Total marks – 120
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of $\frac{\sqrt{12.35-8.68}}{6.5+2.9}$ correct to 2 decimal places. **2**

(b) Simplify $\frac{2x}{3}-\frac{x+2}{5}$ **2**

(c) Rationalise the denominator of the expression $\frac{5}{1-\sqrt{3}}$. **2**

(d) Differentiate x^3-7x **1**

(e) Solve $|x-1|=2x-1$ **3**

(f) A girl deposits $1200 in a bank account, which pays 9% per annum. How much will be in the account after 10 years, if interest is calculated monthly? **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate

(i) $\sqrt{2x+1}$ **2**

(ii) $x \ln x$ **2**

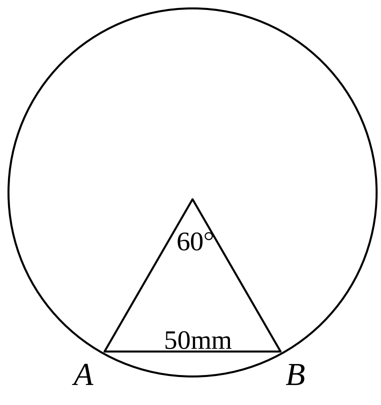
(iii) $\dfrac{\ln x}{\sin x}$ **2**

(b) Find

(i) $\int 3e^{3x} dx$ **1**

(ii) $\int \sec^2 3x dx$ **1**

(c) A circle has a chord of 50 mm, which subtends an angle of $60°$ at the centre. Find, to 1 decimal place, the length of the arc cut off by the chord AB. **2**



(d) Show that $\tan\beta - \sin^2\beta\tan\beta = \sin\beta\cos\beta$. **2**

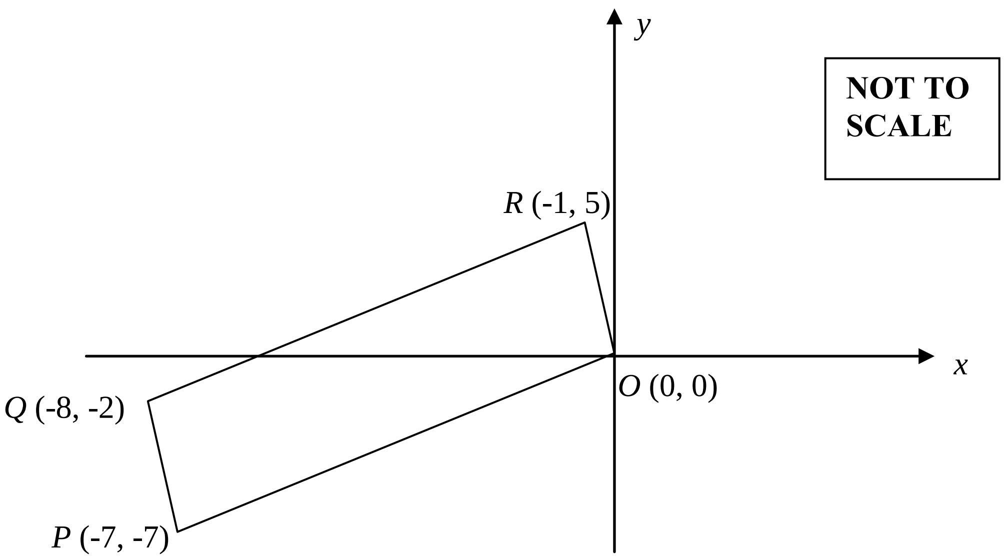
Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) For the equation $x^2 + (k+6)x - 2k = 0$:

(i) Prove the discriminant is $k^2 + 20k + 36$ **2**

(ii) Find the alues of k for which this equation has real roots. **2**

(b) In the diagram, $OPQR$ is a quadrilateral.



(i) Find the midpoint of the interval joining PR. **1**

(ii) Find the gradient of QR. **1**

(iii) Show that the equation of QR is $x - y + 6 = 0$ **1**

(iv) Find the exact length of QR. **1**

(v) Show that QR is parallel to PO. **1**

(vi) Find the exact perpendicular distance from O to QR. **2**

(vii) Hence find the area of parallelogram $OPQR$. **1**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) How many terms are there in the sequence: **2**

$-8, -3, 2, \ldots.., 122$?

(ii) What is the sum of this sequence? **2**

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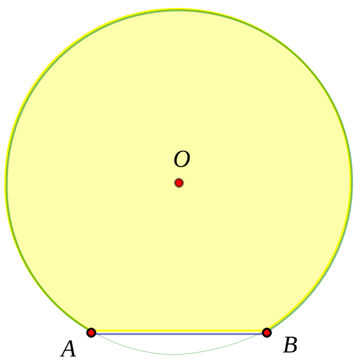
(b)



A new hallway table (shaded) is in the shape of a circle with a small segment removed as shown. The circle has a centre O and radius 80 centimetres. The length of the straight edge *AB* is also 80 centimetres.

(i) Explain why $\angle AOB = \dfrac{\pi}{3}$. **1**

(ii) Find the area of the hallway table. **3**

(c) The point *P* (*x*, *y*) moves in the *XY*-plane such that its distance from the point *R* (–1, 0) is always twice its distance from the point *S* (2, 0).

(i) Find the equation of the locus of point *P*. **2**

(ii) Describe the locus geometrically. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\sum_{n=2}^{6} n^3$ **1**

(b) The table provides values for the continuous function $f(x)$ that has no turning points in the interval $3 \le x \le 7$. **3**

Use all the data to approximate $\int_3^7 f(x).dx$ using the Trapezoidal Rule.

x	3	4	5	6	7
y	1.21	2.83	4.15	5.32	6.21

(c) Solve the following equation for x: **3**

$$e^{2x} + 3e^x - 10 = 0$$

(d) A bag contains 7 red, 3 white, and 8 blue marbles.

(i) One marble is drawn at random from the bag. What is the probability that it is not blue? **1**

(ii) If 2 are drawn in succession with replacement, find the probability that the outcome will be a red followed by a blue. **2**

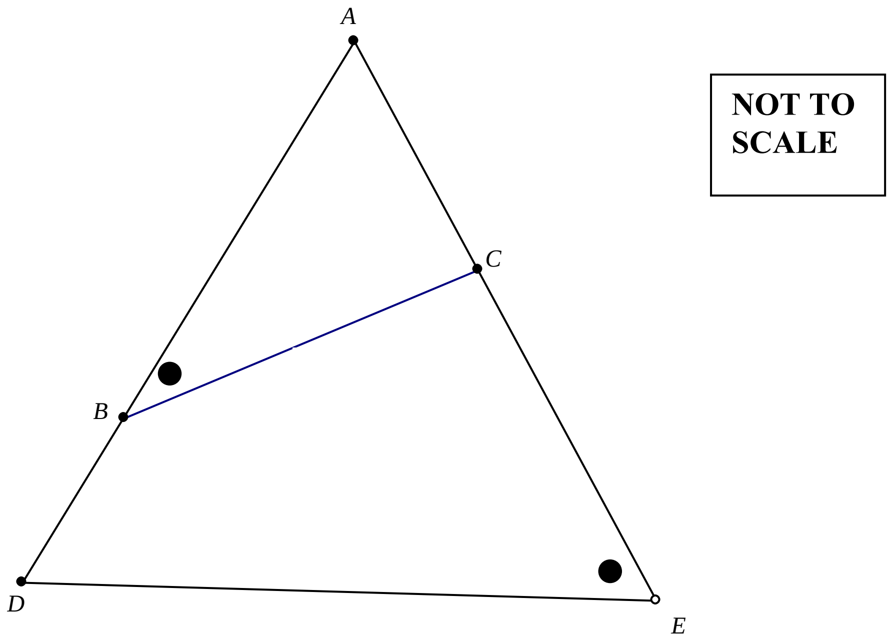
(iii) If 3 marbles are drawn without replacement, find the probability that they are all red. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Write down the exact value of $\sin 225°$. **1**

(ii) Find all possible values of θ, if $3\tan^2\theta - 1 = 0$ and $0 \le \theta \le 2\pi$. **3**

(b)



(i) Copy the diagram into your booklet.

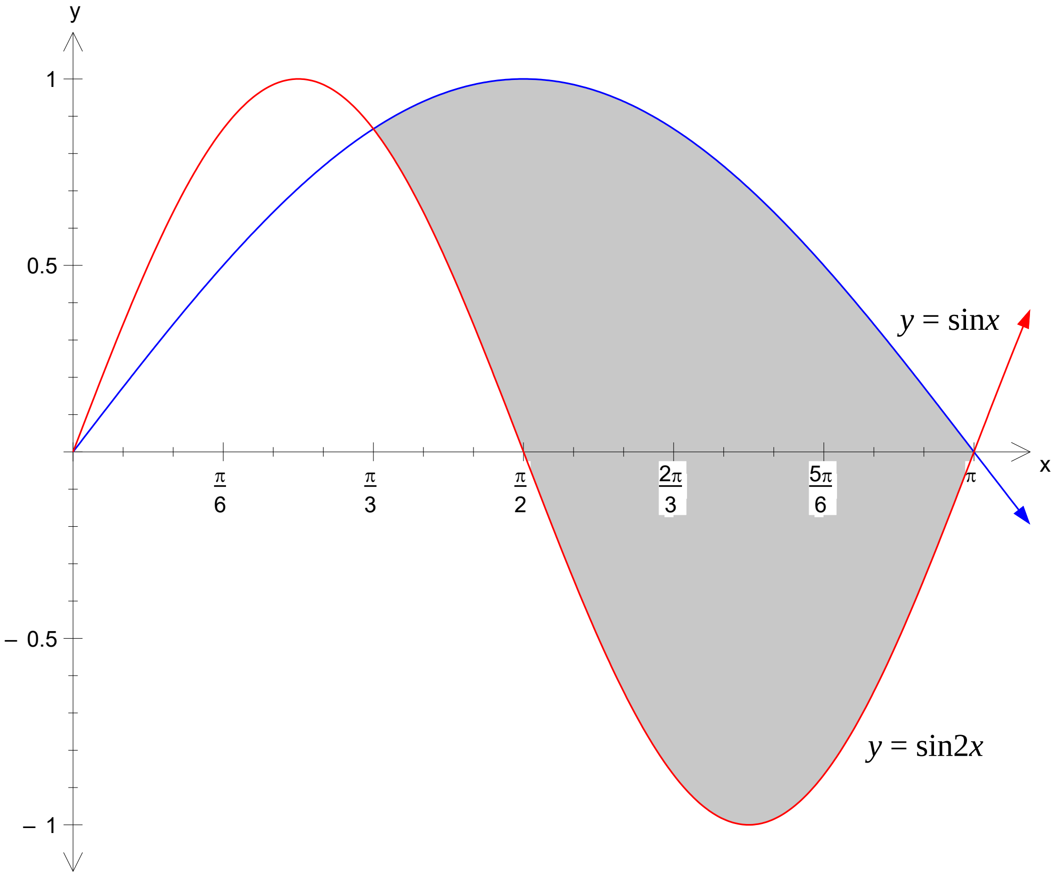
(ii) Prove that $\Delta ABC \,|||\, \Delta AED$. **2**

(iii) Given AB = 6 cm, BC = 5 cm, AC = 4 cm and CE = 8 cm, find the length of BD. **2**

Question 6 continues on next page

Question 6 (continued)

(c)



(i) Show that $x = \dfrac{\pi}{3}$ is a solution to the equation $\sin x = \sin 2x$. **1**

(ii) Find the exact shaded area in the diagram above. **3**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A curve is given by the function:

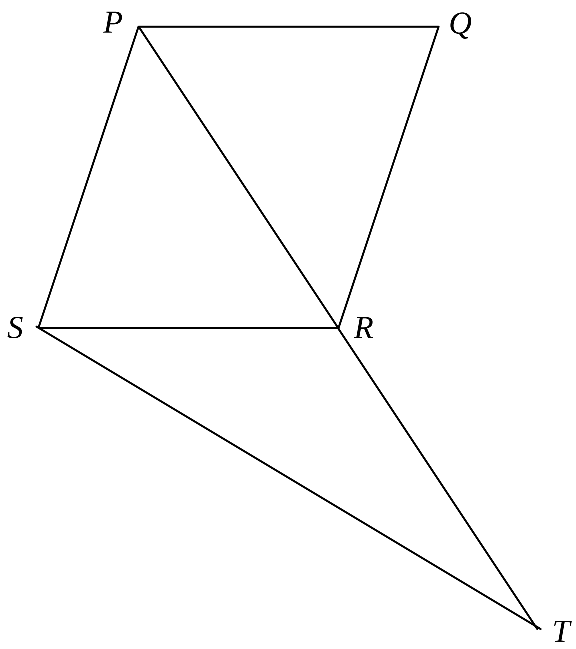
$$f(x) = x^3 - 2x^2 - 4x + 8$$

(i) Find the x-intercepts of the curve. **2**

(ii) Find the turning points of the curve and determine their nature. **3**

(iii) For what values of x is the function concave down? **1**

(b) *PQRS* is a rhombus. *PR* is produced to *T* such that $SR = TR$.



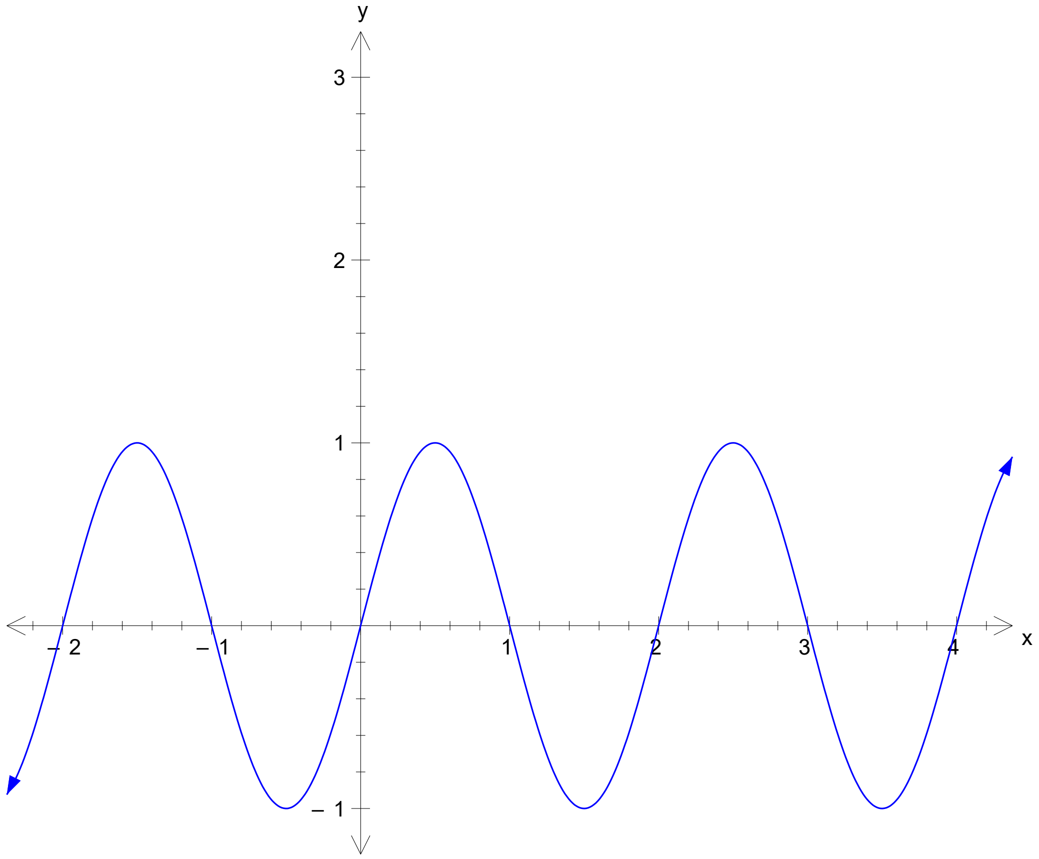
(i) Show that $\angle SPQ = 4\angle STR$. **3**

(ii) Show that R is the midpoint of PT, given that $\angle PST = 90°$. **3**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Give, in simplest form, the exact value of $\int_1^e \frac{dx}{3x}$ **3**

(b) The graph below is of the curve $y = \sin \pi x$.



(i) Find the axis of symmetry for $y = x^2 - 4x + 3$ **1**

(ii) Copy the graph into your answer booklet and on the same number plane graph the curve $y = x^2 - 4x + 3$ **2**

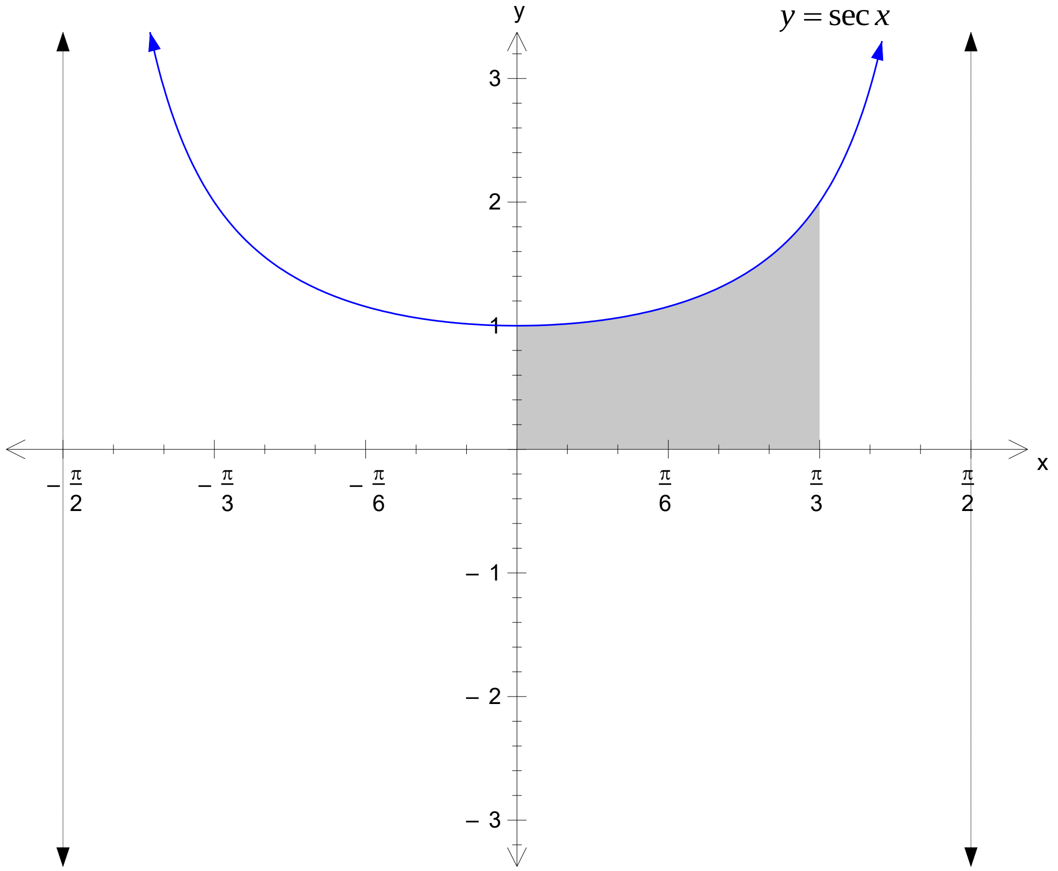
(iii) Determine the range of $y = x^2 - 4x + 3$ **2**

(iv) Hence find the number of real solutions to the equation $\sin \pi x = x^2 - 4x + 3$, for $0 \le x \le 2$. **1**

Question 8 continues on next page

Question 8 (continued) **Marks**

(c)



The shaded region that lies between the x-axis and the curve $y = \sec x$, from $x = 0$ to $x = \frac{\pi}{3}$, is rotated about the x-axis to form a solid. **3**

Find the volume of the solid correct to 1 decimal place.

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the equation of the normal to the curve $y = x\sin x$ at the point where $x = \frac{\pi}{2}$. **3**

(b) Solve $\ln(8x - 12) = 2\ln x$ **2**

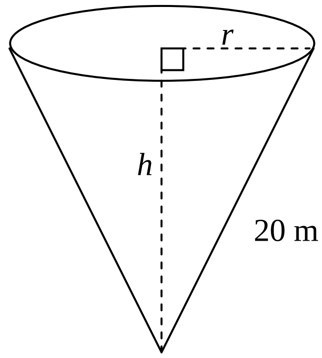
(c) (i) Show that $\frac{3x-5}{x^2-25} = \frac{1}{x-5} + \frac{2}{x+5}$ **2**

(ii) Hence, find $\int \frac{3x-5}{x^2-25}\,dx$ **2**

(d) A little green frog jumps 0.3 metres. It then jumps 0.2 metres, and on each subsequent jump it travels half of the distance of the previous jump. Find the total distance through which the little green frog jumps. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A cone shaped storage unit is constructed with a slant height of 20 metres.



(i) Show that the volume can be expressed as **3**

$$V = \frac{\pi}{3}(400h - h^3)$$

where h is the perpendicular height.

(ii) Hence find the value of h, such that the cone will have a maximum volume. **3**

(b) John borrows \$300 000 at 6% p.a. interest rate. He aims to pay the loan back in equal monthly instalments of \$$M$ over 25 years.

(i) How many instalments will John make? **1**

(ii) Show that immediately after making his first monthly instalment, John owed **1**

$$A_1 = \$[300000 \times 1.005] - M$$

where A_1 is the amount owing after 1 month.

(iii) Show that immediately after making his third monthly instalment, John owed **2**

$$A_3 = \$[300000 \times 1.005^3 - M(1 + 1.005 + 1.005^2)]$$

(iv) Calculate the value of M. **3**

End of Examination