THE KING'S SCHOOL

2009
Higher School Certificate Course
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

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Attempt Questions 1-10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of $12e$ to 2 decimal places. **2**

(b) Solve $7 - 8x < -9$ **2**

(c) Simplify $\dfrac{1 + x + x^{11} + x^{12}}{1 + x}$ **2**

(d) Find the perpendicular distance from the point $\left(\frac{1}{2},\ 1\right)$ to the line $6x + 8y + 1 = 0$ **2**

(e) $f(x) = \dfrac{12 + x^2}{10}$

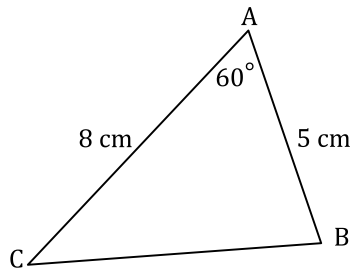
Find $f'(60)$ **2**

(f) Show on a number line the solutions of the inequality $x(2 - x) > 0$ **2**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



(i) Find the exact area of ΔABC **2**

(ii) Find the length of side BC **2**

(b) Differentiate

(i) $(1 + 2x)^3$ **1**

(ii) $x\ln x$ **2**

(c) The curve $y = f(x)$ passes through the point P(1, 3) where $f'(x) = 1 + 2x - 3x^2$

(i) Show that P(1, 3) is a stationary point. **1**

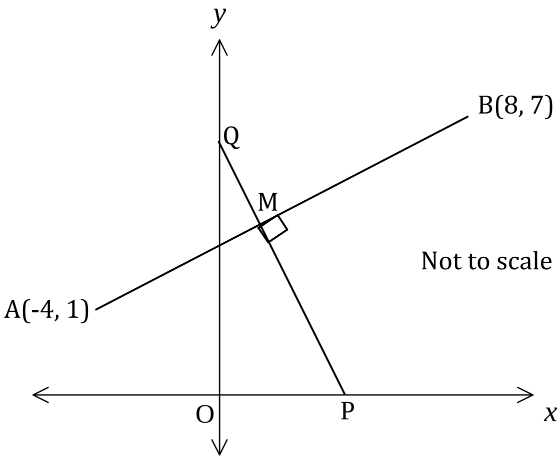
(ii) Determine the precise type of stationary point at P(1, 3). **2**

(iii) Find $f(x)$ **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



A(-4, 1) and B(8, 7) are two points in the number plane.

M is the midpoint of AB and the perpendicular bisector of AB meets the *x* axis at P and the *y* axis at Q.

(i) Find the coordinates of M. **1**

(ii) Show that the equation of the perpendicular bisector of AB, i.e. line QMP, is $2x + y = 8$ **2**

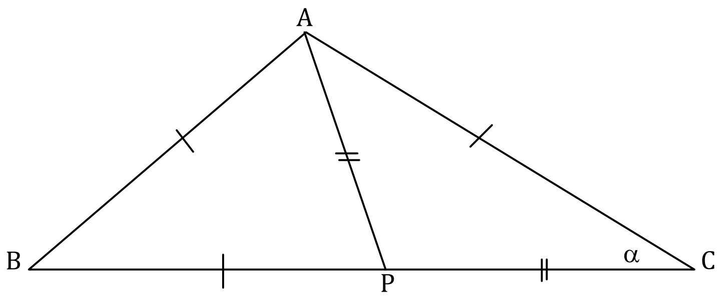
(iii) Hence find the coordinates of the points P and Q. **1**

(iv) Show that the quadrilateral APBQ is a rhombus. **2**

(v) Hence, or otherwise, find the area of the rhombus. **2**

Question 3 continues on the next page

(b)



ΔABC is isosceles where AB = AC

P is the point on BC where AB = PB and AP = PC

Let $\angle ACP = \alpha$

(i) Prove that $\angle APB = 2\alpha$ **2**

(ii) Find the value of α **[YOU DO NOT NEED TO GIVE REASONS IN PART (ii)]** **2**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find a primitive function of $\dfrac{12x^3}{x^4 + 1}$ **2**

(b) Use Simpson's Rule with 3 function values to find an approximate value of the integral $\displaystyle\int_0^2 \sqrt{3x^3 + 1}\ dx$ **3**

(c) (i) State the vertex of the parabola $(x - 1)^2 = -8(y + 2)$ **1**

(ii) Find the equation of the directrix of this parabola. **2**

(d) $x^2 - 4x + 2 = 0$ has the roots $\alpha,\ \beta$

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$ **2**

(ii) Hence find a quadratic equation with the roots $3\alpha,\ 3\beta$ **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the series $30 + 21 + \ldots$

(i) If the series is arithmetic, find the sum of the first 14 terms. **2**

(ii) If the series is geometric, find the limiting sum. **2**

(b) Let $f(x) = 4\cos 2x$

(i) State the period of the function. **1**

(ii) Sketch the graph of the function for $0 \leq x \leq \pi$ **2**

(iii) Find the area of the region in the first quadrant bounded by the curve and the coordinate axes. **3**

(c) (i) Sketch the graph of $y = |x - 2|$ **1**

(ii) Hence, or otherwise, evaluate $\int_0^4 |x - 2|\, dx$ **1**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A particle moves on the x axis according to the displacement equation of motion

$$x = t + \frac{16}{t+1} - 8 \quad , \quad t \geq 0$$

t is time in seconds and x is measured in metres.

(i) Find the initial displacement. **1**

(ii) Find the times when the particle stops. **2**

(iii) Find the distance travelled by the particle during the first 7 seconds. **2**

(iv) Find the maximum acceleration. **2**

(b) The population P of a town is increasing exponentially so that

$\frac{dP}{dt} = kP$, k a positive constant, t time in years

(i) Show that $P = Ae^{kt}$ is a solution of $\frac{dP}{dt} = kP$, where A is a constant. **1**

(ii) Initially the population was 4 000 and 12 years later it was 5 400.

Find the value of k correct to 2 significant figures. **3**

(iii) What was the initial rate of increase of the population in people/year? **1**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Joni borrows $\$P$ for an overseas holiday. This loan plus interest is to be repaid in equal monthly instalments of \$232 for 5 years.

Interest of 6% p.a. is charged monthly on the balance owing at the start of each month.

Let $\$A_n$ be the amount owing at the end of n months.

(i) Show that $A_1 = P(1.005) - 232$ **1**

(ii) Show that $A_2 = P(1.005)^2 - 232(1 + 1.005)$ **1**

(iii) Write down a similar expression for A_n and deduce that

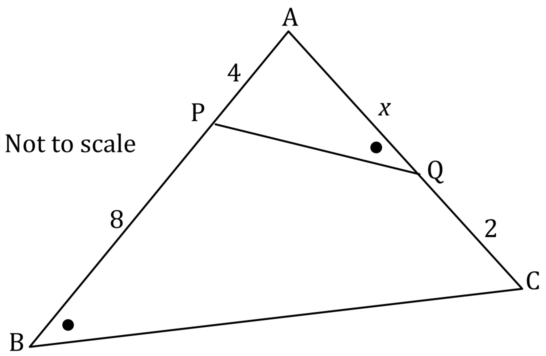
$$A_n = P(1.005)^n - 46\,400(1.005^n - 1)$$ **2**

(iv) Find, in whole dollars, the amount that Joni borrowed. **2**

(v) How much will Joni owe after 1 year of monthly instalment payments? **1**

Question 7 continues on the next page

(b)



In the diagram, AP = 4, PB = 8, QC = 2 and $\angle ABC = \angle AQP$

Let AQ = x

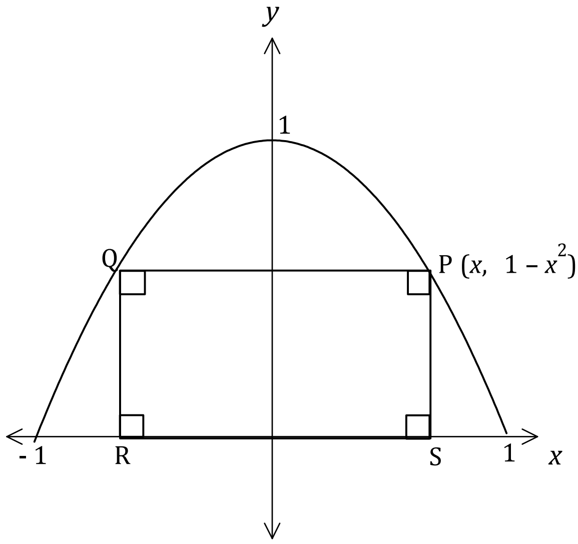
(i) Prove that ΔABC is similar to ΔAPQ. **2**

(ii) Find x. **3**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



The sketch shows the parabolic arc $y = 1 - x^2, \quad -1 \leq x \leq 1$

Let P (x, $1 - x^2$) be a point on this arc where $x > 0$

The rectangle PQRS is constructed as shown in the diagram.

(i) Show that the area A of the rectangle is given by $A = 2x - 2x^3$ **1**

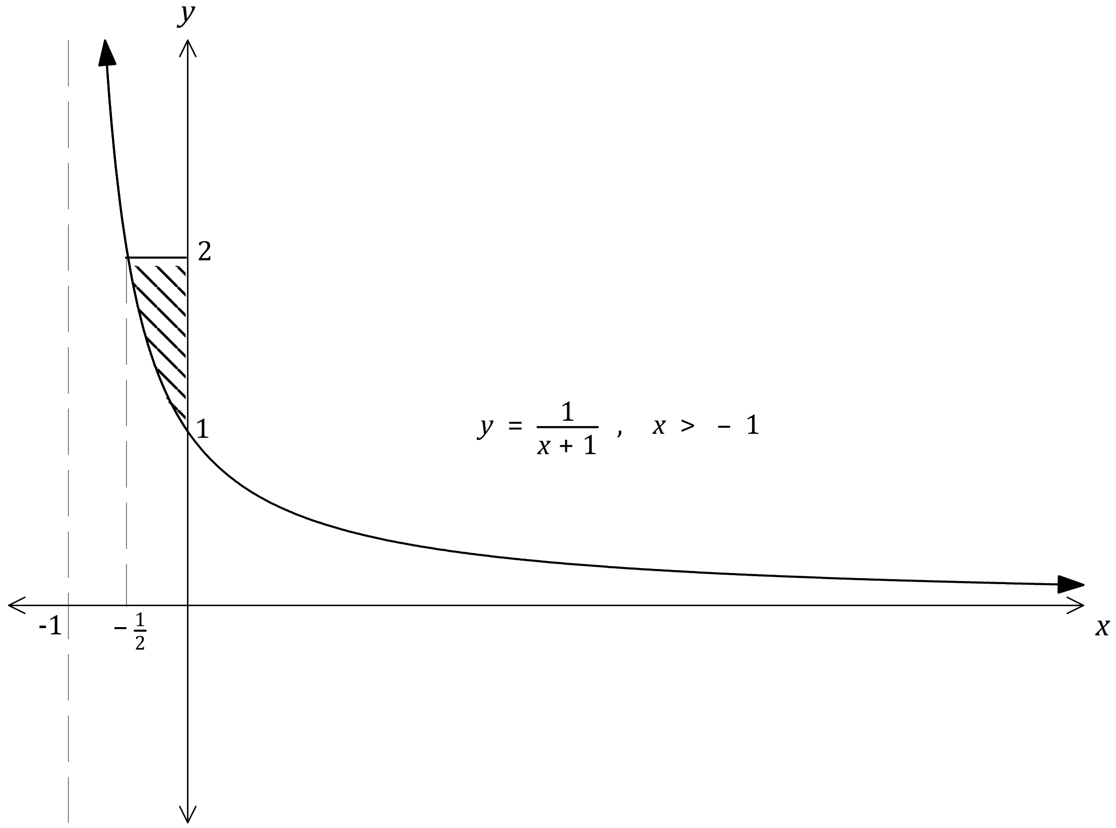
(ii) Find the exact value of x in simplest form if PQRS is a square. **2**

(iii) Prove that the maximum area of the rectangle is $\frac{4\sqrt{3}}{9}$ **4**

(iv) Hence, or otherwise, find the minimum area enclosed between the parabolic arc and the rectangle. **1**

Question 8 continues on the next page

(b)



The shaded region bounded by the hyperbola $y = \frac{1}{x+1}$ and the *y* axis between $y = 1$ and $y = 2$ is revolved about the *y* axis.

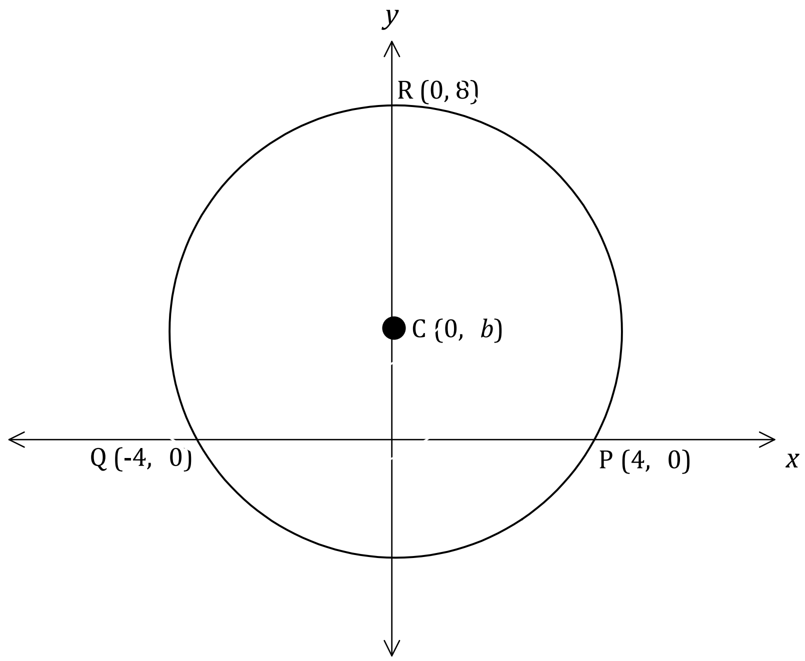
Find the volume of the solid of revolution. **4**

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the values for k for which the equation $x + \frac{1}{x} = 2k$ has no real solutions for x. **3**

(b)



The sketch shows the circle with centre C (0, b) passing through the points P(4, 0), Q(-4, 0) and R(0, 8).

(i) Find the radius of the circle. **2**

(ii) Hence, or otherwise, find the length of the minor arc PQ. Give your answer to 1 decimal place. **2**

Question 9 continues on the next page

(c) A tank contains 2009 L of water. It is leaking so that after t hours the volume V litres of water is decreasing at a rate of R L/h where $R = 2t - 90$.

The rate formula for R applies until the tank is empty.

(i) Find the rate at which the volume is decreasing after 20 hours. **1**

(ii) Find the volume of water in the tank after 20 hours. **2**

(iii) When is the tank empty? **2**

End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Sketch $y = \sec x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, showing any intercepts made with the coordinate axes. **1**

(b) Sketch $y = \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and hence, or otherwise, evaluate

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x \; dx$$ **2**

(c) Let $f(x) = (1 + \tan x)^2$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(i) Solve $f(x) = 0$ **1**

(ii) Find any stationary points on $y = f(x)$ and determine their nature. **3**

(iii) Sketch the graph of $y = f(x)$ **2**

(iv) Find the area bounded by the curve and the x axis from $x = -\frac{\pi}{4}$ to $x = \frac{\pi}{4}$ **3**

End of Examination

Standard Integrals

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

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Mathematics

Question	Algebra and Number	Geometry	Functions	Trigonometry	Differential Calculus	Integral Calculus	Total
1	(a)(b)(c)(f) **8**		(d) **2**		(e) **2**		**/ 12**
2				(a) **4**	(b)(c) **8**		**/ 12**
3		(b) **4**	(a) **8**				**/ 12**
4			(c)(d) **7**			(a)(b) **5**	**/ 12**
5			(a) **4**	(b) **6**		(c) **2**	**/ 12**
6					**12**		**/ 12**
7		(b) **5**	(a) **7**				**/ 12**
8					(a) **8**	(b) **4**	**/ 12**
9	(a) **3**		(b) **4**		(c) **5**		**/ 12**
10				(a) **1**	(c)(i)-(iii) **6**	(b)(c)(iv) **5**	**/ 12**
Total	**11**	**9**	**32**	**11**	**41**	**16**	**/120**

TKS MATHEMATICS TRIAL SOLUTIONS 2009

Question 1

(a) 32·62

(b) $-8x < -16$

$\therefore x > 2$

(c) $\frac{1+x}{1+x} + x^{11}\frac{(1+x)}{1+x} = 1 + x^{11}$

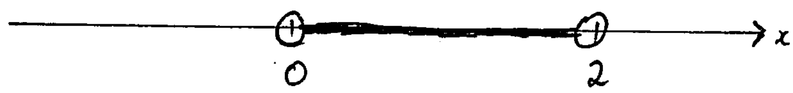
OR, of course, $\frac{1+x + x^{11}(1+x)}{1+x} = \frac{(1+x)(1+x^{11})}{1+x} = 1+x^{11}$

(d) $d = \frac{3 + 8 + 1}{\sqrt{6^2+8^2}} = \frac{12}{10} = 1\cdot 2$

(e) $f'(x) = \frac{2x}{10} = \frac{120}{10}$ for $x = 60$

i.e. $f'(60) = 12$

(f)



i.e. $0 < x < 2$

Question 2

(a) (i) Area $= \frac{1}{2} \cdot 8 \cdot 5 \sin 60^\circ \text{ cm}^2 = 20 \cdot \frac{\sqrt{3}}{2} \text{ cm}^2 = 10\sqrt{3} \text{ cm}^2$

(ii) $BC^2 = 8^2 + 5^2 - 2 \cdot 8 \cdot 5 \cos 60^\circ$

$= 64 + 25 - 40 = 49$

$\therefore BC = 7 \text{ cm}$

(b) (i) $3(1+2x)^2 \cdot 2 = 6(1+2x)^2$

(ii) $x \cdot \frac{1}{x} + \ln x \cdot 1 = 1 + \ln x$

(c) (i) $f'(1) = 1 + 2 - 3 = 0$

$\therefore P(1, 3)$ is a stationary point

(ii) $f''(x) = 2 - 6x$

$f''(1) = -4 < 0 \Rightarrow \frown$ ie. maximum turning point

(iii) $f(x) = x + x^2 - x^3 + c$

$\therefore f(1) = 1 + 1 - 1 + c = 3 \Rightarrow c = 2$

$\therefore f(x) = x + x^2 - x^3 + 2$

Question 3

(a) (i) $M = \left(\frac{4}{2}, \frac{8}{2}\right) = (2, 4)$

(ii) Gradient of $AB = \frac{6}{12} = \frac{1}{2}$

$\therefore$ Gradient $QMP = -2$

$\therefore$ QMP is $y - 4 = -2(x-2)$

or $y - 4 = -2x + 4$

ie $2x + y = 8$

(iii) $P = (4, 0)$, $Q = (0, 8)$

(iv) The mid-point of $PQ = \left(\frac{4}{2}, \frac{8}{2}\right) = (2, 4) = M$

$\therefore$ diagonals AB, PQ bisect each other at right angles

$\Rightarrow$ $APBQ$ is a rhombus

[ALTERNATIVES, OF COURSE]

(v) $AB = \sqrt{12^2 + 6^2} = 6\sqrt{5}$

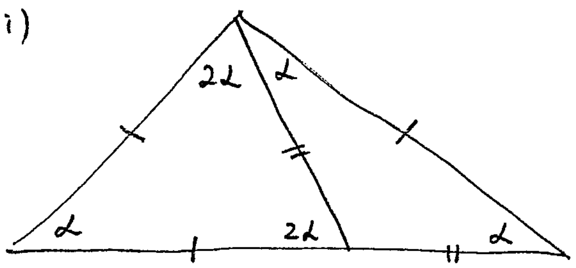
$PQ = \sqrt{4^2 + 8^2} = 4\sqrt{5}$

$\therefore$ Area $= \frac{1}{2} \cdot 6\sqrt{5}\ 4\sqrt{5} = 60$

(b) (i) $\angle CAP = \angle PCA = \alpha$, base $\angle$s of isos ΔAPC

$\therefore$ $\angle APB = \angle CAP + \angle PCA = 2\alpha$, ext. $\angle$ theorem in ΔAPC

(iii)



$\therefore$ $5\alpha = 180$

$\alpha = 36$

Question 4

(a) $I = 3\int \frac{4x^3}{x^4+1}\,dx = 3\ln(x^4+1)$

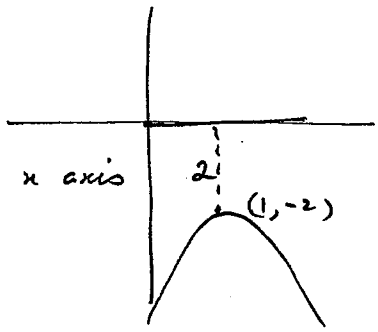
(b) $I \approx \frac{1}{6}\cdot 2\left(1 + 5 + 4\times 2\right) = \frac{14}{3}$

(c) (i) $V = (1, -2)$

(ii) $a = 2$ $\therefore$

$\Rightarrow$ directrix is the x axis

i.e. $y = 0$



(d) (i) $\alpha + \beta = 4$

$\alpha\beta = 2$

(ii) $3\alpha + 3\beta = 3(\alpha+\beta) = 12$

$3\alpha \cdot 3\beta = 9\alpha\beta = 18$

$\therefore$ Equation is $x^2 - 12x + 18 = 0$

Question 5

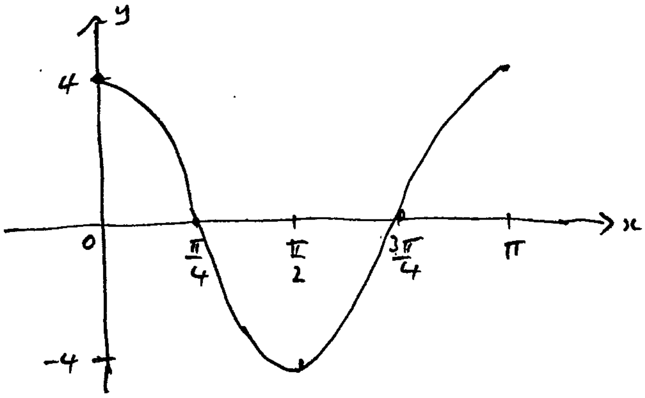
(a) (i) $d = -9$

$$\therefore S_{14} = \frac{14}{2}\left(60 - 9 \times 13\right) = -399$$

(ii) $r = \frac{21}{30} = \frac{7}{10}$ $\quad \therefore S_\infty = \frac{30}{1-\frac{7}{10}} = 100$

(b) (i) period $= \pi$

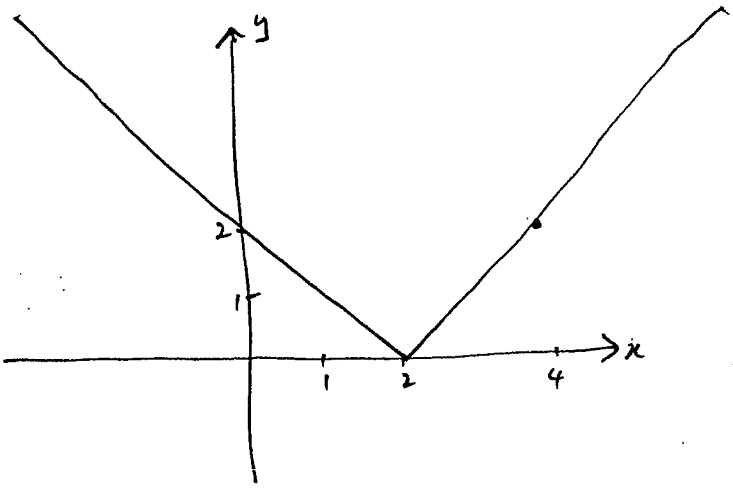
(ii)



(iii) $$A = \int_0^{\frac{\pi}{4}} 4\cos 2x \, dx$$

$$= 4\left[\frac{\sin 2x}{2}\right]_0^{\pi/4} = 2(1-0) = 2$$

(c) (i)



(ii) $$\int_0^4 |x-2| \, dx = 2 \cdot \tfrac{1}{2} \cdot 2 \cdot 2 = 4$$

Question 6

(a) (i) $t=0, \quad x = 16 - 8 = 8$ (cm)

(ii) $x = t + 16(t+1)^{-1} - 8$

$\therefore \dot{x} = 1 - 16(t+1)^{-2} = 1 - \dfrac{16}{(t+1)^2}$

Stops when $\dot{x} = 0 \Rightarrow (t+1)^2 = 16$

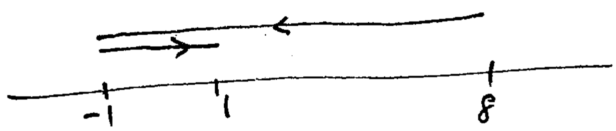
$\Rightarrow t+1 = 4$ since $t \geq 0$

ie. stops at $t = 3$

(iii) $t = 3, \quad x = 3 + 4 - 8 = -1$

$t = 7, \quad x = 7 + 2 - 8 = 1$

$\Rightarrow$



distance travelled is 11 (cm)

(iv) $\ddot{x} = 32(t+1)^{-3} = \dfrac{32}{(t+1)^3}$ has maximum value when $t = 0$

$\Rightarrow \max \ddot{x} = 32$ (cm/s²)

(b) (i) If $P = Ae^{kt}$ then $\dfrac{dP}{dt} = Ae^{kt}.k = k(Ae^{kt}) = kP$

(ii) $P = Ae^{kt}$ $\quad t=0, P = 4000$
$\quad t = 12, P = 5400$

$\therefore P = 4000e^{kt}$

$\therefore 5400 = 4000e^{12k}$

$\Rightarrow e^{12k} = \dfrac{54}{40} = 1.35$

$\therefore 12k = \ln 1.35$

$k = \frac{1}{12}\ln 1.35 = 0.025$, 2 sig figs

(iii) at $t = 0$, $\dfrac{dP}{dt} = 0.025 \times 4000 = 100$ people / yr

Question 7

(a) (i) 6% p.a $= \frac{6}{12}\%$ p.month $= 0.005$ p.month

$\therefore A_1 = P(1+.005) - 232 = P(1.005) - 232$

(ii) $A_2 = A_1(1.005) - 232 = P(1.005)^2 - 232 \times 1.0005 - 232$

$= P(1.005)^2 - 232(1 + 1.005)$

(iii) $A_n = P(1.005)^n - 232(1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1})$

$= P(1.005)^n - 232\left(\frac{1.005^n - 1}{1.005 - 1}\right)$

$= P(1.005)^n - 46400(1.005^n - 1)$

(iv) Now $A_{60} = 0$

$\therefore P(1.005)^{60} = 46400(1.005^{60} - 1)$

$P = \frac{46400(1.005^{60} - 1)}{1.005^{60}} = \$12000$, whole dollars

(v) Owes $A_{12} = 12000(1.005)^{12} - 46400(1.005^{12} - 1)$

$= \$9878.28$ or $\$9878$ whole dollars

(b) (i) $\angle A$ is common

$\angle B = \angle Q$, given

$\therefore \Delta ABC \parallel\!| \Delta APQ$, equiangular

(ii) $\therefore \frac{x}{12} = \frac{4}{x+2}$

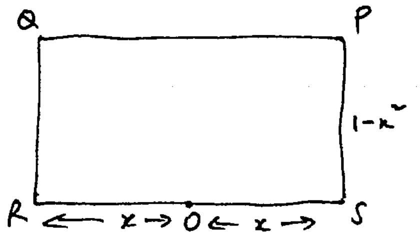
$\Rightarrow x^2 + 2x = 48$

$x^2 + 2x - 48 = 0$

$(x+8)(x-6) = 0 \Rightarrow x = 6$ since $x > 0$

Question 8

(a) (i)



$\therefore A = 2x(1-x^2)$

$= 2x - 2x^3$

(ii) Here $2x = 1 - x^2$

or $x^2 + 2x - 1 = 0$

$\therefore x = \dfrac{-2 + \sqrt{4+4}}{2}$ since $x > 0$

$= \dfrac{-2 + 2\sqrt{2}}{2} = \sqrt{2} - 1$

(iii) $\dfrac{dA}{dx} = 2 - 6x^2 = 0$ if $x^2 = \dfrac{1}{3}$ i.e. $x = \dfrac{1}{\sqrt{3}}$ since $x > 0$

$\dfrac{d^2A}{dx^2} = -12x < 0$ if $x = \dfrac{1}{\sqrt{3}}$ $\Rightarrow \cap$ i.e. maximum area when $x = \dfrac{1}{\sqrt{3}}$

$\therefore \max A = 2 \cdot \dfrac{1}{\sqrt{3}}\left(1 - \dfrac{1}{3}\right) = \dfrac{4}{3\sqrt{3}} = \dfrac{4\sqrt{3}}{9}$

(iv) Min area $= \dfrac{1}{6} \cdot 2(4) - \dfrac{4\sqrt{3}}{9} = \dfrac{4}{3} - \dfrac{4\sqrt{3}}{9}$

(b) $V = \pi \displaystyle\int_1^2 x^2 \, dy$ where $y = \dfrac{1}{x+1}$ or $x + 1 = \dfrac{1}{y}$

$\therefore x = \dfrac{1}{y} - 1$

$\therefore V = \pi \displaystyle\int_1^2 \left(\frac{1}{y} - 1\right)^2 dy$

$= \pi \displaystyle\int_1^2 \frac{1}{y^2} - \frac{2}{y} + 1 \, dy$

$= \pi \displaystyle\int_1^2 y^{-2} - \frac{2}{y} + 1 \, dy = \pi \left[-\frac{1}{y} - 2\ln y + y\right]_1^2$

$= \pi\left(-\dfrac{1}{2} - 2\ln 2 + 2 - (-1 - 0 + 1)\right)$

$= \pi\left(\dfrac{3}{2} - 2\ln 2\right)$

Question 9

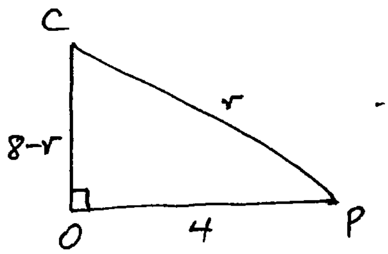
(a) $\Rightarrow \quad x^2 + 1 = 2kx$

$\therefore \quad x^2 - 2kx + 1 = 0$

$\Delta = 4k^2 - 4 < 0$ for no roots

$\Rightarrow \quad k^2 < 1 \quad$ i.e. $\quad -1 < k < 1$

(b) (i)

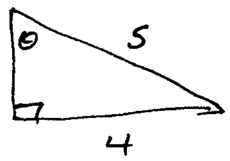


$\therefore \; r^2 = (8-r)^2 + 16 = 64 - 16r + r^2 + 16$

$\therefore \quad 16r = 80$

$r = 5$

(ii) $\therefore$



$\sin\theta = \frac{4}{5} \;, \quad \theta = \cdot 927 \ldots\ldots$

$\therefore \angle PCQ = 2 \times \cdot 927 \ldots\ldots$

$\therefore$ arc $PQ = 5 \times 2 \times \cdot 927 \ldots\ldots = 9.3$, 1 d.p.

(c) (i) $R = 2 \times 20 - 90 = -50$ i.e. decreasing at 50 L/h

(ii) $R = \frac{dV}{dt} = 2t - 90$

$\therefore \; V = t^2 - 90t + c$

$: \; 2009 = 0 + c, \quad c = 2009$

$\therefore \; V = t^2 - 90t + 2009$

$\therefore \; t = 20, \; V = 20^2 - 1800 + 2009 = 609$ L

or $\frac{dV}{dt} = 2(t - 45)$

$\therefore \; V = (t-45)^2 + c$

$: \; 2009 = 45^2 + c, \quad c = -16$

$\therefore \; V = (t-45)^2 - 16$

$t = 20, \; V = 25^2 - 16 = 609$ L

(iii) Empty when $V = 0$

$\Rightarrow t^2 - 90t + 2009 = 0$

$(t - 41)(t - 49) = 0$

$\Rightarrow t = 41 \quad$ since $41 < 49$

i.e. empty after 41 hours

$V = 0$

$\Rightarrow (t-45)^2 = 16$

$\therefore t - 45 = 4$ or -4

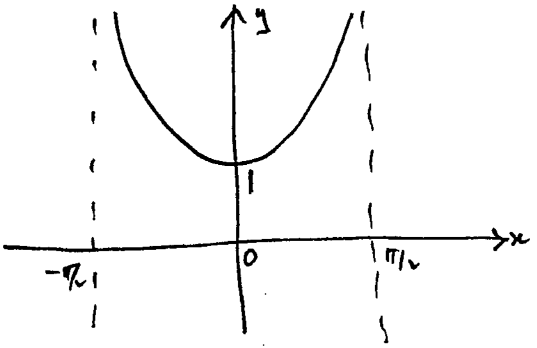
$t = 49$ or 41

$\Rightarrow t = 41 < 49$

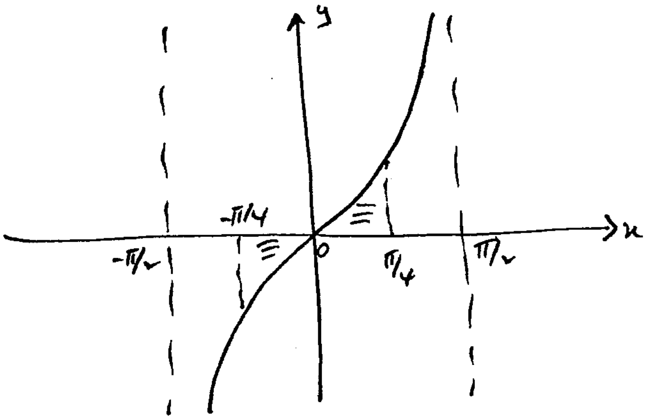
$\therefore \ldots$

Question 10

(a)



(b)



From sketch,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x \, dx = 0$$

(c) (i) $f(x) = 0 \Rightarrow 1 + \tan x = 0$

i.e. $\tan x = -1 \Rightarrow x = -\frac{\pi}{4}$ since $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(ii) $f'(x) = 2(1 + \tan x)\sec^2 x = 0$ if $1 + \tan x = 0$

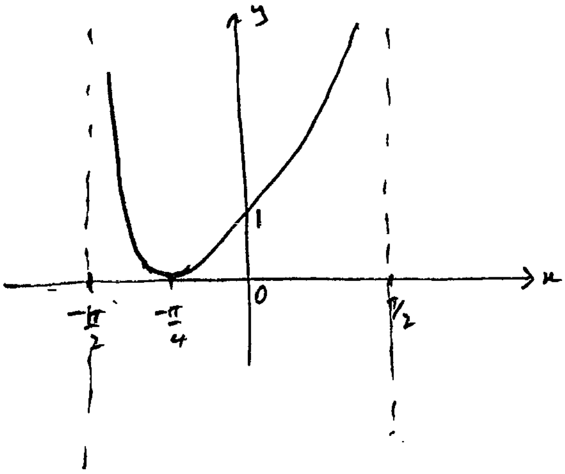
i.e. $x = -\frac{\pi}{4}$ from (i)

≈ -0.8

x	-0.9	-0.8	-0.7
$f'(x)$	<0	0	>0

$\Rightarrow$ min turning point $\left(-\frac{\pi}{4}, 0\right)$

(iii)



(iv) $A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1 + \tan x)^2 \, dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1 + 2\tan x + \tan^2 x \, dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x + 2\tan x \, dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \, dx$ since $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x \, dx = 0$ from (b)

$= \left[\tan x\right]_{-\frac{\pi}{4}}^{\pi/4} = 1 + 1 = 2$

MATHEMATICS MARKING SCHEME 2009

Qn 1 (a) 1 for correct 2 d.p.

Qn 2 (b) (i) Allow 1 for $3(1+2x)^2 2$

(ii) Allow 2 for $x \cdot \frac{1}{x} + \ln x \cdot 1$

(c) (ii) Allow 1 for $f''(1) = -4$

[Alternative solutions exist, of course]

(iii) Allow 1 for $f(x) = x + x^2 - x^3 + c$

Qn 3 (a) (i) Allow 1 for $\left(\frac{4}{2}, \frac{8}{2}\right)$

(ii) Allow 1 for grad $PQ = -2$

(b) Alternative solutions, of course

Allow 1 for distances AB, PQ

Qn 4 (a) Allow 1 for $3\int \frac{4x^3}{x^4+1}\,dx$. No need for $+c$

(b) Allow a sensible decimal approximation

Allow 2 for $\frac{1}{6} \cdot 2\left(\sqrt{1} + \sqrt{25} + 4\sqrt{4}\right)$

(c) (ii) Allow 1 for $a = 2$

(d) (ii) Allow 1 for $3\alpha + 3\beta = 12$ and $3\alpha \cdot 3\beta = 18$

Qn 5 (a) (i) Allow 1 for $d = -9$

(ii) Allow 1 for $r = \frac{21}{30}$

Don't allow 2 for $S_\infty = \frac{30}{1 - \frac{7}{10}}$

(b) (iii) Allow 1 for $\int_0^{\frac{\pi}{4}} 4\cos 2x\,dx$

1 for $4\left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$

Qn 6 (a) Ignore units

(ii) Allow 1 for $\dot{x} = 1 - 16(t+1)^{-2} = 0$

(iii) Allow 1 for $t = 3, x = -1$ and $t = 7, x = 1$

(iv) Allow 1 for $\ddot{x} = 32(t+1)^{-3}$

(b) (ii) Allow 1 for $A = 4000$ quoted or otherwise

Allow 0 for $5400 = 4000e^{12k}$

Allow 1 for $12k = \ln\left(\frac{54}{40}\right)$

Allow 1 for 2 significant figures

Question 7 (a) (ii) Need to indicate $A_2 = A_1(1.005) - 232$

(iii) Allow 1 for $A_n = P(1.005)^n - 232(1 + 1.005 + \dots + 1.005^{n-1})$

(iv) Allow 2 for \$12000·33

Allow 1 for $A_{60} = 0$

(b) (ii) Allow 1 for $\frac{x}{12} = \frac{4}{x+2}$ or equivalent

Allow 1 for $x^2 + 2x - 48 = 0$

Allow 3 if they determine $x = 6$ by "trial" with clear reasons

Qu 8 (a) (i) Allow 1 for $A = 2x(1-x^2)$

(ii) Allow 1 for $x = \frac{-2 \pm \sqrt{4+4}}{2}$

IGNORE UNITS

(iii) Allow 1 for $\frac{dA}{dx} = 2 - 6x^2 = 0$

Allow 1 for $x = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$, of course

Deduct 1 if check for maximum not shown

(iv) Allow 1 for correct use of Simpson's rule

or Min area $= \int_{-1}^{1} 1 - x^2 \, dx - \frac{4\sqrt{3}}{9}$ or equivalent

(b) Allow 1 for $V = \pi \int_1^2 x^2 \, dy$

Allow 1 for $x = \frac{1}{y} - 1$ or equivalent

Allow 1 for $V = \pi \int_1^2 \frac{1}{y^2} - \frac{2}{y} + 1 \, dy$

Don't deduct marks for decimal approximation [0.3572....]

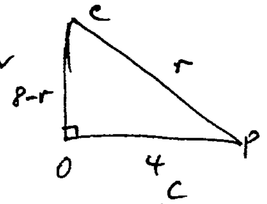
IGNORE UNITS

<u>Qu 9</u>

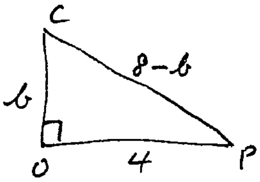
(a) Allow 1 for $x^2 - 2kx + 1 = 0$

Allow 1 for indicating $\Delta < 0$

(b) (i) Allow 1 for



or equivalent



(ii) Allow 1 for $\angle PCQ = 1.854\ldots$

(most will use cos rule)

Allow 2 for arc $PQ = 5 \times 1.854\ldots$

ie ignore 1 d.p. instruction

(c)(i) Allow 1 for -50

(ii) Allow 1 for $V = t^2 - 90t + c$ or $\frac{2(t-45)^2}{2} + c$

(iii) Allow 1 for $t = 41, 49$

<u>Qu 10</u>

(c) (ii) Allow 1 for $f'(x) = 2(1 + \tan x)\sec^2 x$

Allow 1 for $\Rightarrow 1 + \tan x = 0$

Need to do check of some sort to show <u>min</u>

(iii) Don't deduct 1 if y intercept not shown

(iv) Allow 1 for $A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1 + 2\tan x + \tan^2 x \, dx$

Allow 1 for $A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x + 2\tan x \, dx$