Question 1 **(12 marks)** **Use a separate writing booklet** **Marks**

(a) The base length x, of a square pyramid of volume v and perpendicular height h, is given by

$$x = \sqrt{\frac{3v}{h}}$$ **2**

Find x correct to 3 significant figures if $v = 750$ and $h = 8.45$

(b) Find $\int (2x^5 - 4)dx$ **2**

(c) If $5^{2x} = 50$, find x correct to 2 decimal places **2**

(d) Differentiate $(4 - 3x^2)^6$ with respect to x **2**

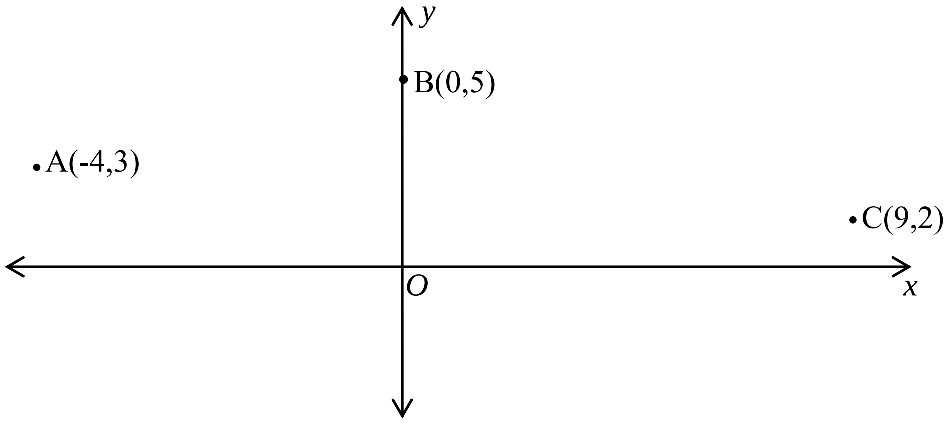
(e) Solve $\dfrac{x-5}{4} = \dfrac{6}{x}$ **2**

(f) Solve the pair of simultaneous equations

$$8x + 2y + 7 = 0$$

$$x - 2y + 2 = 0$$ **2**

Question 2 (12 marks) Use a separate writing booklet **Marks**



The diagram shows the origin O and the coordinates of the points A(-4, 3), B(0, 5) and C(9, 2).

Copy this diagram into your writing booklet.

(a) Find the exact length of the interval BC. **2**

(b) Show that the equation of the line k, drawn through A and parallel to BC is $x + 3y - 5 = 0$. Clearly indicate the line k on your diagram. **3**

(c) Find the coordinates of D, the point where the line k meets the x axis. **1**

(d) Prove ABCD is a parallelogram. **2**

(e) Find the perpendicular distance from the point B to the line k. **2**

(f) Hence, or otherwise, find the area of ABCD. **2**

Question 3 (12 marks) Use a separate writing booklet

Marks

(a) $\dfrac{\sqrt{6}-\sqrt{3}}{\sqrt{10}-\sqrt{5}} = \dfrac{a\sqrt{15}}{b}$ where a and b are rational numbers. **3**

Find the value of a and b.

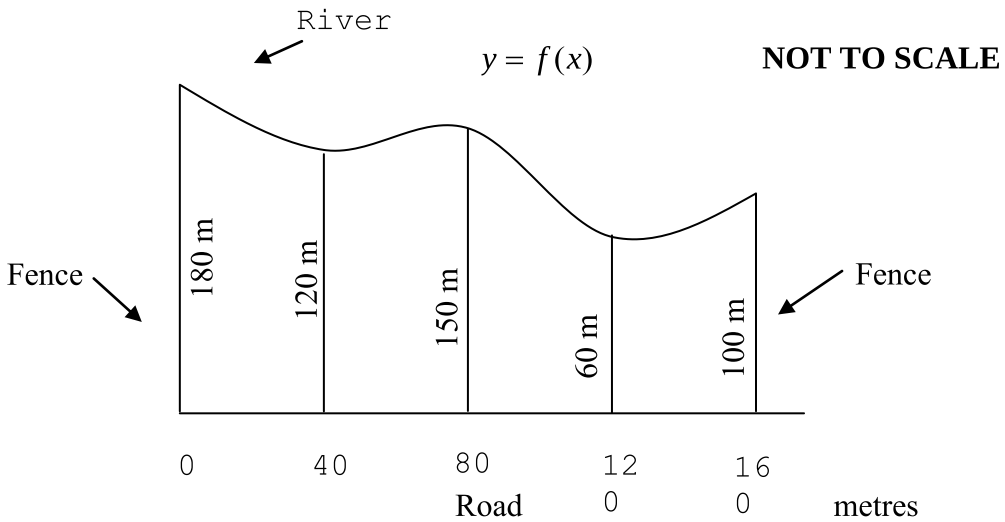
(b) Solve for real values of x

$$4^x - 12(2^x) + 32 = 0$$ **3**

(c) Solve

$$2\log_5 3 = \log_5 x - \log_5 6$$ **2**

(d)



The diagram represents a paddock bounded by a river, a straight road and two fences perpendicular to the road.
The perpendicular distances from the river to the road have been measured at intervals of 40 metres.

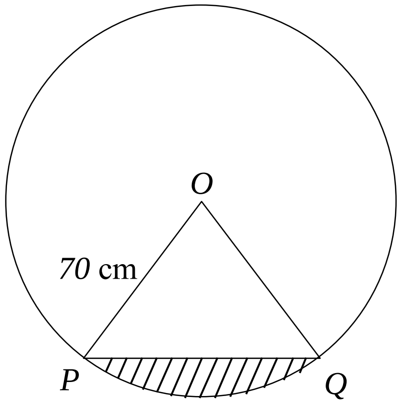
(i) Copy and complete the following table of values in your exam booklet. **1**

x	0	40	80	120	160
$y = f(x)$					

(ii) Estimate the area of the paddock using Simpson's Rule with five function values (1 decimal place). **3**

Question 4 (12 marks) Use a separate writing booklet **Marks**

(a) In the diagram the length of the arc PQ is 30 cm. The radius of the circle is 70 cm.



NOT TO SCALE

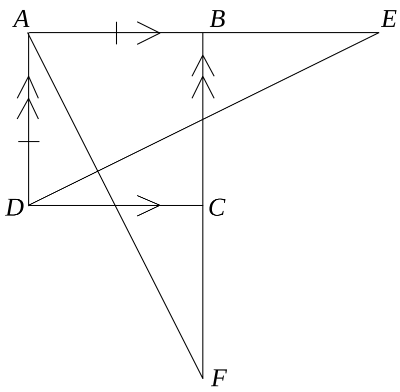
(i) Show that $\angle POQ = 25^{o}$ to the nearest degree. **2**

(ii) Find the shaded area correct to the nearest cm^2 **2**

(b) Sketch a graph of $y = 3\cos x$ for $0 \leq x \leq 2\pi$ **2**

(c) Find all possible values of θ if $\tan\theta = -\sqrt{3}$ and $0 \leq \theta \leq 2\pi$ **2**

(d) In the diagram $ABCD$ is a square. AB is produced to E so that $AB = BE$ and BC is produced to F so that $BC = CF$.



NOT TO SCALE

(i) Copy the diagram onto your answer page.

(ii) Prove $\Delta AED \equiv \Delta BFA$. **2**

(iii) Hence prove $\angle AED = \angle BFA$. **2**

Question 5 (12 marks) Use a separate writing booklet

Marks

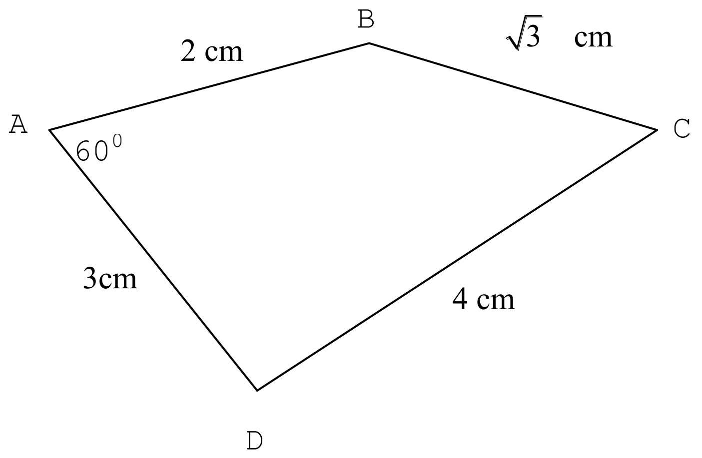
(a) Find the values of $a, b \text{ and } c$, given that

$a(x^2-1)+b(x-1)^2+c(x+1)\equiv 2x^2-6$ **3**

(b) Find the equation of the tangent to the curve

$y=3x+e^{-x}$ at the point where $x=0$ **3**

(c) **NOT TO SCALE**



ABCD is a quadrilateral with AB = 2 cm, BC = $\sqrt{3}$ cm, CD = 4 cm,

AD = 3 cm and $\angle A = 60^o$

(i) Find the exact length of diagonal BD. **2**

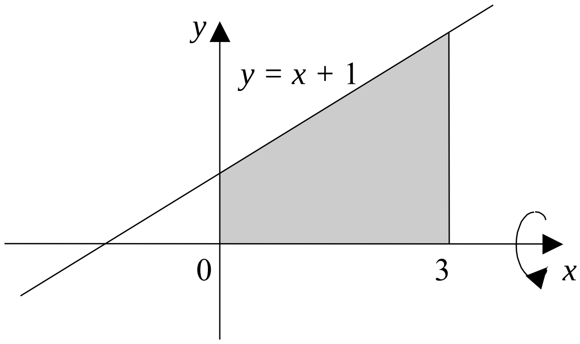
(ii) Find the size of $\angle C$ to nearest degree. **2**

(iii) Hence, find the area of the quadrilateral ABCD (to the nearest cm^2). **2**

Question 6 (12 marks) Use a separate writing booklet **Marks**

(a) Find $\int 2e^{3x-5}\,dx$ **2**

(b)



The region which lies between the x-axis and the line $y = x + 1$ from $x = 0$ to $x = 3$ is rotated about the x-axis to form a solid. Find the exact volume of the solid. **3**

(c) For the equation $x^2 + (k+6)x - 2k = 0$ find:

(i) the discriminant in terms of k. **1**

(ii) the values of k for which this equation has real roots. **3**

(d) For the parabola $(x+2)^2 = -8(y-4)$

Draw a neat sketch of the curve clearly indicating the:

(i) coordinates of the vertex;

(ii) coordinates of the focus;

(iii) equation of the directrix. **3**

Question 7 (12 marks) Use a separate Writing Booklet

Marks

(a) Give, in simplest form, the exact value of **2**

$$\int_1^4 \frac{dx}{1+2x}$$

(b) The curve $y = f(x)$ has gradient function

$$\frac{dy}{dx} = 3x^2 - 2x + 1$$ **2**

The curve passes through the point $Q(2,3)$.
Find the equation of the curve.

(c) Consider the curve whose equation is

$$y = 7 + 4x^3 - 3x^4$$

(i) Find the coordinates of the two stationary points. **2**

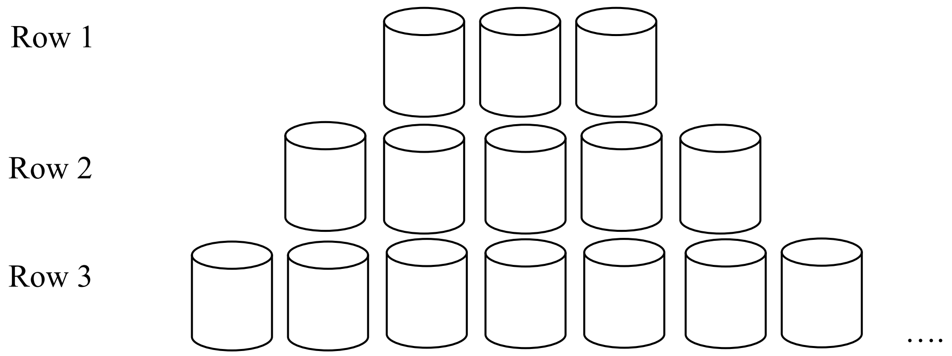
(ii) Find all values of x for which $\frac{d^2y}{dx^2} = 0$. **1**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve for the domain $-1 \le x \le 2$. **3**

Question 8 (12 marks) Use a separate writing booklet **Marks**

(a) Cans of fruit in a supermarket display are stacked so that there are 3 cans in the top row, 5 in the next row, 7 in the next and so on.



If there are 10 rows in the display find

(i) The number of cans in the bottom row (10^{th} row) **2**

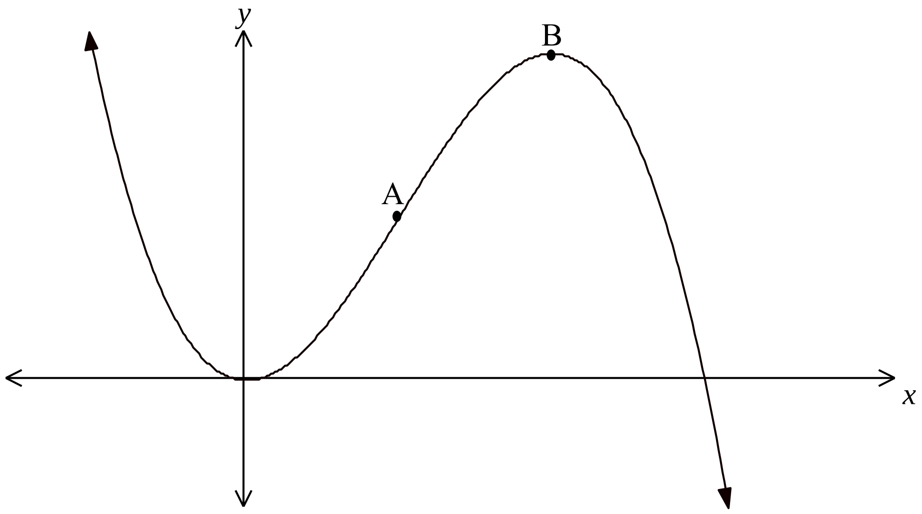
(ii) The total number of cans in the display **2**

(b) If $A=(-4, 4)$, $B=(6,-2)$ and $P(x,y)$.

(i) P moves so that PA is perpendicular to PB
Show that the locus of P is the circle : $x^2+y^2+2x-2y-32=0$ **2**

(ii) Find the centre and the radius of this circle. **2**

(c) Below is the graph of the function $y=f(x)$



(i) Describe what is happening at A and B. **2**

(ii) Draw a possible sketch of $y=f'(x)$ **2**

Question 9 (12 marks) Use a separate writing booklet

Marks

(a) How many terms of the series

$$6 + 3 + \frac{3}{2} + \ldots.$$

must be taken to give a sum of $11\frac{13}{16}$? **3**

(b) By using appropriate series techniques, evaluate the expression below, giving your answer as a rational number

$$\frac{1 + 2 + 3 + \ldots\ldots\ldots\ldots + 10}{1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\ldots\ldots\ldots\frac{1}{512}}$$ **3**

(c) (i) For what range of values of x does the series

$$1 + (x-2) + (x-2)^2 + \ldots\ldots\ldots\ldots$$

have a sum to infinity? **2**

(ii) Find an expression for this infinite sum. **1**

(d) Show that $\frac{d}{dx} x \log_e x = 1 + \log_e x$ **1**

Hence, find the exact value of

$$\int_1^5 \log_e x \, dx$$ **2**

Question 10 (12 marks) Use a separate writing booklet **Marks**

(a) The shaded region *OABC* is bounded by the lines $x = 0$, $x = 5$, the curve $y = 3x^2$, the line $y = 4 - x$ and the x axis, as in the diagram.

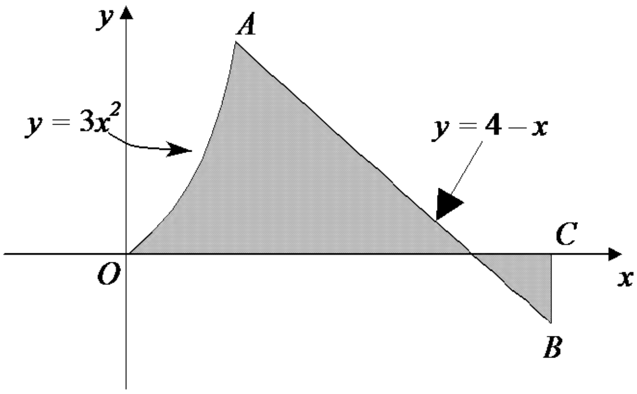
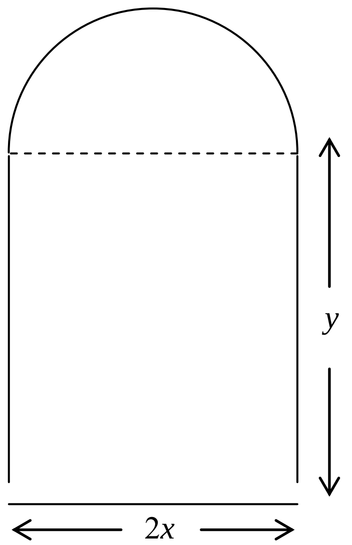


FIGURE NOT TO SCALE

i. Show that A has coordinates (1 , 3) **1**

ii. What is the area of the shaded region *OABC*? **3**

(b)



A window is in the shape of a rectangle and a semi circle as shown. The dimensions of the rectangle are $2x$ metres and y metres. The perimeter of the window is 4 metres.

(i) Show that $y = 2 - x - \dfrac{\pi x}{2}$ **2**

(ii) Show that the area , A, of the window is given by $A = 4x - 2x^2 - \dfrac{\pi x^2}{2}$ **2**

(iii) Find the exact value of x which will make the area of the window a maximum. **2**

(iv) Show that the maximum occurs when $x = $ y **2**