Killara HIGH SCHOOL

Student Number

2018

Mathematics

HSC Course Assessment Task 4

HSC Trial Assessment Block

General Instructions

- Reading time – 5 minutes
- Working time **3** hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- In Questions 11-16 show relevant mathematical reasoning and/or calculations
- Answer each question in a separate writing booklet
- This paper must not be removed from the examination room
- A reference sheet is provided separately
- Diagrams are NOT to scale

Total Marks – 100

Section I Pages 2 - 5

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 6 - 14

90 marks

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

Multiple Choice	**/10**
Question 11	**/15**
Question 12	**/15**
Question 13	**/15**
Question 14	**/15**
Question 15	**/15**
Question 16	**/15**

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 40% of the HSC Course Assessment

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10 (Detach from paper)

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1 Evaluate $\sqrt[3]{\dfrac{651}{4\pi}}$ correct to four significant figures.

(A) 3.7278
(B) 3.727
(C) 3.728
(D) 3.730

2 The volume of Saturn is $10^{10}\,km^3$. Expressed in scientific notation, this is:

(A) $10\times10^{10}\,km^3$
(B) $1\times10^{10}\,km^3$
(C) $10\times10^{9}\,km^3$
(D) $1\times10^{30}\,km^3$

3 The derivative of $\dfrac{x}{\sin x}$ is:

(A) $\dfrac{\sin x + x\cos x}{\sin^2 x}$

(B) $\dfrac{\sin x - x\cos x}{\sin^2 x}$

(C) $\dfrac{x\sin x - \cos x}{\sin^2 x}$

(D) $\dfrac{x\sin x + \cos x}{\sin^2 x}$

4 The gradient of the line $5x+3y-7=0$ is:

(A) $\frac{3}{5}$

(B) $-\frac{3}{5}$

(C) $\frac{5}{3}$

(D) $-\frac{5}{3}$

5 $\int_0^1 \frac{dx}{x+2}$ is:

(A) $\frac{1}{6}$

(B) ln 6

(C) ln 1.5

(D) ln 3

6 A pack of 52 cards consists of 4 suits with 13 cards in each suit. One card is selected at random from the pack and placed on a table. A second card is then selected and placed next to the first card.

The probability that the second card is from a different suit to the first card is:

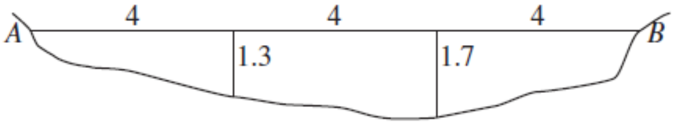
(A) $\frac{3}{4}$

(B) $\frac{12}{51}$

(C) $\frac{39}{52}$

(D) $\frac{39}{51}$

7 The diagram shows the cross-section of a gully. The depths of the gully are shown at 4 metre intervals in metres.



Use the trapezoidal rule to find an approximate value for the area of the cross-section of the gully.

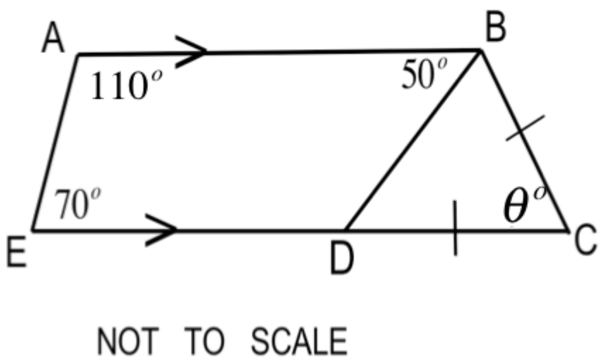
(A) $3\ m^2$

(B) $6\ m^2$

(C) $9.2\ m^2$

(D) $12\ m^2$

8 Calculate the value of angle θ in the diagram below.



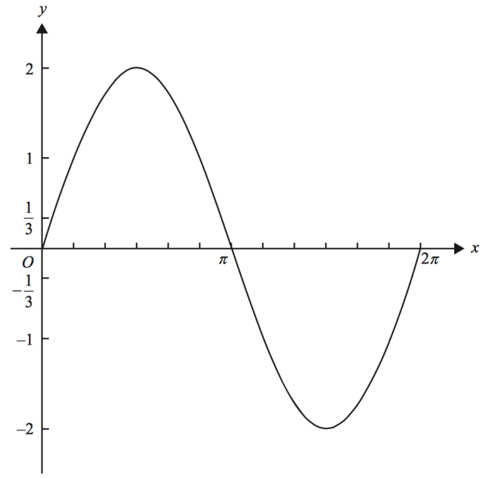
(A) 50^o

(B) 80^o

(C) 110^o

(D) 70^o

9 The graph $y = 2\sin x$ is shown in the graph below. Sketch the graph $y = \cos 2x$ on the same graph.



The number of solutions to the equation $2\sin x = \cos 2x$ is:

(A) 0
(B) 1
(C) 2
(D) 3

10 If $\int_0^5 f(x)\,dx = 20$ and $\int_0^5 \left[2f(x) + ax\right]dx = 90$, the value of a is:

(A) 0
(B) 4
(C) 2
(D) -3

Section II
90 marks
Attempt Questions 11-16
Allow about 2 hours 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Simplify $\dfrac{x^2-1}{x+1} \div \dfrac{x^2-2x+1}{2x+2}$ **3**

(b) Write $\dfrac{1}{2+\sqrt{5}}$ in the form $a+b\sqrt{5}$ and hence find the values of *a* and *b*. **3**

(c) Differentiate $\dfrac{2-x}{x-3}$, giving your answer in simplest form. **3**

(d) Differentiate $\cos(x^2)$ **2**

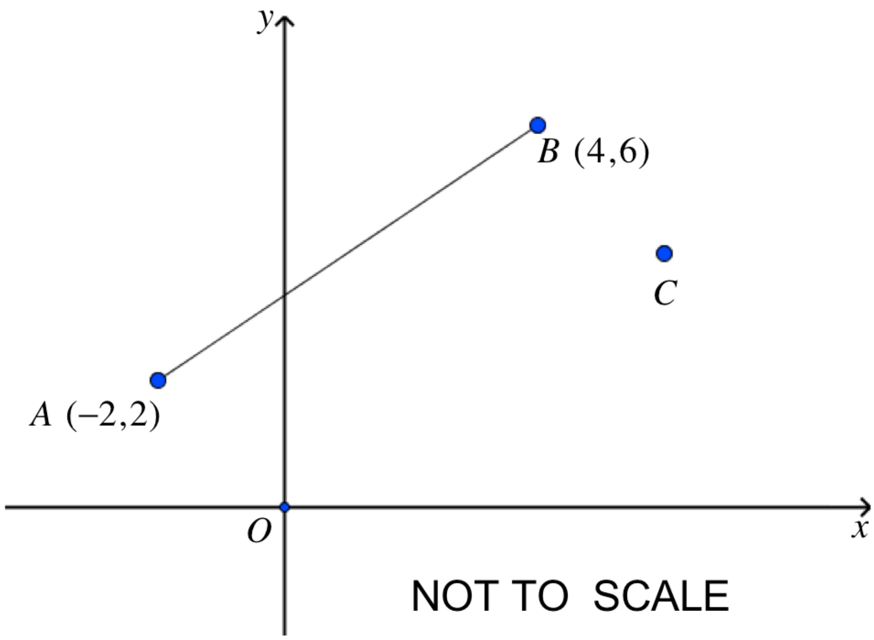
(e) Simplify $\dfrac{3}{x+5} - 2 + \dfrac{3}{x-5}$ **2**

(f) Find $\displaystyle\int \frac{4x}{x^2-25}\,dx$ **2**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Solve $\sin\theta = \dfrac{1}{\sqrt{2}}$ for $0 \le \theta \le 2\pi$ **2**

(b)



The diagram shows the points $A(-2,2)$, $B(4,6)$, $O(0,0)$ and C for the parallelogram $OABC$. The equation of line AB is $2x-3y+10=0$. Do **NOT** prove this

(i) Show that the length of AB is $2\sqrt{13}$. **1**

(ii) Calculate the perpendicular distance from O to the line AB. **2**

(iii) Calculate the area of the parallelogram $OABC$. **1**

(c) Differentiate $x^3 \log_e x$ with respect to x **2**

(d) Let α and β be the solutions of the quadratic function: $3x^2+2x-8=0$

Find $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ **2**

(e) (i) Find the coordinates of the focus, S, of the parabola $y = x^2 + 4$ **2**

(ii) The graphs of $y = x^2 + 4$ and the line $y = x + k$ have only one point of intersection, P. Show that the x-coordinate of P satisfies **1**

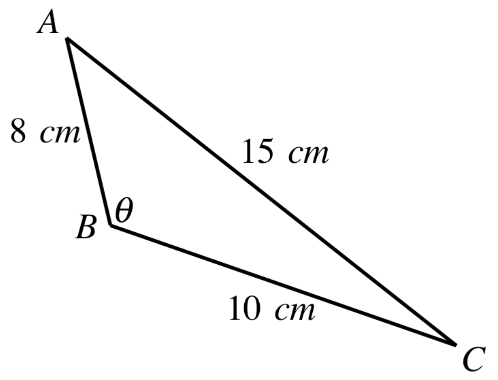
$$x^2 - x + 4 - k = 0$$

(iii) Using the discriminant, or otherwise, find the value of k. **1**

(iv) The coordinates of P are $\left(\frac{1}{2}, \frac{17}{4}\right)$. Show that SP is parallel to the directrix of the parabola. **1**

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Find the obtuse angle θ correct to the nearest minute **2**



(b) Consider the function $f(x)=\sqrt{2-x}$

(i) Find the domain of the function. **1**

(ii) Find the range of the function. **1**

(c) Solve the inequality $\sqrt{2-x}\le x$ and plot the solution on a number line. **3**

(d) Consider the function $g(x)=x^2(x^2-4)$

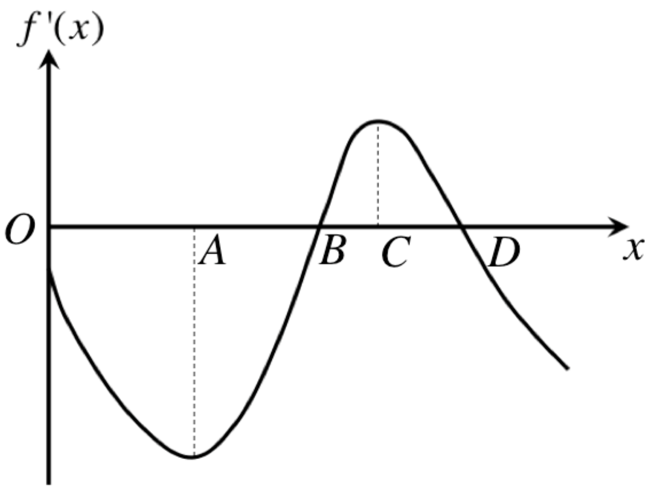
(i) Find all stationary points and determine their nature. **3**

(ii) Find the points of inflection. **2**

(iii) Graph the function $g(x)$ showing all important features. **3**

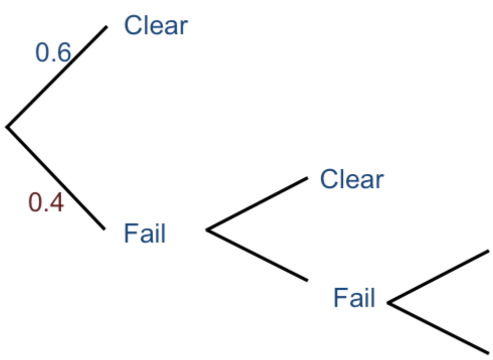
Question 14 (15 marks) Use a SEPARATE writing booklet

(a) The graph below shows the function $f'(x)$. Copy the diagram into your writing books and draw the primitive graph on the same axes. **2**



(b) Kurtis is competing in the pole vault at an athletics competition. He has three attempts to clear each height. When he clears the bar, he does not have a second attempt at that height.
When the bar is set at 5.20 metres, the probability that Kurtis will clear the bar on his first attempt is 0.6. If he fails to clear the bar on any attempt, the probability he will clear on his second attempt is 0.3.

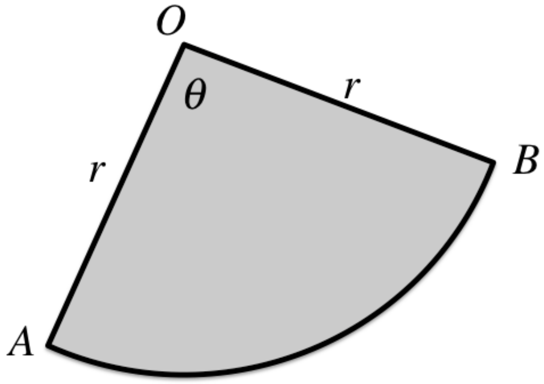
(i) Copy the tree diagram into your writing book. Complete the tree diagram to show the probability on each branch. **1**



(ii) What is the probability that Kurtis fails to clear the bar on his third attempt at 5.20 metres. **1**

(iii) What is the probability that he clears the bar on at least one attempt at this height **2**

(c) A rotating sprinkler rotates in a circular arc and covers the area shown. The arc AB subtends the angle θ (radians) at point O.



(i) If the perimeter of the sector is 12 metres, show that the radius, r, is given by **1**

$$r = \frac{12}{2+\theta}$$

(ii) Show that the area of the sector is given by **2**

$$A = \frac{72\theta}{(\theta+2)^2}$$

(iii) Hence determine the angle, θ, for which the area of the sector is a maximum. **3**

(d) By letting $u = x^{\frac{1}{3}}$, or otherwise, solve $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0$ **3**

Give your answer as an exact value.

Question 15 (15 marks) Use a SEPARATE writing booklet

(a) A small and mildly fluffy mammal lives along Badgery's Creek. Its population is declining due to habitat loss, and scientists estimated there are only 500 left in 2010. The population can be given by

$$P(t) = Ae^{kt}$$

where A and k are constants, and t is the number of years since scientists first estimated its population.

(i) Show that $P(t)$ satisfies $\dfrac{dP}{dt} = kP$ **1**

(ii) The scientists estimate the population again in 2018 and determine there are only 240 left. Show that the value of k is -0.0917, to 4 decimal places. **2**

(iii) The species is declared endangered if their population falls below 100. In how many years will the species become endangered if no recovery measures are taken? **2**

(b) (i) Use Simpson's Rule with 3 function values to find the area of the semi-circle given by the formula $y = \sqrt{9 - x^2}$ **2**

(ii) Calculate the exact value of $\int_{-3}^{3} \sqrt{9 - x^2}\,dx$ **1**

(c) A water tank has been installed at a house and, after a period of rain, is full. When the water tank is being drained the volume of water remaining in the tank, V, in litres, after time t seconds, is given by:

$$V = 5000\left(1 - \frac{t}{3600}\right)^2$$

(i) How much water is initially in the tank? **1**

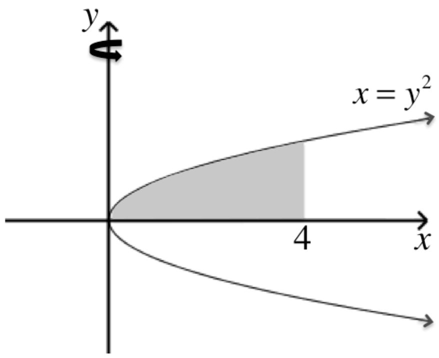
(ii) After how many seconds is the water tank half full? **2**

(iii) At what rate is the water draining out when the tank is half full? **2**

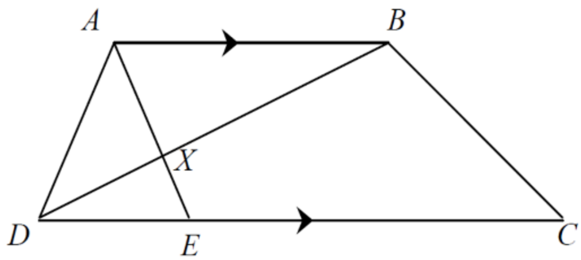
(d) Find the area contained by the curve $y = 2\sin x$ and the x axis for $0 \leq x \leq \pi$ **2**

Question 16 (15 marks) Use a SEPARATE writing booklet

a) Find the volume of the solid generated by rotating the shaded region about the y-axis. **3**



b)



ABCD is a trapezium where $AB \parallel DC$. Also *BD* is perpendicular to *BC* and *AE* is perpendicular to *BD* at *X*. Given that $DX = 9$cm and $AX = 40$cm.

i) What type of quadrilateral is *ABCE*? Give a reason. **2**

ii) Show that $\Delta DXE \ ||| \ \Delta DBC$. **1**

iii) Hence show that $BX \times XE = 360$ **3**

c) Emma borrows \$150 000 from the bank. The interest rate charged is 4.8% p.a. compounded monthly. She repays the loan in equal monthly instalments of \$1200 at the end of each month.

i) Calculate the amount of money Emma owes after the first repayment is made. **1**

ii) Let A_n be the amount Emma owes after the n^{th} payment is paid. Show that **2**

$$A_n = 300000 - 150000 \times 1.004^n$$

iii) The amount owing at the end of the 11th year is \$45937.04. **DO NOT PROVE THIS.** **3**

At the end of the 11^{th} year, Emma's interest rate is increased to 7.2%. How long will it take for her to repay her loan if the repayments remain the same.

Let B_n be the amount Emma owes after the n^{th} payment is made at the new rate.

Section I

10 marks
Attempt Questions 1-10
Allow about ** minutes for this section**
Use the multiple-choice answer sheet for questions 1-10 (Detach from paper)

---

1 Evaluate $\sqrt[3]{\frac{651}{4\pi}}$ correct to four significant figures.

(A) 3.7278

(B) 3.727

[redacted] 3.728

(D) 3.730

2 The volume of Saturn is $10^{10}km^3$. Expressed in scientific notation, this is:

(A) $10\times10^{10}km^3$

[redacted] $1\times10^{10}km^3$

(C) $10\times10^{9}km^3$

(D) $1\times10^{30}km^3$

3 The derivative of $\frac{x}{\sin x}$ is:

(A) $\frac{\sin x + x\cos x}{\sin^2 x}$

[redacted] $\frac{\sin x - x\cos x}{\sin^2 x}$

(C) $\frac{x\sin x - \cos x}{\sin^2 x}$

(D) $\frac{x\sin x + \cos x}{\sin^2 x}$

4 The gradient of the line $5x+3y-7=0$ is:

(A) $\frac{3}{5}$

(B) $-\frac{3}{5}$

(C) $\frac{5}{3}$

[redacted] $-\frac{5}{3}$

5 $\int_0^1 \frac{dx}{x+2}$ is:

(A) $\frac{1}{6}$

(B) $\ln 6$

(C) $\ln 1.5$

[redacted] $\ln 3$

6 A pack of 52 cards consists of 4 suits with 13 cards in each suit. One card is selected at random from the pack and placed on a table. A second card is then selected and placed next to the first card.
The probability that the second card is from a different suit to the first card is:

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(B) $\frac{12}{51}$

(C) $\frac{39}{52}$

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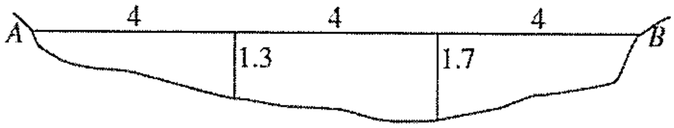
7 The diagram shows the cross-section of a gully. The depths of the gully are shown at 4 metre intervals in metres.



Use the trapezoidal rule to find an approximate value for the area of the cross-section of the gully.

(A) 3 m^2

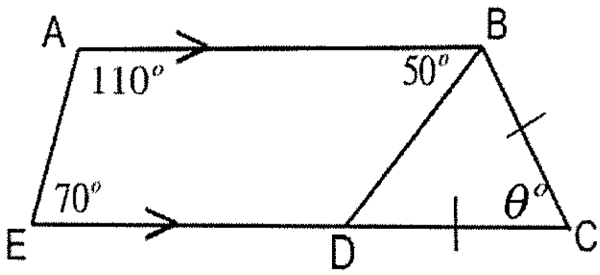
(B) 6 m^2

(C) 9.2 m^2

[redacted] 12 m^2

$$\frac{h}{2}[0+2\times1.3+2\times1.7+0]$$
$$=2[2.6+3.4]$$
$$=12m^2$$

8 Calculate the value of angle θ in the diagram below



NOT TO SCALE

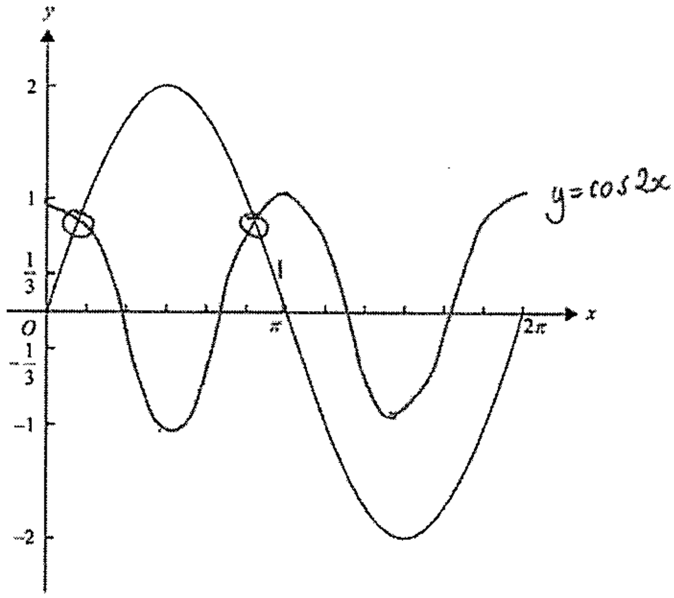
(A) 50°

[redacted] 80°

(C) 110°

(D) 70°

9 The graph $y = 2\sin x$ is shown in the graph below. Sketch the graph $y = \cos 2x$ on the same graph.



The number of solutions to the equation $2\sin x = \cos 2x$ is:

(A) 0

(B) 1

[redacted] 2

(D) 3

10 If $\int_0^5 f(x)dx = 20$ and $\int_0^5 [2f(x) + ax]dx = 90$, the value of a is:

(A) 0

[redacted] 4

(C) 2

(D) -3

$$2\int_0^5 f(x) + \int_0^5 ax\,dx = 90$$
$$2 \times 20 + \int_0^5 ax\,dx = 90$$
$$\int_0^5 ax\,dx = 50$$
$$\left[\frac{ax^2}{2}\right]_0^5 = 50$$
$$\frac{25a}{2} = 50$$
$$a = 4$$

Section II - Question 11 Common Errors.

Question 11(a)

Simplify $\dfrac{x^2-1}{x+1} \div \dfrac{x^2-2x+1}{2x+2}$

3 marks – Expression correctly factorised and simplified with all working shown.

2 marks – Expression correctly factorised but not simplified or 1 error. 1 mark was taken off if they expressed numerator as ***2X + 2***, but not if done after expressing it as ***2(X + 1)***

1 mark – Expression partly factorised but not simplified or 2 error.

$$\frac{x^2-1}{x+1} \div \frac{x^2-2x+1}{2x+2}$$

$$= \frac{(x-1)(x+1)}{(x+1)} \text{ X } \frac{2x+2}{(x-1)(x-1)}$$

$$\frac{\cancel{(x-1)(x+1)}}{\cancel{(x+1)}} X \frac{2x+2}{\cancel{(x-1)}(x-1)}$$

$$= \frac{\mathbf{2(x+1)}}{\mathbf{(x-1)}}$$

FEEDBACK

Must got this question correct.

Common mistakes:-

- Failed to realise that: $(X^2 - 1) = (X - 1)(X + 1)$
- Failed to realise that: $(X^2 - 2X + 1) = (X - 1)(X + 1)$
- Very small number did not cancel terms between numerator and denominator appropriately.

Question 11(b)

Write $\dfrac{1}{2+\sqrt{5}}$ in the form $a+b\sqrt{5}$ hence find the values for $\boldsymbol{a}$ and $\boldsymbol{b}$.

3 marks – Multiplied by conjugate, simplified and correctly defined *a* and *b*.

2 marks – Multiplied by conjugate, simplified correctly but failed to define *a* and *b*

or multiplied by conjugate, simplified and defined *a* and *b with 1 error.*

1 mark – Multiplied by conjugate, simplified but failed to define *a* and *b and 1 error*

or multiplied by conjugate, simplified and defined *a* and *b with 2 error.*

$$\frac{1}{2+\sqrt{5}} \text{ X } \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$=\frac{2-\sqrt{5}}{4-5}$$

$$=\frac{2-\sqrt{5}}{-1}$$

$$=-2+\sqrt{5}$$

$$\mathbf{a}=-\mathbf{2} \qquad \mathbf{b}=\mathbf{1}$$

FEEDBACK

Must got this question correct.

Common mistakes:-

- Failed to realise that the conjugate was required.
- Some mistakes with negative √5 by making: $\frac{2-\sqrt{5}}{-1}=-2-\sqrt{5}$
- Some had: ***(2 + √5) x (2 - √5) = 4 + 5 = 1***

Question 11(c)

Differentiate $\dfrac{2-x}{x-3}$ giving answer in simplest form.

3 marks – Correctly applied quotient rule and fully simplified.

2 marks – Quotient rule is correct but applies with single error or has correct answer but fails to fully simplify.

1 mark – Quotient rule is correct but applies with 2 errors and they fully simplify or quotient rule is correct but applies with 1 errors and fails to fully simplify.

$$y = \frac{2-x}{x-3}$$

$$u = 2 - x \qquad v = x - 3$$

$$u' = -1 \qquad v' = 1$$

$$\frac{dy}{dx} = \frac{(x-3)(-1) - (2-x)(1)}{(x-3)^2}$$

$$= \frac{-x+3-2+x}{(x-3)^2}$$

$$= \frac{1}{(x-3)^2}$$

FEEDBACK

Common mistakes:-

- Failed to realise that the quotient rules was required, one used the product rule.
- One person got the U and V back to front.
- Some mistakes with multiplying negatives, common one was making:

-1 x (2 – x) = -2 - x

Question 11(d)

Differentiate $\cos(x^2)$

2 marks – Correctly differentiates expression and expresses with ***-2x*** at the front of sin.

1 mark – Appears to correctly apply ***dy/dx = dy/du x du/dx*** but makes 1 error or expressed answer as ***$-sin(x^2)$. 2x***

$$y = \cos(x^2)$$

$$\frac{dy}{dx} = -\sin(x^2)\ .\ 2x$$

$$= -2x.\sin(x^2)$$

FEEDBACK

Must got this question correct.

Of those that got it wrong, which was very few, mistake was in:-

- Understanding of: ***dy/dx = dy/du x du/dx***
- Some stated that: ***$cos(x^2) = (cos\ x)^2$***
- Which followed onto them stating that: ***dy/dx = -2.sin(x)***
- Some had final answer as: ***-2x.sin(x)***

Question 11(e)

Simplify $\dfrac{3}{x+5} - 2 + \dfrac{3}{x-5}$

2 marks – Correct evaluation of fraction and answer simplified. It did need to be factorised. Any of the following was accepted:

$$= \frac{6x - 2x^2 + 50}{(x+5)(x-5)} = \frac{2(3x - x^2 + 25)}{(x+5)(x-5)} = \frac{-2(x^2 - 3x - 25)}{(x+5)(x-5)} = \frac{6x - 2x^2 + 50}{(x^2 - 25)}$$

1 mark – 1 error or failed to apply common denominator to -2.

$$\frac{3}{x+5} - 2 + \frac{3}{x-5}$$

$$= \frac{3(x-5)}{(x+5)(x-5)} - \frac{2(x+5)(x-5)}{(x+5)(x-5)} + \frac{3(x+5)}{(x-5)(x+5)}$$

$$= \frac{3x - 15 - 2x^2 + 50 + 3x + 15}{(x+5)(x-5)}$$

$$= \frac{6x - 2x^2 + 50}{(x+5)(x-5)}$$

FEEDBACK

Common mistakes:-

- Did not find the common denominator for: ***-2*** and would just leave it at the end.
- Quite a few minor errors in expanding and brackets and cancelling terms. This did not indicate any major problem with understanding, just processing errors.

Question 11(e)

Find $\displaystyle\int \frac{4}{x^2 - 25}\,dx$

2 marks – Correct evaluation with |absolute bracket| and + C.

1 mark – Failed to put in |absolute brackets| or +C or 1 error.

$$\int \frac{4}{x^2 - 25}\,dx = \int 2.\frac{2}{(x^2 - 25)}\,dx$$

$$= 2\int \frac{2}{x^2 - 25}\,dx$$

$$= 2 \ln|x^2 - 25| + C$$

FEEDBACK

Must got this question correct with exception to the ***absolute brackets***.

Of those that got it further wrong:-

- Some stated:

$$\int \frac{4}{x^2 - 25}\,dx = \frac{1}{2}\int \frac{4}{x^2 - 25}\,dx$$

- And then stated the answer to be: $\frac{1}{2}\ln(x^2 - 25) + C$
- Small number failed to add: ***+ C***
- Some differentiated using the quotient rule.

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Solve $\sin\theta = \frac{1}{\sqrt{2}}$ for $0 \le \theta \le 2\pi$ **2**

Solution:

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ where sine is positive in the first and second quadrants

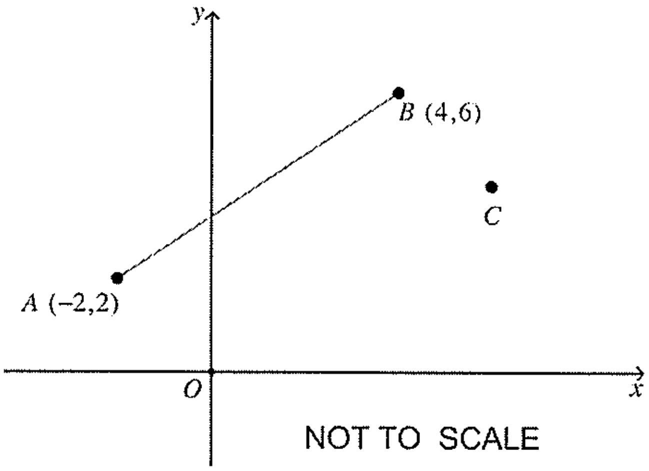
$= \frac{\pi}{4}$ or $\frac{3\pi}{4}$ ✓

$= 45^\circ$ or 135°

2 marks- correctly identifies both angles and shows radians 1 mark - correctly identifies one angle and/or shows degrees

Marker's comments:
Very well done overall. Candidates who did not disregard two of the solutions were given 1 mark for four angles.

(b)



The diagram shows the points $A(-2,2)$, $B(4,6)$ and $O(0,0)$. The point C is the fourth vertex of the parallelogram $OABC$. The equation of line AB is $2x - 3y + 10 = 0$

(i) Show that the length of AB is $2\sqrt{13}$. **1**

Solution:

$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-(-2))^2 + (6-2)^2}$ ✓

$= \sqrt{36+16} = \sqrt{52}$

$= 2\sqrt{13}$

1 mark - correctly shows substitution into distance formula and calculates value

Marker's comments:
Very well done overall.

(ii) The graphs of $y=x^2+4$ and the line $y=x+k$ have only one point of intersection, P. Show that the x-coordinate of P satisfies **1**

$$x^2-x+4-k=0$$

Solution:

$$x^2+4=x+k$$

$$x^2-x+4-k=0$$

1 mark - sets equations equal to each other

Marker's comments:
Very well done. Most candidates who set the equations equal to one another were able to rearrange to obtain the equation.

(iii) Using the discriminant, or otherwise, find the value of k. **1**

Solution:

$\Delta=b^2-4ac$ $\quad\quad$ $\Delta=0$ when one solution.

$$0=(-1)^2-4(1)(4-k)$$
$$=1-16+4k$$
$$15=4k$$
$$k=\frac{15}{4}$$

1 mark - correctly calculates k, must state Δ= 0 for one solution

Marker's comments:
Very well done. Candidates who did not recognise the need to use the discriminant to find one solution.

(iv) The coordinates of P are $\left(\frac{1}{2},\frac{17}{4}\right)$. Show that SP is parallel to the directrix of the parabola. **1**

Solution:

P is $\left(\frac{1}{2},4\frac{1}{4}\right)$ and S is $\left(0,4\frac{1}{4}\right)$

$\overline{SP}=y=4\frac{1}{4}$ with $m_{SP}=0$

Directrix is below the vertex, a focal length below

$y=\left(4-\frac{1}{4}\right)=3\frac{3}{4}$ with $m_{\text{directrix}}=0$

$m_{SP}=m_{\text{directrix}}$

1 mark - determines the gradient of both lines is 0

Marker's comments:
Very well done. Candidates who mis-calculated the focus in part (i) found this part difficult, however, the "*show that*" element should have indicated the error in (i).

(d) Let α and β be the solutions of the quadratic function:

$$3x^2 + 2x - 8 = 0$$

Find $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ **2**

Solution:

$\cancel{3x^2 + 2x - 8 = 0}$

$\cancel{(3x-4)(x+2)=0}$

$\cancel{x=-2 \text{ and } \frac{4}{3}}$

$\cancel{\frac{1}{\alpha}+\frac{1}{\beta}=-\frac{1}{2}+\frac{3}{4}=-\frac{2}{4}+\frac{3}{4}}$

$\cancel{=\frac{1}{4}}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta}{\alpha\beta}+\frac{\alpha}{\alpha\beta}$

$=\frac{\alpha+\beta}{\alpha\beta}$

$\alpha+\beta=\frac{-b}{a}=\frac{-2}{3}$

$\alpha\beta=\frac{c}{a}=\frac{-8}{3}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\dfrac{\frac{-2}{3}}{\frac{-8}{3}}$

$=\frac{-2}{\cancel{3}}\times\frac{\cancel{3}}{-8}$

$=\frac{1}{4}$

2 marks- correctly factorises to obtain roots and calculates fraction sum 1 mark - correctly calculates fraction sum given incorrect factorisation

Marker's comments:

Very well done. Few candidates could not rearrange the fractions to obtain a relationship between the sum and product of roots.

(e) (i) Find the coordinates of the focus, S of the parabola $y = x^2 + 4$ **2**

Solution:

$y = x^2 + 4$

$y = 4\left(\frac{1}{4}\right)(x-0)^2 + 4$ (vertex/focus form of parabola)

vertex is $(0,4)$

focal length is $a = \frac{1}{4}$

or

$4a = 1$

$a = \frac{1}{4}$

$\therefore$ the focus S is $\left(0, 4\frac{1}{4}\right)$

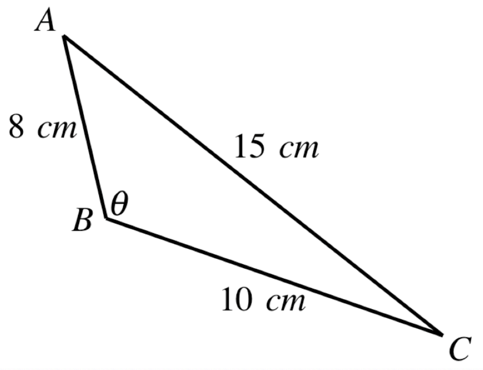
2 marks- correctly rearranges equation to vertex/focus form or otherwise finds the focus 1 mark - rearranges formula into vertex/focus form or finds vertex but not the focal length.

Marker's comments:

Very well done. Few candidates were unable to rearrange the parabola into vertex/focus form or identify the vertex using the axis of symmetry and focal length.

Question 13

(a) Find the obtuse angle θ rounded to the nearest minute **2**



Solution:

$$\cos\theta = \frac{8^2 + 10^2 - 15^2}{2(8)(10)}$$

$$\theta = \cos^{-1}\left(\frac{8^2 + 10^2 - 15^2}{2(8)(10)}\right) = 112.41° \sim 112°25'$$

2 marks - correctly substituted values into the cosine rule and showed correct rounding 1 mark - correctly substituted values into the cosine rule

Marker's comments
Students who substituted into the version of the formula above were more successful. Students who used the other version tended to make careless algebraic errors.

(b) Consider the function: $f(x) = \sqrt{2-x}$

(i) Find the domain of the function. **1**

(ii) Find the range of the function **1**

Solution:

(i) Domain:

$$2 - x \geq 0$$
$$x \leq 2$$

(ii) Range:

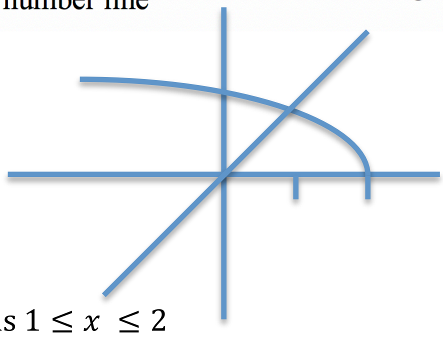
$$y \geq 0$$

1 mark - correct solution for the Domain 1 mark - correct solution for the Range

Marker's comments
Many students did this well. Some neglected to realise that both Domain and Range was > or = 0. Some students believed this was a semi-circle

(c) Solve the inequality $\sqrt{2-x} \leq x$ and plot the answer on a number line **3**

Solution:



$\sqrt{2-x} \leq x$

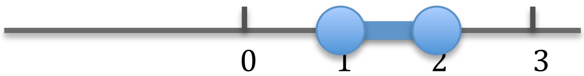
$$2 - x \leq x^2$$

$x^2 + x - 2 \geq 0$

$$(x+2)(x-1) \geq 0$$

$$x \leq -2 \ \ or \ \ x \geq 1$$

but $x \leq 2$ (from (b)) so the correct solution is $1 \leq x \ \leq 2$



3 marks – Correctly solved the inequality and displayed the solution on a number line
2 marks – Correctly solve the inequality (showing working) and graphed the solution
1 mark – Squared both sides of the inequality and correctly solved that inequality

Marker's comments

Very few students received 3 marks as most forgot to consider the restrictions on the domain. Students who drew the graph were above to easily see the solution. Many students neglected to show working for the solution to the inequality so could not receive marks.

(d) Consider the function: $x^2(x^2-4)$

(i) Find all stationary points and determine their nature. **3**

(ii) Find the points of inflection. **2**

(iii) Graph the function $f(x)$ showing all important features. **3**

Solution:

(i)

$$f(x) = x^4 - 4x^2$$
$$f'(x) = 4x^3 - 8x$$
$$f''(x) = 12x^2 - 8$$

When $f'(x) = 0$: $\quad 4x^3 - 8x = 0 \Longrightarrow 4x(x^2-2) = 0$

Therefore the Stationary points are located at:

$$x = 0, \sqrt{2}\,, -\sqrt{2}$$

$f(0) = 0$	$f''(0) = -8 < 0$	$(0,0)$ is a maximum
$f(\sqrt{2}) = -4$	$f''(\sqrt{2}) = 16 > 0$	$(\sqrt{2}, -4)$ is a minimum
$f(-\sqrt{2}) = -4$	$f''(-\sqrt{2}) = 16 > 0$	$(-\sqrt{2}, -4)$ is a minimum

3 marks – Correctly found the stationary points and determined the nature using $f''(x)$
2 marks – Correctly calculated $f'(x)$ and solved $f'(x) = 0$
1 mark – Correctly calculated $f'(x)$ and solved for one stationary point

Solution:
(ii)

$$f''(x) = 0$$
$$12x^2 - 8 = 0$$
$$x^2 = \frac{8}{12}$$
$$x = \pm\frac{\sqrt{2}}{\sqrt{3}}$$

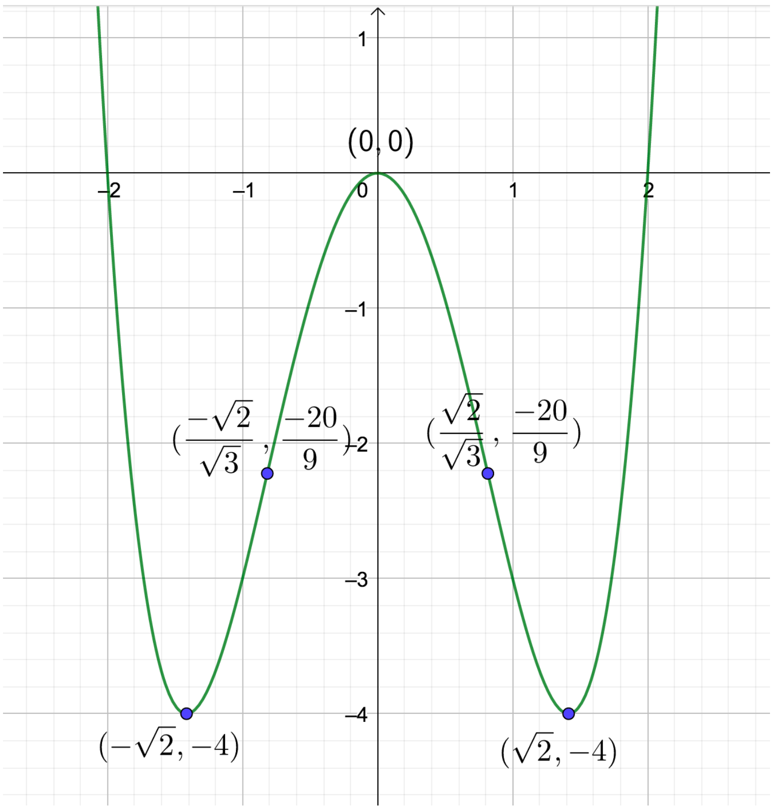
$$f\left(\frac{\sqrt{2}}{\sqrt{3}}\right) = -\frac{20}{9} \qquad f\left(\frac{-\sqrt{2}}{\sqrt{3}}\right) = -\frac{20}{9}$$

Inflection Points:

$$\left(\frac{\sqrt{2}}{\sqrt{3}}, -\frac{20}{9}\right) \quad and. \ (\frac{-\sqrt{2}}{\sqrt{3}}, -\frac{20}{9})$$

2 marks – Correctly solved $f''(x) = 0$ and found the corresponding y-value for the x-values and showed the change in concavity.
1 mark – Correctly solved $f''(x) = 0$ and found y values.

Solution:
(iii)



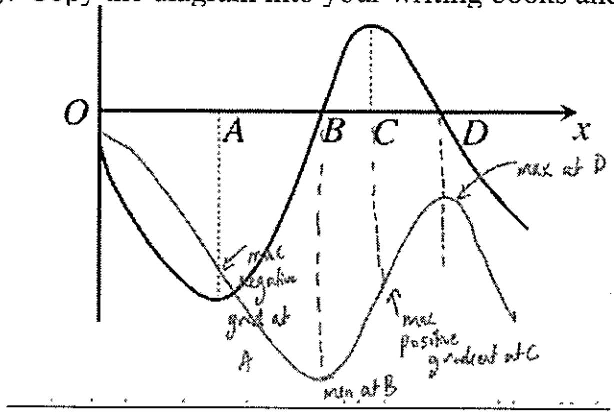
Marker's comments

3 marks – Correct shape with turning points and inflexion points labelled
2 marks – Correct shape with turning points labelled
1 mark – Correct shape of the graph

Students are reminded that they must show a change in concavity for inflexion points.
Students are reminded that when using the first derivative test for stationary points and the second derivative change of concavity test they must label the tabled and show the substitutions.
Many students lost marks for neglecting to label points on the curve and neglecting to mark off a consistent scale.

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) The graph below shows the function $f'(x)$. Copy the diagram into your writing books and draw the primitive graph on the same axes. **2**

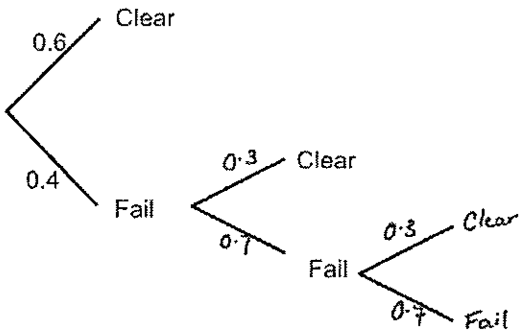


2 marks Correctly draws primitive with min at C, max at D, max negative slope at A, max negative slope at C 1 mark Correctly draws a minimum at C and a max at point D

(b) Kurtis is competing in the pole vault at an athletics competition. He has three attempts to clear each height. When he clears the bar, he does not have a second attempt at that height.
When the bar is set at 5.20 metres, the probability that Kurtis will clear the bar on his first attempt is 0.6. If he fails to clear the bar on any attempt, the probability he will clear on his second attempt is 0.3.

(i) Copy the tree diagram into your writing book. Complete the tree diagram to show the probability on each branch. **1**

1 mark completed tree diagram



(ii) What is the probability that Kurtis fails to clear the bar on his third attempt at 5.20 metres. **1**

1 mark Correct answer must have working

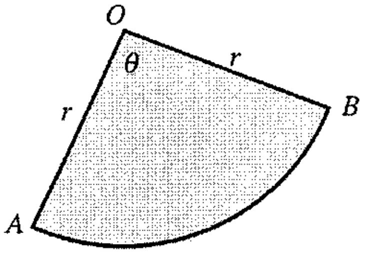
$$P(3rd) = 0.4 \times 0.7 \times 0.7$$
$$= 0.196$$

(iii) What is the probability that he clears the bar on at least one attempt at this height **2**

2 marks correct answer with working 1 mark Valid attempt at correct answer with working

$$P(\text{at least one}) = 1 - P(FFF)$$
$$= 1 - 0.4 \times 0.7 \times 0.7$$
$$= 0.804$$

(c) A rotating sprinkler rotates in a circular arc and covers the area shown. The arc AB subtends the angle θ (radians) at point O.



(i) If the perimeter of the sector is 12 metres, show that the radius, r is given by **1**

$$r = \frac{12}{2+\theta}$$

$$P = r + r + r\theta$$
$$12 = 2r + r\theta$$
$$12 = r(2+\theta)$$
$$r = \frac{12}{2+\theta}$$

(ii) Show that the area of the sector is given by **2**

$$A = \frac{72\theta}{(\theta+2)^2}$$

$$A = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times \frac{144\theta}{(2+\theta)^2}$$
$$= \frac{72\theta}{(2+\theta)^2}$$

(iii) Hence determine the angle, θ for which the area of the sector is a maximum **3**

3 marks	Correct tests A' to demonstrate A is a max
2 marks	simplified expression for A' and value for $\theta = 2$
1 mark	correct expression for derivative

$$A' = \frac{(\theta+2)^2 72 - 72\theta \times 2(\theta+2)}{(\theta+2)^4}$$
$$= \frac{(\theta+2)[72(2+\theta) - 144]}{(\theta+2)^4}$$
$$= \frac{72(\theta+2)(\theta-2)}{(\theta+2)^4}$$

a max for $A' = 0$ when $\theta = 2$ as $\theta \neq 2$

Show A' is a max. Test A'

x	1	2	3
A'	72	0	−72

(d) By letting $u = x^{\frac{1}{3}}$, or otherwise, solve $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0$ **3**

Give your answer as an exact value.

3 marks	2 correct value for pronumerals
2 marks	correct value of 1 pronumeral
1 mark	correct factorising of quadratic

$$(x^{\frac{1}{3}} + 3)((x^{\frac{1}{3}} - 2) = 0$$
$$x^{\frac{1}{3}} = -3, \quad x^{\frac{1}{3}} = 2$$
$$x = -27, \quad x = 8$$

Q14a) A number of students differentiated and not integrated. Students need to ensure they use the labels provided and keep their graphs neat.

Students needed to identify a min at point B and a max at point D for 1 mark

Point A should correspond to a max negative gradient for the primitive, point C should correspond to a max positive gradient for the primitive.

b) (i) Done well
(ii) Done well. Working must be shown.
(iii) Done well. Working must be shown.

c) (i) Done well. A show that requires full working
(ii) Done well. A show that requires full working
(iii) Students are advised to use the quotient rule and not use the product rule.
Many students failed to square the denominator resulting in $(\theta+2)^4$ *and lost a mark*
Some students failed to work through using the denominator and received no marks
Students who cancelled the $(\theta+2)$ *lost a mark unless they stated that* $\theta \neq 2$

$$A' = \frac{72\cancel{(\theta+2)}(\theta-2)}{(\theta+2)^{\cancel{4}}} \quad \text{as } \theta \neq 2$$

a max for $A' = 0$ when $\theta = 2$

d) *Mostly done well. A common error was*

$x^{\frac{1}{3}} = -3$ so no solution

A few students failed to substitute back and did not solve for x.

A number of students made the question harder by trying to solve a log equation with little success.

Question 15

(a) A small and mildly fluffy mammal lives along Badgery's Creek. Its population is declining due to habitat loss, and scientists estimated there are only 500 left in 2010. The population can be given by

$$P(t) = Ae^{kt}$$

Where A and k are constants, and t is the number of years since scientists first estimated its population.

(i) Show that $P(t)$ satisfies $\frac{dP}{dt} = kP$ **1**

1 mark for correct and full working

$$P(t) = Ae^{kt}$$

$$\frac{dP}{dt} = kAe^{kt}$$

$$= kP(t)$$

`(ii) The scientists estimate the population again in 2018 and determine there are only 240 left. Show that the value of k is -0.0917 to 4 decimal places. **2**

2 marks correct substitution with correct working and answer
1 mark correct substitution with error in working or correct use of ln with incorrect substitution

$A = 500$ for initial amount at $t = 0$

$$P(8) = 240 = 500e^{8k}$$

$$0.48 = e^{8k}$$

$$ln(0.48) = 8k$$

$$k = \frac{ln(0.489)}{8}$$

$$= -0.09174614689$$

(iii) The species is declared endangered if the number falls below 100. In how many years will the species come endangered if no recovery measures are taken? **2**

2 marks for correct solution with full working
1 mark for correct substitution or equivalent

$$100 = 500e^{kt}$$
$$\frac{1}{5} = e^{kt}$$
$$t = \frac{ln(\frac{1}{5})}{k}$$
$$t = 17.5$$

Therefore the species will become endangered in 17.5 years.

(b) (i) Use Simpson's Rule with 3 function values to find the area of the semi-circle given by the formula $y = \sqrt{9 - x^2}$ **2**

2 marks for correct substitution into Simpson's rule and correct solution
1 mark for either correct substitution or a correct table of values

x	-3	0	3
y	0	3	0

$$A = 1(0 + 4 \times 3 + 0)$$
$$= 12$$

therefore the area is approximately 12 units squared

(ii) Calculate the exact value of $\int_{-3}^{3} \sqrt{9 - x^2}\,dx$ **1**

1 mark for finding the value of the integral with sufficient reasoning or working

The integral represents a semi circle of radius 3, so it follows:

$$\int_{-3}^{3} \sqrt{9 - x^2} dx = \frac{\pi \times 3^2}{2}$$
$$= \frac{9\pi}{2}$$

(c) A water tank has been installed at a house and after a period of rain, is full. When the water tank is being drained the volume of water remaining in the tank V, in litres, after time t seconds, is given by

$$V = 5000\left(1 - \frac{t}{3600}\right)^2$$

(i) How much water is initially in the tank? **1**

1 mark for correct answer from correct working

$$V = 5000\left(1 - \frac{t}{3600}\right)^2$$

initially t = 0

$$V = 5000(1-0)^2$$
$$= 5000L$$

(ii) After how many seconds is the water tank half full? **2**

2 marks for full working and correct solution 1 mark for working correct to taking the square root to get 2 answers

$$V = 5000\left(1 - \frac{t}{3600}\right)^2$$

$$\frac{1}{2} = \left(1 - \frac{t}{3600}\right)^2$$

$$\pm\frac{1}{\sqrt{2}} = 1 - \frac{t}{3600}$$

$$\frac{t}{3600} = 1 \pm \frac{1}{\sqrt{2}}$$

$$t = 3600\left(1 \pm \frac{1}{\sqrt{2}}\right)$$

$$= 1054.415588 or 6145.584412$$

therefore the tank is half full after 1054 seconds

(iii) At what rate is the water draining out when the tank is half full? **2**

2 marks for correct substitution and correct drain rate from previous answer
1 mark for correct substitution from previous answer or equivalent

$$\frac{dV}{dt} = \frac{-10000}{3600}\left(1 - \frac{t}{3600}\right)$$

$$= -\frac{1273}{648}$$

$$= -1.9645 \ldots Ls^{-1}$$

(d) Find the area contained by the curve $y = 2\sin x$, the x axis for $0 \le x \le \pi$ **2**

2 marks for correct answer with full working
1 mark for correct integration of sin to -cos

$$A = \int_0^{\pi} 2\sin x dx$$

$$= [-2\cos x]_0^{\pi}$$

$$= -2\cos\pi + 2\cos 0$$

$$= -2(-1) + 2(1)$$

$$= 2 + 2$$

$$= 4$$

hence 4 units squared

Question 15
(a)
(i) Was done well

(ii) largely done well, where mistakes occurred they were due to problems manipulating logs.

(iii) Some errors crept into this question, with students out by a factor of 10. Familiarity with logs again was a weakness for some students.

(b)
(i) Where students knew the rule and constructed a table, full marks were common. Where students attempted to use function notation and calculations within the formula, mistakes were more common place.

(ii) Most students failed to identify the area as a half circle, and therefore missed the area of a circle formula, by far the quickest and easiest method of solving this question.
Those who tried to integrate most often lost the mark.
A large number of students seemed happy with a solution of 0, which is clearly wrong when considered graphically.

(c)
(i) Students needed some working, flat answer was insufficient

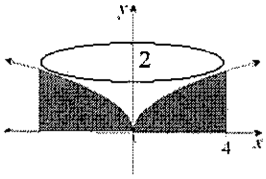
(ii) Most students forgot or ignored the plus/minus, and therefore missed one solution. It needed to be found and discarded.

(iii) Most students were able to get full marks in this part, though some were let down by their differentiation.

(d) Most student were able to get full marks for this section, assuming they knew the integral of sin.

2018-2Units Trial Q16 Answer

a) (a)



$$V_{shaded\ of\ area} = V_{cylinder} - \pi \int_0^2 y^4\, dy \qquad \textbf{(1 mark)}$$

$$= \pi \times 4^2 \times 2 - \pi \int_0^2 y^4\, dy \qquad \textbf{(1 mark – correct substitution)}$$

$$= 32\pi - \pi \left[\frac{y^5}{5}\right]_0^2 = \frac{128\pi}{5} \text{ units cube.} \qquad \textbf{(1 mark – correct answer)}$$

b)

i) $150000 \times 1.004 - 1200 = \$149400$ **(1 mark - correct value)**

ii) $B_2 = 149400 \times 1.004 - 1200$

$B_3 = (149400 \times 1.004 - 1200 \times 1.004 - 1200$
$= 149400 \times 1.004^2 - 1200 \times 1.004 - 1200$

(1 mark - correct expressions of B_2 & B_3)

$B_n = 15000 \times 1.004^n - 1200 \times 1.004^{n-1} - 1200 \times 1.004^{n-2} \ldots\ldots -1200$
$= 15000 \times 1.004^n - \frac{1200(1.004^n - 1)}{.004}$
$= 15000 \times 1.004^n - 300000 \times 1.004^n + 300000$
$= 300000 - 150000 \times 1.004^n$ **(1 mark - correct answer)**

iii) Using the formula $15000 \times 1.004^n - \frac{1200(1.004^n-1)}{.004}$ from part (ii)

$A_n = 45937.04 \times 1.006^n - \frac{1200(1.006^n - 1)}{.006} = 0$ **(1 mark – correct sub. into formula)**

$\rightarrow 1.006^n(45937.04 - 200000) = -200000$

$1.006^n = \frac{-200000}{45937.04 - 200000}$

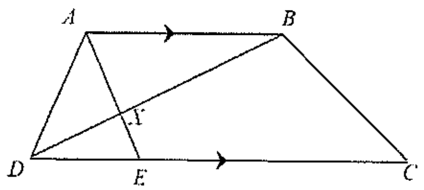
$n = \frac{\ln\left(\frac{200000}{200000 - 45937.04}\right)}{ln 1.006} = 43.62$ months **(1mark-correct answer)**

$43.62 + 11 \times 12 = 175.62$ months

$\therefore$ The loan takes 175.62 months or 14.64 years to pay off the loan

(1mark – final correct answer)

c)



NOT TO SCALE

i) $<CBX = <BXA = 90^o$ (BD is perpendicular to BC and AE is perpendicular to BD)

$\therefore$ $AE // BC$ (alternate angles are equal)

Also $AB // EC$ (opposite sides of trapezium)

(1 mark – 2 pair of opposite sides parallel)

Hence $ABCE$ is a parallelogram (opposite sides are parallel)

(1 mark – correct answer with reason)

ii) In ΔDXE and ΔDBC **(1 marks – 3 steps with reasons)**

$< XDE = < BDC$ (common)

$< CBX = < BXE = 90^o$ (As explain above)

$\therefore \Delta DXE /// \Delta DBC$ (equiangular)

iii) Hence corresponding sides are in same ratio

$\therefore \quad \frac{XE}{BC} = \frac{9}{9+XB}$ **(1 mark – correct ratio with reason)**

$XE(9 + XB) = 9BC$ (1)

But $BC = XE + 40$ (opposite sides of parallelogram)

(1 mark – length of BC)

Sub. into (1)

$\rightarrow 9XE + XE.XB = 9(XE + 40)$ **(1 mark – correct answer)**

$\therefore \quad XE.XB = 360$

Q16

(a) Poorly done
Most student find the answer by calculating the volume bounded by the y-axis between 0 to 2, or between 0 to 4.

b)

(i) well done. A few did not evaluate to get the answer.

(ii) Students must show both B_2 & B_3 to obtain 1 mark. Few students showed only B_2

This is the SHOW question, however, some students wrote down the formula without giving the extended expression for A_n. They also did not show the simplification process to get to the final answer.

(iii) Most students repeated the lengthy process in part (ii) to come to the final formula, instead of replacing the corresponding values in the formula would save them time. They also forgot to add on 11 years for final answer.

c)

(i) Poorly done. Students did not remember the test for parallelogram.

(ii) Well done

(iii) Poorly done. Students forgot to write down the reason for each step.
Alternately, by proving the Δ DXE /// Δ AXB the ratio can be achieved easily.