R.J.B D.

KNOX GRAMMAR SCHOOL

VIRILE AGITUR

TRIAL H.S.C. EXAMINATION

1994

MATHEMATICS

2 UNIT

YEAR 12

Time allowed: Three hours
(plus five minutes reading time)

INSTRUCTIONS

ALL questions should be attempted.
ALL questions are of equal value.
ALL necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved calculators may be used.
Each question should be started on a new page.

The papers are to be handed in the following manner:

Part A	**Questions 1-2**
Part B	**Questions 3-4**
Part C	**Questions 5-6**
Part D	**Questions 7-8**
Part E	**Questions 9-10**

Name: ______________________________ **Class:** ________

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PART A

QUESTION 1 12 marks

(a) Given $R = \dfrac{\pi(D+3K)}{K-2D}$ where $D = 0.0034$ and $K = 0.087$, find the value of R correct to three significant figures.

(b) Write down the gradient of the line $2x + 3y = 5$.

(c) Solve $16^2 = 2^{2t-3}$ for t.

(d) Solve the pair of simultaneous equations:

$$2x + 3y = 9$$
$$x - 2y = 1.$$

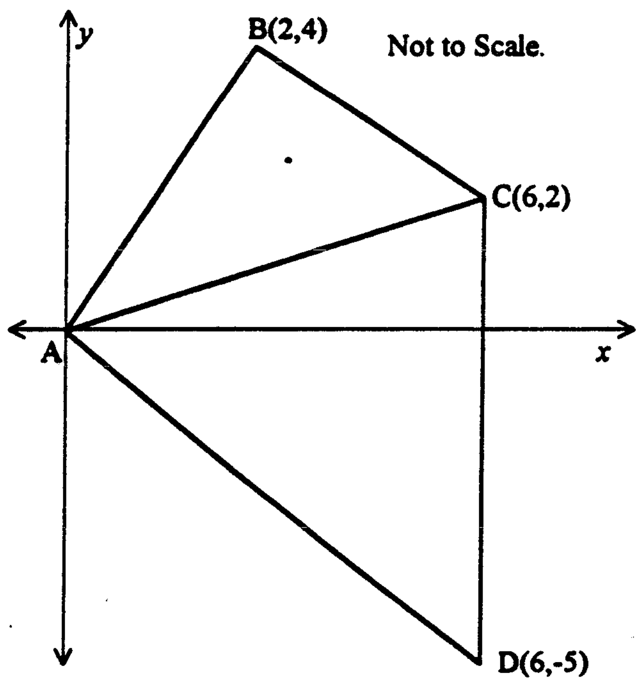
(e) Graph on a number line the solution set of $-12 \le \frac{3}{2}x < 3$.

(f) Solve the equation $5x^2 - 4x - 4 = 0$ giving each solution correct to two decimal places.

QUESTION 2 12 marks

Start a new page

In the diagram A, B, C and D have coordinates (0,0), (2,4), (6,2) and (6,-5) respectively.



(a) Show ΔABC is a right-angled isosceles triangle.

(b) Find the gradient of AC.

(c) Find the equation of AC.

(d) Find the angle which the line AC makes with the positive x- axis.

(e) Find the length of AC.

(f) Find the perpendicular distance of the point D from the line AC.

(g) Find the area of the quadrilateral $ABCD$.

PART B

QUESTION 3 **12 marks** **Start a new page**

(a) Differentiate:

(i) $3\tan(4x - \frac{\pi}{4})$

(ii) $x^2 e^{-2x}$

(iii) $\dfrac{\log_e 3x}{x}$

(b) Evaluate $\int_0^1 5x + \sin(5x)\, dx$ to 2 decimal places.

(c) Evaluate $\int_1^e \frac{2}{x} + \frac{x}{2}\, dx$ leaving your answer in exact form.

QUESTION 4 **12 marks** **Start a new page**

(a) The vertex of a parabola is at $V(2,-1)$ and the focus is at $S(2,3)$.

(i) Sketch the parabola showing the directrix, vertex and focus.

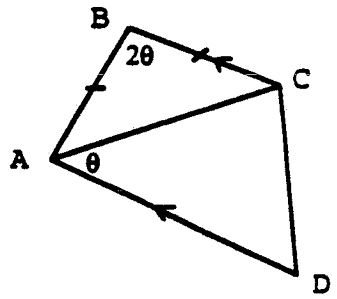
(ii) Write down the focal length and the equation of the directrix.

(iii) Find the equation of the parabola.

(b) In the diagram $ABCD$ is a quadrilateral with $AB = BC$,

$BC \parallel AD$, $\angle ABC = 2\theta$ and $\angle CAD = \theta$.

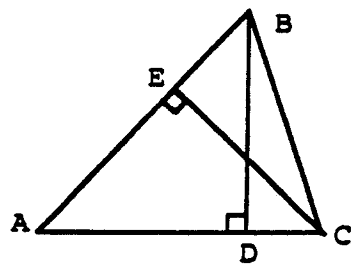
Copy this diagram on your answer sheet and find the value of θ giving reasons.



(c) In the diagram $BD \perp AC$ and $CE \perp AB$.

(i) Copy this diagram on your answer sheet and prove that ΔECA is similar to ΔDBA.

(ii) If $AB = 10$cm, $BD = 7$cm and $AC = 6$cm, find the length of CE.



PART C

QUESTION 5 **12 marks** **Start a new page**

(a) Consider the curve $y = x + \sin(2x)$ for $0 \le x \le \pi$.

(i) Find the two turning points for $0 \le x \le \pi$.

(ii) Determine the nature of the turning points.

(iii) Show their is one point of inflection for x between 0 and π.

(b) Sketch the curve of $y = e^{-2x}$ and shade the region bounded by this curve the x-axis, the y-axis and the line $x = 2$.

Find the volume generated when this area is rotated about the x-axis (leaving your answer in terms of π).

QUESTION 6 **12 marks** **Start a new page**

(a) The speed of the cyclists in the Commonwealth Games was recorded every ten minutes. A table was drawn up of the time, in minutes, and the corresponding speeds S, in km/hr.

Time (min)	0	10	20	30	40	50	60
Speed (km/h)	0	52	49	50	52	53	55

Use the Trapezoidal Rule to find the approximate value of $\int_0^1 S\,dt$ (where t is in hours).

(b) Find the values of m for which the equation $2x^2 - mx + 18 = 0$ has real roots.

(c) If α and β are the roots of the equation $2x^2 - 3x - 5 = 0$, find the value of

(i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $(\alpha + 1)(\beta + 1)$

(d) David borrows \$ 20 000 at 18% p.a. reducible interest, and pays it off in equal monthly instalments over 5 years. Calculate the size of each instalment to the nearest cent.

PART D

QUESTION 7 **12 marks** **Start a new page**

(a) Explain why the series $\frac{9}{8}, \frac{3}{4}, \frac{1}{2}, \ldots$ has a limiting sum. Find this limiting sum.

(b) Given the sequence $U_n = 2n + 3$.

(i) Find U_5, U_6, U_7 and U_{20}.

(ii) Is this sequence a G.P. or an A.P.? Give reasons.

(iii) Find $\sum_{n=5}^{20}(2n+3)$.

(c) Find a number 'n' which when added to each of 2, 5 and 9 will give a set of three numbers in geometric progression.

QUESTION 8 **12 marks** **Start a new page**

(a) A particle initially at rest at the origin moves in a straight line with velocity v metres per second, such that $v = 3t(4-t)$, where t is the time elapsed in seconds. Find

(i) the acceleration of the particle at the end of 1 second,

(ii) an expression for the displacement x of the particle in terms of t,

(iii) the particle's displacement when it is next at rest,

(iv) the velocity of the particle when it returns to the origin,

(v) the time taken for the particle to reach its greatest velocity,

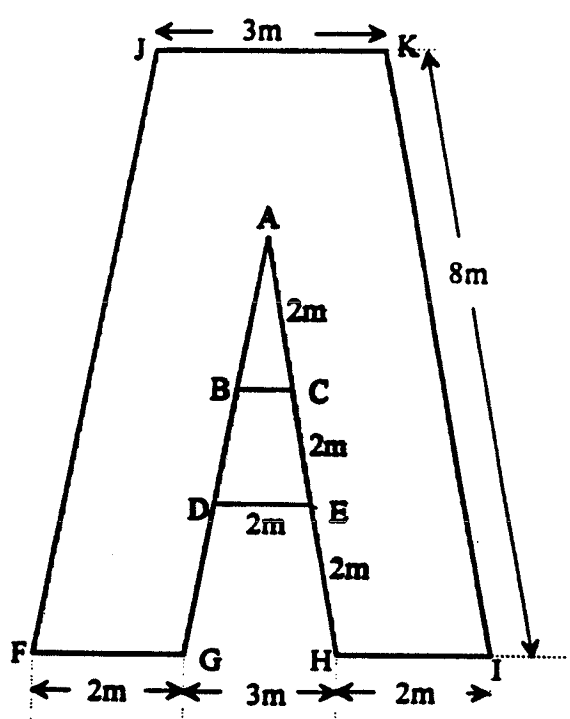
(vi) the distance travelled by the particle in the first 5 seconds.

(b) The point $P(x,y)$ moves such that its distance from the origin is twice its distance from the point $A(3,3)$. Represent this information on a number plane and show that the locus of P is given by $(x-4)^2 + (y-4)^2 = 8$.

PART E

QUESTION 9 **12 marks** **Start a new page**

(a) The letter **A** is to be painted on a billboard. The drawing opposite, not to scale, gives the dimensions of the letter : JF = IK = 8m, JK = 3m, and AB, AC, BD, CE, DG, EH, FG, DE, HI all equal 2m.



(i) Find the size of $\angle$ DAE, to the nearest minute, using the cosine rule.

(ii) Find the length of BC.

(iii) Show the vertical height between BC and DE is $\frac{\sqrt{15}}{2}$ m.

(iv) Show the vertical height of the letter **A** is $2\sqrt{15}$ m.

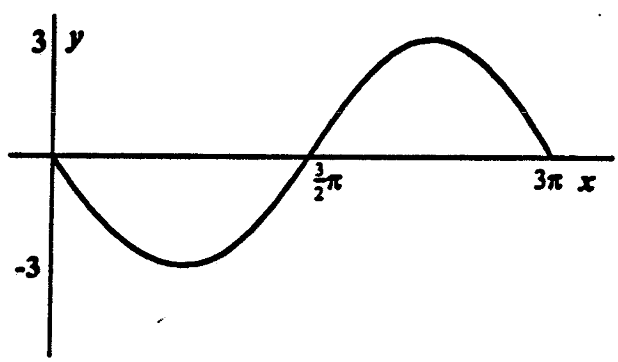
(v) Find the area of the letter **A** as a surd.

(b) The diagram below represents a possible sine or cosine curve.

(i) Give the amplitude.

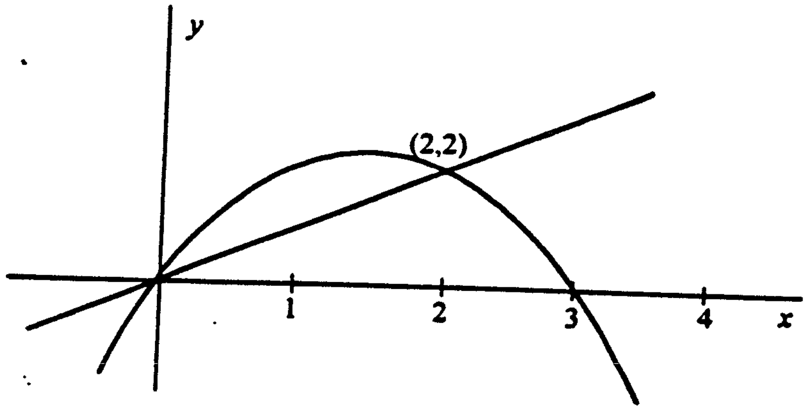
(ii) Give the period.

(iii) Write down a possible equation for the curve.



QUESTION 10 **12 marks** **Start a new page**

(a) The graphs, below, are represented by $y = x(3-x)$ and $y = x$.



(i) Find the area between the curve $y = x(3-x)$ and the x - axis.

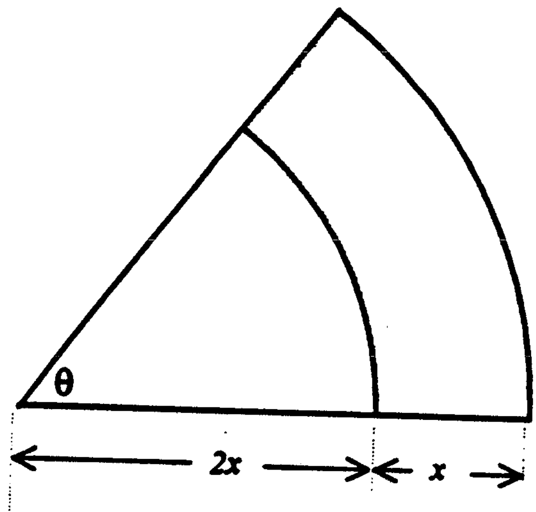
(ii) Find the area between the curve $y = x(3-x)$ and the line $y = x$.

(iii) What percentage of the area in part (i) is the area in part (ii)?

(b) (i) Write down the formulae for the length l of an arc of a circle in terms of r and θ.

(ii) Write down the formulae for the area A of a sector of a circle in terms of r and θ

(c) The shaded area in the diagram represents a portion of an annulus.



(i) Given the perimeter of the shaded region is 20 units, show the area of the shaded region is given by the formulae $A = 10x - x^2$.

(ii) Find the maximum area of this region.

End of Paper.

K.G.S. Trial HSC. 2U. 1994.

Q1. a) $R = 10\cdot357 = 10\cdot4$ (3 S.F.).

b) $m = -\frac{2}{3}$

c) $2t-3=8 \quad \therefore t = \frac{11}{2}$ or $t = 5\frac{1}{2}$.

d) $2x+3y=9$
$2x-4y=2.$
$7y=7$
$\therefore y=1$ and $x=3$.

e) $-12 \le \frac{3}{2}x < 3 \quad \therefore -8 \le x < 2.$

f) $x = -\frac{-4}{10} \pm \frac{\sqrt{16+80}}{10} = \frac{4}{10} \pm \frac{\sqrt{96}}{10}.$

$x = 0\cdot4 + 0\cdot97979$ or $x = 0\cdot4 - 0\cdot97979$
$x = 1\cdot38$ (2dp) or $x = -0\cdot58$ (2dp)

Q2. a) $m_{AB} = \frac{4-0}{2-0} = 2 \qquad m_{BC} = \frac{2-4}{6-2} = -\frac{1}{2}.$

$m_{AB}.m_{BC} = 2.\frac{-1}{2} = -1 \quad \therefore AB \perp BC.$

$AB = \sqrt{2^2+4^2} = \sqrt{20} \qquad BC = \sqrt{(2-6)^2+(4-2)^2} = \sqrt{20}.$

$\therefore \triangle ABC$ is a right-angled isosceles $\triangle$.

b). $m_{AC} = \frac{2-0}{6-0} = \frac{1}{3}.$

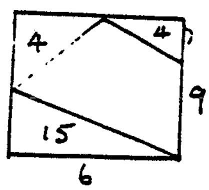
c) $y-0 = \frac{1}{3}(x-0) \quad \therefore y = \frac{1}{3}x$ or $x-3y=0.$

d) $\tan\theta = \frac{1}{3} \quad \therefore \theta = 18°26'$ or $\theta = 18\cdot435°.$

e) $AC = \sqrt{6^2+2^2} = \sqrt{40}$ or $2\sqrt{10}$

f) $d = \left|\frac{1(6)-3(-5)}{\sqrt{1^2+(-3)^2}}\right| = \frac{21}{\sqrt{10}}$

g)



$A = 54 - (15+4+4) = 31$ units^2

or

$A = \frac{1}{2}\sqrt{20}\times\sqrt{20} + \frac{1}{2}2\sqrt{10}\times\frac{21}{\sqrt{10}}.$

$= 10 + 21 = 31$ units^2.

Q3. a) (i) $\frac{d}{dx}(3\tan(4x-\frac{\pi}{4})) = 12\sec^2(4x-\frac{\pi}{4}).$

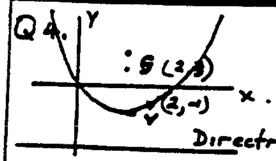
(ii) $\frac{d}{dx}(x^2e^{-2x}) = (2x)(e^{-2x}) + (-2e^{-2x})(x^2)$
$= (2x-2x^2)e^{-2x}.$

(iii) $\frac{d}{dx}\left(\frac{\log 3x}{x}\right) = \frac{x(\frac{3}{3x}) - (1)\log 3x}{x^2}$
$= \frac{1-\log 3x}{x^2}.$

b). $\int_0^1 5x + \sin 5x\, dx = \frac{5}{2}x^2 - \frac{1}{5}\cos 5x\Big]_0^1$
$= \left[\frac{5}{2} - \frac{1}{5}\cos 5\right] - \left[0 - \frac{1}{5}\cos 0\right] = \frac{5}{2} - 0\cdot0567 + \frac{1}{5}$
$= 2\cdot64$ (2dp).

c) $\int_1^e \left(\frac{2}{x} + \frac{x}{2}\right)dx = 2\log x + \frac{1}{4}x^2\Big]_1^e$
$= \left[2\log e + \frac{e^2}{4}\right] - \left[2\log 1 + \frac{1}{4}\right] = 2 + \frac{1}{4}e^2 - \frac{1}{4}$
$= \frac{1}{4}(7+e^2)$ or $1\frac{3}{4} + \frac{1}{4}e^2.$

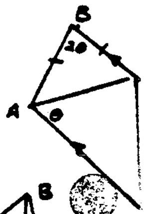
Q4.



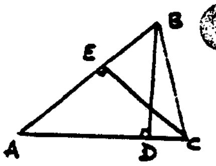
(ii) focal length = 4.
Directrix $y = -5$.

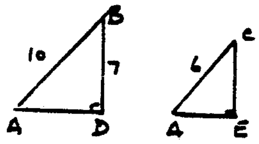
(iii) $(x-2)^2 = 16(y+1)$

b) $B\hat{C}A = \theta$ (alternate angles of $\parallel$ lines)
$B\hat{A}C = \theta$ base angles of isosc $\triangle$'s.
$2\theta+\theta+\theta = 4\theta = 180$ sum angles of $\triangle$
$\therefore \theta = 45°$



c) in $\triangle BDA$ and $\triangle CEA$.
$B\hat{D}A = C\hat{E}A$ Both right angles.
$D\hat{A}B = E\hat{A}C$ common angle.
$\therefore \triangle BDA \,|||\, \triangle CEA$ A.A. or (2 angles).





$\frac{CE}{6} = \frac{7}{10}$

$\therefore CE = \frac{42}{10}$ or $4\cdot2$ cm.

Q5. $y = x + \sin(2x) \quad \frac{dy}{dx} = 1 + 2\cos 2x \quad \frac{d^2y}{dx^2} = -4\sin$

i) SP. $\frac{dy}{dx}=0 \quad \therefore \cos 2x = -\frac{1}{2}, \; 2x = \frac{2\pi}{3}$ or $2x = \frac{4\pi}{3}.$

$\therefore$ SP. $x = \frac{\pi}{3}, y = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ and $x = \frac{2\pi}{3}, y = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

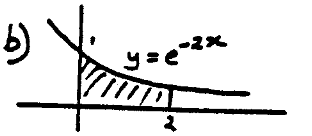
ii) at $x = \frac{\pi}{3} \quad \frac{d^2y}{dx^2} = -4\sin\frac{2\pi}{3} = -2\sqrt{3} \;\therefore$ c.c down $\therefore$ M... T.P

at $x = \frac{2\pi}{3}, \; \frac{d^2y}{dx^2} = -4\sin\frac{4\pi}{3} = 2\sqrt{3} \;\therefore$ c.c. up $\therefore$ Min TP

(iii) $\frac{d^2y}{dx^2} = 0 \quad \therefore \sin 2x = 0 \quad \therefore 2x = 0$ or $2x = \pi.$
$\therefore x = 0$ or $x = \frac{\pi}{2} \quad \therefore$ for $0 < x < \pi \quad x = \frac{\pi}{2}$

at $x = \frac{\pi}{2}^+, \; \frac{d^2y}{dx^2} = \sin\pi^+ < 0.$
at $x = \frac{\pi}{2}^-, \; \frac{d^2y}{dx^2} = \sin\pi^- > 0.$
$\}\therefore$ change in concavity.
$\therefore$ pt of inflection at x =

b)



$V = \pi\int_0^2 (e^{-2x})^2 dx.$
$= \pi\int_0^2 e^{-4x}dx.$

$\therefore V = \pi\left(\frac{-1}{4}e^{-4x}\right)\Big]_0^2.$
$= -\frac{\pi}{4}e^{-8} + \frac{\pi}{4}e^0$
$\therefore Vol = \frac{\pi}{4}\left[1 - e^{-8}\right]$ units^3

Q6. a) $h = \frac{1}{6}$.

$\int_0^1 s\,dt \doteq \frac{1}{12}\left[0 + 55 + 2(52+49+50+52+53)\right]$.

$\doteq \frac{1}{12}\left[55 + 512\right) \doteq$ **47.25 km.**

b) Real roots $\Delta = b^2 - 4ac \geq 0$, $m^2 - 4 \times 2 \times 18 \geq 0$.

$\therefore (m-12)(m+12) \geq 0 \quad \therefore$ **$m \leq -12$ or $m \geq 12$.**

c) (i) $\alpha + \beta = \frac{3}{2}$ (ii) $\alpha\beta = \frac{-5}{2}$.

(iii) $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1 = -\frac{5}{2} + \frac{3}{2} + 1 = 0$

d) $A_1 = PR - m$

$A_2 = PR^2 - m(1+R)$

$A_3 = PR^3 - m(1+R+R^2)$

$\therefore A_n = PR^n - m\left(\frac{R^n - 1}{R-1}\right)$

$\therefore 0 = 20000(1.015)^{60} - m\left(\frac{1.015^{60}-1}{0.015}\right)$

$\therefore m = \frac{20000(1.015)^{60} \times 0.015}{1.015^{60} - 1}$.

$\therefore$ **m = \$571.43.**

Q7. a) $r_1 = \frac{3}{4} \div \frac{9}{8} = \frac{2}{3}$, $r_2 = \frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$ $\therefore$ r is common.

$|r| < \frac{2}{3}$ $\therefore$ Limiting sum of a GP Exists.

$S_\infty = \frac{a}{1-r} = \frac{9}{8} \div \frac{1}{3} = \frac{27}{8} \simeq 3\frac{3}{8}$.

b) (i) $U_5 = 13$, $U_6 = 15$, $U_7 = 17$, $U_{20} = 43$.

(ii) $U_6 - U_5 = 2$, $U_7 - U_6 = 2$. $\therefore$ common difference

$\therefore$ A.P.

(iii) $\sum_{n=5}^{20} (2n+3) = \frac{n}{2}(a+l) = \frac{16}{2}(13+43) =$ **448.**

c) $U_1 = 2+n$, $U_2 = 5+n$, $U_3 = 9+n$.

$\frac{5+n}{2+n} = \frac{9+n}{5+n}$

$(5+n)^2 = (2+n)(9+n)$

$\therefore 25 + 10n + n^2 = 18 + 11n + n^2$

$\therefore$ **n = 7**

Q8. a) $v = 3t(4-t) = 12t - 3t^2 \quad a = 12 - 6t$

(i) at $t = 1$ acc $= 6\ \text{m.sec}^{-2}$.

(ii) $x = \int v\,dt = 6t^2 - t^3 + c$. $\therefore x = 6t^2 - t^3$

(iii) $v = 0 \quad t = 4 \quad x = 6(4)^2 - 4^3 = 2(16) = 32\ m$.

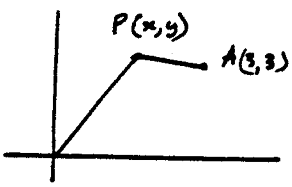
(iv) $x = 0 \quad t = 6. \quad v = 18(-2) = -36\ \text{m.sec}^{-1}$.

(v) max V when acc = 0 $\therefore 12 - 6t = 0$ $\quad t = 2$ Sec.

(vi) when $t = 5$ $x = 6(5^2) - 5^3 = 25\ m$.

$\therefore$ Distance $= 32 + (32-25) =$ **39 m**

b)



$PO = 2PA$.

$PO^2 = 4(PA)^2$

$x^2 + y^2 = 4(x-3)^2 + 4(x-3)^2$

$x^2 + y^2 = 4x^2 - 24x + 36 + 4y^2 - 24y + 36$

$3x^2 - 24x + 36 + 3y^2 - 24y + 36 = 0$.

$x^2 - 8x + y^2 - 8y = -24$.

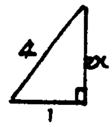
$(x-4)^2 + (y-4)^2 = 32 - 24 = 8$.

Q9. a) (i) $\cos\theta = \frac{4^2 + 4^2 - 2^2}{2 \times 4 \times 4} = \frac{28}{32} = \frac{7}{8}$.

$\therefore \theta = 28°57'$

(ii) BC = 1 m.

(iii)

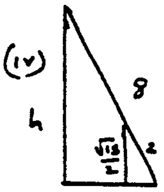


$(2x)^2 = 4^2 - 1$

$4x^2 = 15$

$\therefore x = \frac{\sqrt{15}}{2}$.

(iv)



$\frac{h}{8} = \frac{\sqrt{15}}{4}$

$\therefore h = 2\sqrt{15}$

(v) $A = \frac{3+7}{2} 2\sqrt{15} - \frac{1}{2}\left(\frac{\sqrt{15}}{2}\right) - \frac{2+3}{2}\left(\frac{\sqrt{15}}{2}\right)$.

$\therefore$ area $= \frac{17}{2}\sqrt{15}\ m^2$

b) (i) amp = 3 . (ii) Period = 3π.

(iii) $y = -3\sin\left(\frac{2}{3}x\right)$.

Q10.

(i) $A = \int_0^3 3x - x^2\,dx = \frac{3}{2}x^2 - \frac{1}{3}x^3\Big]_0^3 = \frac{27}{2} - \frac{27}{3} = \frac{9}{2}$ u.

(ii) $A = \int_0^2 [(3x - x^2) - x]\,dx = \int_0^2 2x - x^2\,dx$.

$= x^2 - \frac{1}{3}x^3\Big]_0^2 = 4 - \frac{1}{3}(8) = \frac{4}{3}$ units^2.

(iii) $\% = \frac{4}{3} \div \frac{9}{2} \times 100\% = 29.63\% \simeq 30\%$.

b) (i) $l = r\theta$ (ii) $A = \frac{1}{2}r^2\theta$.

c) $P = 20 = x + x + 2x\theta + 3x\theta \therefore 2x + 5x\theta = 20$.

$\therefore \theta = \frac{20-2x}{5x}$.

(i)

$A = \frac{\theta}{2}(3x)^2 - \frac{\theta}{2}(2x)^2$

$= \frac{\theta}{2} 5x^2$

$A = \frac{5x^2}{2}\left(\frac{20-2x}{5x}\right)$.

$= x(10-x)$

$\therefore A = 10x - x^2$

(ii) $\frac{dA}{dx} = 10 - 2x = 0$.

$\therefore x = 5$

$\frac{d^2A}{dx^2} = -2$ $\therefore$ cc down.

$\therefore$ max T.P.

$\therefore$ MAX Area

$A = 50 - 25$

Area is 25 units^2.