Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2019 | Mathematics

Trial Examination

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks:

Section I – 10 marks

- Allow about 15 minutes for this section

Section II – 90 marks

- Allow about 2 hours and 45 minutes for this section

Section I (10 marks)	Multiple Choice	/10
Section II (90 marks)	Question 11	/15
	Question 12	/15
	Question 13	/15
	Question 14	/15
	Question 15	/15
	Question 16	/15
	Total	**/100**

This question paper must not be removed from the examination room.
This assessment task constitutes 30% of the course.

Section I

10 marks
Allow about 15 minutes for this section

Use the multiple-choice sheet for Question 1–10

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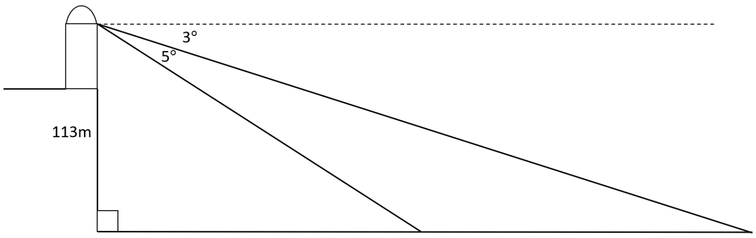
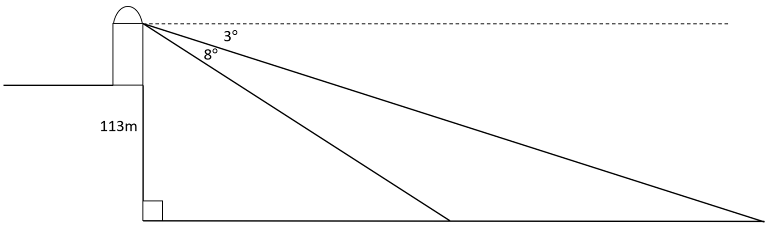
1 George Worthylake is watching from his lighthouse 113m above sea-level. He sees a boat heading straight towards him. He initially calculates the angle of depression to be 3°, a short while later he measures the angle of depression to be 8°. Which diagram most faithfully represents this information?

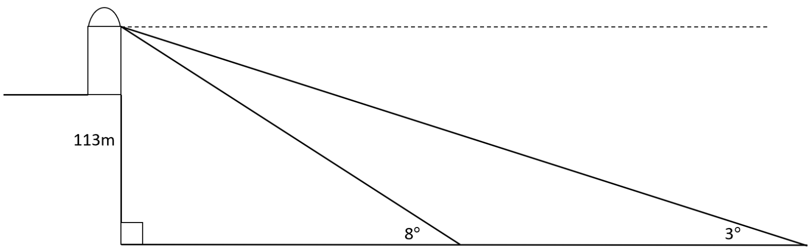
(A)



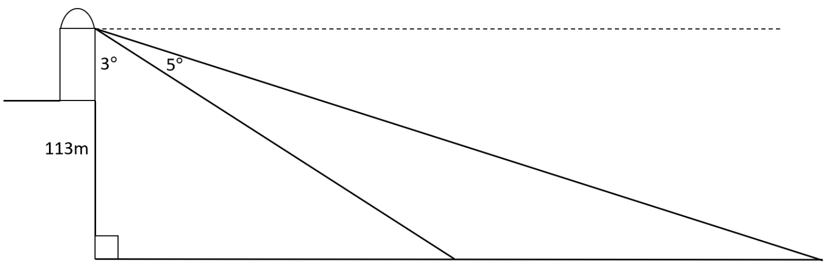
(B)



(C)



(D)



2 In a raffle 30 tickets are sold, how many tickets would you need to have purchased to have at least a 6% chance of winning.

(A) 2

(B) 5

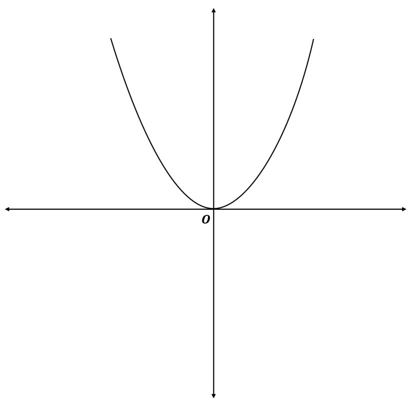
(C) 1.8

(D) 1

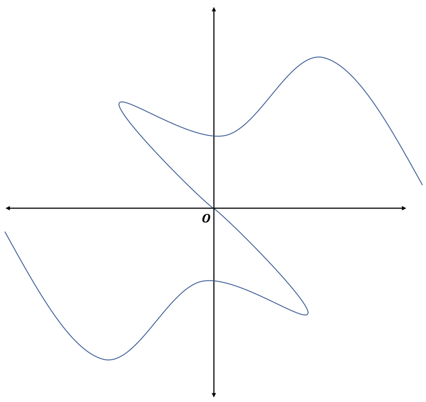
Section I continues on next page

3 Which of the following is an odd function?

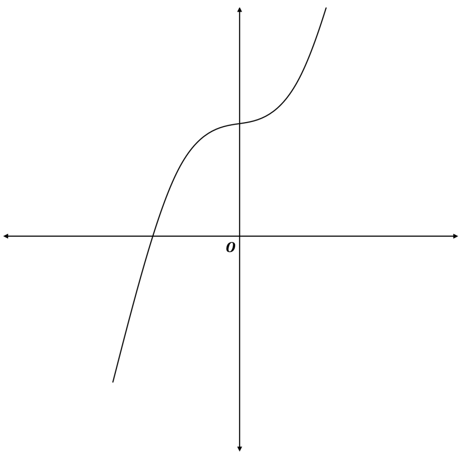
(A)



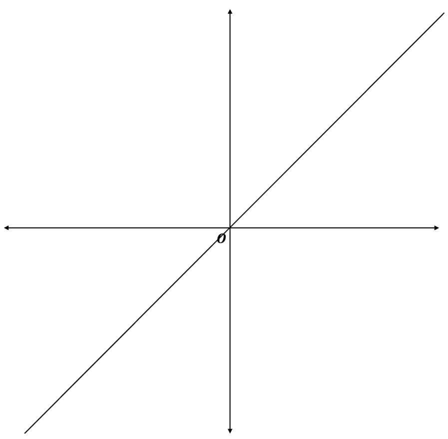
(B)



(C)

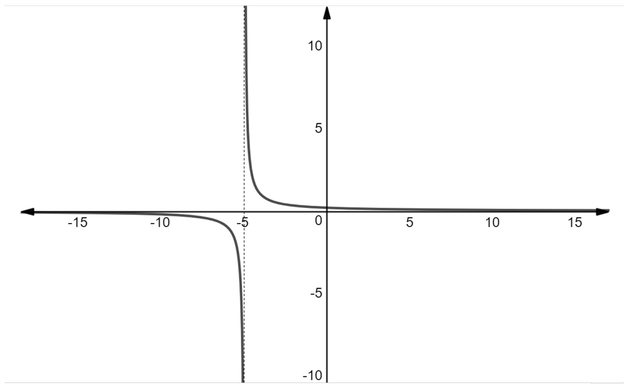


(D)

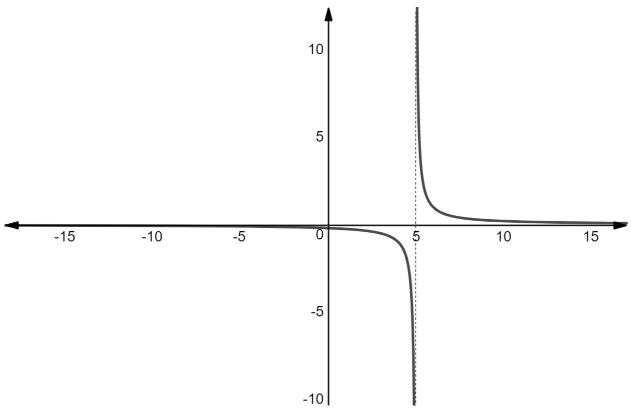


4 Which of the following graphs best represents $f(x) = \frac{1}{x+5}$?

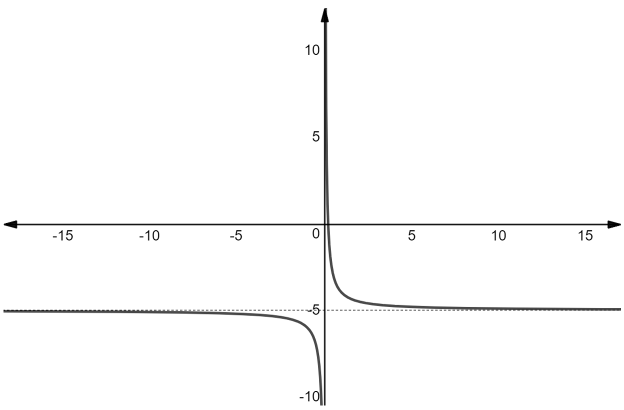
(A)



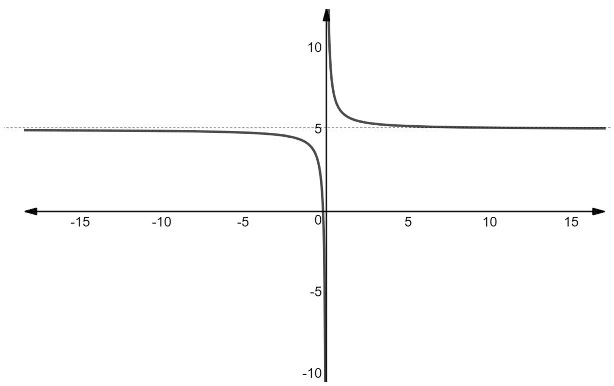
(B)



(C)



(D)



5 Differentiate $\log_e(2x + 16)^2$.

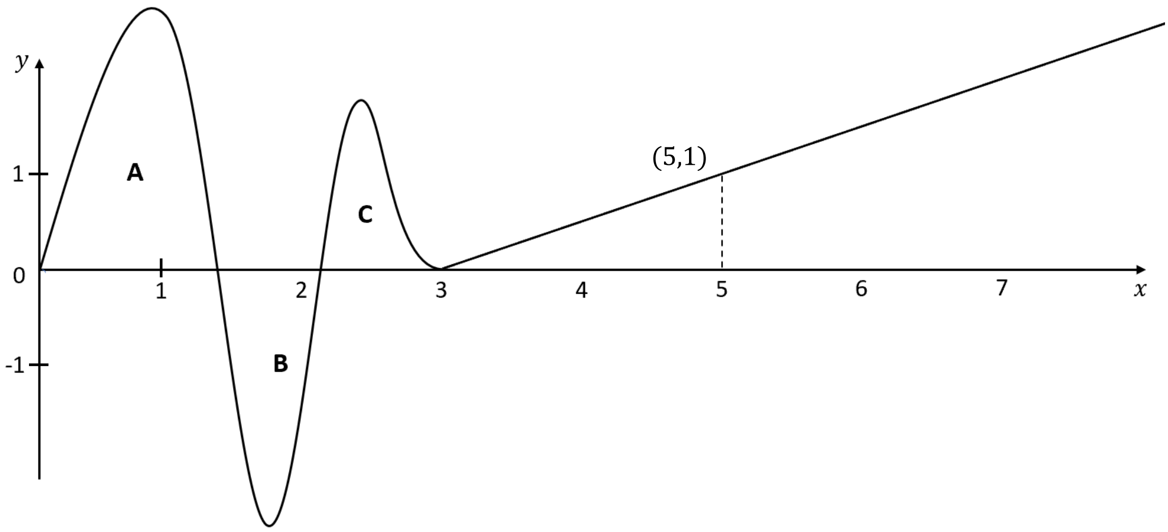
(A) $\frac{1}{2x+16}$

(B) $\log_e 2(2x + 16)$

(C) $\frac{2(2x+16)}{(2x+16)^2}$

(D) $\frac{2}{x+8}$

6 The function f is shown in the diagram below. The area bounded by the curve and the x-axis are labelled. The area of A is 3 square units, the area of B is 1.5 square units, and the area of C is 1. Evaluate $\int_0^5 f(x)dx$.



(A) 3.5 square units

(B) 4.5 square units

(C) 5.5 square units

(D) 6.5 square units

7 Find the area bounded by $y = |x + 7|$, the x-axis and the lines $x = -9$ and $x = 1$.

(A) 30 square units

(B) 32 square units

(C) 34 square units

(D) 36 square units

8 Differentiate $x^2 \sin(x^2)$.

(A) $2x \sin(2x)$

(B) $2x \sin(x^2) - 2x^3 \cos(x^2)$

(C) $2x \sin(x^2) + 2x^3 \cos(x^2)$

(D) $2x \sin(x^2) + x^2 \cos(2x)$

9 How many solutions are there to $\sin(x) = \frac{1}{2}x$ given $-\pi < x < \pi$?

(A) 1

(B) 2

(C) 3

(D) 4

10 What are the values a and b for which $\int_a^b \sin(x)\ dx < \int_a^b \cos(x)\, dx$ is true?

(A) $a = 0$ and $b = 2\pi$

(B) $a = 0$ and $b = \pi$

(C) $a = \frac{\pi}{2}$ and $b = \frac{3\pi}{2}$

(D) $a = \pi$ and $b = 2\pi$

End of Section I

Section II

In Questions 11 – 16, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Sketch the graph with equation: **2**

$$(x-3)^2+(y+5)^2=16$$

(b) Simplify the following: **2**

$$\frac{3x^2-3x+3}{x^2-4x+3}\times\frac{x^2-1}{x^3+1}$$

(c) Differentiate the following with respect to x: **2**

$$\frac{3x}{x-5}$$

(d) Solve $|x-3|<4x$. **2**

(e) Find the points of intersection between $y=2x^2+5x-12$ and $y=-x-4$. **3**

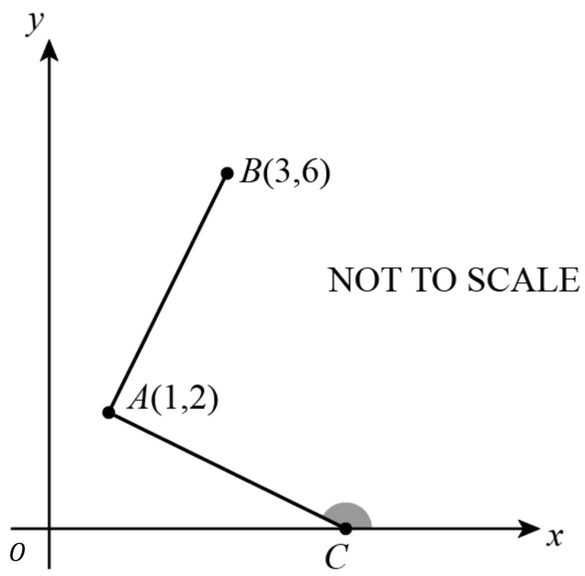
(f) Find the equation of the tangent to $f(x)=\cos\,2x$ at $x=\frac{\pi}{4}$. **2**

(g) Solve $\sin(\theta)=\frac{-\sqrt{3}}{2}$ for $0\le\theta\le 2\pi$. **2**

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) The diagram shows the points A (1, 2), B (3, 6) and C. Point C lies on the x-axis and line AC has equation $x + 2y - 5 = 0$.



(i) Find the coordinates of point C. **1**

(ii) Find the exact length of segment AB. **1**

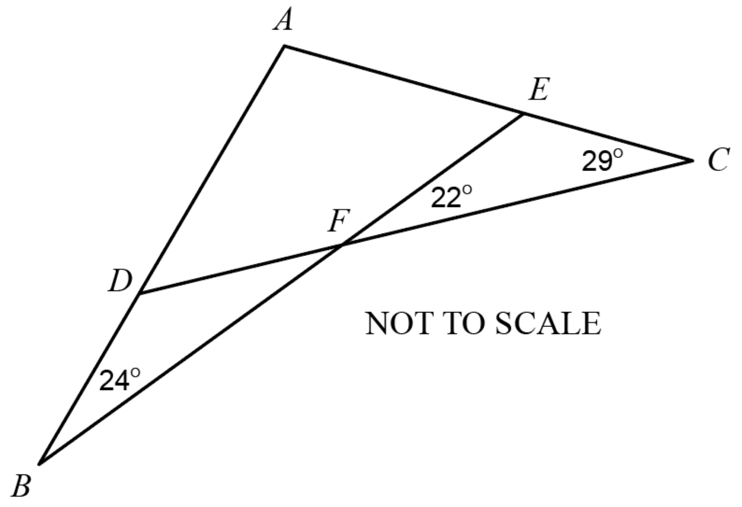
(iii) Find the equation of line AB in general form. **2**

(iv) Show that $\angle BAC = 90°$ **1**

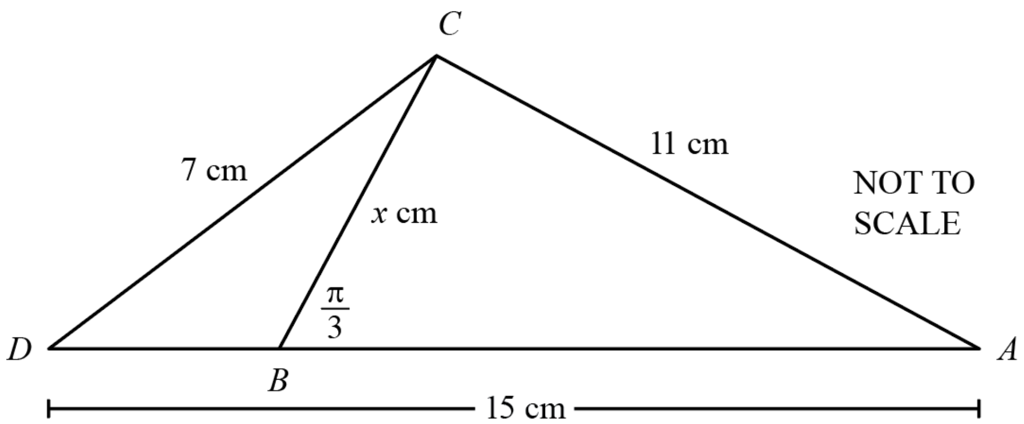
(v) What is the size of the obtuse angle that AC makes with the x-axis? (to the nearest degree) **1**

Question 12 continues on page 11

(b) Copy the diagram below into your answer booklet. Find the size of $\angle BAC$, giving reasons. **2**



(c) The diagram shows ΔACD with sides AC = 11cm, CD = 7cm and AD = 15cm. Point B lies on AD such that $\angle ABC = \frac{\pi}{3}$ and BC = $x$cm.



Copy the diagram into your answer booklet.

(i) Show that $\cos A = \frac{9}{10}$. **1**

(ii) By finding the exact value of $\sin A$, or otherwise, determine the exact length of BC. **2**

(d) Find $\frac{d}{dx}(e^{\sqrt{3}x^2})$. **2**

(e) (i) Differentiate $(6x-5)^3$. **1**

(ii) Hence evaluate $\int (6x-5)^2 dx$. **1**

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) For the curve $y = \frac{1}{3}x^3 - 9x + 2$,

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Sketch the curve labelling the stationary points (x-intercepts are NOT required). **2**

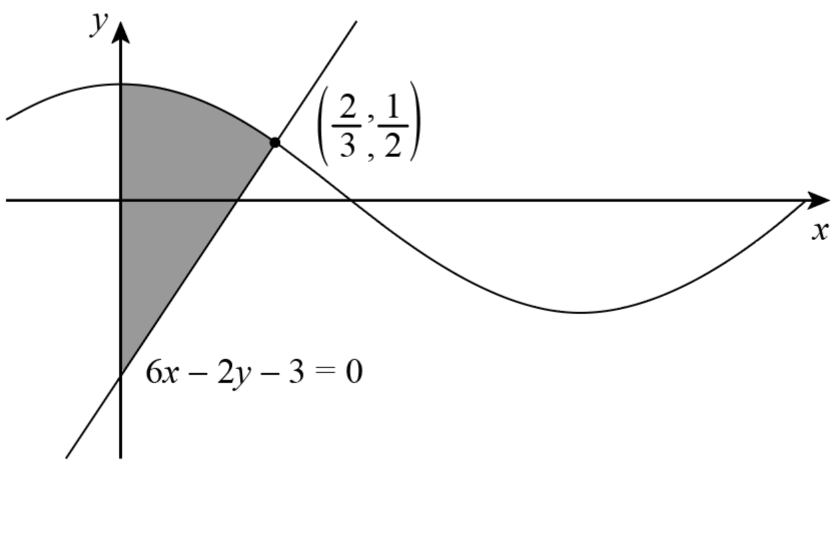
(b) A parabola has equation $y^2 - 6y - 3 = 12x$.

(i) Write the equation of the parabola in the form $(y-k)^2 = 4a(x-h)$ **1**

(ii) Determine the coordinates of the vertex. **1**

(iii) Determine the coordinates of the focus. **1**

(c) The graph shows the functions $y = \cos\left(\frac{\pi x}{2}\right)$ and the line $6x - 2y - 3 = 0$. **2**
Calculate the shaded area as an exact value.



Question 13 continues on page 13

Question 13 continued

(d) The quokka population on the West Australian Mainland was decimated by a bushfire in 2015. The fire destroyed a significant amount of forest and the population on 1st February 2015 dropped to 39.

Recent surveys have shown that the quokka population is increasing according to the equation $P = Ae^{kt}$ where A and k are constants and t is time measured in **months** after 1st February 2015. On 1st February 2018 the population had increased to 115.

(i) Calculate the value of A and show that the value of $k = 0.03$ (to 2 decimal places). **2**

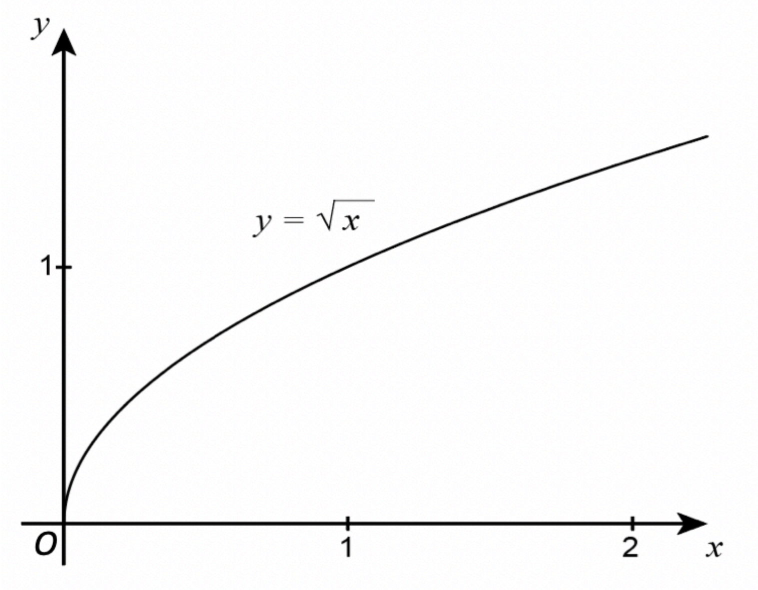
(ii) If this trend continues, how many quokkas will be in Western Australia on 1st February 2023? **1**

(iii) What was the rate of increase on 1st February 2019? **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Given the graph of $y = \sqrt{x}$ below.



(i) Estimate the area under the curve from $x = 0$ to $x = 2$ using two applications of the trapezoidal rule. **1**

(ii) Estimate the area under the curve from $x = 0$ to $x = 2$ using one application of Simpson's rule. **1**

(iii) Which of these estimations will be more accurate, and why? **1**

(b) A bacteria population grows according to the formula $T_n = 3^n + 2n$, where T_n is the increase in the number of bacteria per day and n is the number of days since the bacteria appeared.

(i) How many bacteria were added to the population on each of the first 3 days? **1**

(ii) Find the total number of bacteria in the population after 15 days? **2**

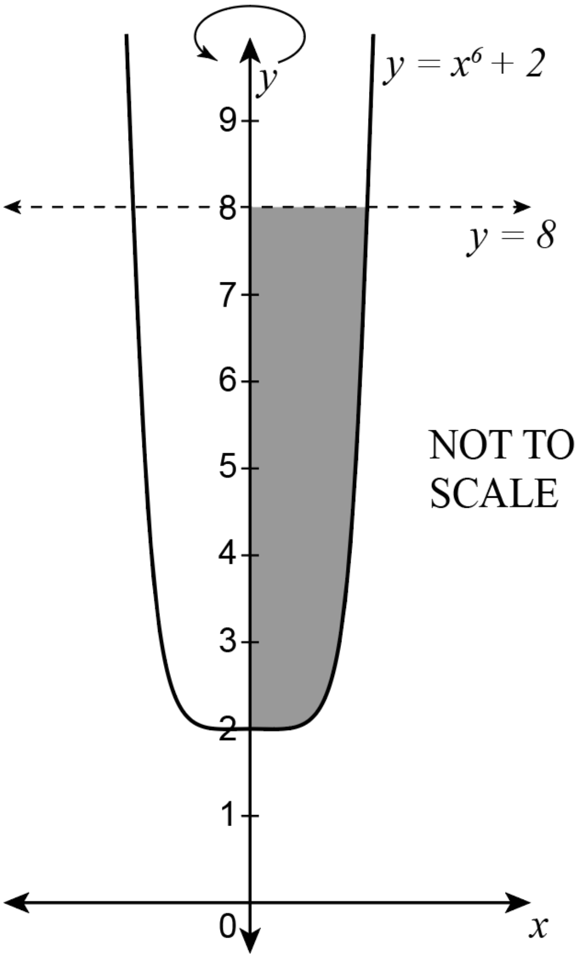
Question 14 continues on page 15

(c) There are three red, four blue and seven green marbles in a bag.
Two marbles are drawn one after another from the bag, without replacement.

(i) What is the probability of drawing one red and one green marble in any order? **1**

(ii) What is the probability that at least one green marble is drawn? **2**

(d) The shaded region below is between the curve $y = x^6 + 2$, the y-axis and the line $y = 8$.

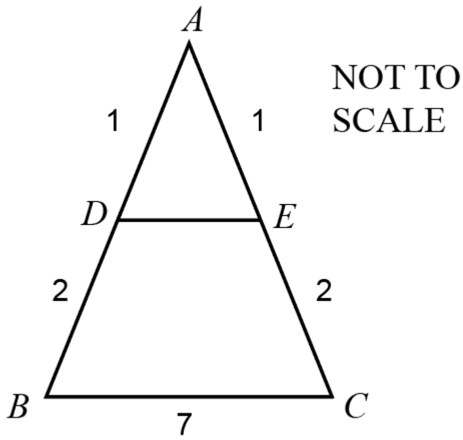


Find the volume of the solid of revolution when the shaded region is rotated about the y-axis. **3**

Question 14 continues on page 16

Question 14 continued

(e) Using the diagram below:



(i) Prove that $\Delta ADE\ |||\ \Delta ABC$. **2**

(ii) Calculate the length DE. **1**

End of Question 14

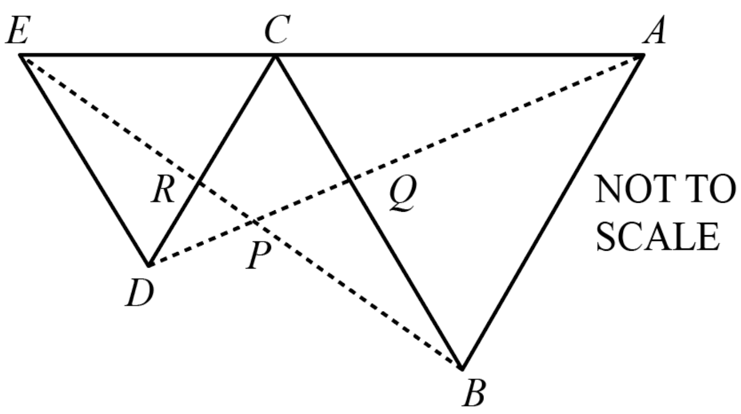
Question 15 (15 marks) Use the Question 15 Writing Booklet.

(a) A closed cylindrical can of radius r cm and height h cm is to be made from a sheet of metal with area $300\pi\ cm^3$. There is 10% wastage of the sheet in manufacturing the can.

(i) Show that $h = \frac{135-r^2}{r}$. **2**

(ii) Find the value of r which maximises the volume of the can. **3**

(b) Triangles ABC and CDE are equilateral triangles. BE intersects AD at P and CD at R. A, C and E are collinear. **3**



Copy the diagram into your writing booklet and prove that $\Delta ACD \equiv \Delta ECB$.

(c) A tank initially containing 18 000 litres of water is to be drained. After t minutes, the rate at which the volume of water is decreasing is given by:
$\frac{dV}{dt} = -40(30 - t)$

(i) Derive a formula for the volume of water remaining after t minutes. **2**

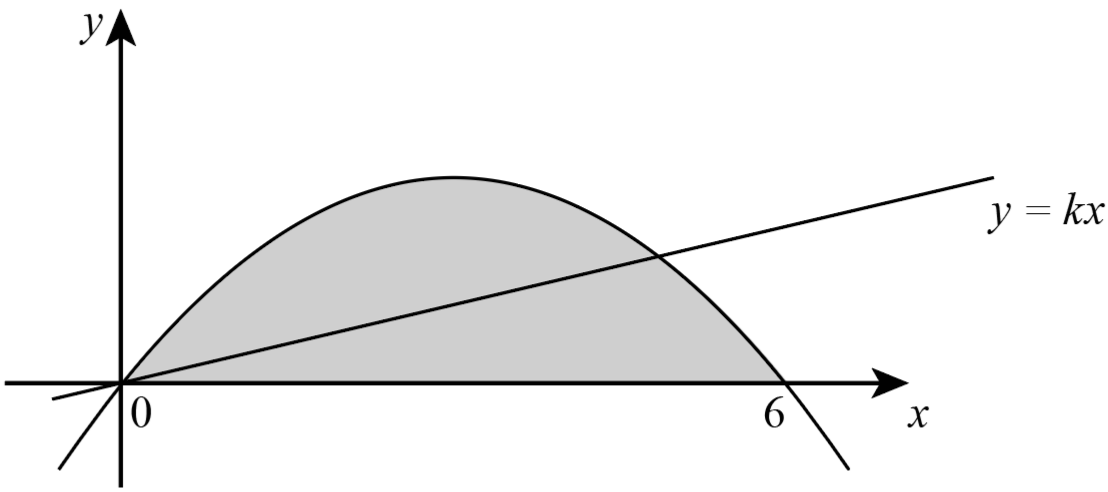
(ii) How long will it take the tank to empty? **1**

(iii) By using the expression for $\frac{dv}{dt}$ or otherwise, sketch the volume – time graph. **1**

Question 15 continues on page 18

Question 15 continued

(d) **3**



The diagram shows the area bounded by the parabola $y = 6x - x^2$ and the x axis. Find the value of k such that $y = kx$ cuts this area in half.

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

(a) Show that $\frac{d}{dx}(a^x) = a^x \log_e a$ **1**

(b) Elliott deposits \$300 000 in an account which earns interest of 6% p.a. compounded monthly.
At the end of the first month, immediately after the interest is calculated, Elliott withdraws \$900.
Each subsequent month, immediately after the interest is calculated, Elliott withdraws 2% more than what he did the previous month.

(i) Show that the amount in the account, immediately after the third withdrawal could be expressed as: **2**
$A_3 = \$300000 \times 1.005^3 - \$900(1.005^2 + 1.005 \times 1.02 + 1.02^2)$

(ii) Show that the amount in Elliott's account after the n^{th} withdrawal is: **3**

$$A_n = \$60\,000(6 \times 1.005^n - 1.02^n)$$

(iii) Find the time at which the amount A_n in Elliott's account is a maximum. **3**
(You may use the result in (a))

(c) Consider a pair of identical biased coins. When tossed the coins land more often on heads than on tails. The probability that a biased coin lands heads up is p.

(i) Draw a tree diagram to show all possible outcomes when the pair of biased coins are tossed together. Write the probabilities on each branch of the tree. **1**

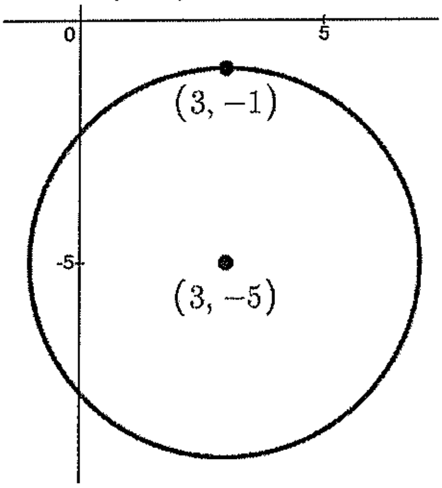
(ii) When a specific pair of biased coins are tossed, 30% of the time they land showing a head and a tail in any order. **3**
Determine the probability that, on the next toss of the pair of coins, they will land with at least one of the coins showing a head.

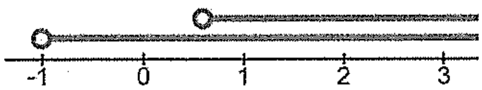
(iii) Let the probability that a pair of biased coins will land showing one head and one tail in any order be k. **2**
Prove that it is impossible for the value of k to be greater than 50%.

End of paper

1	A	
2	A	$P(win) = \frac{x}{30} = 0.06$ $x = 30 * 0.06$ $x \approx 1.8$ ∴2 tickets must be purchased in order to have at least a 6% chance of winning
3	D	The straight line is the only one with rotational symmetry centred on the origin. B is not a function and C is not centred at the origin.
4	A	Compared to $g(x) = \frac{1}{x}$, $\frac{1}{x+5}$ is translated left 5
5	D	$f(x) = (2x+16)^2$ $f'(x) = 2(2x+16) \times 2$ $= 4(2x+16)$ $\therefore \frac{d}{dx}\ln(2x+16)^2 = \frac{4(2x+16)}{(2x+16)^2}$ Which must be cancelled to give D You could also have used your log laws to simplify. $\ln(2x+16)^2 = 2\ln(2x+16)$
6	A	Use area of a triangle to find area from 3 to 5.
7	C	Can integrate by parts, or take the area of the two triangles created by the critical point x=-7
8	C	$f(x) = x^2 sinx^2$ $u = x^2$ $v = sinx^2$ $u' = 2x$ $v' = 2xcosx^2$ $f'(x) = 2xsinx^2 + 2x^3cosx^2$
9	C	There is a solution x=0.
10	D	Between 0 and 2π both functions have an integral of 0. Between 0 and π the integral of the sine function is positive (2), cosine is 0 Between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ the integral of the sine function is 0 and the integral of the cosine function is negative (-2). This means that the answer must be π and 2π, where the sine function is negative (-2) and cosine is 0.

Question 11 (15 marks)

(a)	$(x-3)^2+(y+5)^2=16$ Centre at $(3,-5)$ and radius of 4 	2 marks for good shape and correct centre and radius 1 mark deducted for poor shape, incorrect centre or incorrect radius	Most students were able to sketch the curve with correct radius and centre. Students should try to sketch their curves with no pointy bits.
(b)	$\frac{3x^2-3x+3}{x^2-4x+3}\times\frac{x^2-1}{x^3+1}$ $=\frac{3(x^2-x+1)}{(x-3)(x-1)}\times\frac{(x-1)(x+1)}{(x+1)(x^2-x+1)}$ $=\frac{3}{x-3}$	2 marks for correct factorising and simplifying 1 mark for factorising at least two algebraic expressions	Mostly done well. Students should revise factorising sum and difference of two cubes.
(c)	$\frac{d}{dx}\left(\frac{3x}{x-5}\right)=\frac{vu'-uv'}{v^2}$ $u=3x$ $u'=3$ $v=x-5$ $v'=1$ $\frac{d}{dx}\left(\frac{3x}{x-5}\right)=\frac{(x-5)\ \times 3-3x\times 1}{(x-5)^2}$ $=\frac{3x-15-3x}{(x-5)^2}$ $=-\frac{15}{(x-5)^2}$	2 marks for correctly differentiating and simplifying. 1 mark for correctly applying the quotient rule but incorrectly simplifying	Mostly done well. Careless errors included expanding the numerator incorrectly

(d)	$\lvert x-3\rvert < 4x$ Case 1 $x-3<4x$ $-4x+x<3$ $-3x<3$ $x>-1$ Case 2 $-(x-3)<4x$ $x-3>4x$ $5x>3$ $x>\frac{3}{5}$  Test: $x=0$ $LHS = \lvert 0-3\rvert$ $= \lvert -3\rvert$ $=3$ $RHS = 4x$ $=4(0)$ $=0$ $LHS \neq RHS$ $\therefore x>\frac{3}{5}$	2 marks for correctly solving and checking 1 mark for correctly solving the inequality	Most students were able to solve for the two cases but many students did not pay attention to the direction of the inequality symbol and chose the interval $-1<x<\frac{3}{5}$ or $x>-1$. Most students lost an easy one mark for not **testing** a point and confirming the solution as $x>\frac{3}{5}$. This is where most students lost a mark and their opportunity for full marks in question 11.
(e)	$y=2x^2+5x-12$ ...... (1) $y=-x-4$ ...... (2) (1) = (2) $-x-4=2x^2+5x-12$ $2x^2+6x-8=0$ $x^2+3x-4=0$ $(x+4)(x-1)=0$ $\therefore x=-4, x=1$ ...... (3) and (4) (3) in (2) $y=-(-4)-4$ $y=0$ (4) in (2) $y=-1-4$ $y=-5$ Points of intersection are $(-4,0)$ and $(1,-5)$	4 marks for all correct 1 mark deducted if only x values found or only one coordinate correct	Mostly done well, however student should take care to collect like terms carefully and substitute correctly.

(f)	$f(x) = \cos(2x)$ $f'(x) = -2\sin(2x)$ $f'\left(\frac{\pi}{4}\right) = -2\sin\left(2 \cdot \frac{\pi}{4}\right)$ $= -2\sin\left(\frac{\pi}{2}\right)$ $= -2$ $\therefore m = -2$ $f\left(\frac{\pi}{4}\right) = \cos\left(2 \cdot \frac{\pi}{4}\right)$ $= \cos\left(\frac{\pi}{2}\right)$ $= 0$ coordinates of the point is $\left(\frac{\pi}{4}, 0\right)$ $y - 0 = -2\left(x - \frac{\pi}{4}\right)$ $y = -2x + \frac{\pi}{2}$	2 marks if correct gradient and equation of the line found. 1 mark deducted if gradient found but the equation of the line was incorrect 1 mark also deducted if incorrect gradient found, but applied correct process to find the equation of the line.	Mostly done well Some students did not correctly differentiate $f(x)$. Some students correctly differentiated the function but incorrectly evaluated $f'\left(\frac{\pi}{4}\right)$ or $f\left(\frac{\pi}{4}\right)$.
(g)	$\sin(\theta) = -\frac{\sqrt{3}}{3}$ acute $\theta = \frac{\pi}{3}$ $\theta = \pi + \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$ $= \frac{4\pi}{3}, \frac{5\pi}{3}$	2 marks for each correct answer	Mostly done well.

Question 12 Solutions and Feedback

Wednesday, 31 July 2019 3:18 PM

ai) For AC: $x + 2y - 5 = 0$

Let $y = 0$

$x + 2(0) - 5 = 0$

$x - 5 = 0$

$x = 5$

$\therefore$ Coordinates of C: $(5, 0)$

ii) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3-1)^2 + (6-2)^2}$

$= \sqrt{(2)^2 + (4)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$

iii) $m_{AB} = \frac{rise}{run}$

$= \frac{6-2}{3-1}$

$= \frac{4}{2}$

$= 2$ - 1 mark for gradient

$y - y_0 = 2(x - x_0)$

Sub $(1, 2)$

$y - 2 = 2(x - 1)$

$= 2x - 2$

$2x - y = 0$ — 2 marks for correct general form

iv) $M_{AB} = 2$

$m_{AC} = \frac{0-2}{5-1}$

$= -\frac{2}{4}$

$= \frac{-1}{2}$

$m_{AB} \times m_{AC} = 2 \times -\frac{1}{2}$

$= -1$

$\therefore AB \perp AC$

$\therefore \angle BAC = 90°$

v) $\tan\theta = m$

$m_{AC} = -\frac{1}{2}$

$\tan\theta = -\frac{1}{2}$

$\theta = -26°33'54''$

For obtuse angle

$180 - 26°33'54''$

$= 153°26'6''$

$\doteqdot 153°$ to nearest degree

Feedback

ai) Some students confused x and y coordinates.

ii) Should always try to simplify surds, although marks were not lost in this instance.

iii) A lot of marks lost for "general form".

$ax + by + c = 0$

or

$ax + by = c$

'x' term must be first

'a' must be positive

'a', 'b' and 'c' must be integers

iv) The important line of working here is:

$\boxed{m_1 \times m_2 = -1}$ ← Must write this

Many students attempted alternate methods:

Pythagorean theorem

- Was accepted although many did not know how to use working to "show" $a^2+b^2=c^2$. It left many with the bizarre final line of: $40 = 40$ ← What does this prove???

Instead try this

RTP $a^2+b^2=c^2$

$LHS = a^2+b^2$

$=$

$=$

$= 40$

$= RHS$

Also, this method would be a lot messier if the numbers were not so simple.

Perpendicular Distance

- Poorly applied
- Many assumed perpendicularity in order to apply the formula

Overall

A significant review of perpendicular lines in coordinate geometry is necessary for this cohort

v) Poorly done.

This is a simple question if you know and understand the formula $\boxed{m = \tan\theta}$

b) $\angle FEA = \angle EFC + \angle ECF$ (exterior angle of triangle is equal to the sum of the interior opposite angles)

$= 22 + 29$

$= 51$

$\angle BAC = 180 - \angle ABE - \angle BEA$ (angle sum of triangle)

$= 180 - 24 - 51$

$= 105°$

1 mark for correct angle

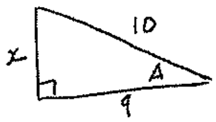
2 marks for correct angle with proper use of reasons

ci) $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{11^2+15^2-7^2}{2\times11\times15}$

$= \frac{9}{10}$ as required

ii) As $\cos A = \frac{9}{10}$



$x = \sqrt{10^2-9^2}$

$= \sqrt{19}$

$\sin A = \frac{\sqrt{19}}{10}$

Using Sine rule in $\triangle ABC$

$\frac{x}{\sin A} = \frac{11}{\sin\frac{\pi}{3}}$

1 mark for $\sin A = \frac{\sqrt{19}}{10}$ OR $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\frac{x}{\frac{\sqrt{19}}{10}} = \frac{11}{\frac{\sqrt{3}}{2}}$

$\frac{x}{\sqrt{19}} = \frac{11}{5\sqrt{3}}$

$x = \frac{11\sqrt{19}}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{11\sqrt{57}}{15}$

2 marks for exact form

$BC = \frac{11\sqrt{57}}{15}$ cm

d) $\frac{d(e^{f(x)})}{dx} = f'(x)e^{f(x)}$

$f(x) = \sqrt{3}x^2$

$f'(x) = 2\sqrt{3}x$

2 marks for correct derivative

$\frac{d}{dx}(e^{\sqrt{3}x^2}) = 2\sqrt{3}x \times e^{\sqrt{3}x^2}$

$= 2\sqrt{3}e^{\sqrt{3}x^2}x$

1 mark for single mistake in $f'(x)$ or power of e

b) - A lot of poor "Reasons" Please review the list of formal, acceptable reasons

- Many overcomplicated the question which would have wasted a lot of time. For geometry questions, consider your strategy BEFORE you start finding angles. When practising, look for the most efficient methods
- Don't forget this property (Exterior angle of triangle is equal to the sum of the interior opposite angles)

Ci) Need to review/memorise this form of the cosine rule.

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

Don't skip any steps for "show" questions

ii) Very poorly done. If you have one trig ratio and want a different ratio using the same angle DRAW A TRIANGLE

Make sure Sine rule is all in the same triangle.

Put all working, even if you don't get to the final answer. Many would have gained a mark for substituting $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

d) Poorly done

See formula sheet

e) i) $\frac{d}{dx}(6x-5)^3 = 3(6x-5)^2 \times 6$
$= 18(6x-5)^2$

ii) $\int (6x-5)^2\,dx = \frac{1}{18}\int 18(6x-5)^2\,dx$
$= \frac{1}{18}(6x-5)^3 + C$

e i) Chain rule.
Multiply by derivative of inner function

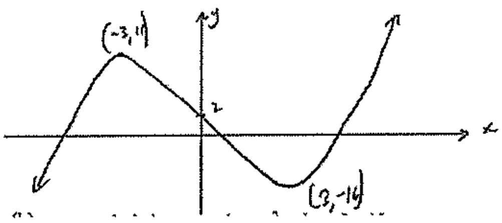
ii) Although this question was fairly simple and could be done without part (i) many wrong answers completely ignored the connection to part (i).

For "Differentiate... hence integrate" style questions, look for the connection.

ALSO... +C

Question 13

(a) For the curve $y = \frac{1}{3}x^3 - 9x + 2$,

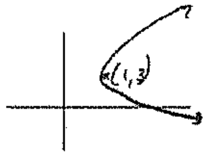
(i) Find the coordinates of the stationary points and determine their nature. $y = \frac{1}{3}x^3 - 9x + 2$ $y' = x^2 - 9$ $y' = 0$ for $x = 3 \text{ or } -3$ $y'' = 2x$ when $x = 3$ $y = \frac{27}{3} - 27 + 2 = -16$ $y'' = 6 > 0$ minimum at (3,–16) when $x = 3$ $y = -9 + 27 + 2 = 20$ $y'' = -6 < 0$ maximum at (-3,11)	3 marks correct stationary points and nature 2 marks correct stationary points and incorrect nature or Correct nature and incorrect y-coord 1 mark correct expression for y'=0	Done well. This questions did not ask for the point of inflection. Many students calculated this unnecessarily.
(ii) Sketch the curve labelling the stationary points (x-intercepts are NOT required). 	2 marks correct graph with correct max and min shown. Graph match part (i) -1 mark for: • Incorrect shape • Incorrect max/ min • Incorrect y-int	Marks were deducted for no y-int, poor shape, and no labelling. A dot on the graph is not labelling! The axes should have a scale.

(b) A parabola has equation $y^2 - 6y - 3 = 12x$.

(i) Write the equation of the parabola in the form $(y-k)^2 = 4a(x-h)$ $y^2 - 6y - 3 = 12x$ $y^2 - 6y + 9 = 12x + 3 + 9$ $(y-3)^2 = 12(x+1)$ $(y-3)^2 = 4(3)(x+1)$	1 mark correct equation no working required	Several errors in completing the square.

(ii) Determine the coordinates of the vertex.

vertex at (1,3)



1 mark correct location no working required

Students should draw a diagram to help determine the focus when vertex is known. Many students assume it was concave up not a sideways parabola.

(iii) Determine the coordinates of the focus.

$a = 3$

focus at (4,3)

1 mark correct location no working required

(c) The graph shows the functions $y = \cos\left(\frac{\pi x}{2}\right)$ and the line $6x - 2y - 3 = 0$.
Calculate the shaded area as an exact value.

$$A = \int_0^{\frac{2}{3}} \cos\frac{\pi x}{2} - (3x - \frac{3}{2})dx$$

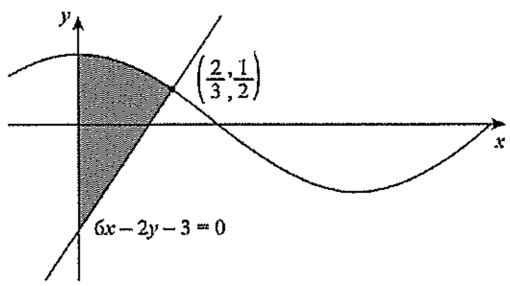
$$= \left[\frac{2}{\pi}\sin\frac{\pi x}{2} - \frac{3x^2}{2} + \frac{3}{2}x\right]_0^{\frac{2}{3}}$$

$$= \left[\frac{2}{\pi}\sin\frac{\pi x}{2} - \frac{3x^2}{2} + \frac{3}{2}x\right]_0^{\frac{2}{3}}$$

$$= \left[\frac{2}{\pi}\sin(\frac{\pi}{2} \times \frac{2}{3}) - \frac{3}{2} \times (\frac{4}{9}) + \frac{3}{2} \times \frac{2}{3} - 0\right]$$

$$= \frac{2}{\pi} \times \frac{\sqrt{3}}{2} - \frac{2}{3} + 1$$

$$= \frac{\sqrt{3}}{\pi} + \frac{1}{3} \quad or \quad \frac{\pi + 3\sqrt{3}}{3\pi}$$



2 marks correct simplified exact value with working

1 mark correct integrated expression

Integration of $\int \cos\frac{\pi x}{2} = \frac{2}{\pi}\sin\frac{\pi x}{2}$ Was a problem. Students who evaluated the integral as a single expression did better than those that tried to break it up into triangles. Some students incorrectly tried to calculate the area under the curve using areas of sectors.

(d) The quokka population on the West Australian Mainland was decimated by a bushfire in 2015. The fire destroyed a significant amount of forest and the population on 1st February 2015 dropped to just 39.
Recent surveys have shown that the quokka population is increasing according to the equation $P = Ae^{kt}$ where A and k are constants and t is time measured in months. On 1st February 2018 the population had increased to 115.

(i) Calculate the value of A and show that the value of $k = 0.03$ (to 2 decimal places).

$P = Ae^{kt}$

$t = 0 \quad P = 39$

$\therefore A = 39$

1st Feb 2018 $\;t = 36$ months

$39e^{36k} = 115$

$36k = \ln(\frac{115}{39})$

$k = 0.03$

2 marks correct value of A and k with working

1 mark, 1 correct variable

Calculation of value of k is a "show that" question. All working must be shown. Many students skipped a step and so lost a mark.

(ii) If this trend continues, how many quokkas will be in Western Australia on 1st February 2023?

In 8 years from 2015 t=96 months

$P = Ae^{kt}$

$P = 39e^{0.03\times96}$

$= 694.75$

$P = 695$ quokkas

1 mark correct value from correct working

The unrounded answer should be given
$P = 694.75$.......
$P = 695$ quokkas

(iii) What was the rate of increase on 1st February 2019?

$\frac{dP}{dt} = 39e^{0.03t} \times 0.03$

$= 1.17e^{0.03t}$

when $t = 48$ months

$\frac{dP}{dt} = 1.17e^{0.03\times48}$

$= 4.94$

$\therefore 4.9$ quokkas per month

2 marks correct answer from correct working

1 mark correct value $\frac{dP}{dt} = 1.17e^{0.03t}$

Students who differentiated did well.
Many students trued to calculate this using percentages unsuccessfully.

End of Question 13

Q 14 Feedback

a i & ii Many students went straight to decimals without clearly stating the exact answer first. You should only answer in decimals if the question requests it.

Many didn't 2 applications of trapezoidal rule, and used Simpson's Rule incorrectly.

iii Only 60% of students realised Simpson's Rule is better and could explain why. Please answer the Q'n

b i generally well done, some didn't answer Q'n

ii more than half got this incorrect as they didn't answer the Q'n.

c Those who drew a probability tree did well.

i most got this correct

ii many forgot cases, and didn't show sufficient correct working to gain marks.

d Many made mistakes here. Some rotated about x-axis NOT the y-axis. Some didn't use $V = \pi \int x^2 dy$.
Many struggled to get x^2 correctly, and integrated fractional powers poorly. As per part a, many students answered in decimals without giving a clear exact answer.

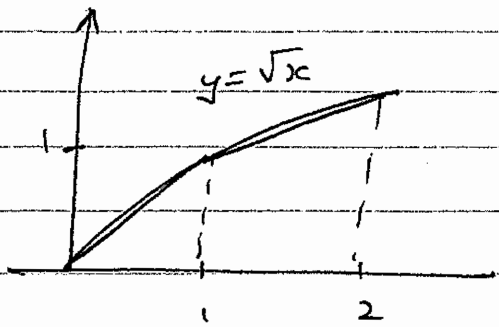
e i Mostly done well but many lost marks due to assuming too much or incorrect reasoning.

ii Mostly done well, but a number wrote $\frac{7}{2}$ instead of $\frac{7}{3}$

Many would have done better if they drew ΔADE, ΔABC separately first...

Q14

a



i $A \approx \frac{h}{2}\{f(a) + f(b)\}$

$h = 1$

$\therefore A = \frac{1}{2}\{f(0) + 2f(1) + f(2)\}$ as 2 trapeziums

$A = \frac{1}{2}(0 + 2 + \sqrt{2})$

$A = \frac{2+\sqrt{2}}{2}$ units^2 [1]

ii $A \approx \frac{h}{3}\{df + 4dm + dl\}$

$h = 1$

$A \approx \frac{1}{3}\{f(0) + 4f(1) + f(2)\}$

$A \approx \frac{1}{3}(0 + 4 + \sqrt{2})$

$A \approx \frac{4+\sqrt{2}}{3}$ [1]

iii Simpson's Rule will be more accurate as it will use a concave down parabolic arc. It will be accurate (exact) or close to.
Trapezoidal Rule as shown in diagram above will underestimate the area, as tops of trapeziums under the curve.
[1]

6 $3^n + 2n$

i day 1: $3^1 + 2\times 1 = 5$

day 2: $3^2 + 2\times 2 = 9 + 4 = 13$

day 3: $3^3 + 2\times 3 = 27 + 6 = 33$ [1]

ii After 15 days
we have 2 series

Arithmetic $a = 2,\ d = 2,\ n = 15$

$S_{15_A} = \frac{15}{2}\{2\times 2 + 14\times 2\} = 240$

Geometric $a = 3,\ r = 3,\ n = 15$

either a.p or g.p correct [1]

$$S_{15_G} = \frac{a(r^n - 1)}{r - 1}$$

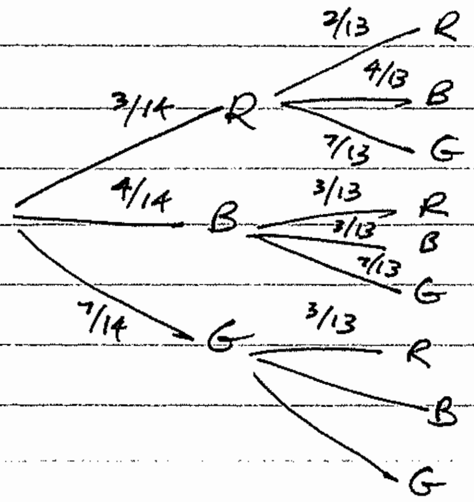
$$= \frac{3(3^{15} - 1)}{3 - 1} = 21{,}523{,}359$$

$\therefore$ Total $= S_{15_A} + S_{15_G}$

$= 21{,}523{,}599$ [1]

c   3R, 4B, 7G

total = 14 marbles



i   P(RG) + P(GR) = P(draw 1R, 1G any order)

$= \frac{3}{14} \times \frac{7}{13} + \frac{7}{14} \times \frac{3}{13}$

$= \frac{3}{13}$ [1]

ii   P(at least one green)

= 1 − P(no green)   in this case just as easy to leverage part i

P(at least one green) = P(RG) + P(BG) + P(G) on 1st draw

$= \left(\frac{3}{14} \times \frac{7}{13}\right) + \left(\frac{4}{14} \times \frac{7}{13}\right) + \frac{7}{14}$

$= \frac{3}{26} + \frac{2}{13} + \frac{1}{2}$ [1]

$= \frac{10}{13}$ [1]

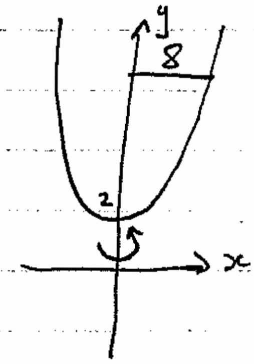
OR   1 − P(RR + RB + BR + BB)

$= 1 - P\left(\frac{3}{14} \times \frac{2}{13} + \frac{3}{14} \times \frac{4}{13} + \frac{4}{14} \times \frac{3}{13} \times 2\right)$ [1]

$= 1 - \frac{3}{13}$

$= \frac{10}{13}$ [1]

d $y = x^6 + 2$



$A = \pi \int x^2 . dy$

need x^2

$y = x^6 + 2$

$(x^6 = y - 2)$ cube root all

$x^2 = \sqrt[3]{y-2}$ [1]

$\therefore A = \pi \int_2^8 (y-2)^{1/3} dy$

$A = \pi \left[\frac{3(y-2)^{4/3}}{4} \right]_2^8$ [1] integrate o.k

$A = \frac{3\pi}{4} \left[(y-2)^{4/3} \right]_2^8$

$A = \frac{3\pi}{4} \left(6^{4/3} - 0 \right)$

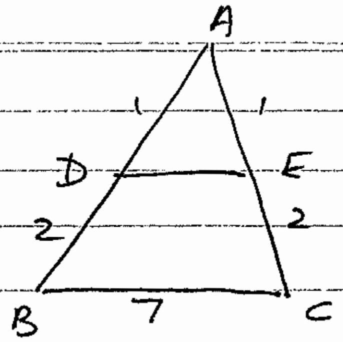
← should be exact answer

[1] $A = \frac{3\pi}{4} . \sqrt[3]{6^4}$ $\approx 8.177\pi$ (incorrect as approximate)

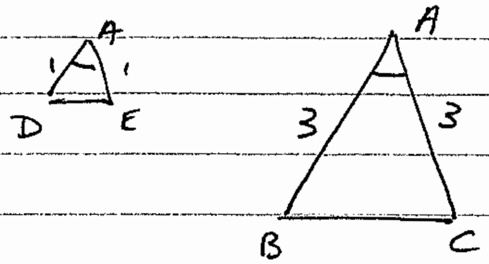
$A = 25.6889...$ ⇐ only if Q'n wanted 4 sig figs, not in final exam Q'n ...

$A = 25.69$ units^3

e



i



in ΔADE and ΔABC

$\angle DAE = \angle BAC$ (common)

$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{3}{1} = 3$$ [1]

$\therefore \Delta ADE \;|||\; \Delta ABC$ (2 pairs of sides in the same ratio and included angle equal) [1]

ii $\quad \dfrac{DE}{BC} = \dfrac{AD}{AB}$ (corresponding sides of similar Δ's are in the same ratio)

$$\frac{DE}{7} = \frac{1}{3}$$

$\therefore DE = \dfrac{7}{3}$ units. [1, with working]

Question 15

(a) (i)

$$SA = 2\pi r^2 + 2\pi rh = \frac{300\pi}{100} \times 90 \text{ (1 mark)}$$
$$2\pi r^2 + 2\pi rh = 270\pi$$
$$r^2 + rh = 135$$
$$rh = 135 - r^2$$
$$h = \frac{135 - r^2}{r} \text{ (1 mark)}$$

> 1 mark for initial statement
> 2 marks clear and thorough working to get the result for t

(ii)

$$V = \pi r^2 h$$
$$= \pi r^2 \left(\frac{135 - r^2}{r}\right) \text{ (1 mark)}$$
$$= \pi(135r - r^3)$$
$$V' = \pi(135 - 3r^2)$$
$$V' = 0 \textit{ (for stationary points)}$$
$$\pi(135 - 3r^2) = 0$$
$$3r^2 = 135$$
$$r^2 = 45$$
$$r = \pm\sqrt{45}$$
$$= 3\sqrt{5} \textit{ (must be positive)} \text{ (1 mark)}$$

Test for maximum:

$$V'' = \pi(-6r)$$
$$V''(3\sqrt{5}) = \pi(-6(3\sqrt{5}))$$
$$\therefore V''(3\sqrt{5}) < 0$$

$$\therefore r = 3\sqrt{5} \text{ gives a maximum (1 mark)}$$

1 mark for first substitution
2 marks with correct answer
3 marks for test of correct answer

(b) In $\triangle ACD$ and $\triangle ECB$:
$CA = CB$ (equilateral $\triangle ABC$, sides equal)
$EC = CD$ (equilateral $\triangle CDE$, sides equal)
$\angle ECB = 180 - 60 = 120$ ($\angle$ on a straight line add to 180 and $\angle ACB = 60$ as equilateral triangle)
Similarly $\angle ACD = 120$
$\therefore \angle ECB = \angle ACD$
$\therefore \triangle ACD \equiv \triangle ECB$ (SAS)

+1 mark for factual statements
+1 mark for reasoning
+1 mark for conclusion and reason

(c) (i)

$$\int \frac{dV}{dt} dt = \int -40(30 - t) dt$$
$$V = -40\left(30t - \frac{t^2}{2}\right) + C \text{ (1 mark)}$$
$$V(0) = -40\left(30 \times 0 - \frac{0^2}{2}\right) + C = 18000$$
$$\therefore C = 18000$$

$$\therefore V = -40\left(30t - \frac{t^2}{2}\right) + 18000 \text{ (1 mark)}$$
$$V = 20t^2 - 1200t + 18000$$

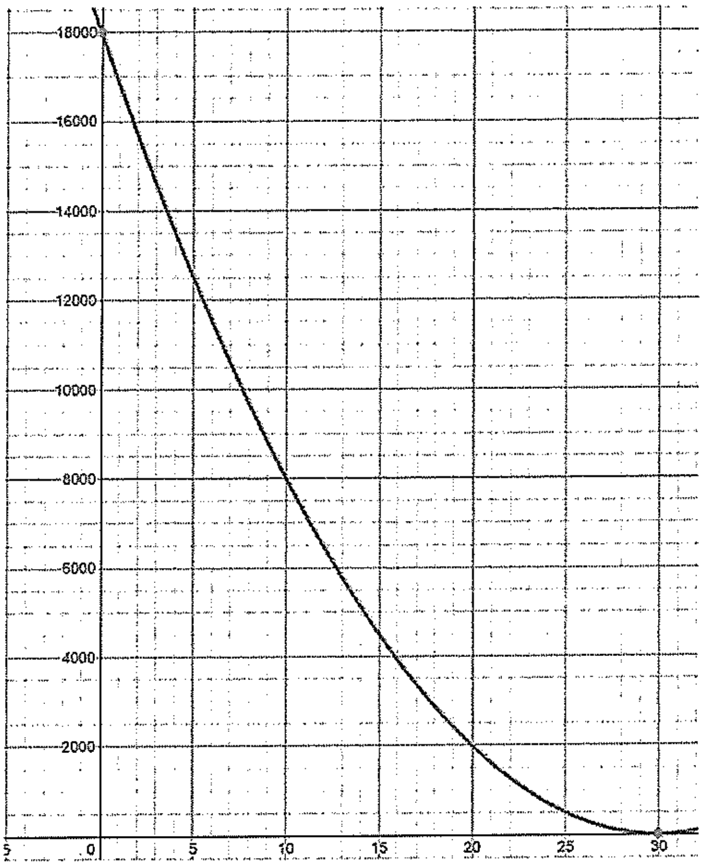
+1 mark for correct integration
+1 mark for final correct solution stated with C calculated

(ii)

$$
\begin{aligned}
V = 0 &= -40\left(30t - \frac{t^2}{2}\right) + 18000 \\
30t - \frac{t^2}{2} &= 450 \\
60t - t^2 &= 900 \\
t^2 - 60t + 900 &= 0 \\
(t-30)^2 &= 0 \\
t - 30 &= 0 \\
t &= 30
\end{aligned}
$$

The tank will take 30 seconds to empty

+1 mark for correct answer with working



(iii) +1 mark for correct shape and endpoints

(d) Total area given by

$$\int_0^6 6x - x^2 dx = \left[\frac{6x^2}{2} - \frac{x^3}{3}\right]_0^6$$
$$= \frac{6 \times 6^2}{2} - \frac{6^3}{3}$$
$$= 36$$

therefore the area of half will be 18 **(1 mark)**

The two lines intercept when:

$$\begin{aligned} y &= kx \\ y &= 6x - x^2 \\ \therefore kx &= 6x - x^2 \\ x^2 + (6-x)x &= 0 \\ x(x+k-6) &= 0 \end{aligned}$$

so $x = 0$ or, more importantly, $x = 6 - x$

$$\begin{aligned} \int_0^{6-k} 6x - x^2 - kxdx &= 18 \textbf{ (1 mark)} \\ \left[\frac{6x^2}{2} - \frac{x^3}{3} - \frac{kx^2}{2}\right]_0^{6-k} &= 18 \\ 3(6-k)^2 - \frac{(6-k)^3}{3} - \frac{k(6-k)^2}{2} &= 18 \\ 18(6-k)^2 - 2(6-k)^3 - 3k(6-k)^2 &= 108 \\ (6-k)^2(18 - 2(6-k) - 3k) &= 108 \\ (6-k)^2(6-k) &= 108 \\ (6-k)^3 &= 108 \\ 6-k &= \sqrt[3]{108} \\ k &= 6 - \sqrt[3]{108} \textbf{ (1 mark)} \end{aligned}$$

+1 mark for identifying the correct area $A = 18$
+1 mark for a correct integral expression equivalent to a correct area
(this can take a few forms)
+1 for correct answer with necessary working

Q16 2019 Mathematics Trial marking scheme

Q#	Solution	Marking criteria	Marker's feedback
16(a)	$\frac{d}{dx}(a^x) = a^x \log_e a$ $\frac{d}{dx}(a^x) = \frac{d}{dx}\left(e^{\log_e a^x}\right)$ 1 mark $= \frac{d}{dx}\left(e^{x \log_e a}\right)$ $=. e^{x \log_e a} \times \log_e a$ $= a^x \times \log_e a$	1 mark: correctly differentiates (students must show all lines of working)	*Very poorly done by almost all students. This is a proof you must learn.*
16 (b) (i)	6% p.a. = $\frac{0.06}{12} = 0.005\ per\ month$ $A_1 = 300000 \times 1.005 - 900$ $A_2 = (300000 \times 1.005 - 900) \times 1.005 - 900 \times 1.02$ $= 300000 \times 1.005^2 - 900 \times 1.005 - 900 \times 1.02.$ **1 mark** $A_3 = (300000 \times 1.005^2 - 900 \times 1.005 - 900 \times 1.02) \times 1.005$ $- 900 \times 1.02^2$ $= 300000 \times 1.005^3 - 900 \times 1.005^2 - 900(1.02)(1.005) - 900 \times 1.02^2$ $= 300000 \times 1.005^3 - 900[1.005^2 + (1.02)(1.005) + 1.02^2]$ **1 mark**		1 mark: gives the correct expression for A_2 1 mark: proves the correct result for A_3
	You need to show all lines of working. Students who wrote the full expression A_1, multiplied by 1.005 and then subtracted the withdrawal amount was more successful than those who simply executed the operation. Please make sure you are showing all the steps. Please remember, the question NOT asking you to STATE A_1, A_2 and A_3. That is you need to DERIVE the expressions.		
16 (b) (ii)	$A_n = 300000 \times 1.005^n - 900[1.005^{n-1} + (1.02)(1.005)^{n-2} + (1.02)^2 (1.005)^{n-3} + \cdots. + (1.02)^{n-2}(1.005) + 1.02^{n-1}]$ **1 mark** $= 300000 \times 1.005^n - 900 \times [1.005^{n-1} + (1.02)(1.005)^{n-2} + (1.02)^2 (1.005)^{n-3} + \cdots. + (1.02)^{n-2}(1.005) + 1.02^{n-1}]$ $= 300000 \times 1.005^n - 900 \times 1.02^{n-1} \times \left[\left(\frac{1.005}{1.02}\right)^{n-1} + \left(\frac{1.005}{1.02}\right)^{n-2} + \cdots + \left(\frac{1.005}{1.02}\right) + 1\right]$ $= 300000 \times 1.005^n - 900 \times 1.02^{n-1} \times 1 \times \frac{1-\left(\frac{1.005}{1.02}\right)^n}{1-\left(\frac{1.005}{1.02}\right)}$ **1 mark** $= 300000 \times 1.005^n - 61200 \times 1.02^{n-1} \times 1 \times \left(1 - \left(\frac{1.005}{1.02}\right)^n\right)$ $= 300000 \times 1.005^n - 61200 \times \frac{(1.02^n - 1.005^n)}{1.02}$ $= 300000 \times 1.005^n - 60000 \times (1.02^n - 1.005^n)$ $= 60000(5 \times 1.005^n - 1.02^n + 1.005^n).$ $= 60000(6 \times 1.005^n - 1.02^n).$ **1 mark**		1 mark: writes the expression for A_n 1 mark: applies the series formula to simplify 1 mark: manipulates to prove the result
	Alternately, $A_n = 300000 \times 1.005^n - 900[1.005^{n-1} + (1.02)(1.005)^{n-2} + (1.02)^2 (1.005)^{n-3} + \cdots. + (1.02)^{n-2}(1.005) + 1.02^{n-1}]$ $= 300000 \times 1.005^n - 900 \times 1.005^{n-1} \times$		*Students need to write the expanded form of A_n to reveal the pattern, so that the common ratio is obvious. If requires write A_4 to understand what is going on.*

$$\left[1+\frac{1.02}{1.005}+\left(\frac{1.02}{1.005}\right)^2+\left(\frac{1.02}{1.005}\right)^3+\cdots+\left(\frac{1.02}{1.005}\right)^{n-1}\right]$$

$$= 300000\times 1.005^n - 900\times 1.005^{n-1}\times 1\times \frac{\left(\frac{1.02}{1.005}\right)^n-1}{\left(\frac{1.02}{1.005}\right)-1}$$

$$= 300000\times 1.005^n - 60300\times 1.005^{n-1}\times 1\times\left(\left(\frac{1.02}{1.005}\right)^n-1\right)$$

$$= 300000\times 1.005^n - 60300\times\frac{(1.02^n-1.005^n)}{1.005}$$

$$= 300000\times 1.005^n - 60000\times(1.02^n-1.005^n)$$

$= 60000(5\times 1.005^n - 1.02^n + 1.005^n)$.
$= 60000(6\times 1.005^n - 1.02^n)$. **1 mark**

Very poorly done due to lack of writing all logical steps

(iii)

$A_n = 60000(6\times 1.005^n - 1.02^n)$.

$\frac{dA_n}{dn} = 60000\left(6\times 1.005^n(ln1.005) - 1.02^n(ln1.02)\right)$.

When A_n is maximum, $\frac{dA_n}{dn} = 0$

$6\times 1.005^n(ln1.005) - 1.02^n(ln1.02) = 0$

$$6\times 1.005^n(ln1.005) = 1.02^n(ln1.02)$$

$$\left(\frac{1.005}{1.02}\right)^n = \frac{\ln(1.02)}{6\times\ln(1.005)} = 0.6617.....$$

$$n\,ln\left(\frac{1.005}{1.02}\right) = \ln(0.6617\ldots.)$$

$$n = \frac{\ln(0.6617\ldots.)}{ln\left(\frac{1.005}{1.02}\right)}$$

= 27.869....

Test:

n	26	27	28
$\frac{dA_n}{dn}$	55.84	26.29	-3.998

Hence, A_n is maximum, between $n = 27$ and $n = 28$
When $n = 26$, $A_n = 309440.35$
$n = 27$, $A_n = 309481.48$
$n = 28$, $A_n = 309492.69$
$n = 29$, $A_n = 309473.23$

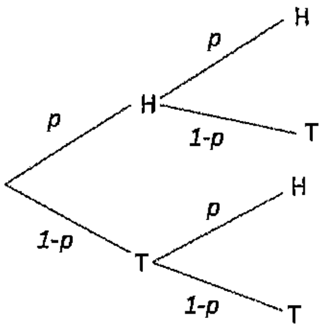
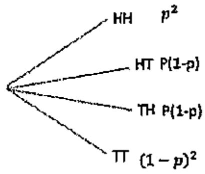
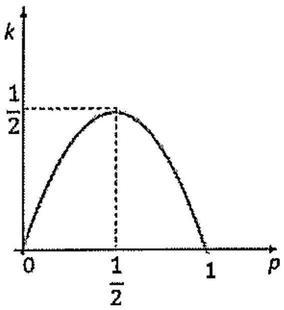
Hence, $n = 28$ *months*

1 mark: correctly differentiates

Very few students realised that you could use the result from (a).
Common errors:
$6\times 1.005^n) = 6.03^n$
$\frac{d}{dx}(1.005^n) = n(1.005)^{n-1}$
Some students let $A_n = 0$ to find the value of n that maximised A.
Very few students tested for max/min.

1 mark: solves $\frac{dA_n}{dn} = 0$

1 mark: proves n = 27 gives the maximum A_n and finds the maximum A_n.

c) (i)	 	1 mark: gives the correct tree diagram  *When the question is clearly asking you to draw a tree diagram, you must show the multi-stages.*
(ii)	$2p(1-p) = \frac{3}{10}$ 1 mark $20p(1-p) = 3$ $20p^2 - 20p + 3 = 0$ $p = \dfrac{20 \pm \sqrt{400 - 4 \times 20 \times 3}}{40}$ $= \frac{20 \pm 4\sqrt{10}}{40} = \frac{5 \pm \sqrt{10}}{10}$ = 0.8162 (4 d.p.); 0.1838 (4 d.p.). 1 mark $P(H) > P(T)$ Hence, $p =$ 0.8162 and $1 - p =$ 0.1838 $P(\text{at least one head}) = 1 - P(TT)$ $= 1 - 0.1838 \times 0.1838$ = 0.9662	1 mark: writes the correct probability equation in terms of p 1 mark: Solves the equation correctly. 1 mark: chooses the correct value for p giving reason $P(H) > P(T)$ and calculates probability at least one head *Many students got the quadratic equation correctly, however poor skills in solving a quadratic equation meant that the solution could not be found.*
16 (c) (iii)	$2p(1-p) = k$ By symmetry of the quadratic function, the maximum value of k occurs when $p = \frac{1}{2}$. Maximum value = $2\left(\frac{1}{2}\right)\left(1 - \frac{1}{2}\right) = \frac{1}{2}$ Hence $k < \frac{1}{2}$  	Proves that the maximum value of k occurs when $p = \frac{1}{2}$ 1 mark: finds the maximum value of k and hence the result *Some students did this question very elegantly, giving strong arguments. However, in general, students substituted the value from (II) and showed that k< ½. This is not a conclusive method.*