Student Number: ______________________

KAMBALA

MATHEMATICS

2 UNIT

TRIAL HSC EXAMINATION

AUGUST 2007

Time Allowed: 3 hours
Reading Time: 5 minutes

INSTRUCTIONS

- This examination contains 10 questions of equal value. Marks for each question are shown.
- Answer all questions in the writing booklets provided. Start each question in a new booklet.
- Calculators may be used.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\dfrac{56.07-\sqrt{39.2}}{7.1}$, correct to 3 significant figures. **2**

(b) Simplify: $\dfrac{3x-4}{2}-\dfrac{5-x}{3}$ **2**

(c) If \$5000 is invested for 10 years and interest is compounded at 9% per annum, what is the final value of the investment? **2**

(d) Find the solution of θ, in radians, in the interval $\dfrac{3\pi}{2}\le\theta\le 2\pi$ if $\cos\theta=\dfrac{\sqrt{3}}{2}$. **2**

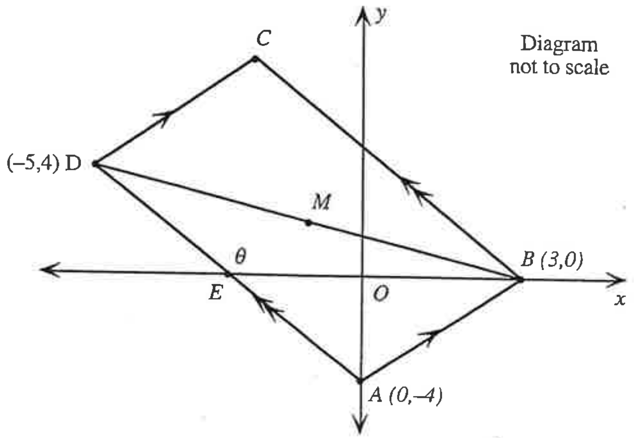
(e) Find $\int \sec^2 5x \ dx$. **2**

(f) Find integers a and b such that $\dfrac{4}{2-\sqrt{3}}=a+b\sqrt{3}$. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet

Marks

In the number plane diagram below, $A(0,-4)$, $B(3,0)$ and $D(-5,4)$ are vertices of parallelogram $ABCD$. The side AD meets the x-axis at E. $\angle DEB = \theta$ and M is the midpoint of the diagonal BD.



(i) Find the gradient of the line AD. **1**

(ii) Find θ to the nearest degree. **1**

(iii) Show that AB has equation $4x - 3y - 12 = 0$. **2**

(iv) Find the perpendicular distance between D and the line AB. **2**

(v) Find the area of the parallelogram $ABCD$. **2**

(vi) Find the co-ordinates of M, the midpoint of BD. **2**

(vii) Hence, or otherwise, find the co-ordinates of C. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet

Marks

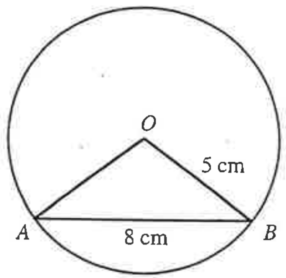
(a) Differentiate: $y = x \sin 5x$ **2**

(b) Differentiate: $y = \dfrac{e^{5x+4}}{2x^2}$ **2**

(c) (i) Given that $y = \sqrt{4x+9}$, show that $\dfrac{dy}{dx} = \dfrac{2}{3}$ when $x = 0$. **2**

(ii) Hence determine the equation of the normal to the curve $y = \sqrt{4x+9}$ at the point $(0,3)$ on it. **2**

(d)



AB is a chord of length 8 cm in a circle centre O of radius 5 cm.

(i) Using the cosine rule, find the size of $\angle AOB$. **2**

(ii) Find the length of the minor arc AB. Give your answer correct to 2 decimal places. **2**

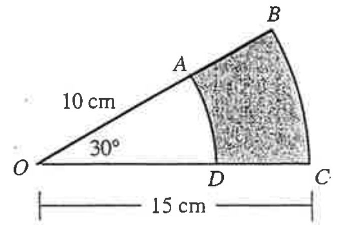
Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find $\int x - 5\sin 3x \; dx$. **2**

(b) Evaluate the definite integral $\int_0^1 (e^{2x} + 1) \; dx$, leaving the answer in exact form. **2**

(c) Consider the equation $2x^2 - (k+3)x + 2 = 0$. For what values of k does the equation have real roots? **3**

(d) **2**



In the diagram above, AD and BC are arcs of concentric circles with O as centre. Calculate, in terms of π, the area of the shaded region $ABCD$.

(e) Consider the parabola $(x-1)^2 = 12y$.

(i) Find the co-ordinates of its vertex and focus. **2**

(ii) Illustrate this information in a neat sketch. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) By completing the square, sketch the graph of $x^2 + 6x + y^2 = 0$. **2**

(ii) State the range of the graph in part (i). **1**

(b) For the function $y = x^3 - 3x^2 - 9x + 6$:

(i) Find the co-ordinates of any stationary points and determine their nature. **3**

(ii) Find the co-ordinates of any points of inflection. **2**

(iii) At what point does the curve cut the y-axis? **1**

(iv) For what values of x is the curve concave up? **1**

(v) Sketch the curve, showing all essential features. **2**

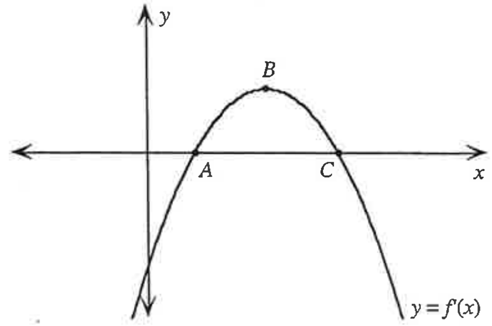
Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate: $\int_0^{\ln 2} e^{-x} \; dx$ **2**

(b) Evaluate: $\sum_{n=3}^{6} (n+1)^2$ **1**

(c) Solve: $e^{2x} + 5e^x - 6 = 0$ **3**

(d) **2**



The diagram above shows the graph of $y = f'(x)$, where $f'(x)$ is the derivative of $f(x)$. What point (A, B or C) on this graph corresponds to a maximum turning point of the function $y = f(x)$? Give reasons for your answer.

(e) Josie accepted a job that pays an initial salary of \$35 000 per annum. After each year of service she will receive an increment of \$5000 until she reaches a maximum salary of \$105 000.

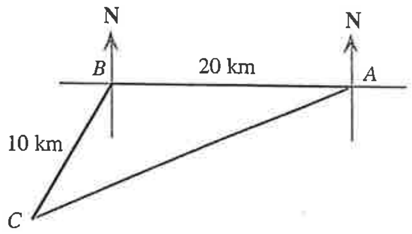
(i) What will be her salary after 5 years of service? **2**

(ii) How long will she have to work before she has earned a total of \$540 000? **2**

Question 7 (12 marks) Use a SEPARATE writing booklet

Marks

(a)



In the diagram above, a hiker walks 20 km due west from A to B and then another 10 km in a direction 220°T to C.

(i) Explain why $\angle ABC = 130°$. **1**

(ii) Find the distance and bearing of the hiker from A. **3**

(b) At the beginning of summer, a dam is 56% full. Due to evaporation and use by consumers it loses 8% of the water in the dam each week. There is no further inflow of water during the season.

(i) What percentage of the full dam remains after 4 weeks? **2**

(ii) If water rationing will be introduced when the dam is 30% full, find when rationing commences. **2**

(c) Consider the function $y = \ln(x+3)$ for $x > -3$.

(i) Sketch the function, showing its essential features. **2**

(ii) Use Simpson's Rule with three function values to find an approximation to $\int_{-1}^{1} \ln(x+3)\,dx$. **2**

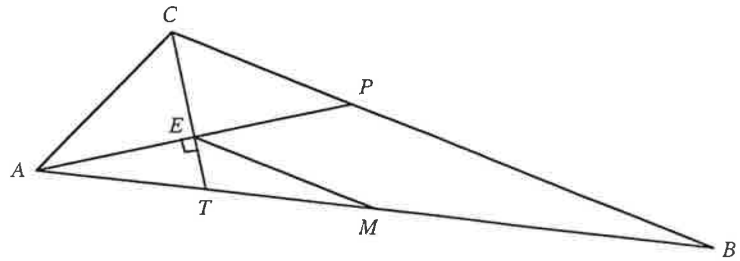
Question 8 (12 marks) Use a SEPARATE writing booklet

Marks

(a) Calculate the exact volume generated when the region enclosed by the curve $y = 2 + 3e^{-x}$ for $0 \le x \le 2$ is rotated about the x-axis. **3**

(b) A tap that empties the water from an urn is turned on slowly so that the volume flow rate, $\frac{dV}{dt}$, varies with time according to the relation $\frac{dV}{dt} = qt^2$, where $t > 0$ and q is a constant. Calculate the total volume of water that flows through the tap in the first 5 seconds if $q = 3.6\ \text{m}^3\text{s}^{-2}$. **2**

(c)



In the diagram above, CT bisects $\angle ACB$. M is the midpoint of AB and AE is perpendicular to CT. AE produced meets BC at P.

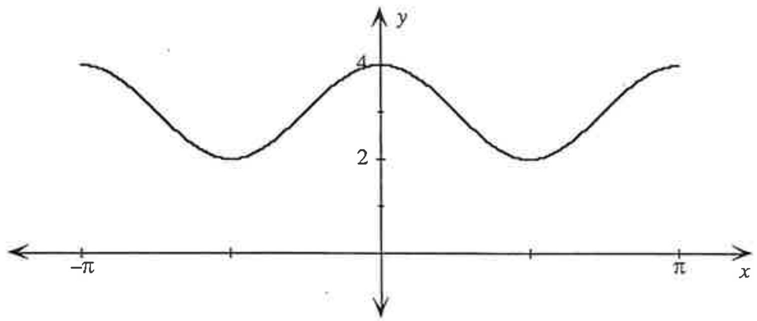
(i) Copy the diagram into your answer book and mark in all the given information.

(ii) Prove that ΔACE is congruent to ΔPCE. **2**

(iii) Explain why $AE = EP$. **1**

(iv) State why ΔAEM is similar to ΔAPB and hence prove that EM is parallel to PB. **2**

(d) **2**



The diagram above shows the graph of $y = \cos 2x + 3$ from $-\pi \le x \le \pi$. Find the area under the curve $y = \cos 2x + 3$ between $x = 0$ and $x = \frac{7\pi}{12}$. Express your answer in terms of π.

Question 9 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The number of bacteria (N) in a culture is growing exponentially according to the formula $N = 80e^{kt}$.

(i) What is the initial number of bacteria? 1

(ii) After eight hours the number of bacteria has doubled. Calculate the value of k. Give your answer correct to 3 decimal places. 2

(iii) How many bacteria, correct to the nearest ten, will there be after 12 hours? 1

(iv) At what rate will the bacteria be increasing after 12 hours? 1

(b) The position of a particle moving along the x-axis is given by $x = 2t + e^{-2t}$ where t is the time in seconds and x is measured in centimetres.

(i) Show that the particle is at rest when $t = 0$. 2

(ii) What is the acceleration of the particle after 1 second? 2

(iii) What is the limiting velocity of the particle? 1

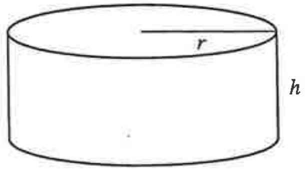
(c) If $y = \log_e x$:

(i) express x in terms of y and e. 1

(ii) express $\log_{10} x$ in terms of y. 1

Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) A closed cylindrical can of radius r cm and height h cm has a capacity of $16\pi\,\text{cm}^3$.



(i) Show that the height can be expressed as $h = \dfrac{16}{r^2}$. 1

(ii) Show that the surface area of the cylinder is given by $S = 2\pi r^2 + \dfrac{32\pi}{r}$. 1

(iii) What are the radius of the base and the height of the cylinder for the total surface area to be a minimum? 3

(b) Alicia borrows \$100 000 at a reducible interest rate of 7.2% p.a., compounded monthly. The debt is to be repaid with monthly repayments over 10 years.

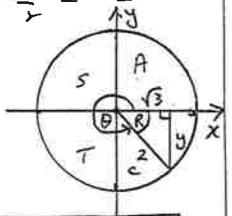
(i) If Alicia pays \$$P$ per month, show that after 2 months, the amount owing is \$$[100\,000(1.006)^2 - P(1.006) - P]$. 2

(ii) The monthly repayment is \$1172. How much will Alicia still owe after 3 years? 2

(iii) After 3 years, the bank tells Alicia that the interest rate has increased by 0.6% to 7.8% p.a. How much will her new monthly payment need to be in order to still pay out the loan at the end of the 10 years? 3

END OF EXAMINATION

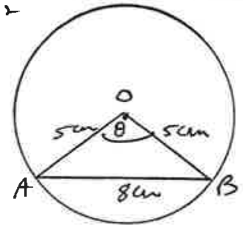
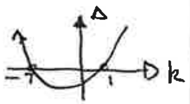
Year 12 Mathematics – Trial HSC August, 200

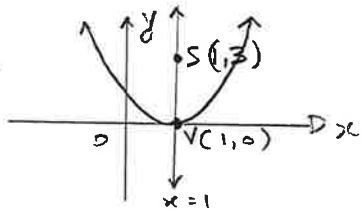
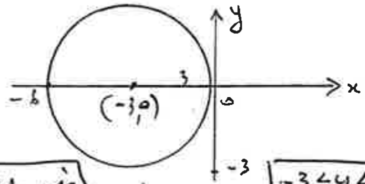
Qn	Solutions	Marks	Comments+Criteria
1(a)	$\dfrac{56.07-\sqrt{39.2}}{7.1} = \dfrac{49.809\ldots}{7.1}$ $= 7.01535$ $= \boxed{7.02}$	 1 1	 correct answer rounding
(b)	$\dfrac{3x-4}{2} - \dfrac{5-x}{3} = \dfrac{3(3x-4)-2(5-x)}{6}$ $= \dfrac{9x-\cancel{12}\,6-10+2x}{6}$ $= \boxed{\dfrac{11x-22}{6}}$	1 1	
(c)	$P = \$5000$, $n = 10$ yrs, $r = 9\%$ pa $A = P\times(1+r)^n$ $= 5000\times(1.09)^{10}$ $\boxed{FV = \$11,836.82}$	 1 1	
(d)	$\cos\theta = \dfrac{\sqrt{3}}{2}$, $\dfrac{3\pi}{2} \le \theta \le 2\pi$ $\cos 30° = \sqrt{3}/2$ $\therefore \theta = 2\pi - \pi/6$ $\boxed{\theta = \dfrac{11\pi}{6}}$ 	1 1	1 for link to acute angle 1 for correct angle in radians
(e)	$\int \sec^2 5x\,dx = \boxed{\dfrac{1}{5}\tan 5x + C}$		1 for tan 1 for correct coefficients
(f)	$\dfrac{4}{2-\sqrt{3}} = a+b\sqrt{3}$ $\dfrac{4}{2-\sqrt{3}}\times\dfrac{2+\sqrt{3}}{2+\sqrt{3}} = a+b\sqrt{3}$ $\dfrac{8+4\sqrt{3}}{1} = a+b\sqrt{3}$ $\therefore \boxed{a=8,\ b=4}$		 1 for rationalising 1 for equating coefficents

Qn	Solutions	Marks	Comments+Criteria
2(i)	$A=(0,-4)$, $D=(-5,4)$ $\quad x_1\ y_1 \qquad x_2\ y_2$ $m = \dfrac{y_2-y_1}{x_2-x_1}$ $= \dfrac{4--4}{-5-0}$ $\boxed{m = -8/5}$	 1	 correct answer
(ii)	$m = \tan\theta$ $\tan\theta = -8/5$ $\therefore \theta = 180° - 58°$ $\boxed{\theta = 122°}$	 1	
(iii)	$A(0,-4)$, $B(3,0)$ $\quad x_1\ y_1 \qquad x_2\ y_2$ $\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$ $\dfrac{y+4}{x-0} = \dfrac{0+4}{3-0}$ $3y+12 = 4x$ $\therefore AB$ is $\boxed{4x-3y-12=0}$		1 for gradient 1 for substitution of point into formula
(iv)	$D = (-5,4)$ $d_{D\to AB} = \dfrac{\lvert 4(-5)-3(4)-12\rvert}{\sqrt{(4)^2+(-3)^2}}$ $\boxed{d_{D\to AB} = \dfrac{44}{5} \text{ units}}$	 1 1	 substitution into formula correct answer
(v)	$d_{AB} = \sqrt{(3-0)^2+(0+4)^2}$ $d_{AB} = 5$ $\therefore \text{Area}(ABCD) = b\times h$ $= 5\times 44/5$ $\boxed{\text{Area} = 44\ u^2}$	1 1	working such as d_{AB} correct answer

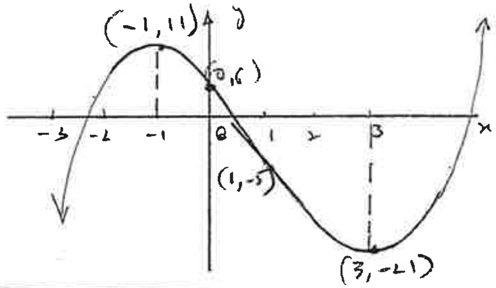
Qn	Solutions	Marks	Comments+Criteria
2(vi)	$B=(3,0)$, $D=(-5,4)$		
	$M=\left(\frac{3+-5}{2}, \frac{0+4}{2}\right)$	1	substitution
	$\boxed{M=(-1,2)}$	1	correct answer
(vii)	$M(AC)=M(BD)$ Let $C=(x,y)$		
	$\left(\frac{x+0}{2}, \frac{y-4}{2}\right)=(-1,2)$		
	$\therefore \frac{x+0}{2}=-1$		1 for working 1 for answer
	$x=-2$		
	$\frac{y-4}{2}=2$		
	$y-4=4$		
	$y=8$		
	$\therefore \boxed{C=(-2,8)}$		

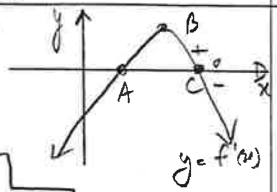
Qn	Solutions	Marks	Comments+Criteria
3(a)	$y=\overset{u}{x}\cdot\overset{v}{\sin 5x}$		1 for use of product rule
	$y'=\sin 5x\cdot 1+x\cdot 5\cos 5x$	2	1 for correct substitution
	$y'=\sin 5x+5x\cos 5x$		
(b)	$y=\frac{e^{5x+4}}{2x^2}$ — u — v		
	$y'=\frac{(2x^2)\cdot 5e^{5x+4}-e^{5x+4}\cdot(4x)}{4x^4}$	2	1 for use of quotient rule 1 for correct substitution
	$=\frac{\cancel{2x}\,e^{5x+4}(5x-2)}{\cancel{4x^4}\;2x^3}$		
	$\boxed{y'=\frac{e^{5x+4}(5x-2)}{2x^3}}$		
(c)(i)	$y=\sqrt{4x+9}=(4x+9)^{1/2}$		
	$y'=\frac{1}{\cancel{2}}(4x+9)^{-\frac{1}{2}}\cdot\cancel{4}\,2$	1	
	$y'=\frac{2}{\sqrt{4x+9}}$		
	when $x=0$,		
	$\boxed{y'=\frac{2}{\sqrt{9}}=\frac{2}{3}}$	1	
(ii)	Normal is $y-y_1=m(x-x_1)$		1 for gradient of normal
	At $(0,3)$, $m=-\frac{3}{2}$ — $y-3=-\frac{3}{2}(x-0)$	1	1 for substitution into equation of line
	$2y-6=-3x$		
	$\boxed{3x+2y-6=0}$		

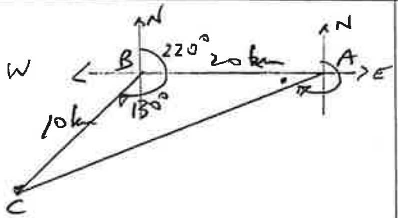
Qn	Solutions	Marks	Comments+Criteria
3(d)(i)	$\cos\theta = \frac{b^2+c^2-a^2}{2bc}$ 		
	$\cos\theta = \frac{5^2+5^2-8^2}{2\times5\times5}$	1	substitution into formula
	$\quad " = \frac{-14}{50}$		
	$\therefore \boxed{\theta = 106^\circ 16'}$	1	angle (rounding ignored)
(ii)	$\theta = 106^\circ 16' = 1.85459^c\ldots$		1 for conversion to radians
	$l = r\times\theta$		
	$\quad " = 5\times 1.85459\ldots$		1 for substitution into formula (ignore rounding)
	$\quad " = 9.27295\ldots$		
	$\boxed{l = 9.27\text{ cm}}$		
4(a)	$\int x - 5\sin 3x\,dx = \boxed{\frac{x^2}{2} + \frac{5}{3}\cos 3x + C}$	2	1 for $\frac{x^2}{2}$ 1 for $\frac{5}{3}\cos 3x$ ignore $+C$ integration
(b)	$\int_0^1 (e^{2x}+1)\,dx = \left[\frac{1}{2}e^{2x}+x\right]_0^1$	1	
	$\quad " = \left(\frac{1}{2}e^2+1\right) - \left(\frac{1}{2}+0\right)$		
	$\quad " = \boxed{\frac{1}{2}(e^2+1)}$ or $\frac{1}{2}e^2+\frac{1}{2}$	1	Must be in exact form
(c)	$2x^2 - (k+3)x + 2 = 0$		
	For real roots $\Delta \geq 0$.	1	
	$\Delta = b^2-4ac$		
	$\quad " = [-(k+3)]^2 - 4(2)(2)$		
	$\quad " = k^2+6k+9-16$		
	$\Delta = k^2+6k-7$	1	
	$\Delta \geq 0$ $(k+7)(k-1) \geq 0$ 		
	$\therefore \boxed{k \leq -7,\ k \geq 1}$	1	

Qn	Solutions	Marks	Comments+Criteria
4(d)	$A_s = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta$		
	$\quad = \frac{1}{2}(15)^2\frac{\pi}{6} - \frac{1}{2}(10)^2\frac{\pi}{6}$	1	
	$\quad = \frac{225\pi}{12} - \frac{100\pi}{12}$		
	$\boxed{A_s = \frac{125\pi}{12}\text{ cm}^2}$	1	
(e)	$(x-1)^2 = 12y$		
	$(x-1)^2 = 4(3)(y-0)$		
(i)	$\boxed{V = (1,0)}$	1	
	$a = 3$		
	$\boxed{S = (1,3)}$	1	
(ii)		1	correct graph showing answers above
5(a)(i)	$x^2+6x \quad +y^2 = 0$		
	$x^2+6x+9 \quad +y^2 = 9$		
	$\therefore (x+3)^2 + (y-0)^2 = 9$	1	
	This is a circle, centre $(-3,0)$ and radius 3 units		
		1	graph from above answer
(ii)	Range is $\boxed{-3 \leq y \leq 3}$	1	

Qn	Solutions	Marks	Comments+Criteria
5(b)	$y = x^3 - 3x^2 - 9x + 6$ (i) $y' = 3x^2 - 6x - 9$ For a stat. pt., $y' = 0$ $0 = 3x^2 - 6x - 9$ $0 = x^2 - 2x - 3$ $0 = (x-3)(x+1)$ $\therefore x = 3, -1$ when $x = 3$, $y = \cancel{27} - \cancel{27} - 27 + 6$, $y = -21$ when $x = -1$, $y = -1 - 3 + 9 + 6$, $y = 11$ $\therefore \boxed{\text{stat pts are } (3,-21), (-1,11)}$ $y'' = 6x - 6$ when $x = 3$, $y'' = 18 - 6$, $y'' = 12 > 0$ $\therefore$ $(3,-21)$ is a MIN TURN PT when $x = -1$, $y'' = -6 - 6$, $y'' = -12 < 0$ $\therefore$ $(-1,11)$ is a Max Turn. PT (ii) For an inflection pt, $y'' = 0$ and changes sign. $y'' = 6x - 6$ $6x - 6 = 0$ $\therefore x = 1$ when $x = 1$, $y = 1 - 3 - 9 + 6$, $y = -5$ Test: x: 0, 1, 2; y'': -6, 0, $+6$ $\therefore \boxed{\text{Inflection pt. at } (1,-5)}$		1 for x-values 1 for y-values 1 for testing 1 for co-ordinate 1 for testing

Qn	Solutions	Marks	Comments+Criteria
5(iii)	$y = x^3 - 3x^2 - 9x + 6$ when $x = 0$, $y = 6$. $\boxed{(0,6)}$	1	
(iv)	For concave up, $y'' > 0$. $y'' = 6x - 6$ $\therefore 6x - 6 > 0$ $\therefore \boxed{x > 1}$	1	
(v)			1 for shape 1 for key points
6(a)	$\int_0^{\ln 2} e^{-x}\,dx = \left[-e^{-x}\right]_0^{\ln 2}$ $= -\left[e^{-\ln 2} - e^0\right]$ $= -\left[e^{\ln 1/2} - 1\right]$ $= -\left[\frac{1}{2} - 1\right]$ $= \boxed{1/2}$	1 1	
(b)	$\sum_{n=3}^{6} (n+1)^2 = 4^2 + 5^2 + 6^2 + 7^2$ $= \boxed{126}$	1	
(c)	$e^{2x} + 5e^x - 6 = 0$ Let $m = e^x$ $m^2 + 5m - 6 = 0$ $(m+6)(m-1) = 0$ $\therefore m = -6, m = 1$ $\therefore e^x \neq -6$, $\therefore e^x = 1$ $\therefore \boxed{x=0}$	2 1	

Qn	Solutions	Marks	Comments+Criteria
6(d)	For a max. turn. pt, $y'=0$ and $y' = + \; 0 \; -$  $\therefore$ $\boxed{\text{Max. Turning pt for } y=f(x) \text{ is at } C}$		1 point 1 reason
(e)	AP $\begin{cases} a = \$35{,}000 \\ d = \$5{,}000 \end{cases}$	1	1 for recognising AP
	(i) $T_5 = a + 4d$ $= \$35{,}000 + 4 \times \$5000$ $\boxed{T_5 = \$55{,}000}$	1	
	(ii) $S_n = \$540{,}000$ $\frac{n}{2}[2a+(n-1)d] = 540{,}000$ $\frac{n}{2}[70{,}000 + (n-1)5000] = 540{,}000$ $\frac{n}{2}[5000n + 65000] = 540{,}000$ $n(5000n + 65{,}000) = 1{,}080{,}000$ $\cancel{5000}n^2 + 65{,}\cancel{000}n - 1{,}080{,}\cancel{000} = 0$ $5n^2 + 65n - 1080 = 0$ $n^2 + 13n - 216 = 0$ $n = \frac{-13 \pm \sqrt{13^2 - 4.1.-216}}{2}$ $= \frac{-13 \pm \sqrt{1033}}{2}$. 9.57 or -22.57 $\boxed{n \geq 0 \text{ so after 10 years}}$	1 2	2 mks for soln to $n^2+13n-216=0$

Qn	Solutions	Marks	Comments+Criteria
7(a)			
	(i) $\angle ABC = 220° - 90°$ $\boxed{\angle ABC = 130°}$	1	
	(ii) $AC^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \cos 130°$ $\boxed{AC = 27.52 \text{ km}}$ $\frac{\sin A}{10} = \frac{\sin 130°}{27.52}$ $\sin A = \frac{10 \times \sin 130°}{27.52}$ $\therefore A = 16° 10'$ $\therefore$ Bearing of Hiker from A is Bearing $= 270° - 16°$ $\boxed{\text{Bear.} = 254° T}$	1 1 1	
(b)	(i) Beginning 56% full let amount remaining after n weeks be A_n $A_1 = 0.56 \times 0.92$ $A_2 = 0.56 \times 0.92 \times 0.92$ $A_3 = 0.56 \times 0.92^3$ $A_4 = 0.56 \times 0.92^4$ $= 0.40118$ $\therefore 40.1\%$	1 1	

Qn	Solutions	Marks	Comments+Criteria

7(b) (ii) $A_n = 0.56 \times 0.92^n = 0.3$

$\ln(0.92)^n = \ln\left(\frac{0.3}{0.56}\right)$

$n = \dfrac{\ln\left(\frac{0.3}{0.56}\right)}{\ln 0.92}$ — 1

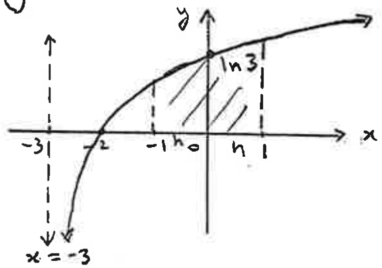
$= 7.49$

After 7 weeks still more than 30%

∴ Rationing introduced after 7.5 weeks (to nearest tenth) — 1

(c) $y = \ln(x+3), \; x > -3$

(i)



1 for shape & asymptote

1 for intercepts

(ii) $\int_{-1}^{1} \ln(x+3)\,dx \doteq \frac{h}{3}\left[y_0 + 4y_1 + y_2\right]$

$y = \ln(x+3)$

x	-1	0	1
y	0.6931	1.0986	1.3863
	y_0	y_1	y_2

$\therefore \int_{-1}^{1} \ln(x+3)\,dx \doteq \frac{1}{3}(0.6931 + 4 \times 1.098 + 1.3863)$ — 1

$\int_{-1}^{1} \ln(x+3)\,dx \doteq 2.158$ — 1

Qn	Solutions	Marks	Comments+Criteria

8(a) $y = 2 + 3e^{-x}, \quad 0 \le x \le 2$

$V = \pi \int_0^b y^2\,dx$

$= \pi \int_0^2 (2 + 3e^{-x})^2\,dx$

$= \pi \int_0^2 (4 + 12e^{-x} + 9e^{-2x})\,dx$ — 1

$= \pi \left[4x + \frac{12e^{-x}}{-1} + \frac{9e^{-2x}}{-2}\right]_0^2$ — 1

$= \pi\left[\left(8 - 12e^{-2} - \frac{9}{2}e^{-4}\right) - \left(0 - 12 - \frac{9}{2}\right)\right]$

$= \pi\left[24\frac{1}{2} - 12e^{-2} - \frac{9}{2}e^{-4}\right]$

$V = \frac{\pi}{2}\left[49 - 24e^{-2} - 9e^{-4}\right] u^3$ — 1

(b) $\frac{dV}{dt} = qt^2 = 3.6t^2$

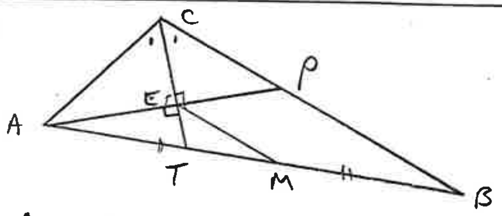
$V = \int_0^5 3.6t^2\,dt$ — 1

$= \left[\frac{3.6t^3}{3}\right]_0^5$

$= \left[1.2t^3\right]_0^5$

$V = 150\,m^3$ — 1

(13)

Qn	Solutions	Marks	Comments+Criteria
8(c)(i)			
(ii)	In Δ's ACE, PCE, $\angle ACE = \angle PCE$ (data) A $\angle CEA = \angle CEP = 90°$ (data) A CE = CE (common) S $\therefore$ $\Delta ACE \equiv \Delta PCE$ (AAS)	2	
(iii)	Since $\Delta ACE \equiv \Delta PCE$ (proved in (ii)) $\therefore$ AE = EP (matching sides of congruent Δ's are =)	1	
(iv)	Since ~~AE~~=EP (from part iii) $\therefore$ E is midpt of AP. But M is midpt of AB (given) $\therefore$ EM ∥ PB and $EM = \frac{1}{2}PB$. $\therefore$ In Δ's AEM, APB, $\angle AEM = \angle APB$ (corresp $\angle$s = in ∥ lines EM, PM) A $\angle AME = \angle ABP$ (") A $\angle EAM = \angle PAB$ (common) A $\therefore$ $\Delta AEM \parallel\!\!\!| \Delta APB$ (AAA)		1 Sim triangle reason 1 Proof that EM ∥ PB.

(13a)

Q8 c) (iv) Alternative solution

In ΔAEM and ΔAPB

AE = EP from (iii)

$\therefore$ AE : AP = 1:2

AM = MB given M is midpt

$\therefore$ AM : AB = 1:2

$\angle PAB$ is common

$\therefore$ $\Delta AEM \parallel\!\!\!| \Delta APB$ 2 sides in same ratio and included angle equal.

$\therefore$ $\angle AME = \angle ABP$ corresponding angles similar Δs

$\therefore$ EM ∥ PB since corresponding angles equal.

Qn	Solutions	Marks	Comments+Criteria
8(d)	$A = \int_0^{7\pi/12} (\cos 2x + 3)\, dx$		
	$= \left[\frac{1}{2}\sin 2x + 3x\right]_0^{7\pi/12}$	1	
	$= \left(\frac{1}{2}\cdot \sin\frac{7\pi}{6} + \frac{7\pi}{4}\right) - (0+0)$		
	$= \frac{1}{2}\left(-\frac{1}{2}\right) + \frac{7\pi}{4}$		
	$\therefore A = \frac{7\pi}{4} - \frac{1}{4}$		
	$\boxed{A = \frac{1}{4}(7\pi - 1)\ u^2}$	1	
9(a)	$N = 80e^{kt}$		
	(i) when $t=0$, $N = 80e^0$		
	$\boxed{N = 80}$	1	
	(ii) when $t=8$, $N=160$.		
	$\therefore \cancel{160}^{2} = \cancel{80}\, e^{8k}$	1	
	$e^{8k} = 2$		
	$8k = \ln 2$		
	$k = \frac{\ln 2}{8} = 0.08664$		
	$\therefore k = 0.087$ (3dp)	1	
	(iii) when $t=12$,		
	$N = 80e^{0.087\times 12}$		
	$= 226.27$		
	$\boxed{N = 230}$ (tnt)	1	Ignore rounding
	(iv) $dN/dt = k\times N$		
	$= 0.087 \times 230$	1	
	$\boxed{\text{rate} = 19.6\ b/h}$ (using exact values)		
	or 20.01 b/h (using answers from (ii) + (iii))		

Qn	Solutions	Marks	Comments+Criteria
9(b)	$x = 2t + e^{-2t}$		
	(i) $\frac{dx}{dt} = 2 - 2e^{-2t}$	1	
	when $t=0$, $\frac{dx}{dt} = 2 - 2\times e^0$		
	$= 2-2$		
	$\boxed{\frac{dx}{dt} = 0}$	1	
	$\therefore$ particle is at rest when $t=0$.		
	(ii) $\frac{d^2x}{dt^2} = +4e^{-2t}$	1	
	when $t=1$, $\boxed{\frac{d^2x}{dt^2} = 4e^{-2}\ cm/s^2}$	1	
	(iii) $\lim_{t\to\infty} \frac{dx}{dt} = \lim_{t\to\infty}\left(2 - \frac{2}{e^{2t}}\right)$		
	$= 2 - 0$		
	$\therefore \boxed{\text{lim vel} = 2\ cm/s}$	1	
(c)	$y = \log_e x$		
	(i) $\boxed{x = e^y}$	1	
	(ii) $\log_{10} x = \log_{10} e^y$		
	$\therefore \boxed{\log_{10} x = y\log_{10} e}$	1	

Qn	Solutions	Marks	Comments+Criteria
10(a)(i)	$V = \pi r^2 h$		
	$V = 16\pi$		
	$\therefore \cancel{\pi} r^2 h = 16\cancel{\pi}$	1	
	$\therefore \boxed{h = \frac{16}{r^2}}$		
	(ii) $S = 2\pi r^2 + 2\pi r h$		
	$\quad = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$	1	
	$\therefore \boxed{S = 2\pi r^2 + \frac{32\pi}{r}}$		
	(iii) For a min, $S' = 0$, $S'' > 0$.		
	$S = 2\pi r^2 + 32\pi r^{-1}$		
	$S' = 4\pi r - \frac{32\pi}{r^2}$		
	$0 = 4\pi r - \frac{32\pi}{r^2}$		
	$\therefore 4\cancel{\pi} r = \frac{32\cancel{\pi}}{r^2}$		
	$r^3 = 8$		
	$\therefore \boxed{r = 2 \text{ cm}}$	1	
	$S' = 4\pi r - 32\pi r^{-2}$		
	$\therefore S'' = 4\pi + 64\pi r^{-3}$		
	$\quad = 4\pi + \frac{64\pi}{r^3}$		
	$r=2$: $S'' = 4\pi + \frac{64\pi}{8}$		
	$S'' = 12\pi > 0 \quad \therefore$ min S.		
	$r = 2, \quad h = \frac{16}{4} = 4$	1	1 for testing
	$\therefore \boxed{r = 2\text{cm}, h = 4\text{cm} \text{ for min } S}$		

Qn	Solutions	Marks	Comments+Criteria
10(b)	$P = \$100,000$		
	$r = 7.2\%$ pa $= 0.072$ pa		
	$r = 0.006$		
	$n = 10$ yrs $= 10 \times 12 = 120$ mths		
	(i) A_n = amount owing after n months.		
	$A_1 = 100,000(1.006) - P$	1	
	$A_2 = A_1(1.006) - P$		Derivation of A_2 had to be clear
	$\quad = (100,000(1.006) - P)(1.006) - P$	1	
	$\boxed{A_2 = 100,000(1.006)^2 - P(1.006) - P}$		
	(ii) $n = 3$ years $= 36$ months.		
	$A_{36} = 100,000(1.006)^{36} - 1172 \times (1 + 1.006 + 1.006^2 + \dots + 1.006^{35})$	1	
	$A_{36} = 100,000(1.006)^{36} - 1172 \times S_{36}$		
	$S_{36} = \frac{1.[(1.006)^{36} - 1]}{1.006 - 1}$		
	$S_{36} = 40.050$		
	$\therefore A_{36} = 100,000(1.006)^{36} - 1172 \times 40.050$		
	$A_{36} = \$77,091.25$		
	$\therefore$ She still owes $\$77,091.25$	1	

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Qn	Solutions	Marks	Comments + Criteria
10 (b)	(iii) After 3 years $A_{36} = 77\,091.25$		
	She has 7 more years to go		
	Let amount owing n months after the 3 years be $\$B_n$ $\quad r = 0.078 \div 12 = 0.0065$		
	$B_n = 77091.25(1.0065)^n - M(1 + 1.0065 + \ldots 1.0065^{n-1})$		
	7 years later $B_{84} = 0$		
	$77091.25(1.0065)^{84} - M(\underbrace{1 + 1.0065 + \ldots 1.0065^{83}}_{S}) = 0$	1	
	$S = \frac{a(r^n - 1)}{r - 1} = \frac{1(1.0065^{84} - 1)}{1.0065 - 1} = 111.27$		
	$77091.25(1.0065)^{84} - MS = 0$		
	$M = \frac{77091.25(1.0065)^{84}}{S} = 1193.89$	1	
	∴ Her repayment needs to be $1193.89	1	