Student Number ____________________

2012

Year 12 Mathematics

Trial HSC Examination

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen

  Black pen is preferred
- Board - approved calculators may be used
- Show all necessary working in questions 11 – 16

Total marks – 100

Section I	Pages 2 to 4

10 marks

- Attempt Questions 1 – 10
- Answers on the multiple choice sheet provided
- Allow about 15 minutes for this section

Section II	Pages 5 to 10

90 marks

- Attempt Questions 11 – 16
- Each Question is worth 15 marks
- Each question must begin on a separate page(s), stapled to a completed cover sheet.
- Your student number should be written on every page.
- Full marks may not be awarded for messy or incomplete solutions.
- Allow about 2 hours 45 minutes for this section

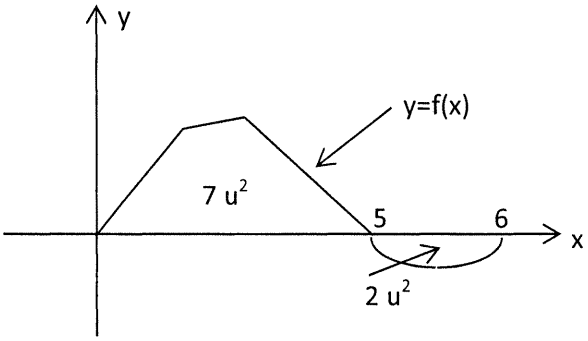
Teacher Setting Paper: Mrs B Brideoake

Head of Department: Mrs M Hill

Section I Multiple choice Use the multiple choice answer sheet provided

Question 1 Given $f(x)$ is an even function and the areas indicated in the diagram below, evaluate $\int_{-6}^{6} f(x)$

Not to scale



(A) 0

(B) 10

(C) 18

(D) Not enough information supplied to determine

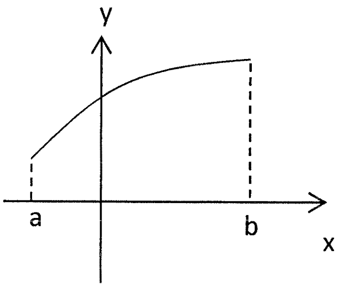
Question 2 For $y = f(x)$ in the diagram below, which is true for a < x < b?

(A) $f'(x) > 0 \quad f''(x) > 0$

(B) $f'(x) > 0 \quad f''(x) < 0$

(C) $f'(x) < 0 \quad f''(x) > 0$

(D) $f'(x) < 0 \quad f''(x) < 0$



Question 3 Evaluate $\sum_{k=2}^{5} (-1)^k \left(\frac{1}{k}\right)$

(A) $\frac{13}{60}$

(B) $1\frac{17}{60}$

(C) $-\frac{13}{60}$

(D) $-1\frac{17}{60}$

Question 4 $\int xe^{5x^2}\,dx =$

(A) $\frac{1}{10}e^{5x^2}+k$

(B) $\frac{x}{10}e^{5x^2}+k$

(C) $10e^{5x^2}+k$

(D) $10xe^{5x^2}+k$

Question 5 If $a+\sqrt{b}=4\left(7+\sqrt{5}\right)$ and *a* and *b* are integers, then

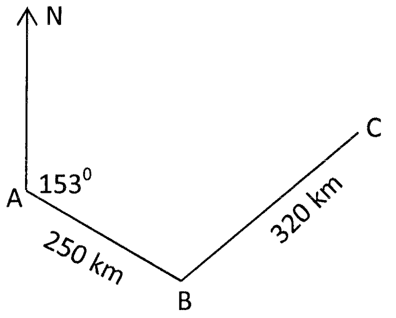
(A) $a=28$ and $b=4\sqrt{5}$

(B) $a=28$ and $b=4$

(C) $a=28$ and $b=20$

(D) $a=28$ and $b=80$

Question 6 A ship sailed 250 km from Port A on a bearing of 153^0 and arrived at Port B to pick up some passengers. It then progressed to its destination Port C, a distance of 320 km on a bearing of 071^0 from B.



The distance of AC can be calculated by:

(A) $AC^2=250^2+320^2-2\left(250^2\right)\left(320^2\right)\cos 98^0$

(B) $AC^2=250^2+320^2-2(250)(320)\cos 98^0$

(C) $AC^2=250^2+320^2-2(250)(320)\cos 71^0$

(D) $\frac{AC}{\sin 98^0}=\frac{320}{\sin 153^0}$

Question 7 Find the centre and radius of the circle with general equation $x^2+2x+y^2-6y-6=0$

(A) $r=6$ and centre $(-1,3)$

(B) $r=\sqrt{6}$ and centre $(-1,3)$

(C) $r=16$ and centre $(-1,3)$

(D) $r=4$ and centre $(-1,3)$

Question 8 A biased coin has heads twice as likely to land upper face upwards when tossed. What is the probability that two heads land upper face upwards when two of these biased coins are tossed?

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{2}{9}$

(D) $\frac{4}{9}$

Question 9 The rate at which a perfumed ball loses its scent over time is given by $A=-\frac{2}{t+1}$ where *t* is measured in days. If the initial amount of perfume in the ball is 6.8 g, how long before the perfume ball has run out (answer to the nearest day).

(A) 2 days

(B) 29 days

(C) 31 days

(D) 80 days

Question 10 Which of the following is NOT true, given $\sec^2\theta-2\tan\theta$?

(A) $\tan^2\theta+1-\frac{2\sin\theta}{\cos\theta}$

(B) $\frac{1}{\sin^2\theta}-2\frac{\sin\theta}{\cos\theta}$

(C) $\frac{1-2\sin\theta\cos\theta}{\cos^2\theta}$

(D) $(\tan\theta-1)^2$

Section II

Question 11 15 Marks Start a new booklet

a) Differentiate with respect to x:

i) $3x^4 + \sin 2x$ **2 marks**

ii) $\dfrac{1}{(3x+2)^2}$ **2 marks**

iii) $4e^{2x}$ **2 marks**

b) Find:

i) $\displaystyle\int \frac{3x+1}{3x^2+2x}\,dx$ **2 marks**

ii) $\displaystyle\int_0^3 \sqrt{x}$ dx **2 marks**

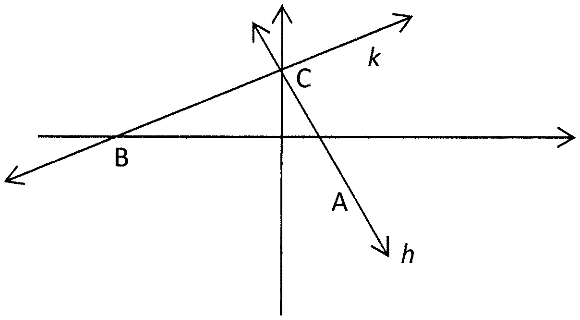
c) Solve $\tan x = \dfrac{1}{\sqrt{3}}$ for $0 \le x \le 2\pi$ **3 marks**

d) Solve $|2x-1| \le 5$ **2 marks**

Question 12 15 Marks Start a new booklet

a) If the roots of $px^2 - x + q = 0$ are -2 and 5, find the values of *p* and *q*. **3 marks**

b) In the diagram, the lines *h* and *k* are drawn. The co-ordinates of A, B and C are (4, -6), (-18,0) and (0,6) respectively. D is the midpoint of AB. Copy the diagram into your answer booklet.



Not to scale

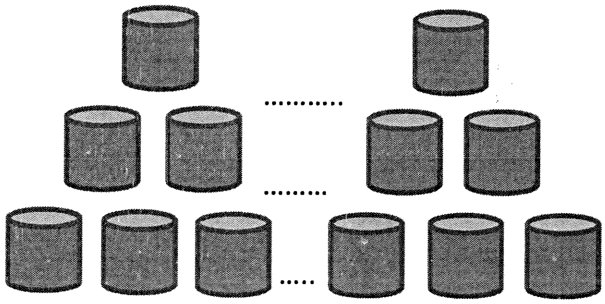
i) Show that the D is (-7,-3). **1 mark**

ii) Calculate the length of DC. Leave your answer in exact form. **1 mark**

iii) Show that the equation of the line *h* is given by $3x+y-6=0$ **2 marks**

iv) Show that the line *h* is perpendicular to the line *k*. **2 marks**

c) Paint cans are stacked such that there are 38 cans on the bottom row, 35 cans on the next row, 32 cans on the next row and so on until a total of 253 cans are stacked.



Not to scale

i) Show that the number of cans in each row forms an arithmetic sequence. **1 mark**

ii) Write down a formula for the number of cans in the *n*th row. **1 mark**

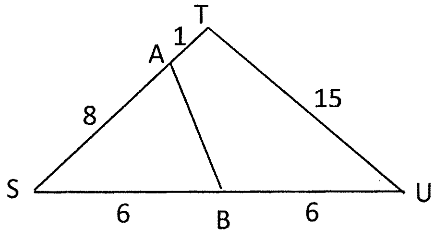
iii) How many cans are there in the 10th row? **1 mark**

iv) How many rows are there in this stack? **2 marks**

v) How many cans are there in the final row of this stack? **1 mark**

Question 13 15 Marks Start a new booklet

a) In the diagram given below,



Not to scale

i) prove $\Delta SAB \mid\mid\mid \Delta SUT$. **2 marks**

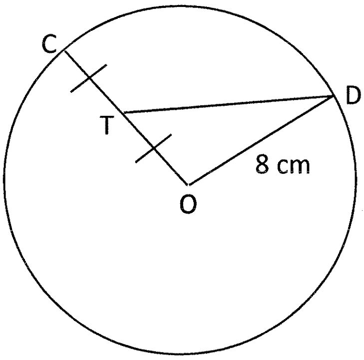
ii) Hence, find AB. **2 marks**

b) In the diagram below, O is the centre of the circle and $\angle COD = \frac{2\pi}{3}$.

Find the <u>exact perimeter</u> of CTD.

3 marks

Not to scale



c) Find the vertex and focus of the parabola $y^2 = 8(x+2)$ **2 marks**

d) If $y = x^4 - 8x^2 + 16$,

i) show that $\frac{dy}{dx} = 4x(x-2)(x+2)$. **1 mark**

ii) Find the stationary points and determine their nature. **3 marks**

iii) Sketch $y = x^4 - 8x^2 + 16$ **2 marks**

Question 14 **15 Marks** **Start a new booklet**

a) Given $y = 3\cos 2x$,

i) state the period. **1 mark**

ii) Sketch $y = 3\cos 2x$ for $-\pi \le x \le \pi$ **2 marks**

b) The die in a new game has 20 faces. Each face has different letter of the alphabet. However, the letters Q, U, V, X, Y and Z have not been used.
The die is rolled twice.

i) What is the probability that the letter B appears both times? **1 mark**

ii) What is the probability that the same letter appears both times? **1 mark**

c) An insurance company has calculated the probability of a woman being alive in 40 years to be 0.8 and the probability that her husband will be alive in 40 years to be 0.7

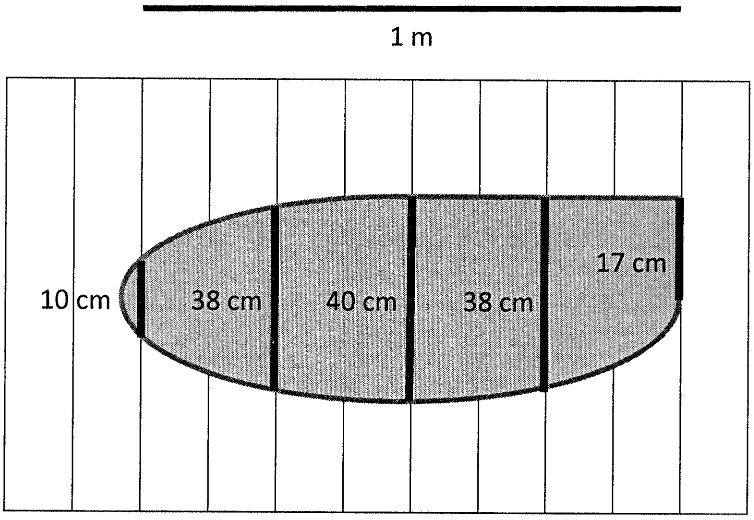
What is the probability that

i) they will both be alive in 40 years? **1 mark**

ii) only one will be alive in 40 years? **2 marks**

d) Use Simpson's Rule to estimate the area of the hole in this fence. **3 marks**

Not to scale



e) In Mr Jones' will, he established a fund for his son where \$1000 earned interest at 5% pa compounded annually.

i) How much will be in the fund after 20 years? **1 mark**

ii) A year after establishment, Mr Jones' son deposits another \$1000 into the fund. If he plans to deposit \$1000 each year for another 19 years, how much will now be in the fund after 20 years? (Assume interest compounded annually at 5%pa) **3 marks**

Question 15 15 Marks Start a new booklet

a) The velocity, V, in m/s of a particle moving in a straight line is given by $V = 4\cos 2t$.

i) Find the initial velocity of the particle. **1 mark**

ii) Find the times when the particle is at rest **2 marks**

iii) Find the acceleration of the particle as a function of time. **1 mark**

iv) If the particle is 3 m to the right of the origin after π seconds, find the particle's displacement as a function of time. **2 marks**

v) Find the exact displacement after $\frac{\pi}{6}$ seconds. **1 mark**

b) Populations cannot increase indefinitely. Environmental and economic factors such as limited food, weather and space control the size of the population. Two thousand kangaroos, each aged 2 years old, are released into the wild on an island. After 3 years there are approximately 1800 kangaroos that inhabit the island. The size of the population, N, after *t* years is predicted by the equation $N = N_0 e^{-kt}$

i) Find the value of N_0. **1 mark**

ii) Find the value of *k* correct to 4 significant figures. **2 marks**

iii) After how many years will the kangaroo population have halved? **2 marks**

c) Find the exact volume of revolution when the area in the first quadrant for $y = 1 - x^2$ is rotated around the x axis. **3 marks**

Question 16 15 Marks Start a new booklet

a) It is assumed that the number of termites, N, in a certain mound at time $t \geq 0$ is given by $N = \dfrac{900000}{2+e^{-t}}$ where t is measured in months.

i) Find the number of termites after 1 month. **1 mark**

ii) How many termites are expected as t gets very large? **1 mark**

iii) Find an expression for the rate at which the number of termites increases at any time. **2 marks**

b) A closed cylindrical can of radius x cm and height y cm is to be made from a sheet of metal with area 435π cm^2. There is 20% wastage of the sheet metal in manufacturing the can.

i) Show that the area of sheet metal required to make the can is 348π cm^2. **1 mark**

ii) Show that $y = \dfrac{174}{x} - x$ **2 marks**

iii) Show that the volume, V, of the can is given by $V = 174\pi x - \pi x^3$ **1 mark**

iv) Find, correct to 1 decimal place, the value of x which gives a maximum volume. **2 marks**

c) Bill borrows $25 000 from his local bank. The loan plus interest and charges are to be repaid at the end of each month with equal monthly installments of $F.
Interest is charged at 6% pa and is calculated on the balance owing at the beginning of each month. Furthermore, at the end of each month a bank charge of $15 is added to the account balance.

Let A_n be the amount owing after n months.

i) Write down an expression for A_1 **1 mark**

ii) Show that the amount owing after 3 months is given by $A_3 = 25000 \times 1.005^3 - (F-15)(1+1.005+1.005^2)$ **2 marks**

iii) If the loan is to be paid off in 5 years, find the monthly installment. **2 marks**

Trial 2U 2012

11 a) i) $\frac{d}{dx}\left[3x^4+\sin 2x\right] = 12x^3 + 2\cos 2x$ ②

ii) $\frac{d}{dx}\left[\frac{1}{(3x+2)^2}\right] = \frac{d}{dx}\ (3x+2)^{-2}$

$= -2(3x+2)^{-3} \times 3$

$= -6(3x+2)^{-3}$ or $\frac{-6}{(3x+2)^3}$ ②

iii) $\frac{d}{dx}\left[4e^{2x}\right] = (4)2e^{2x}$

$= 8e^{2x}$ ②

b) i) $\int \frac{3x+1}{3x^2+2x}\,dx = \frac{1}{2}\log_e(3x^2+2x) + K$ ②

ii) $\int_0^3 \sqrt{x}\,dx = \int_0^3 x^{\frac{1}{2}}\,dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^3$

$= \left(\frac{2}{3}(3)^{3/2}\right) - \left(\frac{2}{3}(0)^{3/2}\right)$

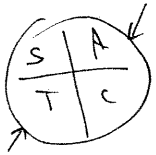
$= \frac{2}{3}\ (3)^{\frac{3}{2}}$ $\left[\text{or } \frac{2}{3}\sqrt{27} = \frac{2}{3}.3\sqrt{3} = 2\sqrt{3}\right]$

$= \frac{2}{3}\ (\sqrt{3})^3$

$= \frac{2}{3}\ 3.\sqrt{3}$

$= 2\sqrt{3}$ ②

c) $\tan x = \frac{1}{\sqrt{3}}$ $0 \le x \le 2\pi$



$30°$ $\frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

d) $|2x-1| \le 5$

$-5 \le 2x-1 \le 5$

$-4 \le 2x \le 6$

$-2 \le x \le 3$ ②

12 a) $px^2 - x + q = 0$ roots $-2, 5$

$(x+2)(x-5) = 0$

$x^2 - 3x - 10 = 0$ but $px^2 - x + q = 0$

$\therefore \frac{1}{3}x^2 - x - \frac{10}{3} = 0$

$\therefore p = \frac{1}{3}$ $q = -\frac{10}{3}$

or roots α, β $px^2 - x + q = 0$

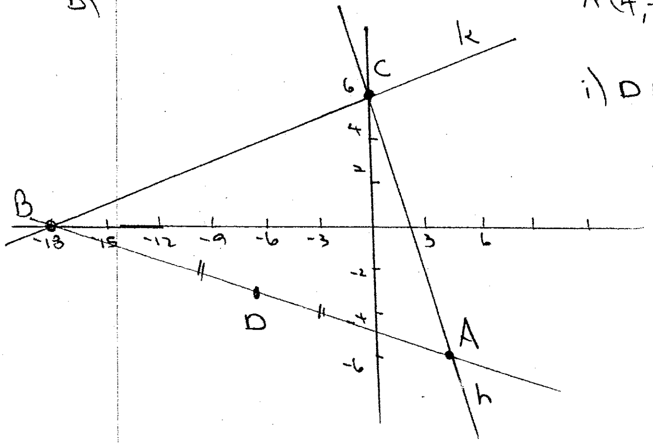
$\alpha = -2$ $\beta = 5$ $a = p$ $b = -1$ $c = q$

sum $\alpha + \beta = -\frac{b}{a}$ $\Rightarrow$ $3 = \frac{1}{p}$ $\Rightarrow$ $p = \frac{1}{3}$

product $\alpha\beta = \frac{c}{a}$ $\Rightarrow$ $-10 = \frac{q}{p}$ $q = -10p$

$\therefore q = -\frac{10}{3}$

b)



$A(4,-6)$ $B(-18,0)$ $C(0,6)$

i) $D\left(\frac{4-18}{2}, \frac{-6+0}{2}\right)$

ie $D(-7,-3)$

$D(-7,-3) \quad C(0,6)$

$DC = \sqrt{(-3-0)^2 + (-7-6)^2}$

$= \sqrt{(-3)^2 + (-13)^2}$

$= \sqrt{9 + 169} \quad = \sqrt{130}$

$= \cancel{\sqrt{178}}$

iii) $h: A(4 -6) \quad C(0 6)$

$\frac{y-6}{x-0} = \frac{-6-6}{4-0}$

$\frac{y-6}{x} = \frac{-12}{4}$

$\frac{y-6}{x} = -3$

$y-6 = -3x$

$3x+y-6 = 0$ AC or line h

iv) h: slope ⟹ $y = -3x+6$

slope $= -3$

k: slope ⟹

$B(-18 \; 0) \quad C(0 \; 6)$

slope $= \frac{6-0}{0+18}$

$= \frac{6}{18}$

$= \frac{1}{3}$

as $-3 \times \frac{1}{3} = -1$ lines h and k perpendicular

c)   Total = 253 cans

row 3	32 cans
row 2	35 cans
row 1	38 cans

i) number of cans each row

38, 35, 32 ... is arithmetic sequence with $a = 38$, $d = -3$

ii) $T_n = a + (n-1)d$

$T_n = 38 + (n-1)-3$

$T_n = 38 - 3n + 3$

$T_n = 41 - 3n$

AP

$a = 38$

$d = -3$

$T_n = a + (n-1)d$

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{n}{2}(2a + (n-1)d)$

iii) $T_{10} = 41 - 3(10)$

$T_{10} = 11$ cans

iv) Total cans = 253   ie $S_n = 253$

$253 = \frac{n}{2}(2(38) + (n-1)-3)$

$253 = \frac{n}{2}(76 - 3n + 3)$

$253 = \frac{n}{2}(79 - 3n)$

$506 = n(79 - 3n)$

$506 = 79n - 3n^2$

$3n^2 - 79n + 506 = 0$

$a = 3 \quad b = -79 \quad c = 506$

$n = \frac{79 \pm \sqrt{6241 - 4(3)(506)}}{6}$

$n = \frac{79 \pm \sqrt{169}}{6}$

$n = \frac{79 \pm 13}{6}$

ie $n = 15.\dot{3}$ or $n = 11$ but n integer

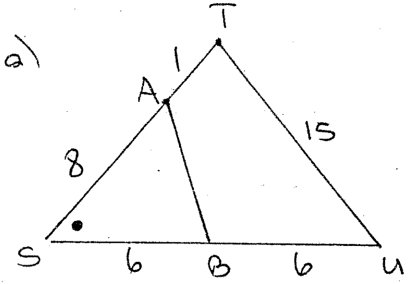
∴ $n = 11$ ∴ there are 11 rows

(or $3n^2 - 79n + 506 = (3n - 46)(n - 11)$)

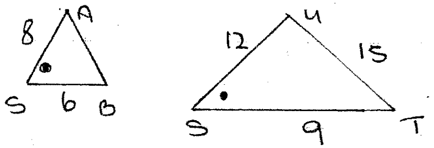
v) $T_{11} = 41 - 3(11)$

$T_{11} = 8$ cans

13) a)



i) ΔSAB ΔSUT



$\frac{SA}{SU} = \frac{8}{12} = \frac{2}{3}$

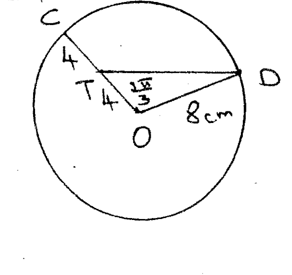
$\frac{SB}{ST} = \frac{6}{9} = \frac{2}{3}$

$\angle ASB = \angle UST$ (common)

$\therefore \Delta SAB ||| \Delta SUT$ (two sides in proportion with included angle equal

ii) $\frac{AB}{15} = \frac{8}{12}$ $\therefore AB = 10$

b)



$\angle COD = \frac{2\pi}{3}$

arc CD : $l = r\theta$

$l = 8 . \frac{2\pi}{3}$

$l = \frac{16\pi}{3}$

perimeter CTD $= 4 + TD + \frac{16\pi}{3}$

ΔTDO $a^2 = b^2 + c^2 - 2bc\cos A$

$TD^2 = 4^2 + 8^2 - 2(4)(8)\cos 120°$ $\frac{2\pi}{3} = 120°$

$TD^2 = 112$

$TD = \sqrt{112}$

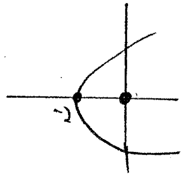
$TD = 4\sqrt{7}$

$\therefore$ perimeter CTD $= 4 + 4\sqrt{7} + \frac{16\pi}{3}$

c) $y^2 = 8(x+2)$ form $(y\ \ )^2 = 4a(x\ \ )$

Vertex $(-2,0)$ $a = 2$

Focus $(0,0)$



d) $y = x^4 - 8x^2 + 16$

$y' = 4x^3 - 16x$

$y'' = 12x^2 - 16$

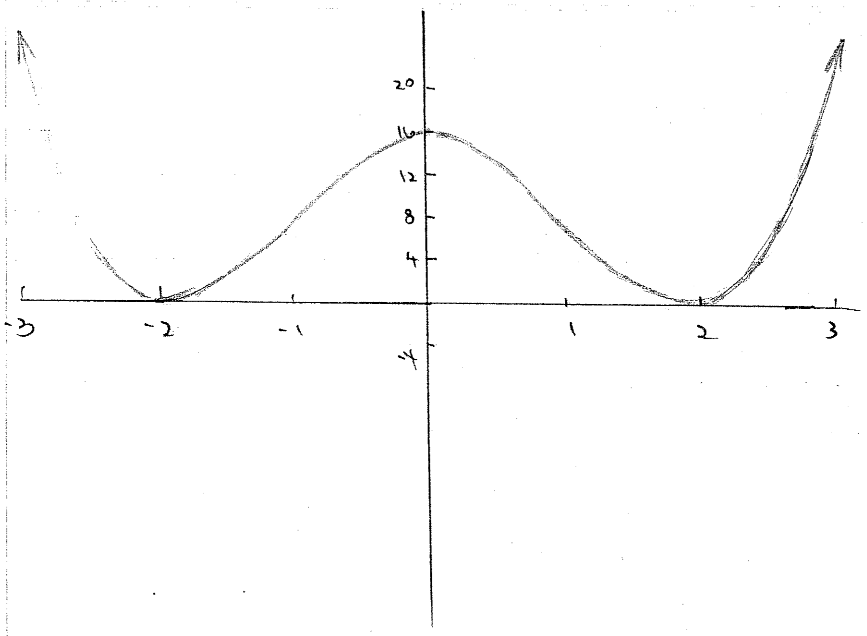
i) $\frac{dy}{dx} = 4x^3 - 16x$

$= 4x(x^2 - 4)$

$\frac{dy}{dx} = 4x(x-2)(x+2)$

ii) Stat pt $y' = 0$ $4x(x-2)(x+2) = 0$

$x = 0$	$x = 2$	$x = -2$
$(0, 16)$	$(2, 0)$	$(-2, 0)$
$\therefore f(0) = 16$	$f(2) = 2^4 - 8.2^2 + 16$	$f(-2) = (-2)^4 - 8(-2$
$f''(0) = 12(0) - 16$	$f''(2) = 12(2^2) - 16$	$f''(-2) = 12(-2)^2 - 16$
$f''(0) < 0$	$f''(2) > 0$	$f''(-2) > 0$
MAX $(0,16)$	MIN $(2,0)$	MIN $(-2,0)$



Intercepts (x=0) y=16

Inflexion $y''=0 \Rightarrow 12x^2-16=0$

$x^2 = \frac{4}{3}$

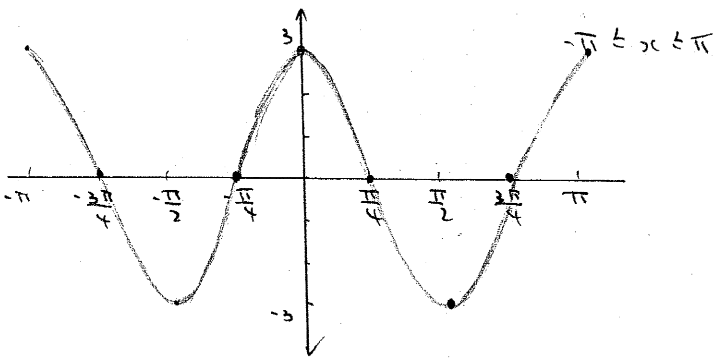
$x = \pm\frac{2}{\sqrt{3}} \approx \pm 1.2$

$f(3)=25 \quad f(-3)=25$

14) a) $y = 3\cos 2x$

i) period $= \frac{2\pi}{2} = \pi$

ii)



b) 20 faces twice

i) $P(BB) = \frac{1}{20}\cdot\frac{1}{20} = \frac{1}{400}$

ii) P(same letter) = P(AA or BB or.....)

$= \frac{1}{400} \times 20$

$= \frac{1}{20}$

c) $P(W) = 0.8 \qquad P(H) = 0.7$

$P(\bar{W}) = 0.2 \qquad P(\bar{H}) = 0.3$

i) $P(WH) = (0.8)(0.7)$

$= 0.56$

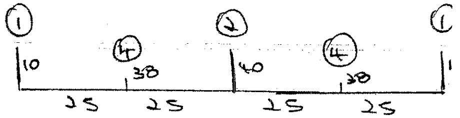
ii) P(only one) $= P(W\bar{H} \text{ or } \bar{W}H)$

$= (0.8)(0.3) + (0.2)(0.7)$

$= 0.24 + 0.14$

$= 0.38$

d) $A = \frac{b-a}{6}\left[f(a)+4f(mid)+f(b)\right]$



twice

$A = \frac{50}{6}\left[10+4(38)+2(40)+4(38)+17\right]$

$= \frac{25}{3}\left[411\right]$

$\frac{h}{3}\left(d_F + 4d_m + d_L\right)$

$A = 3425 \text{ cm}^2$

e) \$1000 5% p.a compounded annually

i) $A = 1000(1.05)^{20}$

$A = \$2653.2977$

$A = \$2653.30$

ii) $A_1 = 1000(1.05)^{20}$

$A_2 = 1000(1.05)^{19}$

$A_{20} = 1000(1.05)$

Total $= 1000\left[1.05 + 1.05^2 + \ldots + 1.05^{20}\right]$

$= 1000\left[\frac{1.05(1.05^{20}-1)}{0.05}\right]$

$= \$34\,719.2518$

$= \$34\,719.25$

GP

$a = 1.05$

$r = 1.05$

$n = 20$

$S_n = \frac{a(r^n-1)}{r-1}$

or $\$2653.2977 + 1000\left[\frac{1.05(1.05)^{19}-1}{0.05}\right]$

$\$2653.2977\ldots + \$32\,065.9541$

$\$34719.2518$

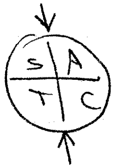
$\$34\,719.25$

15) a) $V = 4\cos 2t$ m/s

i) $t = 0$ $V = 4\cos 0$ ie $V = 4$ m/s

ii) rest $V = 0$ $4\cos 2t = 0$

$\cos 2t = 0$



ie $2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \ldots$ seconds

iii) $V = 4\cos 2t$ m/s

$a = -8\sin 2t$ m/s^2

iv) $t = \pi$ $x = 3$

$V = 4\cos 2t$

$x = 2\sin 2t + K$

$3 = 2\sin 2\pi + K$

$3 = K$

$\therefore x = 2\sin 2t + 3$ m

v) $x = ?$ $t = \frac{\pi}{6}$

$x = 2\sin 2.\frac{\pi}{6} + 3$

$x = 2\sin\frac{\pi}{3} + 3$

$x = 2\frac{\sqrt{3}}{2} + 3$

$x = \sqrt{3} + 3$ m

b) 2000 $t = 3$ $N = 1800$

$N = N_0 e^{-kt}$

i) $N_0 = 2000$ $t = 0$ $N = 2000$

ii) $N = 2000e^{-kt}$ $t = 3$ $N = 1800$

$1800 = 2000e^{-3k}$

$0.9 = e^{-3k}$

$-3k = \log_e 0.9$

$k = \frac{\ln 0.9}{-3}$

ie $k = 0.03512071$

$k \approx 0.03512$

iii) $N = 2000e^{-kt}$ $t = ?$ $N = 1000$

$1000 = 2000e^{-kt}$

$0.5 = e^{-kt}$

$-kt = \log_e 0.5$

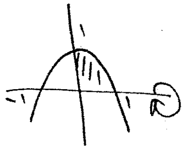
$t = \frac{\ln 0.5}{-k}$

$t = \frac{\ln 0.5}{-0.035120171}$

$t = 19.736..$

$t \approx 20$ years

c) $y = 1 - x^2$



$y^2 = (1-x^2)^2$

$y^2 = 1 - 2x^2 + x^4$

$V = \pi \int y^2\,dx$

$= \pi \int_0^1 1 - 2x^2 + x^4\,dx$

$= \pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \right]_0^1$

$= \pi \left[\left(1 - \frac{2}{3} + \frac{1}{5}\right) - (0) \right]$

$= \frac{8\pi}{15}\ u^3$

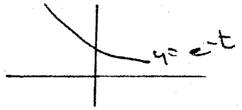
16) a) $N = \frac{900\,000}{2 + e^{-t}}$ $t \geq 0$ months

i) $t = 1$ $N = \frac{900\,000}{2 + e^{-1}}$

$N = 380\,086.9184$

$N \approx 380\,087$ termites

ii) as $t \to \infty$ $e^{-t} \to 0$



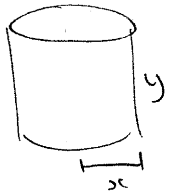
$\therefore N \to \frac{900\,000}{2}$ ie $N \to 450\,000$ kangaroos

iii) $N = 900\,000(2 + e^{-t})^{-1}$

$\frac{dW}{dt} = -900\,000(2 + e^{-t})^{-2} \times -e^{-t}$

$\frac{dN}{dt} = 900\,000e^{-t}(2 + e^{-t})^{-2}$ or $\frac{900\,000}{e^{t}(2 + e^{-t})^2}$

b)



metal 435π cm^2

20% wastage

i) metal needed $= 80\%\ 435\pi$

$= (0.8)\,435\pi$

$= 348\pi$ cm^2

ii) $SA = 2\pi r^2 + 2\pi rh$

ie $SA = 2\pi x^2 + 2\pi xy$

$348\pi = 2\pi x^2 + 2\pi xy$

ie $2\pi xy = 348\pi - 2\pi x^2$

$y = \frac{348\pi}{2\pi x} - \frac{2\pi x^2}{2\pi x}$

$y = \frac{174}{x} - x$

iii) $V = \pi r^2 h$

ie $V = \pi x^2 y$

$V = \pi x^2 \left[\frac{174}{x} - x\right]$

$V = 174\pi x - \pi x^3$ cm³

iv) max V $\quad \frac{dV}{dx} = 0 \quad V = 174\pi x - \pi x^3$

$\frac{dV}{dx} = 174\pi - 3\pi x^2$

$0 = 174\pi - 3\pi x^2$

$3\pi x^2 = 174\pi$

$x^2 = 58$

$x = \pm\sqrt{58}$ but $x > 0$

$\therefore x = \sqrt{58}$

check max $\frac{d^2V}{dx^2} = -6\pi x$

$\frac{d^2V}{dx^2} < 0$ when $x = \sqrt{58}$

$\therefore$ $x = 7.61577$ ie $x \approx 7.6$ cm for max V

c) \$25 000 $\quad$ paid F end month $\quad$ 6% p.a

$r = 0.06 \div 12$

\$15 charge end month $\quad$ ie $r = 0.005$

i) $A_1 = (1.005)25\,000 - F + 15$

ii) $A_2 = (1.005)\left[(1.005)25000 - F + 15\right] - F + 15$

$A_2 = (1.005)^2 25000 - 1.005F + 1.005(15) - F + 15$

$A_2 = (1.005)^2 25000 - F(1.005 + 1) + 15(1.005 + 1)$

$A_2 = (1.005)^2 25000 - (F-15)\left[1.005 + 1\right]$

$A_3 = (1.005)^3 25000 - (F-15)\left[1.005^2 + 1.005 + 1\right]$

iii) paid 5 years $\therefore A_{60} = 0$

$A_{60} = 25000(1.005)^{60} - (F-15)\left[1 + 1.005 + \ldots + 1.005^{59}\right]$

$0 = 25000(1.005)^{60} - (F-15)\left[1 + 1.005 + .. + 1.005^{59}\right]$

series $a = 1$

$r = 1.005$

$n = 60$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$\therefore 0 = 25000(1.005)^{60} - (F-15)\left[\frac{1.005^{60} - 1}{0.005}\right]$

$(F-15)\left[\frac{1.005^{60} - 1}{0.005}\right] = 25000(1.005)^{60}$

$F - 15 = \frac{25000(1.005)^{60}(0.005)}{1.005^{60} - 1}$

$F - 15 = 483.3200..$

$F = \$498.32$

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

Trial 2U 2012

	A	B	C	D
1		X		
2		X		
3	X			
4	X			
5				X
6		X		
7				X
8				X
9		X		
10		X		