get full marks one had to show M_1M_1 M_2M_2 ....etc does not exist

	M_1	M_2	M_3	V_1	V_2	V_3	C_1	C_2
M_1	////	O	O					
M_2	O	////	O					
M_3	O	O	////					
V_1				////	O	O		
V_2				O	////	O		
V_3				O	O	////		
C_1							////	O
C_2							O	////

56 possibe outcome

(ii) p(same category) ie $p(VV) = 6/56$ 1

$p(MM) = 6/56$

$p(CC) = 2/56$

$\therefore$ p(same category is $\frac{14}{56} = \frac{1}{4}$ ⟶ 1 mark

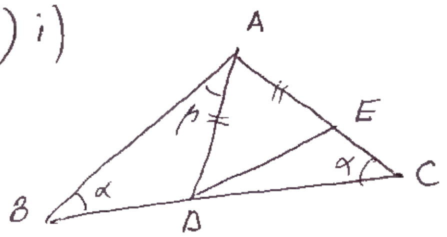
(iii) Amy wishes to order 2 different dishes at random. Find p(ordered exactly the same dishes).

ways of picking V_1V_2 and V_2V_1 is 2

$\therefore \frac{2}{56} = \frac{1}{28}$ ⟶ 1 mark

MATHEMATICS: Question 15 — 11/3

Suggested Solutions	Marks	Marker's Comments
a) i)		



$\angle DAC + \beta + \alpha + \alpha = 180$ (angle sum of $\triangle ABC$)

$\therefore \angle DAC = 180 - 2\alpha - \beta$ — (1)

ii) $\angle ADC = \alpha + \beta$ (exterior angle of $\triangle ABD$)

$\angle DAC + \angle AED + \angle ADE = 180$ (angle sum of $\triangle ADE$)

$\angle ADE = \angle AED$ (equal angles opposite equal sides)

$180 - 2\alpha - \beta + 2ADE = 180$

$2ADE = 180 - 180 + 2\alpha + \beta$

$\angle ADE = \alpha + \frac{\beta}{2}$

$\angle EDC = \angle ADC - \angle ADE$ (angle subtraction)

$= \alpha + \beta - (\alpha + \beta/2)$

$= \beta/2$

1 mark for $\angle EDC$
remaining 2 " 's for $\angle ADC$, $\angle ADE$ with reasoning

b) $R = 3\pi t(12 - t)$

i) stop flowing when $R = 0$

$\therefore t = 0$ or 12 $t > 0$ — (1)

but t is 12 s only

ii) Tank empty.

$\int R = 36\pi t - 3\pi t^2$

$V = 18\pi t^2 - \pi t^3 + C$ — (1)

Suggested Solutions	Marks	Marker's Comments

when $t=0$, $v=0$

$\therefore 0 = 18\pi(0)^2 - \pi(0)^3 + c$ (1) to calculate value of C

$\therefore c = 0.$

So $V = \pi t^2(18-t)$

iii) when $V=0$

$t = 0$ or 18.

when $t = 18$.

$R = 36\pi(18)(12-18)$ — (1)

$= -324\pi \ L/s$

to b value of t in R and must have units.

c) $A = \int_0^3 f'(x)$ given $f(0)=12$

$= [f(x)]_0^3$

$-10 = f(3) - f(0)$

$-10 = f(3) - 12$

$f(3) = 2$

1 mark to show an understanding how to get to $f(7)$

$B = \int_3^6 f'(x) = [f(x)]_3^6$

$-5 = f(6) - f(3)$

$f(6) = -5 + 2$

$= -3.$

$C = \int_6^7 f'(x) = [f(x)]_6^7$

$5 = f(7) - f(6)$

$f(7) = 5 + (-3)$

$= 2$ — (1)

MATHEMATICS: Question 11		3/3
Suggested Solutions	**Marks Awarded**	**Marker's Comments**

ii) $g(x) = (f(x))^2$, $f'(7) = 6$, $f(7) = 2$ — cfe from(i)

Eqn of tangent to $y = g(x)$ at $x = 7$

$$g(x) = (f(x))^2$$

$$g'(x) = 2f(x) \cdot f'(x)$$

when $x = 7$

$$\left.\begin{aligned} g'(x) &= 2(2) \times 6 \\ &= 24 \end{aligned}\right\} \text{— 1}$$

when $x = 7$ $y = (f(7))^2$ (cfe) !!

1 — $= 4$.

must have correct y value squared from (i) ie it has to be only then a student could get 2 marks for this part

Eqn of tangent.

$$y - 4 = 24(x - 7)$$

1 — $y = 24x - 164$

d) $y = 1 + 2\sqrt{x}$ rotate abt y axis.

$\therefore 2\sqrt{x} = y - 1$

$$x = \left(\frac{y-1}{2}\right)^2 = \frac{(y-1)^2}{4}$$

when $x = 4$ at pt B

$y = 1 + 2\sqrt{4}$

$= 5$.

$$V = \pi \int_1^5 \left(\frac{(y-1)^2}{4}\right)^2$$

1 correct limit

1 mark has π, and eqn squared.

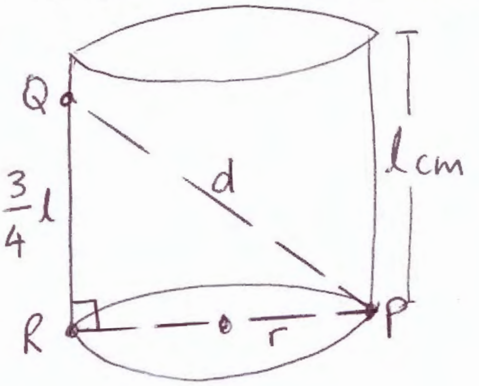
$$= \frac{\pi}{16}\int_1^5 (y-1)^4$$

$$= \frac{\pi}{16}\left[\frac{(y-1)^5}{5}\right]_1^5 = \frac{\pi}{16 \times 5}(4^5 - 0^5)$$

$$= \frac{64\pi}{5} u^3$$

— correct value from substitution.

MATHEMATICS: Question 16 Page 1

Suggested Solutions	Marks	Marker's Comments
		
(i) $V = \pi r^2 l$		
In ΔRPQ $\quad (2r)^2 + \left(\frac{3}{4}l\right)^2 = d^2$ (by Pythagoras' Theorem)		
$4r^2 + \frac{9}{16}l^2 = d^2$		
$4r^2 = d^2 - \frac{9}{16}l^2$	1	
$r^2 = \frac{d^2}{4} - \frac{9}{4\times 16}l^2$		2
$\therefore V = \pi r^2 l$	1	
$= \pi\left(\frac{d^2}{4} - \frac{9}{4\times16}l^2\right)l$		— given answer
$= \frac{\pi}{4}l\left(d^2 - \frac{9}{16}l^2\right)$		
(ii) Maximum volume occurs when $\frac{dV}{dl} = 0$		
$V = \frac{\pi}{4}ld^2 - \frac{9\pi l^3}{4\times16}$		
$\frac{dV}{dl} = \frac{\pi d^2}{4} - \frac{27\pi l^2}{64}$	1	

MATHEMATICS: Question 16 (c) (ii)

Suggested Solutions	Marks	Marker's Comments
Max when $\frac{dV}{dl}=0$ ie $\frac{\pi d^2}{4}-\frac{27\pi l^2}{64}=0$ $\frac{16\pi d^2}{64}-\frac{27\pi l^2}{64}=0$ $16\pi d^2-27\pi l^2=0$	1	
$27\pi l^2=16\pi d^2$ $l^2=\frac{16\pi d^2}{27\pi l^2}$ $l=\frac{4d}{\sqrt{27}}$ $l>0$		3
To check concavity $\frac{d^2V}{dl^2}=-\frac{54\pi}{64}$	1	Needed to check concavity.
Since $d>0$ and $\frac{d^2V}{dt^2}<0$ & $l>0$ $\therefore$ concave down $\therefore$ max volume when $l=\frac{4d}{\sqrt{27}}$ $\left(\text{or } \frac{4d}{3\sqrt{3}}\right)$		

MATHEMATICS: Question ___

Suggested Solutions	Marks	Marker's Comments
(b) $P = P_0 e^{kt}$		
Initially $t=0$ $P=10000$		
$\therefore 10000 = P_0 e^{0k}$		
$\underline{10000 = P_0}$		
$\therefore P = 10000 e^{kt}$	1	needed to this.
Subst $t=8$ & $P=40000$ to find k.		
$40000 = 10000 e^{8k}$		
$e^{8k} = \frac{40000}{10000}$		(2)
$e^{8k} = 4$		
$\log_e 4 = 8k$		
$k = \frac{\log_e 4}{8}$		
$k = \frac{\ln 4}{8}$	1	
$k = \frac{\ln 2^2}{8}$		
$k = \frac{2\ln 2}{8}$		
$k = \frac{\ln 2}{4}$		

MATHEMATICS: Question 16 page 4

Suggested Solutions	Marks	Marker's Comments
(ii) Population exceeds 1million when		
$10000\, e^{\frac{\ln 2}{4} t} > 1000000$	1	
ie $e^{\frac{\ln 2}{4} t} > 100$		
$\log_e e^{\frac{\ln 2}{4} t} > \log_e 100$		
$\frac{\ln 2}{4} \cdot t > \log_e 100$		2
$\ln 2t > 4 \ln 100$		
$t > \frac{4 \ln 100}{\ln 2}$		
$t > 26.6$.	1	
$\therefore$ Popn exceeds 100000 after 27 mths (nearest month).		
(c) $P = 30000$ $\quad 9\% pa = \frac{9}{12 \cdot 100} = \frac{3}{400}$		
5 years = 60 months.		
Let $R = 1 + \frac{3}{400} = \frac{403}{400}$		
Amount owing after 1 month		
$A_1 = 30000 \times R - M$		
$A_2 = (30000 \times R - M)R - M$	1	
$= 30000R^2 - MR - M$		

MATHEMATICS: Question 16 page 5

Suggested Solutions	Marks	Marker's Comments
$A_3 = (30000R^2 - MR - M)R - M$ $= 30000R^3 - MR^2 - MR - M$ $= 30000R^3 - M(R^2 + R + 1)$ $\vdots$ $A_n = 30000R^n - M(R^{n-1} + R^{n-2} + \dots + R^2 + R + 1)$ $= 30000R^n - M\underbrace{(1 + R + R^2 + \dots + R^{n-1})}$ this is a GP where $a = 1,\ r = R$ $S_n = \frac{a(R^n - 1)}{R-1}$ $= \frac{R^n - 1}{R-1}$	1 1	(3)
$\therefore A_n = 30000R^n - M\left(\frac{R^n - 1}{R-1}\right)$ $= 30000 \times R^n - M\left(\frac{R^n - 1}{R-1}\right)$ where $R = \frac{403}{400}$		this answer is given.
(ii) Loan is paid off when $A_n = 0$ ie $A_{60} = 0$ $30000 \times R^n - M\left(\frac{R^n - 1}{R-1}\right) = 0$ $M = \frac{30000R^n(R-1)}{R^n - 1}$ $= \frac{30000\left(\frac{403}{400}\right)^{60}\left(\frac{3}{400}\right)}{\left(\frac{403}{400}\right)^{60} - 1}$ $= \$622.75$	(1)	
(iii) Balance owing after 24th payment $A_{24} = 30000R^{24} - 622.75\left(\frac{R^{24} - 1}{R-1}\right)$ $= 35892.40588 - 622.75 \times 26.18847$ $= \$19583.54$	1 1	(2)