Killara HIGH SCHOOL

STUDENT NUMBER

MATHEMATICS

TRIAL HIGHER SCHOOL CERTIFICATE

Monday 23rd JULY 2012 9:00am

General Instructions

- Reading Time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start a new booklet for each question

Total Marks - 100

- Attempt questions 1 - 16
- Answer questions 1 – 10 on the multiple choice answer sheet provided
- For questions 11-16, start each question in a new booklet

QUESTION NO	MARK
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15
TOTAL	/100

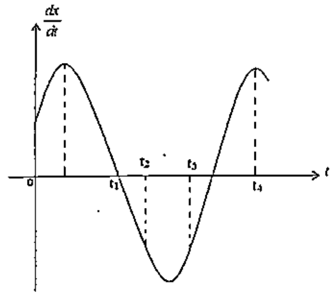
THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 40% of the Higher School Certificate Course Assessment.

Answer questions 1 – 10 on the multiple choice answer sheet provided *1 MARK EACH*

1. The graph shows the velocity $\frac{dx}{dt}$ of a particle as a function of time t.

Initially the particle is at the origin accelerating forward.



NOT TO SCALE

At what time is the displacement from the origin a maximum?

(A) t_1

(B) t_2

(C) t_3

(D) t_4

2. Consider the parabola $8(x-2)^2 = (y+3)$.

Which of the following is NOT true about the parabola?

(A) The focal length is $\frac{1}{32}$ units

(B) The vertex is (2, -3)

(C) The focus is $(2, -2\frac{31}{32})$

(D) The equation of the directrix is $-5\frac{31}{32}$

3. The limiting sum of the geometric series $3 + \frac{3}{\sqrt{3}+1} + \frac{3}{(\sqrt{3}+1)^2} + \ldots\ldots\ldots\ldots$ is:

(A) $\sqrt{3}+1$

(B) $\sqrt{3}+2$

(C) $\sqrt{3}+3$

(D) $\sqrt{3}+4$

4. The equation of the directrix of the parabola $y = x^2 - 2$ is given by

(A) $y = -1\frac{3}{4}$

(B) $y = -2\frac{1}{4}$

(C) $y = 1\frac{3}{4}$

(D) $y = 2\frac{1}{4}$

5. Differentiate: $\frac{1}{x^3}$

(A) $\frac{1}{3x^2}$

(B) $\frac{1}{3x^4}$

(C) $\frac{-3}{x^4}$

(D) $\frac{1-3x^2}{x^6}$

6. The first derivative of $\frac{e^x}{(x+1)^2}$ is:

(A) $\frac{(x+1)^2.e^x - e^x.2(x+1)}{(x+1)^2}$

(B) $\frac{2(x+1).e^x - (x+1)^2.e^x}{(x+1)^4}$

(C) $\frac{(x+1)^2.e^x + e^x.2(x+1)}{(x+1)^4}$

(D) $\frac{(x+1)^2.e^x - e^x.2(x+1)}{(x+1)^4}$

7. The domain of $\log_e(3-x)$ is:

(A) $x > 3$

(B) $x < 3$

(C) $x > 4$

(D) $x < 4$

8. The value of $\int_0^{\frac{\pi}{2}} (\sin x - \cos 2x)\, dx$ is:

(A) -1

(B) 2

(C) $\frac{1}{2}$

(D) 1

9. Which of the following expressions will find the perpendicular distance from the point $(-1,3)$ to the line $3x - 4y + 5 = 0$?

(A) $d = \frac{|3\times(-1)+4\times3+5|}{\sqrt{(-1)^2+3^2}}$

(B) $d = \frac{|3\times(-1)-4\times3+5|}{\sqrt{(-1)^2+3^2}}$

(C) $d = \frac{|3\times(-1)-4\times3+5|}{\sqrt{3^2+(-4)^2}}$

(D) $d = \frac{|3\times(-1)+4\times3+5|}{\sqrt{3^2+(-4)^2}}$

10 In a circle of radius 4.6 units, an angle θ is subtended by an arc length of 6.3 units.

What is the size of θ correct to the nearest degree?

(A) 1

(B) 79

(C) 57

(D) 78

End of multiple choice section

ANSWER QUESTIONS 11 – 16 IN THE ANSWER BOOKLETS PROVIDED

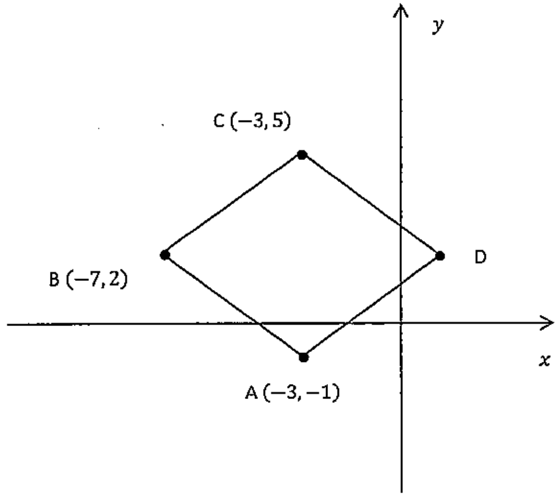
Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate ln 0.75 to 3 significant figures. **2**

(b) Solve $|2m-1| \leq 3$ **2**

(c) Find the equation of the normal to the curve $y = 3\cos(2x)$ at the point where $\left(\frac{\pi}{6}, \frac{3}{2}\right)$ **3**

(d) In the diagram below, ABCD is a rhombus.



COPY OR TRACE THE DIAGRAM INTO YOUR ANSWER BOOKLET

(i) Find the length of AB. **1**

(ii) Find the gradient of BC. **1**

(iii) Find the equation of the line AD and of the line CD; and hence show the coordinates of D are $(1, 2)$. **4**

(iv) Find the angle that AB makes with the positive x axis. **1**

(v) Find the area of ABCD. **1**

End of Question 11

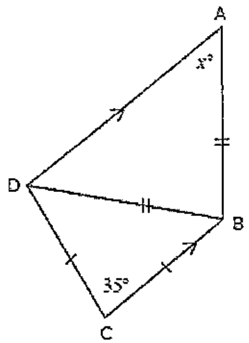
Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate

(i) $(x-\log_e x)^4$ **2**

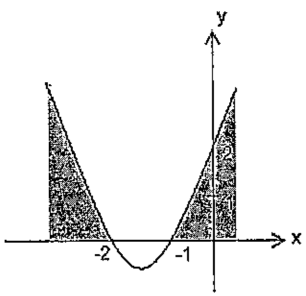
(ii) $x^2 e^{2x}$ **2**

(b) In the diagram, ABCD is a trapezium. Given that BC = CD, $\angle DBC = 35°$, AD // BC and AB = DB, find the value of $x°$. Give reasons for your answer. **2**



NOT TO SCALE

(c) Write the inequality that holds for the shaded region indicated on the Cartesian plane below. **2**



Question 12 is continued on the next page

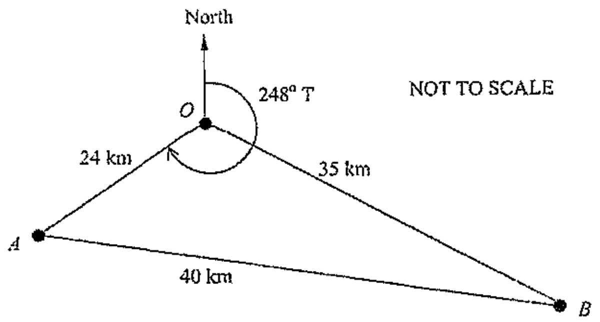
(d) (i) Find: $\int \frac{3x}{1-x^2}\,dx$ **2**

(ii) Find the exact value of: $\int_0^{\frac{\pi}{3}} 3\sin\frac{x}{2}\,dx$ **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram below shows a section of rainforest that is to be designated for a species count. The bearing of landmark A from landmark O is 248° T and A is 24 km from O. The distance from landmark A to B is 40 km and from landmark B to O is 35 km.



COPY OR TRACE THE DIAGRAM INTO YOUR ANSWER BOOKLET

(i) Use the cosine rule to show $\angle AOB$ is 83°. **1**

(ii) Hence, calculate the area of this section of the rainforest. **1**

(iii) What is the bearing of landmark O from landmark B? **2**

(b) Consider the function $f(x) = x^3 - 3x^2$.

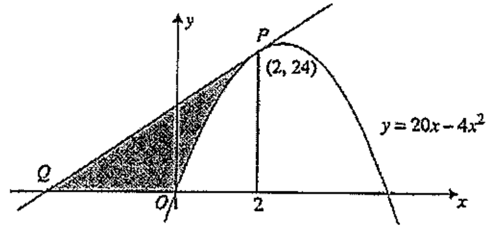
(i) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

(ii) Sketch the curve showing all the major features. **2**

(iii) Find the values of x for which the curve $y = f(x)$ is concave up. **1**

Question 13 is continued on the next page

(c)



In the graph, a tangent is drawn at P (2, 24) on the parabola $y = 20x - 4x^2$. The tangent intersects the x-axis at Q.

(i) Show that the equation of the tangent is $4x - y + 16 = 0$. **2**

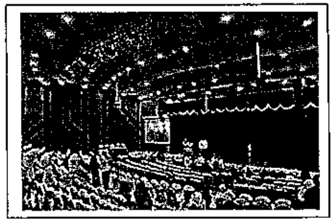
(ii) Find the coordinates of Q. **1**

(iii) Find the area of the shaded region POQ. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a)



In an auditorium, people are to be seated in rows on identical chairs each of width 65 cm. The chairs in each row are placed next to each other, leaving no gaps.

The first row, the closest to the stage is 13 m long. Each of the following rows 2,3, 4, …., 25 has two more chairs than the row in front of it.

After row 25, each row has only one chair more than the row in front of it. The last row has 84 chairs.

(i) How many chairs are in row 25? **2**

(ii) What is the total number of rows in the stadium? **1**

(iii) How many people can be seated in the auditorium, if no chair is empty. **2**

(b) The local council believes that the rabbits in the area have developed immunity to the Calcivirus. At the beginning of 2008, the rabbit population in the local area is believed to be 10000. Four years later, at the beginning of 2012, the rabbit population in the area has risen to 40000.

Assume that the population N is increasing exponentially, and satisfies the equation $N = Ae^{kt}$, where A and k are constants and the time t is measured in years from the beginning of 2008.

(i) Find the values of A and k. **3**

(ii) The local council believes that the rabbits will decimate the local habitat, once the population reaches 640 000 .
Find the time t required for the population N to reach 640 000. **2**

Question 14 is continued on the next page

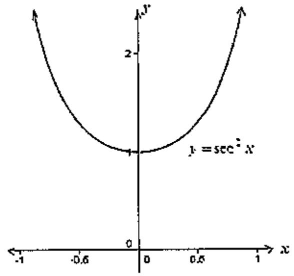
(c) *Joanna is learning about approximate methods of integration in class.*

Her first task is to use the trapezoidal rule to evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$

(i) Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$, using trapezoidal rule with three function values. **2**

Joanna's second task is to use simpson's rule to evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$

(ii) Joanna quickly drew a sketch of $y = \sec^2 x$ as shown below.



Use Simpson's rule with three function values to evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$ **1**

Joanna's teacher said that simpson's rule was a better approximation to $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$ *than the trapezoidal rule.*

(iii) By means of a sketch or otherwise explain why Simpson rule gives a better approximation to $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$ than the Trapezoidal rule. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) A particle moves in a straight line so that at any time $t \geq 0$ its velocity, v ms^{-1}, is given by $v = 4\sin 2t$. At time $t = 0$ the particle has displacement $x = 0$.

(i) Find displacement x, as a function of t. **1**

(ii) For t in the range $0 \leq t \leq 2\pi$, sketch the displacement-time graph $x = f(t)$ **2**

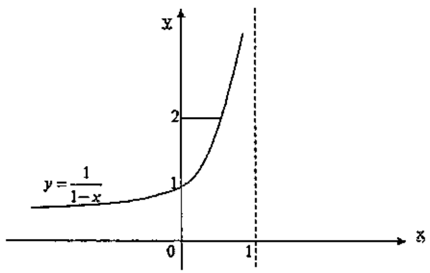
(iii) At what times is the particle at rest for t in the range $0 \leq t \leq 2\pi$? **2**

(iv) Find the acceleration when $t = 1$. **2**

(b) (i) For the equation $y = xe^{2x}$, show that the gradient of the tangent at $x = 1$ is given by $3e^2$. **2**

(ii) Hence find the equation of the tangent in general form to $y = xe^{2x}$ at the point where $x = 1$. **2**

(c) The region bounded by the graph $y = \dfrac{1}{1-x}$, the y-axis and the line $y = 2$ is rotated about the y-axis to form a solid of revolution.



(i) Show that the volume V of the solid is given by $V = \pi \int_1^2 \left(1 - \frac{2}{y} + \frac{1}{y^2}\right) dy$ **2**

(ii) Find the volume V of the solid formed. **2**

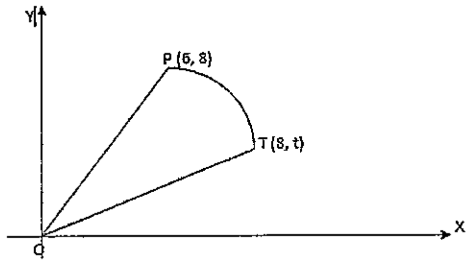
End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $4\sin^2 x + 12\sin x\cos x - \cos^2 x + 5 = 0$ for $0^\circ \leq x \leq 360^\circ$ **2**

Give your answer to the nearest minute.

(b) The figure below shows a sector OPT of a circle, centre O, with its arc joining the points P(6,8) and T(8,t).



COPY OR TRACE THIS FIGURE INTO YOUR ANSWER BOOKLET.

The figure above shows a sector of a circle OPT, centre O, with its arc joining The points P(6,8) and T(8,t).

(i) Find the value of t. **1**

(ii) Show that the size of $\angle TOP$ is 0.28 radians, correct to 2 decimal places. **2**

Question 16 is continued on the next page

(c) On the 1st of Jan 2000, Selina deposits $1000 into an account which pays a fixed interest of $200 per year, credited into the account at the end of every year.

On 1st Jan 2005, Hebe deposits $1000 into an investment account and receives an interest of $100 on 31st Dec 2005. Thereafter, the interest returned at the end of the year is 1.5 times the interest returned in the previous year.
Take year 2000 as the first year.

(i) Write down the amount of savings that Selina has in her account at the end of the n^{th} year. **1**

(ii) Show that the amount of savings that Hebe has in her account at the end of the n^{th} year is $\$[1000 + 200(1.5^{n-5} - 1)]$ **2**

(iii) At the end of the k^{th} year, Hebe saw that her savings finally exceeds Selina's savings for the first time. Find the value of k and the interest that Hebe receives in the k^{th} year. **2**

(d) A soft drink can, in the shape of a cylinder, is to hold 500 millilitres of drink.
The metal used to construct the sides of the can costs $5 cents/100 cm^2$.
The metal used on the top and bottom of the can costs twice as much as the cost of the metal used on the sides.

Let the radius of the can be r cm and its height be h cm and the total cost of the metal used to construct the can be C cents.

(i) Show that $C = \frac{50}{r} + \frac{\pi r^2}{5}$ **2**

(ii) Hence find the dimensions of the can so that the cost of the metal used is the cheapest. **3**

End of paper

Killara HIGH SCHOOL

STUDENT NUMBER: | | | | | | | | | |

Multiple Choice Answer Sheet

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

SOLUTIONS 2012 MATHEMATICS TRIAL

MC 1A 2D 3C 4B 5C 6D 7B 8D 9C 10D

2012 TRIAL MATHEMATICS

Question 11

a) $\ln 0.75 = -0.2876820725$
$= -0.288$ (3 sig fig)

1 mark for answ.
1 mark for sig fig

b) $|2m-1| \leq 3$

$2m-1 \leq 3$ $2m-1 \geq -3$
$2m \leq 4$ $2m \geq -2$
$m \leq 2$ $m \geq -1$

1 mark for removing absolute value signs

$-1 \leq m \leq 2$ ← Many did not write as one expression

1 mark for correct answer expressed as one rule

c) $y = 3\cos(2x)$
$y' = -6\sin(2x)$ *

(sin / −cos, cos / −sin)

$\left(\frac{\pi}{6}, \frac{3}{2}\right)$

1 mark for correct differentiation

$y'\left(\frac{\pi}{6}\right) = -6\sin\left(\frac{\pi}{3}\right)$
$= -6 \times \frac{\sqrt{3}}{2}$
$= -3\sqrt{3}$

$\therefore m = \frac{1}{3\sqrt{3}}$ for normal * or $\frac{\sqrt{3}}{9}$

1 mark for correct gradient

$y - y_1 = m(x - x_1)$

$y - \frac{3}{2} = \frac{1}{3\sqrt{3}}\left(x - \frac{\pi}{6}\right)$ *

$3\sqrt{3}y - \frac{9\sqrt{3}}{2} = x - \frac{\pi}{6}$

$18\sqrt{3}y - 18\sqrt{3} = 6x - \pi$

$6x - 18\sqrt{3}y + 27\sqrt{3} - \pi = 0$

$6\sqrt{3}x - 54y - \pi\sqrt{3} + 81 = 0$

1 mark for correct substitution into point gradient formula

d) i) $(-3, -1)$ (x_1, y_1) $(-7, 2)$ (x_2, y_2)

$d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(-7--3)^2 + (2--1)^2}$
$= \sqrt{16 + 9}$
$= \sqrt{25}$
$= 5$ units.

1 mark

ii) $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $(-7, 2)$ (x_1, y_1) $(-3, 5)$ (x_2, y_2)

$= \frac{5 - 2}{-3 - -7}$

$= \frac{3}{4}$

1 mark.

iii) AD gradient $= \frac{3}{4}$, $A(-3, -1)$.

AD

$y - y_1 = m(x - x_1)$
$y - -1 = \frac{3}{4}(x - -3)$.
$4y + 4 = 3x + 9$.

$3x - 4y + 5 = 0$. —①
$4y = 3x + 5$
$y = \frac{3x}{4} + \frac{5}{4}$

CD

$m_{CD} = m_{BA}$.

$m_{BA} = -\frac{3}{4}$

$y - 5 = -\frac{3}{4}(x - -3)$.
$4y - 20 = -3x - 9$.
$3x + 4y - 11 = 0$. —②
$4y = -3x + 11$
$y = \frac{-3x}{4} + \frac{11}{4}$

1 mark for eqn AD
1 mark for eqn CD

①+② $6x \quad -6 = 0$
$6x = 6$
$x = 1$.

1 mark for removing 1 variable

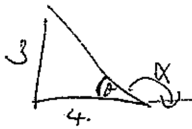
sub into ①.
$3 - 4y + 5 = 0$
$-4y = -8$
$y = 2$

so D is $(1, 2)$

1 mark for answer with full working

Many did not show enough

iv) $-\frac{3}{4}$



$\tan\theta = \left(\frac{3}{4}\right)$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$= 36^\circ 52' 11.63''$

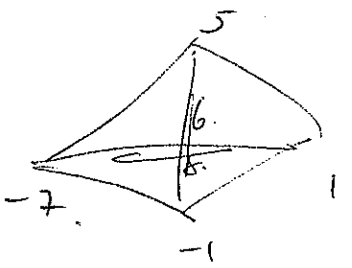
$\alpha = 180 - \theta$

$\boxed{\alpha = 143^\circ 7' 48.37''}$

1 mark

(Students did not understand what the positive x axis is)

v) Area $\frac{1}{2}xy$



$\frac{1}{2}xy = \frac{1}{2}\times 6\times 8$

$= 3\times 8$

$= 24$ units^2

1 mark

MATHEMATICS

MC 7 B 8 D 9

Award

Question 12

a) (i) $\frac{d}{dx}(x-\log_e x)^4 = 4(x-\log_e x)^3.\left(1-\frac{1}{x}\right)$

2 marks correct differentiation
1 mark for $\frac{d\log_e x}{dx} = \frac{1}{x}$

(ii) $\frac{d}{dx}(x^2.e^{2x}) = 2x.e^{2x} + x^2.2e^{2x}$

2 marks correct differentiation
1 mark $\frac{d}{dx}e^{2x} = 2e^{2x}$

* well done generally *

b) $\triangle ABD$ is isosceles $AB = DB$ (Given)

$\therefore \angle A = \angle ADB = x^\circ$ (Angles opposite to equal sides) *

(There is NO BASE ANGLES!!)

$\angle ABD = 180^\circ - 2x^\circ$ ($\angle$ sum of $\triangle$)

$\angle A + \angle ABC = 180^\circ$ (co-interior angles on // lines AD//BC)

(Parallel lines must be specified)

$x^\circ + 180^\circ - 2x^\circ + \angle DBC = 180^\circ$, $\angle DBC = x^\circ$

$\triangle DBC$ $2x^\circ + 35^\circ = 180^\circ$ ($\angle DBC = CBD$ $\angle$'s opposite to = sides)

$x^\circ = 72.5^\circ$

1 mark Finding $\angle DBC$, or $\angle CBD$
1 mark Evaluating x°
1 mark for reasoning

c) Equation of the curve $(x+2)(x+1) = 0$

$x^2 + 3x + 2 = 0.$

The inequality $x^2 + 3x + 2 \geq 0$

1 mark writing the quadratic equation
1 mark writing correct inequality

d) (i) $\int \frac{3x}{1-x^2}dx = -\frac{3}{2}\int\frac{-2x}{1-x^2}dx$

$= -\frac{3}{2}\log_e(1-x^2) + C$

2 marks correct integration
1 mark some working towards solution.

(ii) $\int_0^{\pi/3} 3\sin\frac{x}{2}dx = 3\left[\frac{-\cos\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi/3}$

$= -6\left[\cos\frac{\pi}{6} - \cos 0\right]$

$= -6\left[\frac{\sqrt{3}}{2} - 1\right]$

$= 6 - 3\sqrt{3}$ OR $3(2-\sqrt{3})$

1 mark $3\left[\frac{\cos\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi/3}$
1 mark correct evaluation

MC 1 A 2 D 3 C 4 B 5 C 6 D 7 B 8 D 9 C 10 D

Q13

Start here

a) (i) $\cos(\angle AOB) = \frac{24^2+35^2-40^2}{2\times 24\times 25}$ — well done!

$\angle AOB \doteq \cos^{-1}\left(\frac{24^2+35^2-40^2}{2\times 24\times 25}\right)$

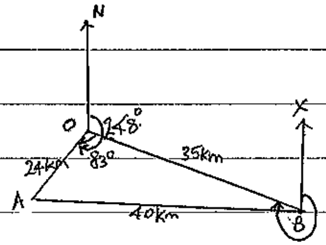
$= \underline{83^\circ}$ — (1 mark) for 83°.

(ii) $A = \frac{1}{2}\times 24\times 35\times \sin 83^\circ$ — well done!

$= 416.8693\ldots$

$\doteq \underline{417\ \text{Km}^2}$ — (1 mark) for 417 km²

(iii)



The angle between the north line and the interval AB is $248 - 83^\circ = \underline{165^\circ}$ — (1 mark) for 165° well done!

$\angle XBO = 180^\circ - 165^\circ = 15^\circ$ (co-interior angles)

∴ The bearing of O from B is $360^\circ - 15^\circ$

$= \underline{345^\circ T}$ — (1 mark) for 345°T ok.

b) (i) $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

stationary points when $f'(x) = 0$

$3x^2 - 6x = 0$ ~~$3x(x-2)=0$~~ ∴ $\underline{x = 0 \text{ or } x = 2}$ — (1 mark) for x=0 or x=2. well done!

At $x = 0$ $y = 0^3 - 3(0)^2 = 0$

At $x = 2$, $y = 2^3 - 3(2)^2 = -4$

∴ stationary points are $(0,0)$ and $(2,-4)$ — 1 mark for 2 stationary points (Missing y-coordinate on stationary points)

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-2-

Method 1. Using first derivative, test $x=0$

x	−1	0	1
$f'(x)$	9	0	−3

∴ Max turning point at $(0,0)$

Test $x = 2$

x	1	2	3
$f'(x)$	−3	0	9

∴ Min turning point at $(2,-4)$ — 1 mark for test x=0 & x=2

Method 2 Using 2nd derivative $f''(x) = 6x - 6$ — Well done!

Test $x = 0$ $f''(0) = 6(0) - 6$

$= \underline{-6}$

i.e. $f''(0) < 0 \Rightarrow$ Concave down

∴ Max turning point at $(0,0)$

Test $x = 2$ $f''(2) = 6(2) - 6$

$= \underline{6}$

i.e. $f''(2) > 0 \Rightarrow$ Concave up

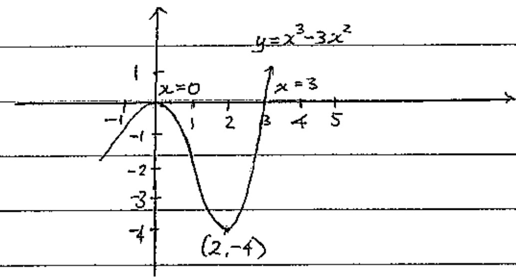
∴ Minimum Turning point at $(2,-4)$

(ii) Cuts x-axis when $y = 0$ — (Missing x-coordinate on x = 3)

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

∴ $x = 0$ or $x = 3$ — 1 mark for both x-intercepts, or else 1 mark deducted



1 mark for other features. (Did not plot x=3 on the graph −1)

(iii) $f''(x) = 6x - 6$

Curve is concave up when $f''(x) > 0$

$6x - 6 > 0$

$6x > 6$ — 1 mark for $x > 1$.

∴ $x > 1$

Additional writing space on back page.

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Q13(C)

(i) $y = 20x - 4x^2$	
$\frac{dy}{dx} = 20 - 8x$	
at $x = 2$, $\frac{dy}{dx} = 20 - 8 \times 2 = \underline{4}$	1 mark for gradient.
Equation of tangent given by	Didn't show how to get the equation of tangent
$y - y_1 = m \cdot (x - x_1)$	
$y - 24 = 4(x - 2)$	
$y - 24 = 4x - 8$	
$\therefore \underline{4x - y + 16 = 0}$	1 mark for equation of the tangent.
(ii) To find Q, Let $y = 0$ in equation of tangent	
$4x - 0 + 16 = 0$	Well done!
$x = -4$	
$\therefore$ Co-ordinates of Q are $\underline{(-4, 0)}$	1 mark for finding the coordinates.
(iii) Area of triangle $= \frac{1}{2} \times 6 \times 24 = 72 \text{ units}^2$	
Area under curve $= \int_0^2 (20x - 4x^2)\,dx$	Didn't separate the area of the region
$= \left[10x^2 - \frac{4}{3}x^3\right]_0^2$	
$= \left(40 - \frac{32}{3}\right) - 0$	
$= 29\frac{1}{3} \text{ units}^2$	1 mark for both triangle ~~and area under curve~~
$\therefore$ Area of region $POQ = 72 - 29\frac{1}{3}$	
$= 42\frac{2}{3} \text{ units}^2$	1 mark for area of region.
— END —	

You may ask for an extra Writing Booklet if you need more space.

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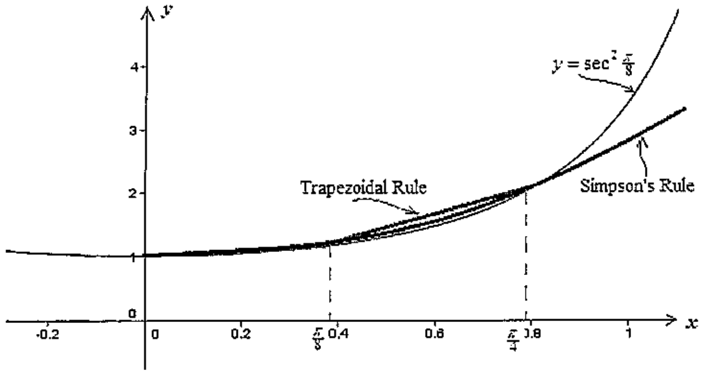
Question 14

14(a) (i)	No: of chairs in the first row = 13÷0.65 = 20 chairs $a = 20$, $d = 2$ $\therefore T_{25} = a + 24d$ $= 20 + 24 \times 2 = 68$ chairs in the 25th row.	1 mark: correctly calculates *a*, the no. of chairs in the first row. 1 mark: uses their value of *a* and d = 2 and correctly evaluates using the correct formula	*Those students who made a series of lengths made it too difficult for themselves.* $R_1 = 13m$ $R_2 = 13 + 1.3m$ $R_3 = 13 + 2 \times 1.3\,m$ ... *It was common for students to make 2 as first term a.*
ii)	Let $F_1 = 69$ $d = 1$ $F_n = 69 + (n-1) \times 1$ $84 = 69 + (n-1)$ $\therefore n = 84 - 69 + 1 = 16$ $\therefore$ total number of rows = 25 + 16 = 41	1 mark: correct answer	*Poorly done. Many students took a = 68 which was already considered in the first series.* *Also, many students failed to realise that there are two different series in operation here.*
iii)	no. of people in the first 25 rows = $\frac{n}{2}(a+l) = \frac{25}{2}(20+68)$ $= 1100$ Total no. of people $= 1100 + \frac{16}{2}(69+84)$ $= 1100 + 1224$ =2324 people in the auditorium	1 mark: correctly evaluates the no. of people in the first 25 rows 1 mark: : correctly evaluates the no. of people in the next 12 rows and finds the total.	*Again, if you started with 68 in the second series, it would mean that you are using row 25 twice.* *Most students got the S_{25} correctly, thus scoring 1 out of 2 marks.*
b)(i)	$N = Ae^{kt}$ t = 0, N = 10000 $10000 = Ae^0$ $\therefore \underline{A = 10000}$ t = 4, N = 40000 $40000 = 10000e^{4k}$ $e^{4k} = 4$ $4k = \ln(4)$ $\therefore \underline{k = \frac{1}{4}\ln(4)}$	1 mark: substitutes t = 0 and correctly evaluates A 1 mark: correctly substitutes and gets the equation $e^{4k} = 4$ 1 mark: solves the equation to evaluate k	*It is a 3 mark question and there are three concepts here. Hence, you needed to set t = 0 and show the evaluation process of A.* *Most students scored 2 out of 3 marks. Generally, well done!*

b)(i)	$N=10000e^{\frac{1}{4}\ln(4)t}$ $640000=10000e^{\frac{1}{4}\ln(4)t}$ $64=e^{\frac{1}{4}\ln(4)t}$ $\frac{1}{4}\ln(4)t=\ln(64)$ $t=\dfrac{\ln(64)}{\frac{1}{4}\ln(4)}$ = 12 years	1 mark: correctly substitutes N = 640000 into the equation and simplifies to $64=e^{\frac{1}{4}\ln(4)t}$ 1 mark: correctly solves the equation to evaluate t.	*Well done.*
c)(i)	x: 0, $\frac{\pi}{8}$, $\frac{\pi}{4}$ y: 1, $\sec^2\frac{\pi}{8}=1.17...$, 2 Trapezoidal rule ≈ $\frac{\frac{\pi}{8}}{2}\left[1+2\times\sec^2\frac{\pi}{8}+2\right]$ = 1.04912.... = 1.049 (3 d.p.)	1 mark: correctly evaluates the points 1 mark: correctly substitutes to Trapezoidal rule and evaluates correctly	*The part(c) was not well-received by the students. Lack of understanding basic concepts as well as inability to learn formulae came into surface through this question. The main shortcomings were:* *# inability to calculate h. many students thought that halfway between 0 and $\frac{\pi}{4}$ is $\frac{\pi}{2}$ rather than $\frac{\pi}{8}$.* *# lack of practise in evaluating $\sec^2\frac{\pi}{8}$ using radian measure* *# ignorance of the formula itself* *# not reading the question properly. A large number of students used five function values, even though the question asked for three.*
(ii)	Using Simpson's rule $\int_0^{\frac{\pi}{4}}\sec^2 x\,dx$ ≈ $\frac{\frac{\pi}{8}}{3}\left[1+4\times\sec^2\frac{\pi}{8}+2\right]$ = 1.00613... = 1.006 (3 d.p.)	1 mark: correctly substitutes to Simpson's rule and evaluates correctly	*For some students "h" changed from Q(i) to Q(ii)* *Also, some students did not remember the formula.*



Exact value = $\int_0^{\frac{\pi}{4}}\sec^2 x\,dx = [\tan x]_0^{\frac{\pi}{4}} = 1-0=1$

Simpson rule assumes a parabola through the three points and hence Simpson's rule gives the area under the parabola.

As shown in the diagram,

Area under the curve $y=\sec^2 x$ < area under the parabola < area under the trapezium.

1 < 1.006 < 1.049

∴ Simpson'rule gives a better approximation to the exact value of $\int_0^{\frac{\pi}{4}}\sec^2 x\,dx$.

(iii)	Evaluates the exact value of the integral and states that Simpson's rule gives the area under the parabola. Compares the three areas and justifies the result	2 marks	*Appalling presentation of deeper understanding of concepts in this case!* *# Simson's rule puts thinner rectangles, whereas Trapezoidal rule puts thicker rectangles under the curve.* *# Trapezoidal rule divides the area in to two rectangles, whereas Simpson rule puts three.*
	Evaluates the exact value of the integral ***or*** states that Simpson's rule gives the area under the parabola and Trapezoidal rule rule gives the area under the trapeziums ***or*** Compares the three areas and justifies the result	Award 1 mark	*Students must realise that this was an "Explain why" question. This means that there are two parts to this question:* *1...Calculate the exact value using integration and compare it with the results from (i) and (ii). This is to establish that indeed, the statement is correct in this case.* *2...Explain, why this could be happening and you can use the graph to establish this.*

Question 15

(a) (i) $x = \int 4\sin 2t \, dt$

$= -2\cos 2t + C$

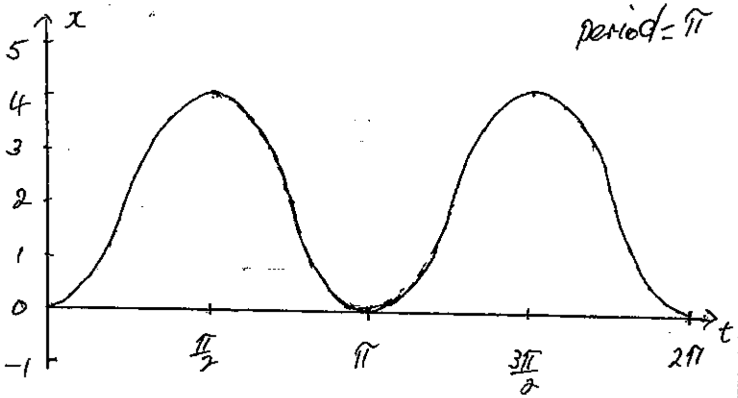
$0 = -2 + C$

$C = 2$

$\therefore x = -2\cos 2t + 2$

i
~~1 for $x = 2\cos 2t$~~
1 for answer

(ii) period $= \pi$



ii
1 for shape, period correct
1 for axes correct and neatness.

(iii) particle is at rest when $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.

iii
all answers 2
1 wrong or not there 1 mark.
2 wrong or not there 0 marks
(depends on graph)

(iv) $v = 4\sin 2t$

$a = 8\cos 2t$

when $t = 1$

$a = 8 \times \cos 2$

$= -3.3 \text{ ms}^{-2}$

iv
1 for $a = 8\cos 2t$
1 for answer

(b) (i) $y = xe^{2x}$

$y' = 2xe^{2x} + e^{2x}$

$M_T = 2e^2 + e^2$ (at $x = 1$)

$= 3e^2$

1 for derivative
1 for arriving at answer. Must show $M_T = 2e^2 + e^2$ after substitution

Question 15

(b) (ii) when $x = 1$, $y = e^2$

Equ^n of tangent

$y - e^2 = 3e^2(x - 1)$

$y - e^2 = 3e^2 x - 3e^2$

$3e^2 x - y - 2e^2 = 0$

1 Must show $y - e^2 = 3e^2(x-1)$ or equivalent
1 for answer

(c) (i) $V = \int_1^2 \pi x^2 \, dy$

$y = \frac{1}{1-x}$

$1 - x = \frac{1}{y}$

$x = 1 - \frac{1}{y}$

$= \pi \int_1^2 \left(1 - \frac{1}{y}\right)^2 dy$

$= \pi \int_1^2 \left(1 - \frac{2}{y} + \frac{1}{y^2}\right) dy$

1 for showing $x = 1 - \frac{1}{y}$.
1 for expanding $\left(1 - \frac{1}{y}\right)^2$, writing the volume in required format

(ii) $V = \pi \int_1^2 \left[1 - 2y^{-1} + y^{-2}\right] dy$

$= \pi \left[y - 2\ln y + \frac{y^{-1}}{-1}\right]_1^2$

$= \pi \left[2 - 2\ln 2 - \frac{1}{2} - (1 - 2\ln 1 - 1)\right]$

$= \left[\frac{3}{2} - 2\ln 2\right]\pi$ units^3

1 for integrating showing $\pi\left[y - 2\ln y - \frac{1}{y}\right]_1^2$
1 for answer (exact)

Question 16

(a) $4\sin^2 x + 12\sin x\cos x - \cos^2 x + 5 = 0$

$9\sin^2 x + 12\sin x\cos x + 4\cos^2 x = 0$

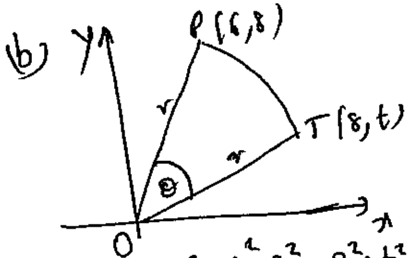
$(3\sin x + 2\cos x)^2 = 0$

$\tan x = -\frac{2}{3}$ $(\cos x \neq 0)$

$x = 146^\circ 19'$ or $x = 326^\circ 19'$

* Answer to the nearest minute
* correct answer

(b)



(i) $r^2 = 6^2 + 8^2 = 8^2 + t^2$

$t^2 = 36$

$t = \pm 6$

Since T is in the 1st quadrant

$t = 6$ and $t \neq -6$

ignore if $t \neq -6$ is not explained.

(ii) Let $\angle POT = \theta$

$\cos\theta = \frac{r^2 + r^2 - PT^2}{2r \cdot r}$

$= \frac{2 \times 10^2 - (2^2 + 2^2)}{2 \times 10 \times 10}$

$= \frac{24}{25}$

$\theta \approx 0.28$ radian

or $\tan\theta = \frac{8/6 - 6/8}{1 + 8/6 \cdot 6/8}$

$= \frac{7}{24}$

$\theta = 0.28$ radian

or any other correct method.

16 (c)

(i) Selina

Interest at end of 1st year = 200

" " " 2nd " = 2 × 200

⋮

" " " nth year = 200n

∴ Total savings at end of nth year

$= \$1000 + 200n$

(ii) Hebe

Interest at the end of 6th year = 100

" " " 7th year = 100 × 1.5

⋮

" " " nth year $= 100 \times 1.5^{n-6}$

∴ Total savings at end of nth year

$= 1000 + 100 + 100 \times 1.5 + \dots + 100 \times 1.5^{n-6}$

$= 1000 + 100[1 + 1.5 + \dots + 1.5^{n-6}]$

$= 1000 + 100\left[\frac{1.5^{n-5} - 1}{1.5 - 1}\right]$

$= 1000 + 200[1.5^{n-5} - 1]$

* need to form the correct geometric ser
* need to be clear about $(n-5)$

(iii) At end of kth year,

Hebe's savings > Selina's saving

$\Rightarrow 1000 + 200(1.5^{k-5} - 1) > 1000 + 200k$

$\Rightarrow 1.5^{n-5} > 1 + k$

$\Rightarrow k > 11$ (by trial and error)

∴ Hebe's savings exceeds Selina's in the 12th year.

ie $k = 12$

(iv) Interest earned in the 12th year

$= 200[1.5^7 - 1]$

$= \$3217.19$

16 (d) Volume $= \pi r^2 h = 500$

$\therefore h = \frac{500}{r^2}$

Cost of metal on the curved surface

$C_1 = \frac{2\pi r h}{100} \times 5$ cents

$= \frac{2\pi r}{100} \times \frac{500}{r^2} \times 5$ ✓

$= \frac{50}{r}$

Cost of metal on top and bottom $= 2\pi r^2 \times \frac{10}{100}$ ✓

$C_2 = \frac{\pi r^2}{5}$

$\therefore$ Total Cost $C = C_1 + C_2$

$= \frac{50}{r} + \frac{\pi r^2}{5}$

$\frac{dC}{dr} = -\frac{50}{r^2} + \frac{2\pi r}{5}$

$\frac{dC}{dr} = 0 \Rightarrow r = \frac{5}{\sqrt[3]{\pi}}$

$\frac{d^2C}{dr^2} = \frac{100}{r^3} + \frac{2\pi}{5}$

$\therefore$ when $r = \frac{5}{\sqrt[3]{\pi}}$, $\frac{d^2C}{dr^2} > 0$ ✓

$\therefore C$ is minimum at $r = \frac{5}{\sqrt[3]{\pi}}$ ✓

when $r = \frac{5}{\sqrt[3]{\pi}}$, $h = \frac{500}{\left(\frac{5}{\sqrt[3]{\pi}}\right)^2} = \frac{20}{\sqrt[3]{\pi}}$ ✓

$\therefore$ For minimum cost

$r = \frac{5}{\sqrt[3]{\pi}}$ cm and $h = \frac{20}{\sqrt[3]{\pi}}$ cm

* correct working must be clearly shown

* need to prove that the cost is minimum