THE KING'S SCHOOL

2006
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

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Attempt Questions 1-10
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Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate, correct to four significant figures $\dfrac{1251.5 \times \pi}{1.4^2}$ **2**

(b) Factorise fully $18 - 32a^2$ **2**

(c) Find a primitive of $4x + \cos 2x$ **2**

(d) Find the exact value of 225° in radians in simplest form. **2**

(e) Solve $|2x - 1| \geq 3$ **2**

(f) Solve $\dfrac{x-5}{3} - \dfrac{x+1}{4} = -5$ **2**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\tan\theta = -\dfrac{1}{\sqrt{3}}$ for $0 \le \theta \le 2\pi$ **2**

(b) Differentiate with respect to x

(i) $2x\cos x$ **2**

(ii) $\dfrac{x^3}{2x + 1}$ **2**

(c) (i) Find $\displaystyle\int \frac{2x^3 + 1}{x^4 + 2x}\,dx$ **2**

(ii) Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \sin 3x\,dx$ **2**

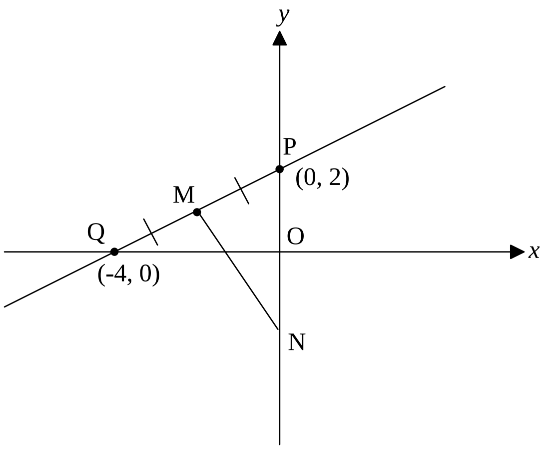
(d) Solve the simultaneous equations

$$x + 2y = -2 \quad \text{and} \quad 2x - y = 11$$ **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



The diagram shows the points P (0, 2) and Q (-4, 0). The point M is the midpoint of PQ. The line MN is perpendicular to PQ and meets the *y* axis at N.

(i) Show that the gradient of PQ is $\frac{1}{2}$. **1**

(ii) Find the coordinates of M. **2**

(iii) Find the equation of the line MN. **2**

(iv) Show that N has coordinates (0, -3). **1**

(v) Find the distance NQ. **1**

(vi) The point R lies in the second quadrant, and PNQR is a rhombus. Find the coordinates of R. **1**

(vii) Find the area of rhombus PNQR. **1**

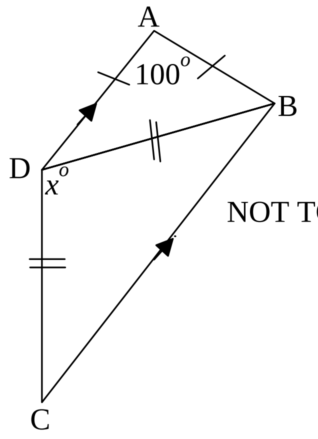
(b) Find the equation of the tangent to the curve $y = e^{x^2}$ at the point where $x = 1$. **3**

End of Question 3

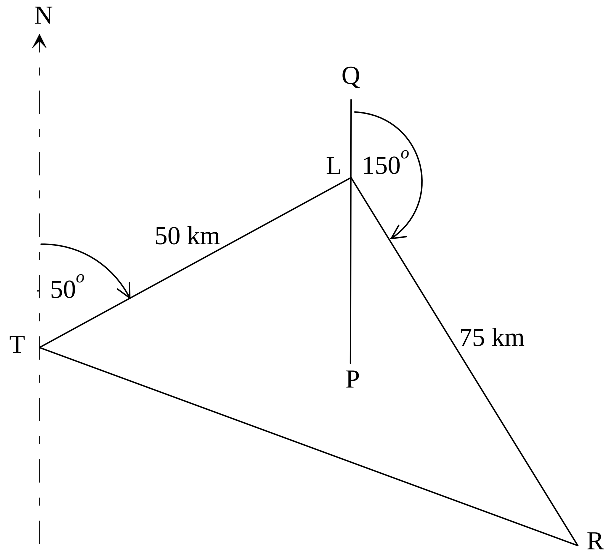
Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of x giving reasons for your answer. **3**

AB = AD
BD = DC and AD || BC



(b)



A motorist drives 50 km from town T to a landmark L on a bearing of 050°.
He then drives 75 km to a resort R on a bearing of 150°.

(i) Find the size of $\angle$ TLR. **1**

(ii) Find the distance TR to the nearest kilometre **2**

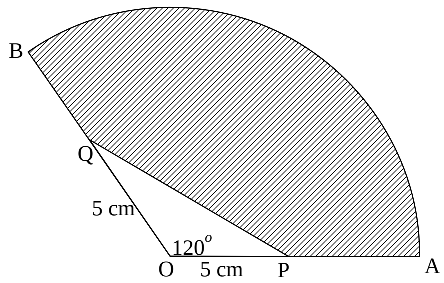
(c) Solve $\log_2 x + \log_2 (x - 1) = 1$ **3**

(d) Find $\frac{dy}{dx}$ when $y = \ln \sqrt{\frac{4x - 1}{2x + 3}}$ **3**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



O is the centre of the circle containing the arc AB. P is the midpoint of OA and Q is the midpoint of OB.

$\angle AOB = 120^\circ \qquad OQ = OP = 5 \text{ cm}$

(i) Find the exact length of the arc AB. **1**

(ii) Find the shaded area PQBA in exact form. **2**

(b) Find, justifying your answer, any point(s) of inflexion on the curve

$$y = \frac{1}{3}x^3 - x^2 + 1$$ **3**

(c) Water is taken out of a tank at the rate $R = 2t + \dfrac{200}{t + 4}$ litres/min.

Initially there were 1000 litres of water in the tank.

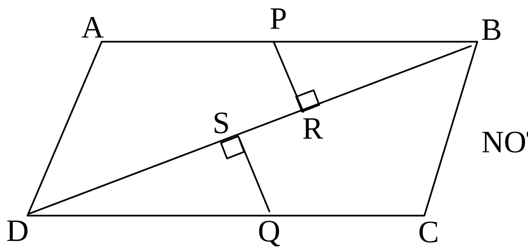
(i) At what time will the rate be a minimum? **3**

(ii) How many litres of water will be left in the tank after $24\frac{1}{2}$ minutes. **3**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



ABCD is a paralellogram and P, Q are the midpoints of the sides AB, DC respectively. The intervals PR and QS are drawn perpendicular to the diagonal BD.

(i) Prove the triangles BPR and DQS are congruent. **2**

(ii) If AB = 10 cm, PR = 3 cm and BD = 14 cm, find the length of SR. **2**

(b) The equation $(x - 2)^2 = -6\left(y - \frac{3}{2}\right)$ represents a parabola.

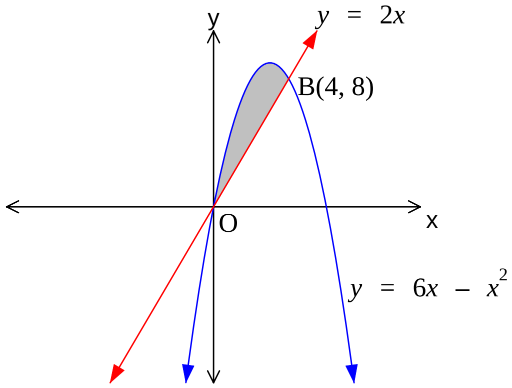
(i) Write down the coordinates of the vertex. **1**

(ii) Find the focal length. **1**

(iii) Find the x-intercepts of the parabola. **2**

(iv) Sketch the graph of the parabola showing the focus and directrix. **1**

(c)



The graphs of $y = 2x$ and $y = 6x - x^2$ intersect at the origin and at point B (4, 8).

Find the shaded area bounded by $y = 6x - x^2$ and $y = 2x$. **3**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The table shows the values where $y = 5xe^{-x}$.

x	0	1	2	3	4
y	0	1.84	1.35		0.37

(i) Find y when $x = 3$ correct to two decimal places. **1**

(ii) Find $\int_0^4 5x\,e^{-x}\,dx$ using Simpson's Rule with five function values, correct to one decimal place.. **2**

(b) The number, N, of bacteria in a culture is growing exponentially according to the formula

$$N = 120e^{kt}$$

(i) What is the initial number of bacteria? **1**

(ii) After eight hours the number of bacteria has tripled. Calculate the value of k to three decimal places. **2**

(iii) How many bacteria, correct to the nearest ten, will there be after twelve hours? **1**

(iv) At what rate will the bacteria be increasing after eighteen hours? **2**

(c) Sketch the curve $y = 1 - \sin 2x$ for $0 \le x \le \pi$ **3**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) **Without** using calculus, sketch the curve $y = e^x - 2$ **1**

(ii) On the same sketch, find graphically the number of solutions of the equation

$$e^x - x - 2 = 0$$

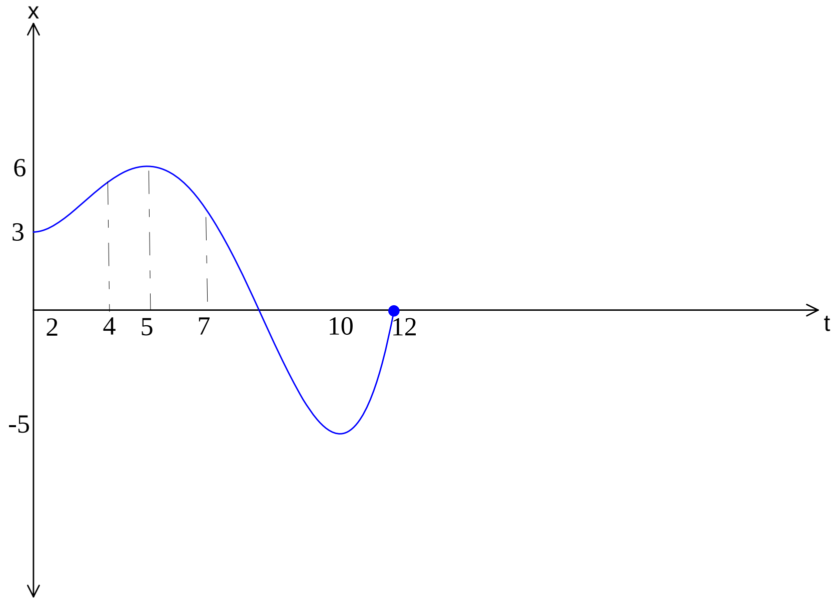
(Do not attempt to solve the equation.) **2**

(b) Some children have 500 identical toy blocks. They wish to build a triangular wall so that the top layer has 1 brick, the second layer has 2 bricks, the third layer has 3 bricks and so on. Assuming that the wall remains stable, find

(i) the maximum number of layers they can complete **3**

(ii) the number of bricks they have left over **1**

(c)



A particle moves along the x axis. The graph shows the displacement x metres of the particle, from a fixed point (origin) at time t seconds.

(i) What is the initial velocity of the particle? **1**

(ii) Between what times is the particle moving in a negative direction? **1**

(iii) How many metres did the particle travel? **1**

(iv) Sketch the velocity as a function of time. **2**

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) What are the values of a and b if $x - 3$ and $x + 7$ are factors of the quadratic expression

$$ax^2 + 12x + b$$ **2**

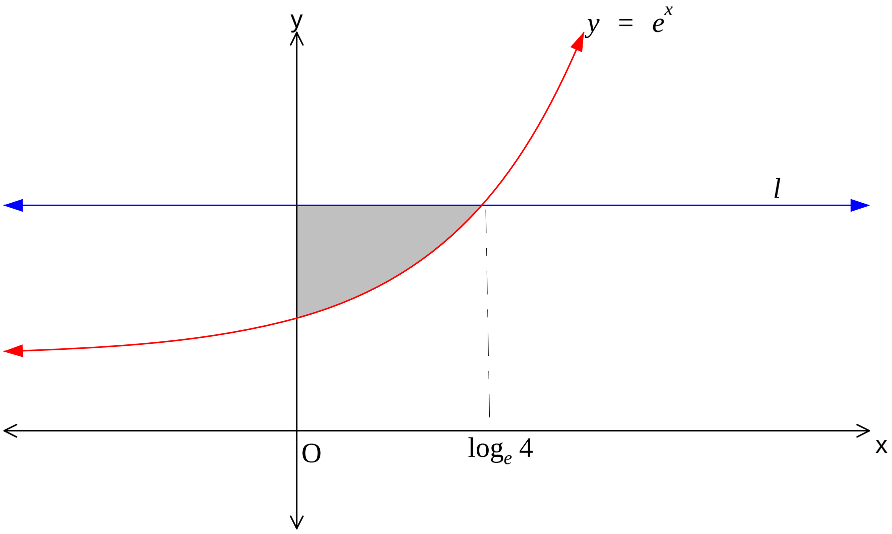
(b) A man sets up a trust fund for his grandson by investing $75 at the beginning of each month. The money is invested at 6% p.a. compounded monthly and it matures at the end of the month, 20 years after the first investment.

(i) After 20 years, what will be the value of the first $75 invested? **1**

(ii) Using geometric series, calculate the final value of the trust fund at the end of the 20 years. **3**

(c) Solve $2\cos^2 x - 3\cos x - 2 = 0$ for $0 \le x \le 2\pi$ **3**

(d)



In the diagram, the shaded region is bounded by the y axis, the curve $y = e^x$ and a horizontal line l that cuts the curve at a point whose x coordinate is $\log_e 4$.

A solid is formed by rotating the shaded region about the y axis.

Find a definite integral in terms of y whose value is the volume of the solid formed. **3**

(Do not attempt to evaluate the integral.)

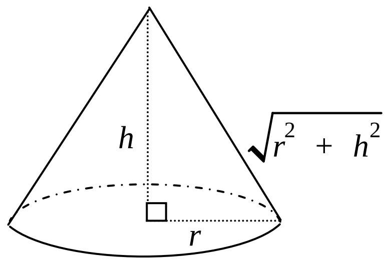
End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Prove that $\frac{d}{d\theta}\left(\tan^3\theta\right) = 3\sec^4\theta - 3\sec^2\theta$ **2**

(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \sec^4\theta \, d\theta$ **2**

(b)



A right circular cone has base radius r and height h.

As r and h vary, its curved surface area $\pi r\sqrt{r^2 + h^2}$ is kept constant. Let $\pi r\sqrt{r^2 + h^2} = K$ where K is a constant.

(i) Show that $V^2 = \frac{1}{9}r^2\left(K^2 - \pi^2 r^4\right)$ where V is the volume of the cone. (Note: $V = \frac{1}{3}\pi r^2 h$) **2**

(ii) Let $Q = V^2$. Show that $\frac{dQ}{dr} = 0$ when $r^4 = \frac{k^2}{3\pi^2}$ **2**

(iii) Show that the maximum value of Q (and hence the maximum value of V) occurs when $h = \sqrt{2}\, r$. **4**

End of Examination

Standard Integrals

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 + x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

Year 12 – 2 Unit Mathematics Aug 2006

Question 1

(a) Exp $= 2005.971$ — 1 mark
$= 2006$ (to 4 sig fig) — 1 mark

(b) $18 - 32a^2 = 2(9 - 16a^2)$ — 1 mark
$= 2(3+4a)(3-4a)$ — 1 mark

(c) $2x^2 + \frac{1}{2}\sin 2x \; (+c)$ 1 off each error

(d) $225^\circ = \frac{\pi}{180} \times 225$ radians — 1 mark
$= \frac{5\pi}{4}$ radians — 1 mark

(e) $2x - 1 \leq -3$ $\quad 2x - 1 \geq 3$
$2x \leq -2$ $\quad 2x \geq 4$ — 1 each
$x \leq -1$ $\quad x \geq 2$
$\therefore x \leq -1$ or $x \geq 2$

(f) $\frac{x-5}{3} - \frac{x+1}{4} = -5$
$4(x-5) - 3(x+1) = -60$ 1 off each error
$4x - 20 - 3x - 3 = -60$
$x - 23 = -60$
$x = -37$.

Question 2

(a) $\tan\theta = -\frac{1}{\sqrt{3}}$
$\therefore$ acute $\theta = \frac{\pi}{6}$ — 1 mark
$\theta = \pi - \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$
$= \frac{5\pi}{6},\ \frac{11\pi}{6}$ — 1 mark

(b) (i) $\frac{d}{dx}(2x\cos x) = \cos x . 2 + 2x . -\sin x$ — 1 mark
$= 2\cos x - 2x\sin x$ — 1 mark

ii) $\frac{d}{dx}\left(\frac{x^3}{2x+1}\right) = \frac{(2x+1).3x^2 - x^3.2}{(2x+1)^2}$

$= \frac{6x^3 + 3x^2 - 2x^3}{(2x+1)^2}$

$= \frac{4x^3 + 3x^2}{(2x+1)^2}$ — 1 off each error

(c) i) $\int \frac{2x^3+1}{x^4+2x}\,dx = \frac{1}{2}\int \frac{4x^3+2}{x^4+2x}\,dx$ — 1 mark

$= \frac{1}{2}\log_e(x^4+2x) + c$ — 1 mark

ii) $\int_0^{\frac{\pi}{3}} \sin 3x\,dx = -\frac{1}{3}\left[\cos 3x\right]_0^{\frac{\pi}{3}}$

$= -\frac{1}{3}\left[\cos\pi - \cos 0\right]$

$= -\frac{1}{3} \times -2$

$= \frac{2}{3}$ — 1 off each error

(d)

$x + 2y = -2 \quad \ldots\ (1)$

$2x - y = 11 \quad \ldots\ (2)$

(2) × 2 $\quad 4x - 2y = 22 \quad \ldots\ (3)$

(1) + (3) $\quad 5x = 20$

$x = 4$

subs in (2) $\quad 8 - y = 11$ — 1 off each error

$y = -3$

$\left\{\begin{array}{l} x = 4 \\ y = -3 \end{array}\right\}$

Question 3

(a) (i) $m = \frac{2-0}{0--4}$

$= \frac{2}{4}$

$= \frac{1}{2}$ — 1 mark

(ii) coords of M $= \left(\frac{0+-4}{2}, \frac{2+0}{2}\right)$

$= (-2, 1)$ — 1 mark each coordinate

(iii) $y - 1 = -2(x - -2)$

$= -2x - 4$ — 1 off each error

$y = -2x - 3$

(iv) cuts y-axis where $x = 0$

$y = 0 - 3$

$= -3$ — 1 mark

coordinates of N are $(0, -3)$

(v) $NQ = \sqrt{(-4-0)^2 + (0--3)^2}$

$= \sqrt{25}$

$= 5$ units — 1 mark

(vi) $PN = QR$

$= 5$ units

Coordinates of R are $(-4, 5)$ — 1 mark

(vii) area $= bh$

$= 4 \times 5$

$= 20\ u^2$ — 1 mark

(b) $\frac{dy}{dx} = e^{x^2} \cdot 2x$ — 1 mark

$= 2x \cdot e^{x^2}$

where $x=1$, $y=e$

, $\frac{dy}{dx}=2e$ — 1 mark

$y-e = 2e(x-1)$

$= 2ex-2e$

$y = 2ex-e$ — 1 mark

Question 4

(a) $\angle ADB = \angle DBC$ (base $\angle$s, isos $\triangle ADB$)

$= 40°$ — 1 mark

$\angle DCB = 40°$ ($\triangle DCB$ is isos) — 1 mark

$\therefore \angle x = 100°$ ($\angle$ sum of $\triangle DCB$) — 1 mark

(b) (i) $\angle TLP = 50°$ (alt $\angle$ NT$\|$QP)

$\angle PLR = 30°$ ($\angle QLP$ st $\angle$)

$\therefore \angle TLR = 80°$ — 1 mark

(ii) $TR^2 = 50^2 + 75^2 - 2 \times 50 \times 75 \times \cos 80°$ — 1 mark

$= 6822.6387$

$\therefore TR = 83$ km (nearest km) — 1 mark

(c) $\log_2 x + \log_2 (x-1) = 1$

$\therefore x(x-1) = 2^1$ — 1 mark

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$ — 1 mark

$\therefore x = 2$, $x \neq -1$ — 1 mark

(d) $y = \ln\sqrt{\frac{4x-1}{2x+3}}$

$= \frac{1}{2}\ln\frac{4x-1}{2x+3}$

$= \frac{1}{2}\{\ln(4x-1) - \ln(2x+3)\}$ — 1 off each error

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{4}{4x-1} - \frac{2}{2x+3}\right)$

$$= \frac{2}{4x-1} - \frac{1}{2x+3}$$

Question 5

(a) i, arc length $= \frac{2\pi}{3} \times 10$ cm

$= \frac{20\pi}{3}$ cm — 1 mark

ii, shaded area $= \frac{1}{2} \times 10^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 5 \times 5 \times \sin \frac{2\pi}{3}$ — 1 mark

$= \left(\frac{100\pi}{3} - \frac{25\sqrt{3}}{4}\right) \text{cm}^2$ — 1 mark

(b) i, $y = \frac{1}{3}x^3 - x^2 + 1$

$\frac{dy}{dx} = x^2 - 2x$

$\frac{d^2y}{dx^2} = 2x - 2$ — 1 mark

where $\frac{d^2y}{dx^2} = 0$; $x = 1$, $y = \frac{1}{3}$. — 1 mark

$x = \frac{1}{2}$, $\frac{d^2y}{dx^2} = 1 - 2 < 0$

$x = 1\frac{1}{2}$, $\frac{d^2y}{dx^2} = 3 - 2 > 0$ — 1 mark

∴ pt of inflexion at $(1, \frac{1}{3})$

(c) (i) $R = 2t + 200(t+4)^{-1}$

$\frac{dR}{dt} = 2 - \frac{200}{(t+4)^2}$ — 1 off each error

where $\frac{dR}{dt} = 0$, $2 = \frac{200}{(t+4)^2}$

$(t+4)^2 = 100$

$t+4 = \pm 10$

$t = 6, \quad t \neq -14.$

$\frac{d^2R}{dt^2} = \frac{400}{(t+4)^3}$

$> 0, \ t > 0$

$\therefore$ min at $t = 6$

ii, Vol emptied $= \int (2t + \frac{200}{t+4}) dt$ — 1 off each error

$= t^2 + 200 \log (t+4) + c.$

where $t = 0$, $v = 0$

$\therefore V = t^2 + 200 \log (t+4) - 200 \log 4$,

where $t = 24\frac{1}{2}$, $V = (24\frac{1}{2})^2 + 200 \log 28\frac{1}{2} - 200 \log 4$

$= 992.97$

$\therefore$ water left $= 7$ litres (nearest litre)

Question 6

(a) i, In $\triangle$s BPR, DQS

BP = DQ (half of equal sides of parm)

$\angle PBR = \angle SDQ$ (alt $\angle$s, AB $\parallel$ DC)

$\angle PRB = \angle DSQ$ (both rt $\angle$s)

$\therefore \triangle BPR \equiv \triangle DQS$ [AAS] — 1 off each error

ii, By Pythagoras $BR^2 = 5^2 - 3^2$

$= 16$

BR = 4 cm

similarly DS = 4 cm — 1 off each error

$\therefore$ SR = 6 cm

(b) (i) vertex $(2, 1\frac{1}{2})$ — 1 mark

(ii) focal length $= 1\frac{1}{2}$ units — 1 mark

(iii) x-intercept where $y = 0$

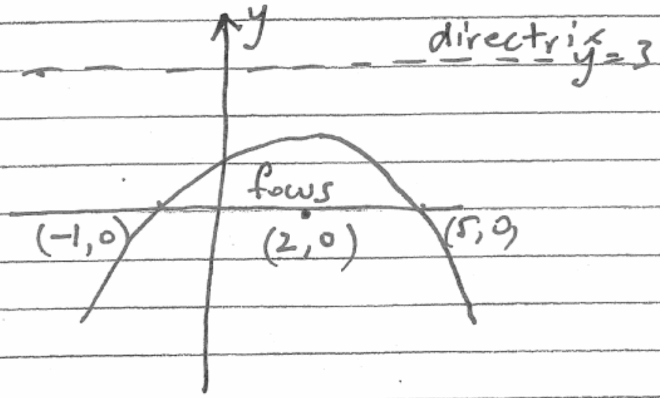
$(x-2)^2 = 9$ — 1 mark

$x - 2 = \pm 3$

$x = 5, -1$

$\therefore$ intercepts at $(5,0)$ and $(-1,0)$ — 1 mark

(iv)



— 1 mark

(c) Reqd area $= \int_0^4 (6x - x^2 - 2x)\,dx$

$= \int_0^4 (4x - x^2)\,dx$ — 1 off each error

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \left[32 - \frac{64}{3}\right] - [0]$

$= 10\frac{2}{3}\ u^2$

Question 7

(a) (i) 0.75 — 1 mark

(ii) $\int_0^4 5xe^{-x}\,dx \doteqdot \frac{1}{3}\{0 + 4\times1.84 + 2\times1.35 + 4\times0.75 + 0.37$ — 1 mark

$= 4.5$ (to 1 dec pl) — 1 mark

(b) (i) where $t=0$, $N=120$. — 1 mark

(ii) $360 = 120e^{8k}$

$3 = e^{8k}$

$\log_e 3 = 8k$ — 1 mark

$k = 0.137$ (to 3 dec pl) — 1 mark

(iii) where $t=12$, $N = 120e^{0.137\times 12}$ OR $N = 120e^{\frac{1}{8}\ln 3 \times 12}$

$= 620$ (nearest 10) — 1 mark

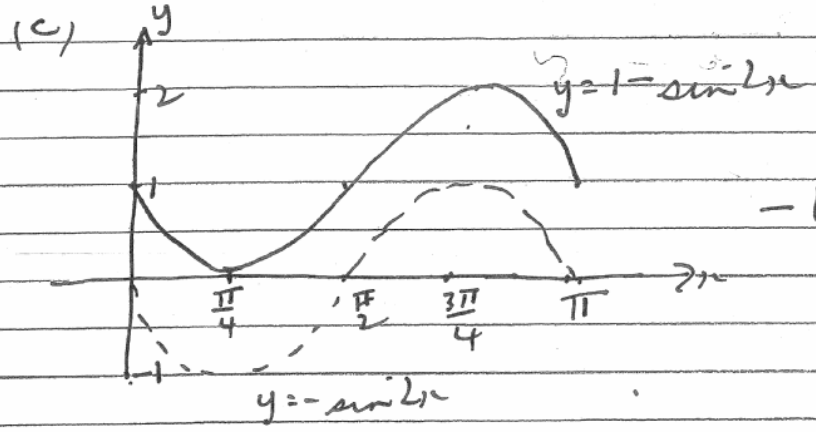
(iv) $\frac{dN}{dt} = 120e^{0.137t} \times 0.137$ OR $\frac{dN}{dt} = 120e^{\frac{1}{8}\ln 3t} \times \frac{1}{8}\ln 3$

— 1 mark

where $t=18$, $\frac{dN}{dt} = 194$ $\quad t=18$ $\frac{dN}{dt} = 195$ (nearest whole number)

— 1 mark.

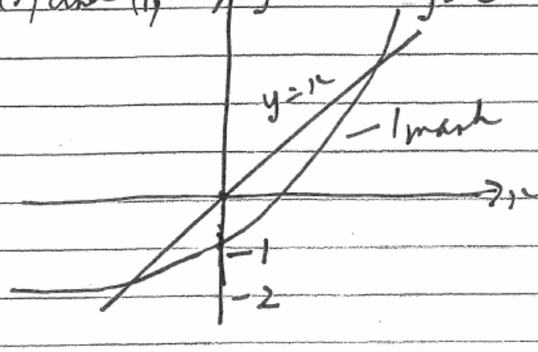
(c)



— 1 off each error

Question 8

(a) (i) and (ii)



— 1 mark

$y = x$

$y = e^x - 2$ — 1 mark

$\therefore e^x - x - 2 = 0$ — 1 mark

$\therefore 2$ solutions

(b) (i) For $S = 500$

$$\frac{n}{2}\left[2 + (n-1)1\right] = 500$$

$$\frac{n}{2}(n+1) = 500$$ — 1 off each error

$$n^2 + n - 1000 = 0$$

$$n = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times -1000}}{2}$$

$$= \frac{-1 \pm \sqrt{4001}}{2}$$

$$\doteqdot 31.13, -32.13$$

∴ but n is a positive integer

∴ $n = 31$.

ie 31 layers.

(ii) $S = \frac{31}{2}(1 + 31)$

$= 496$ — 1 mark

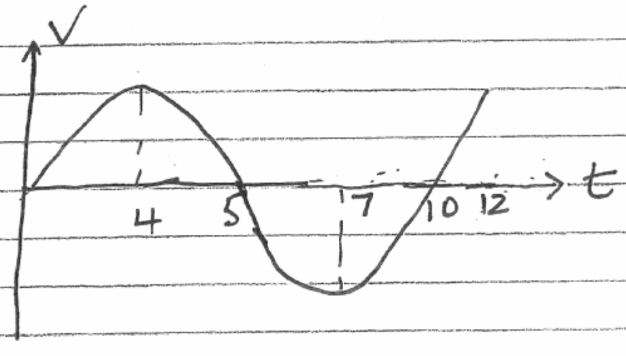
∴ 4 bricks left over

(c) (i) initial velocity = 0 m/s — 1 mark

(ii) between 5th and 10th seconds — 1 mark

(iii) particle moves 19m — 1 mark

(iv)



— 1 mark for basic shape

— 1 mark for intercepts at 5 and 10

Question 9

(a) $ax^2 + 12x + b = a(x-3)(x+7)$

$= ax^2 + 4ax - 21a$ — 1 mark

$\therefore 4a = 12$

$a = 3$

$b = -21a$

$= -63$

$\begin{cases} a = 3 \\ b = -63 \end{cases}$ — 1 mark

(b) i) $A_1 = 75 \times 1.005^{240}$ — 1 mark

$= \$248.27$

ii) $A_s = 75\left[\frac{1.005(1.005^{240} - 1)}{1.005 - 1}\right]$ — 1 off each error

$= \$34826.63$

(c) $2\cos^2 x - 3\cos x - 2 = 0$

$(2\cos x + 1)(\cos x - 2) = 0$ — 1 mark

$\cos x = -\frac{1}{2}$, $\cos x \neq 2$ — 1 mark

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ — 1 mark

(d) where $x = \log_e 4$, $y = e^{\log_e 4}$

$= 4$ — 1 mark off each error

$x = 0$, $y = 1$

$\therefore V = \pi \int_1^4 x^2 \, dy$

$y = e^x$

$\therefore x = \log_e y$

$V = \pi \int_1^4 (\log_e y)^2 \, dy$

Question 10

(a) (i) $\frac{d}{d\theta}(\tan^3\theta) = 3\tan^2\theta\sec^2\theta$ — 1 mark

$= 3\sec^2\theta(\sec^2\theta - 1)$

$= 3\sec^4\theta - 3\sec^2\theta$ — 1 mark

(ii) $\int_0^{\frac{\pi}{4}} \sec^4\theta\, d\theta = \frac{1}{3}\int_0^{\frac{\pi}{4}}(3\sec^4\theta - 3\sec^2\theta)\,d\theta + \int_0^{\frac{\pi}{4}}\sec^2\theta\, d\theta$ — 1 mark

$= \frac{1}{3}\left[\tan^3\theta\right]_0^{\frac{\pi}{4}} + \left[\tan\theta\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{3} + 1$ — 1 mark

$= \frac{4}{3}$

(b) (i) $\pi^2 r^2(r^2 + h^2) = K^2$

$\pi^2 r^4 + \pi^2 r^2 h^2 = K^2$

$h^2 = \frac{K^2 - \pi^2 r^4}{\pi^2 r^2}$ — 1 mark

$V = \frac{1}{3}\pi r^2 h$

$\therefore V^2 = \frac{1}{9}\pi^2 r^4 \times \left(\frac{K^2 - \pi^2 r^4}{\pi^2 r^2}\right)$

$= \frac{1}{9} r^2 (K^2 - \pi^2 r^4)$ — 1 mark

(ii) $Q = \frac{1}{9}K^2 r^2 - \frac{1}{9}\pi^2 r^6$

$\frac{dQ}{dr} = \frac{2}{9}K^2 r - \frac{2}{3}\pi^2 r^5$

$= \frac{2}{9} r (K^2 - 3\pi^2 r^4)$ — 1 mark

where $\frac{dQ}{dr} = 0$, $K^2 - 3\pi^2 r^4 = 0$

$K^2 = 3\pi^2 r^4$ — 1 mark

$r^4 = \frac{K^2}{3\pi^2}$, $r \neq 0$

(iii) $\frac{d^2Q}{dr^2} = \frac{2}{9}k^2 - \frac{10\pi^2}{3}r^4$

when $r^4 = \frac{k^2}{3\pi^2}$ $\frac{d^2Q}{dr^2} = \frac{2}{9}k^2 - \frac{10\pi^2}{3} \times \frac{k^2}{3\pi^2}$

$= \frac{2k^2}{9} - \frac{10k^2}{9}$

< 0

$\therefore$ max at $r^4 = \frac{k^2}{3\pi^2}$ — 1 off each error

but $k^2 = \pi^2 r^2 (r^2 + h^2)$

$\therefore r^4 = \frac{\pi r^2 (r^2+h^2)}{3\pi^2}$

$3r^4 = r^4 + r^2h^2$

$2r^4 = r^2h^2$

$h^2 = 2r^2$

$\therefore h = \sqrt{2}r$, $\because r$ and $h > 0$

THE KING'S SCHOOL

2006
Higher School Certificate
Trial Examination

Mathematics

Question	Algebra and Number	Geometry	Functions	Trigonometry	Differential Calculus	Integral Calculus	Total
1	(a), (b), (e), (f) /8			(d) /2		(c) /2	12
2	(d) /2			(a) /2	(b) /4	(c) /4	12
3			(a) /9		(b) /3		12
4	(c) /3	(a) (3)		(b) /3	(d) /3		12
5				(a) /3	(b) /3	(c) /6	12
6		(a) /4	(b) /5			(c) /3	12
7				(c) /3	(b) /6	(a) /3	12
8	(b) /4		(a) /3		(c) /5		12
9	(b) /4		(a) /2	(c) /3		(d) /3	12
10					(b) /8	(a) /4	12
Marks	/21	/7	/19	/16	/33	/24	120