



2006
INTERNAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

Question One

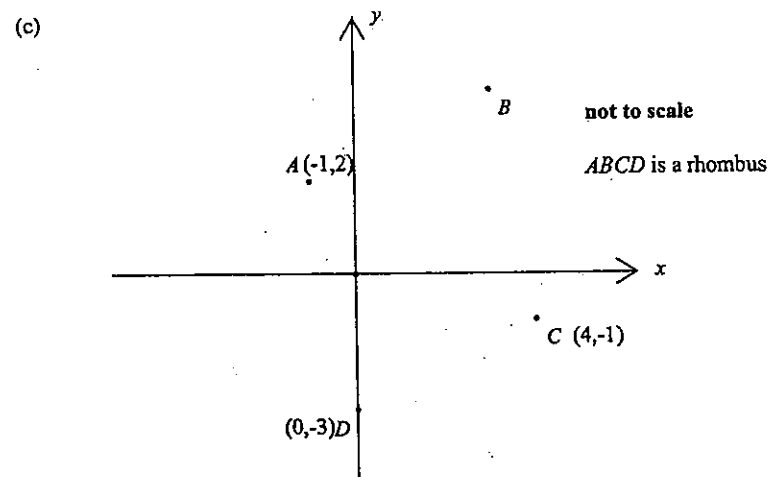
- (a) Evaluate $\sqrt[3]{\frac{315.6}{15.7+21}}$ correct to two decimal places 2
- (b) Factorise $8x^3 + 1$ 2
- (c) Find the integral of $2\sin x + x$ 2
- (d) Express $\frac{x-1}{3} - \frac{4x-1}{2}$ in simplest form. 2
- (e) If $x^2 \leq 16$, find values of x for which this will be true. 2
- (f) For the circle $x^2 + (y+1)^2 = 3$, write down :
(i) the radius 1
(ii) the centre 1

Question Two Use a SEPARATE writing booklet.
Marks

- (a) Solve $\tan \theta = \frac{1}{\sqrt{3}}$ $0 \leq \theta \leq 2\pi$ 2
- (b) Differentiate with respect to x : 2
- (i) $x^2 \cos x$ 2
- (ii) $\frac{x^3}{x+1}$ 2
- (c) (i) Find $\int \frac{8x^3}{x^4-2} dx$ 2
- (ii) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x dx$ (give your answer correct to 3 decimal places). 2
- (d) Find the equation of the tangent to $y = e^x$ at the point $(0,1)$. 2

Question Three Use a SEPARATE writing booklet.
Marks

- (a) Evaluate $\sum_{n=10}^{13} (n^2 + 1)$ 1
- (b) The sides of a triangle are 10cm, 11cm and 12 cm. 2
- (i) Find the size of the smallest angle. 2
- (ii) Find the area of the triangle 1

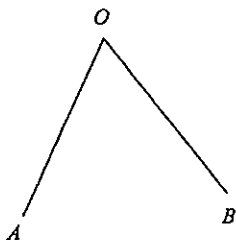


- (i) Show that the gradient of AD is -5 1
- (ii) Show that the equation of AD is $5x + y + 3 = 0$ 2
- (iii) Find the midpoint of AC 1
- (iv) Use your answer in (iii) to find the coordinates of B . 2
- (v) Given that AC has length $\sqrt{34}$ find the area of $ABCD$ 2

Question Four Use a SEPARATE writing booklet.

Marks

- (a) In the diagram $OA = OB = 1.2\text{m}$. The $\angle BAO = 0.7$ radians. O is the centre of the circle. A and B are points on a circle, centred at O .



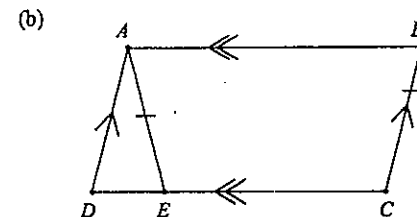
- (i) Find the length of the arc AB . 1
- (ii) Find the straight line distance AB . 2
- (iii) Find the area of the sector OAB . 1

- (b) A function $f(x)$ is defined by $f(x) = (x-2)(x^2-4)$

- (i) Find all solutions of $f(x) = 0$ 2
- (ii) Find any turning points of $y = f(x)$, and determine their nature 3
- (iii) Sketch $y = f(x)$ showing turning points and points where the curve cuts the axes. 2
- (iv) For what values of x is $y = f(x)$ concave down? 1

Question Five Use a SEPARATE writing booklet.
Marks

- (a) Use the change of base law to evaluate $\log_5 9$ correct to two decimal places. 2



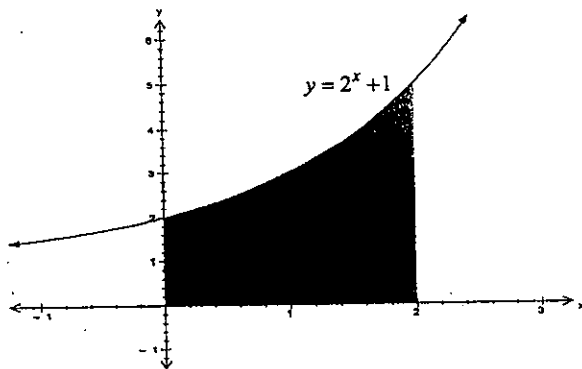
$ABCD$ is a parallelogram and $\angle ABC = 60^\circ$. AE is drawn to meet OD such that $AE = BC$.

- (i) Copy the diagram into your answer booklet
- (ii) Prove that $\triangle AED$ is equilateral. 3
- (c) (i) If $y = \ln(x+1)$, find the gradient function. 1
- (ii) Find any points on the curve $y = \ln(x+1)$ at which the tangent is parallel to $y = \frac{1}{2}x + 3$. 2
- (d) (i) Factorise $2a^2 - 7a + 3$. 1
- (ii) Hence solve $2(\log_2 x)^2 - 7\log_2 x + 3 = 0$ 3

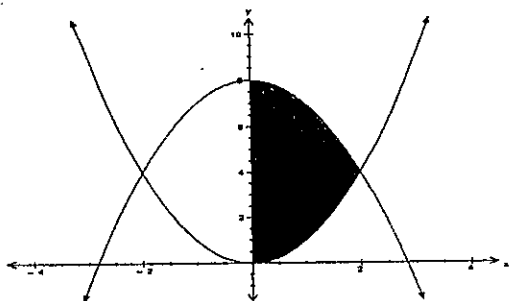
Question Six Use a SEPARATE writing booklet.

Marks

- (a) Use Simpson's Rule with 3 function values to find the shaded area. 3



- (b) The curves $y = x^2$ and $y = 8 - x^2$ are sketched below



- (i) Find the points of intersection of the two curves. 2
- (ii) The shaded area between the curves and the y -axis is rotated about the y -axis. 4
By splitting the shaded area into 2 parts, or otherwise, find the volume of the solid formed
- (c) If $y = \ln\left(\frac{\sqrt{x-1}}{x^3+5}\right)$, find $\frac{dy}{dx}$. 3

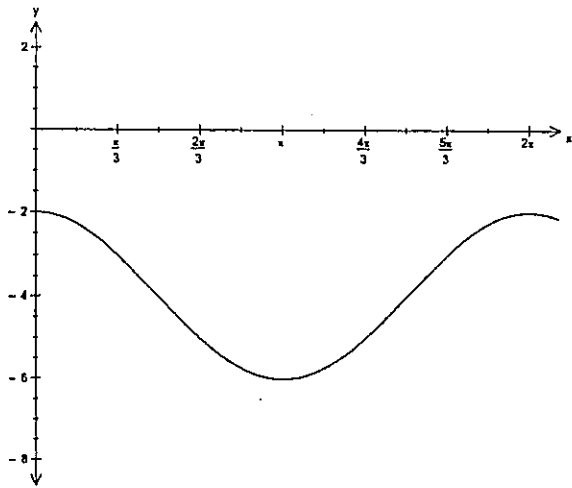
Question Seven Use a SEPARATE writing booklet.

Marks

- (a) (i) Sketch the graph of $y = \sin x$, for $0 \leq x \leq 2\pi$ 1
- (ii) Solve $\sin x = \frac{1}{2}$, for $0 \leq x \leq 2\pi$ 2
- (iii) Hence find the values of x for which $\sin x < \frac{1}{2}$, for $0 \leq x \leq 2\pi$ 2
- (b) In an arithmetic sequence $T_{10} = 35$ and $T_{16} = 59$
- (i) find the value of a . 1
- (ii) find the value of d . 1
- (iii) find T_{100} 1
- (iv) What will be the value of the first term that is greater than 750? 1
- (c) A geometric series has a first term of 4 and a limiting sum of 12. 3
Find the common ratio.

Question Eight Use a SEPARATE writing booklet.
Marks

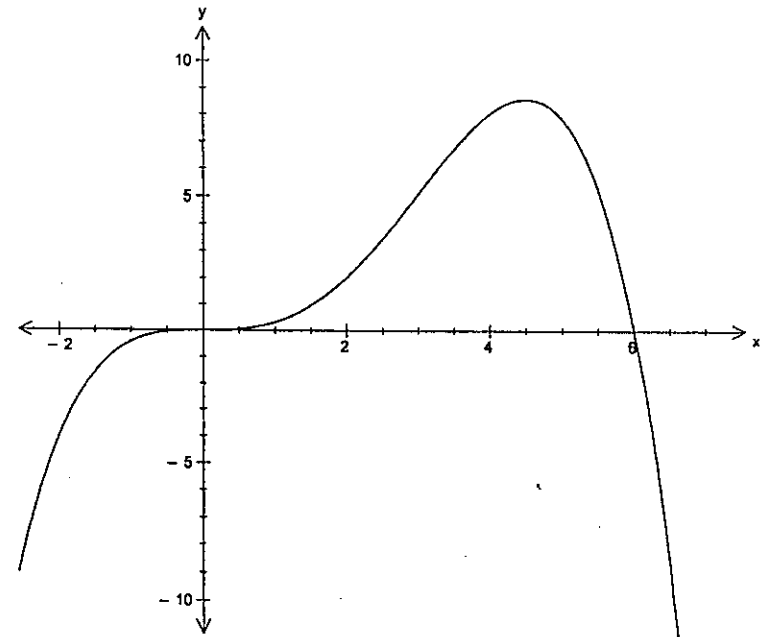
- (a) Find the value(s) of m for which the equation $x^2 + 4mx + 8 - 4m = 0$ has equal roots. 3
- (b) A graduate earns \$48000 per annum in her first year and then in each successive year her salary rises by \$2400.
- (i) What is her salary in the 10th year? 2
- (ii) What are her total earnings over the ten years? 1
- (c) The graph of $A + B \cos Cx$ is given below.



- (i) State the period of the curve. 1
- (ii) Hence or otherwise determine the values of A , B and C . 3
- (d) Over what domain is the function $y = \sqrt{x^2 - 9}$ defined? 2

Question Nine Use a SEPARATE writing booklet. Marks

- (a) A cylindrical can of radius r centimetres and height h centimetres is to be made from a sheet of metal with area 270π square centimetres.
- (i) Show that $h = \frac{135 - r^2}{r}$ 2
- (ii) Show that the volume V is given by $V = \pi r(135 - r^2)$ 1
- (iii) Calculate the maximum volume, justifying your answer. 3
- (b) Given that $y = xe^{-2x}$, prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$. 4
- (c) Copy the graph of $y = f(x)$ into your answers. Sketch the graph of the gradient function. 2



Question Ten Use a SEPARATE writing booklet.
Marks

- (a) (i) Given that $a^2 + b^2 = 23ab$, express $\left(\frac{a+b}{5}\right)^2$ in terms of ab . 2
- (ii) Hence show that $\log\left[\frac{1}{5}(a+b)\right] = \frac{1}{2}(\log a + \log b)$. 2
- (b) Susie borrows \$250 000 to be repaid over a period of 25 years at 6% per annum reducible interest. Each year there are k regular repayments of \$ F . Interest is calculated and charged just before each repayment.
- (i) Write down an expression for the amount owing after two repayments. 2
- (ii) Show that the amount owing after n repayments is 2
$$A_n = 250000\alpha^n - \frac{kF(\alpha^n - 1)}{0.06}, \text{ where } \alpha = 1 + \frac{0.06}{k}$$
- (iii) Calculate the amount of each repayment if the repayments are made quarterly (ie. $k = 4$). 2
- (iv) How much would Susie have saved over the term of the loan if she had chosen to make monthly rather than quarterly repayments? 2



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Question One

(a) Evaluate $\sqrt[3]{\frac{315.6}{15.7+21}} = 2.05$ (2d.p.) correct to two decimal places

(b) Factorise $8x^3 + 1 = (2x+1)(4x^2 - 2x + 1)$

(c) $\int (2 \sin x + x) dx = -2 \cos x + \frac{x^2}{2} + C$

(d) $\frac{x-1}{3} - \frac{4x-1}{2} = \frac{2(x-1) - 3(4x-1)}{6}$
 $= \frac{-10x+1}{6}$

(e) If $x^2 \leq 16$, then $-4 \leq x \leq 4$

(f) For the circle $x^2 + (y+1)^2 = 3$, write down:

(i) the radius = $\sqrt{3}$

(ii) the centre = (0, -1)

Question Two Use a SEPARATE writing booklet.

Marks

(a) $\tan \theta = \frac{1}{\sqrt{3}}$ $0 \leq \theta \leq 2\pi$

related angle = $\frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

(b) Differentiate with respect to x:

(i) $y = x^2 \cos x$ $u = x^2 \rightarrow u' = 2x$ $v = \cos x \rightarrow v' = -\sin x$

$\frac{dy}{dx} = vu' + uv'$

$= 2x \cos x - x^2 \sin x$

(ii) $y = \frac{x^3}{x+1}$ $u = x^3 \rightarrow u' = 3x^2$ $v = x+1 \rightarrow v' = 1$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{3x^2(x+1) - x^3}{(x+1)^2}$

$= \frac{2x^3 + 3x^2}{(x+1)^2}$

(c) (i) $\int \frac{8x^3}{x^4-2} dx = 2 \int \frac{4x^3}{x^4-2} dx$
 $= 2 \ln(x^4-2) + C$

(i) $\int_0^{\frac{\pi}{3}} \sin 2x dx = -\left[\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{3}}$

$= -\frac{1}{2} \left(\cos \frac{2\pi}{3} - \cos 0 \right)$

$= 0.75$

(d) Find the equation of the tangent to $y = e^x$ at the point (0,1).

$y = e^x \rightarrow \frac{dy}{dx} = e^x$ (0,1)

$m = e^0 = 1$

$y - 1 = 1(x - 0)$

$y = x + 1$

Question Three Use a SEPARATE writing booklet.
Marks

$$\text{Area} = \frac{1}{2} \times \sqrt{58} \times \sqrt{34} = \sqrt{493} u^2 \quad \boxed{\checkmark} \boxed{\checkmark}$$

(a) $\sum_{n=10}^{13} (n^2 + 1) = 100 + 1 + 121 + 1 + 144 + 1 + 169 + 1 = 538 \quad \boxed{\checkmark}$

(b) The sides of a triangle are 10cm, 11cm and 12 cm.

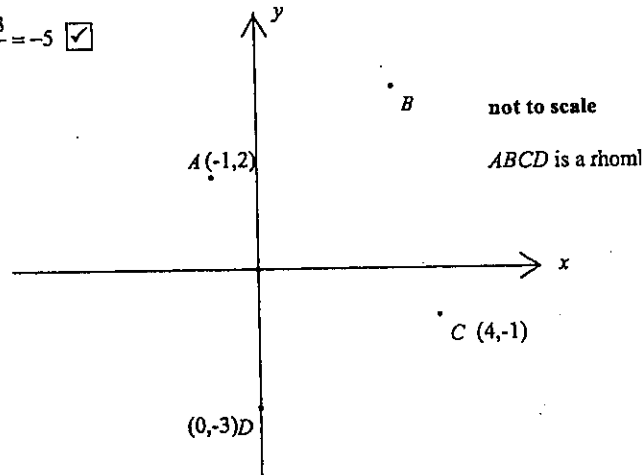
(i) $\cos \theta = \frac{10^2 + 11^2 - 12^2}{2 \times 10 \times 11} \quad \boxed{\checkmark}$

$$\cos \theta = \frac{77}{220}$$

$$\theta = 50^\circ 5' \text{ (nearest minute)} \quad \boxed{\checkmark}$$

(ii) $A = \frac{1}{2} \times 10 \times 11 \times \sin 50^\circ 5' = 42.18 \text{ cm}^2 \text{ (2dp)} \quad \boxed{\checkmark}$

(c) (c) (i) $m(AD) = \frac{2 - -3}{0 - 1} = -5 \quad \boxed{\checkmark}$



(ii) $m = -5, (0, -3) \rightarrow y + 3 = -5(x - 0) \rightarrow 5x + y + 3 = 0 \quad \boxed{\checkmark} \boxed{\checkmark}$

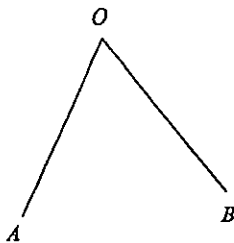
(iii) midpoint of AC = $\left(\frac{-1+4}{2}, \frac{2+-1}{2} \right) = \left(\frac{3}{2}, \frac{1}{2} \right) \quad \boxed{\checkmark}$

(iv) Since this is rhombus the diagonals bisect, so the midpoints of BC and AD are the same point. Let B = (x, y)

$$\frac{x+0}{2} = \frac{3}{2} \rightarrow x = 3, \frac{y+-3}{2} = \frac{1}{2} \rightarrow y = 4 \therefore B = (3, 4) \quad \boxed{\checkmark} \boxed{\checkmark}$$

(v) $BD = \sqrt{(3-0)^2 + (4--3)^2} = \sqrt{58}$

- Question Four** Use a SEPARATE writing booklet. **Marks**
 (a) In the diagram $OA = OB = 1.2\text{m}$. The $\angle BOA = 0.7$ radians. O is the centre of the circle. A and B are points on a circle, centred at O .



- (i) length of the arc $AB = 1.2 \times 0.7 = 0.84\text{ cm}$
 (ii) Let the straight line distance $AB = d$
 $d^2 = 1.2^2 + 1.2^2 - 2 \times 1.2 \times 1.2 \times \cos 0.7$
 $d = 0.83\text{ cm}$ (2d.p.)
 (iii) Area of the sector $OAB = \frac{1}{2} \times 1.2^2 \times 0.7 = 0.504\text{ cm}^2$

- (b) A function $f(x)$ is defined by

(i) $f(x) = (x-2)(x^2-4) = (x-2)(x+2)(x-2) = 0$

$x = 2, x = -2$

(ii) $f(x) = (x-2)(x^2-4) = x^3 - 2x^2 - 4x + 8$

$f'(x) = 3x^2 - 4x - 4$

$f'(x) = (x-2)(3x+2) = 0$ for stationary points

$x = 2, x = -\frac{2}{3}$

$(2, 0)$ and $(-\frac{2}{3}, \frac{256}{27})$

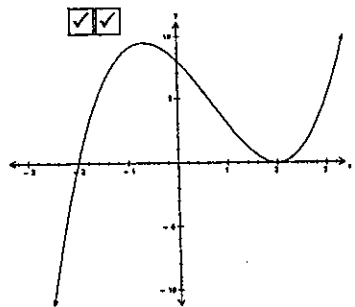
$f''(x) = 6x - 4 \rightarrow f''(2) = 6 \times 2 - 4 > 0$ concave up, minimum

$f''(x) = 6x - 4 \rightarrow f''(-\frac{2}{3}) = 6 \times -\frac{2}{3} - 4 < 0$ concave down, maximum

- (iii)

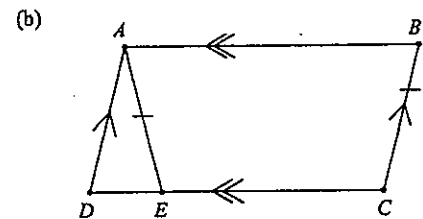
(iv) $f''(x) = 6x - 4 = 0 \rightarrow x = \frac{2}{3}$

concave down if $x < \frac{2}{3}$



- Question Five** Use a SEPARATE writing booklet. **Marks**

(a) $\log_5 9 = \frac{\log_{10} 9}{\log_{10} 5} = 1.37$ (2d.p.)



$ABCD$ is a parallelogram and $\angle ABC = 60^\circ$. AE is drawn to meet OD such that $AE = BC$.

- (ii) Prove that $\triangle AED$ is equilateral

$\angle ADE = 60^\circ$ (opposite angles of a parallelogram are equal)

$BC = AD$ (opposite sides of a parallelogram are equal)

$BC = AE$ (given)

$AD = AE$ so $\triangle AED$ is isosceles and $\therefore \angle ADE = \angle AED = 60^\circ$

$\angle DAE = 60^\circ$ (angle sum of triangle)

so all angles are equal and $\triangle AED$ is equilateral

- (c) (i) If $y = \ln(x+1)$, find the gradient function.

$\frac{dy}{dx} = \frac{1}{x+1}$

- (ii) If $y = \frac{1}{2}x + 3$, then the gradient is $\frac{1}{2}$ and if they are parallel then

$\frac{1}{x+1} = \frac{1}{2} \rightarrow x = 1$

if $x = 1$ then $y = \ln 2$

so the point is $(1, \ln 2)$

- (d) (i) Factorise $2a^2 - 7a + 3 = (2a-1)(a-3)$

(ii) $2(\log_2 x)^2 - 7\log_2 x + 3 = 0$ let $a = \log_2 x$

$2a^2 - 7a + 3 = 0 \rightarrow (2a-1)(a-3) = 0$

$a = \frac{1}{2}, 3$

$\log_2 x = \frac{1}{2}, \log_2 x = 3$

$x = 2^{\frac{1}{2}}, x = 2^3$

$x = \sqrt{2}, 8$

Question Six Use a SEPARATE writing booklet.

Marks

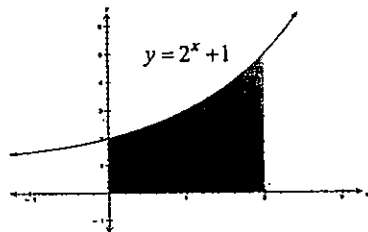
- (a) Use Simpson's Rule with 3 function values to find the shaded area. 3

$$A = \frac{b-a}{6} \left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right)$$

$$A = \frac{2-0}{6} (f(0) + 4f(1) + f(2)) \quad \checkmark$$

$$A = \frac{1}{3} (2 + 4 \times 3 + 5) \quad \checkmark$$

$$A = 6 \frac{1}{3} u^2 \quad \checkmark$$



- (b) The curves $y = x^2$ and $y = 8 - x^2$ are sketched below

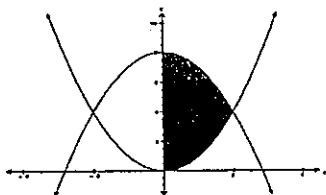
(i) $x^2 = 8 - x^2$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = \pm 2 \quad \checkmark$$

$$(2, 4) \text{ and } (-2, 4) \quad \checkmark$$



(ii) $V_1 = \int_0^2 x^2 dy$

$$= \int_0^2 y dy$$

$$= \left[\frac{y^2}{2} \right]_0^2 = 2 - 0 = 2$$

$$V_1 = \int_0^2 x^2 dy$$

$$= \pi \int_0^2 y dy = \pi \left[\frac{y^2}{2} \right]_0^2 = \pi(2 - 0) = 2\pi \quad \checkmark$$

$$V_2 = \pi \int_2^4 x^2 dy \rightarrow y = 8 - x^2 \rightarrow x^2 = 8 - y$$

$$= \pi \int_2^4 (8 - y) dy = \pi \left[8y - \frac{y^2}{2} \right]_2^4 = \pi(32 - 8 - (16 - 2)) = 10\pi \quad \checkmark$$

$$V = V_1 + V_2 = 2\pi + 10\pi = 12\pi u^3 \quad \checkmark$$

(c) $y = \ln\left(\frac{\sqrt{x-1}}{x^3+5}\right) \rightarrow y = \ln(x-1)^{\frac{1}{2}} - \ln(x^3+5) = \frac{1}{2}\ln(x-1) - \ln(x^3+5) \quad \checkmark$

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{x-1} + \frac{3x^2}{x^3+5}$$

$$= \frac{1}{2(x-1)} + \frac{3x^2}{x^3+5} \quad \checkmark \checkmark$$

Question Seven Use a SEPARATE writing booklet.

Marks

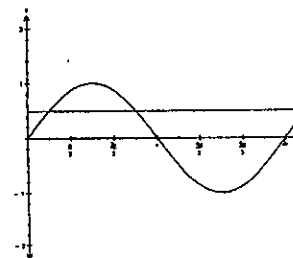
- (a) (i) Sketch the graph of $y = \sin x$, for $0 \leq x \leq 2\pi$

(ii) $\sin x = \frac{1}{2}$

related angle = $\frac{\pi}{6} \quad \checkmark$

$$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad \checkmark$$



(iii) $0 < x < \frac{\pi}{6} \quad \checkmark$ and $\frac{5\pi}{6} < x < 2\pi \quad \checkmark$,

- (b) In an arithmetic sequence $T_{10} = 35$ and $T_{16} = 59$

(i) and (ii) $a + 9d = 35$ and $a + 15d = 59$

$$a + 15d = 59$$

$$\frac{a + 9d = 35}{6d = 24}$$

$$6d = 24$$

$$d = 4 \rightarrow a + 9(4) = 35 \rightarrow d = -1 \quad \checkmark \checkmark$$

(iii) $T_{100} = a + 99d = -1 + 99 \times 4 = 395 \quad \checkmark$

(iv) $T_n = -1 + (n-1)4 > 750$

$$4n - 4 > 751 \rightarrow 4n > 755 \rightarrow n > \frac{755}{4} \rightarrow n = 189 \quad \checkmark$$

- (c) A geometric series has a first term of 4 and a limiting sum of 12. Find the common ratio.

$$S_{\infty} = \frac{a}{1-r}$$

$$12 = \frac{4}{1-r} \quad \checkmark$$

$$12(1-r) = 4$$

$$1-r = \frac{1}{3}$$

$$r = \frac{2}{3} \quad \checkmark \checkmark$$

Question Eight Use a SEPARATE writing booklet.
Marks

(a) Find the value(s) of m for which the equation $x^2 + 4mx + 8 - 4m = 0$ has equal roots.

Equal roots if $b^2 - 4ac = 0$

$(4m)^2 - 4 \times 1 \times (8 - 4m) = 0$

$16m^2 - 32 + 16m = 0$

$m^2 + m - 2 = 0$

$(m+2)(m-1) = 0$

$m = -2, 1$

(b) A graduate earns \$48000 per annum in her first year and then in each successive year her salary rises by \$2400.

(i) What is her salary in the 10th year?

$a = 48000, d = 2400, T_n = a + (n-1)d$

$a = 48000, d = 2400, T_{10} = 48000 + 9 \times 2400 = \69600

(ii) What are her total earnings over the ten years?

$a = 48000, d = 2400, S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = 5(2 \times 48000 + 9 \times 2400) = \588000

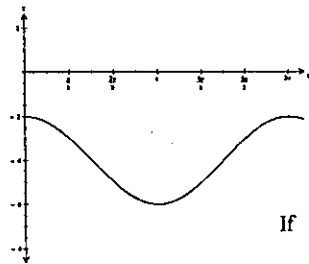
(c) The graph of $A + B \cos Cx$ is given below.

(i) period = 2π

(ii) $A = -4, B = 2$ and $C = 1$

(d) Over what domain is the function $y = \sqrt{x^2 - 9}$ defined?

$y = \sqrt{x^2 - 9}$ then $x^2 - 9 \geq 0 \rightarrow x \leq -3, x \geq 3$



If

Question Nine Use a SEPARATE writing booklet.

Marks

(a) A cylindrical can of radius r centimetres and height h centimetres is to be made from a sheet of metal with area 270π square centimetres.

(i) Show that $h = \frac{135 - r^2}{r}$

$2\pi r^2 + 2\pi r h = 270\pi$

$r^2 + r h = 135$

$r h = 135 - r^2$

$h = \frac{135 - r^2}{r}$

(ii) Show that the volume V is given by $V = \pi r(135 - r^2)$

$V = \pi r^2 h$

$= \pi r^2 \frac{135 - r^2}{r}$

$= \pi r(135 - r^2)$

(iii) Calculate the maximum volume, justifying your answer.

$V = \pi r(135 - r^2)$

$\frac{dV}{dr} = 0$ for max vol

$\frac{d^2V}{dr^2} = \pi(-6r) < 0$

$= \pi(135r - r^3)$

$135 - 3r^2 = 0$

$r = 3\sqrt{5}$ is a max

$\frac{dV}{dr} = \pi(135 - 3r^2)$

$r^2 = 45 \rightarrow r = 3\sqrt{5}$

(b) Given that $y = xe^{-2x}$, prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$.

$y = xe^{-2x} \rightarrow u = x, v = e^{-2x} \rightarrow u' = 1, v' = -2e^{-2x}$

$\frac{dy}{dx} = e^{-2x} \times 1 + x \times -2e^{-2x}$

$= (e^{-2x} - 2xe^{-2x})$

$\frac{d^2y}{dx^2} = -2e^{-2x} - 2(e^{-2x} - 2xe^{-2x})$

$= -4e^{-2x} + 4xe^{-2x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = -4e^{-2x} + 4xe^{-2x} + 4(e^{-2x} - 2xe^{-2x}) + 4xe^{-2x}$

$= -4e^{-2x} + 4xe^{-2x} + 4e^{-2x} - 8xe^{-2x} + 4xe^{-2x}$

$= 0$

Question Ten Use a SEPARATE writing booklet.

Marks

- (a) (i) Given that $a^2 + b^2 = 23ab$, express $\left(\frac{a+b}{5}\right)^2$ in terms of ab .

$$\left(\frac{a+b}{5}\right)^2 = \frac{a^2 + b^2 + 2ab}{25} \quad \checkmark$$

$$= \frac{23ab + 2ab}{25} = ab \quad \checkmark$$

- (ii) Hence show that $\log\left[\frac{1}{5}(a+b)\right] = \frac{1}{2}(\log a + \log b)$.

$$\frac{a+b}{5} = \sqrt{ab}$$

$$\log\left(\frac{a+b}{5}\right) = \frac{1}{2}\log(ab) \quad \checkmark$$

$$\log\left(\frac{a+b}{5}\right) = \frac{1}{2}(\log a + \log b) \quad \checkmark$$

- (b) Susie borrows \$250 000 to be repaid over a period of 25 years at 6% per annum reducible interest. Each year there are k regular repayments of \$ F . Interest is calculated and charged just before each repayment.

- (i) Write down an expression for the amount owing after two repayments.

$$A_1 = 250000 \times \left(1 + \frac{0.06}{k}\right) - F \quad \checkmark$$

$$A_2 = \left(250000 \times \left(1 + \frac{0.06}{k}\right) - F\right) \times \left(1 + \frac{0.06}{k}\right) - F$$

$$A_2 = 250000 \times \left(1 + \frac{0.06}{k}\right)^2 - F \times \left(1 + \frac{0.06}{k}\right) - F$$

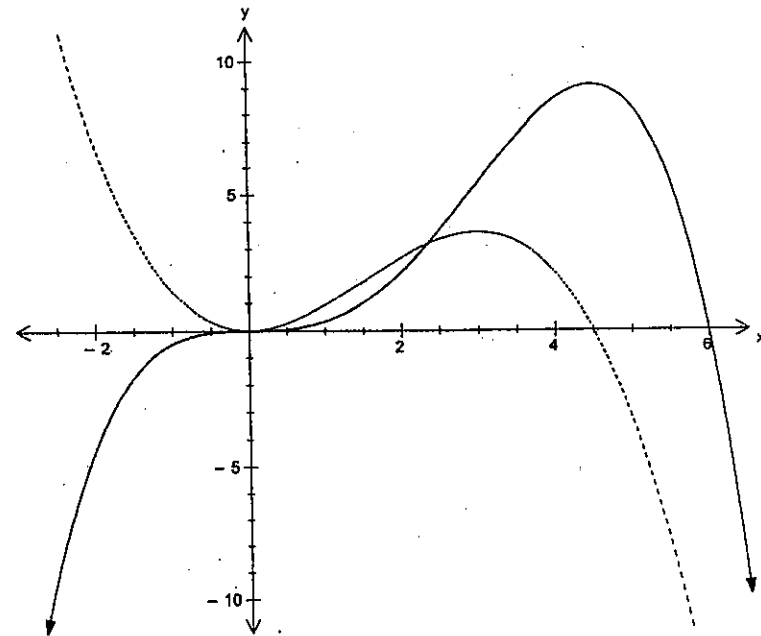
- (ii) Show that the amount owing after n repayments is

$$A_n = 250000 \left(1 + \frac{0.06}{k}\right)^n - F \left(1 + \left(1 + \frac{0.06}{k}\right) + \left(1 + \frac{0.06}{k}\right)^2 + \dots + \left(1 + \frac{0.06}{k}\right)^{n-1}\right) \quad \checkmark$$

$$= 250000 \left(1 + \frac{0.06}{k}\right)^n - F \left(\frac{\left(1 + \frac{0.06}{k}\right)^n - 1}{1 + \frac{0.06}{k} - 1}\right)$$

$$= 250000 \alpha^n - F \left(\frac{\alpha^n - 1}{\frac{0.06}{k}}\right) = 250000 \alpha^n - kF \left(\frac{\alpha^n - 1}{0.06}\right) \quad \checkmark$$

- (c) Copy the graph of $y = f(x)$ into your answers. Sketch the graph of the gradient function.



- (iii) Calculate the amount of each repayment if the repayments are made quarterly (ie. $k = 4$).
 $k = 4$ and $n = 100$ 2

$$A_{100} = 250000 \left(\frac{1+0.06}{4} \right)^{100} - 4F \left(\frac{\left(\frac{1+0.06}{4} \right)^{100} - 1}{0.06} \right) = 0$$

- (iv) How much would Susie have saved over the term of the loan if she had chosen to make monthly rather than quarterly repayments? 2