RJBD

KNOX GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

2001
TRIAL HSC EXAMINATION

Mathematics

- General Instructions
- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 12
- All necessary working should be shown in every question

Total marks (120)

- **Attempt Questions 1–10**
- All questions are of equal value
- Use a SEPARATE Writing Booklet for each question

NAME:__________________________ **TEACHER:**______________________

99

Total marks (120)
Attempt questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE Writing Booklet. Extra Writing Booklets are available.

Question 1 (12 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Find the value of $(2^{1.7}+3)^2$ correct to two decimal places. **2**

(b) Solve $4-3x>10$ and graph the solution on the number line. **2**

(c) Convert 300° to radians in terms of π. **1**

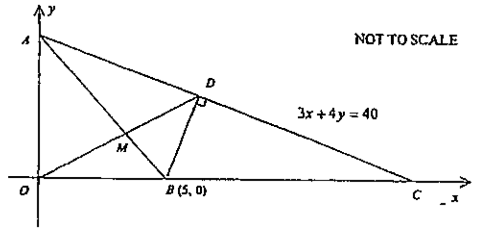
(d) From a pack of fifty-two playing cards a person selects two cards and places them in a row. What is the probability that the second card selected is the ace of hearts? **1**

(e) Solve for x: $\frac{2x}{3}-\frac{x+1}{4}=1$. **2**

(f) Find integers a and b such that $\frac{3}{1-\sqrt{2}}=a-\sqrt{b}$. **2**

(g) Solve $3^{x-3}=\frac{1}{9}$. **2**

Question 2 (12 marks) Use a SEPARATE Writing Booklet. **Marks**



The diagram shows the point A on the y-axis and the points $B(5, 0)$ and C on the x-axis. Point D lies on AC such that BD is perpendicular to AC. The equation of the line AC is $3x + 4y = 40$. Point M is the mid point of OD.

(a) Show that A has coordinates (0, 10). **1**

(b) Find the gradient of the line AB. **1**

(c) Write down the equation of the line AB. **1**

(d) Show that the equation of BD is $4x - 3y = 20$. **1**

(e) By solving the equations of the lines AC and BD simultaneously, show that D has coordinates (8, 4). **2**

(f) Find the coordinates of M, the mid-point of DO. **1**

(g) Find the perpendicular distance of D from the line AB. **2**

(h) Show that M lies on the line AB. **1**

(i) Find the area of quadrilateral $AOBD$. **2**

100

Question 3 (12 marks) Use a SEPARATE Writing Booklet. **Marks**

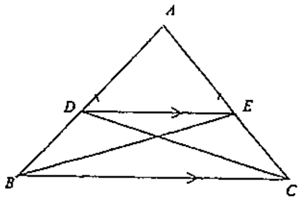
(a) Differentiate the following:

(i) $\sin(3x + 1)$. **1**

(ii) $x^3 \ln x$. **2**

(iii) $\frac{e^x}{x}$. **2**

(b)



NOT TO SCALE

In the diagram, ABC is an isosceles triangle where AB = AC and DE is parallel to BC.

(i) Show that ADE is an isosceles triangle. **2**

(ii) Show that $DB = EC$. **1**

(c) Find:

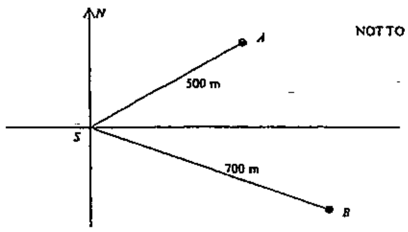
(i) $\int (2x + 3)^5 \, dx$ **1**

(ii) $\int \sin \frac{x}{2} \, dx$ **1**

(iii) $\int_0^1 \frac{dx}{3x + 1}$. **2**

Question 4 (12 marks). Use a SEPARATE Writing Booklet. **Marks**

(a)



From a ship at point S two buoys are observed, one at point A at a distance of 500 metres and a bearing of 043° T, the other at point B at a distance of 700 metres and a bearing of 118° T.

(i) Copy or trace the diagram into your Writing Booklet and mark on your diagram all the given information. Show $\angle ASB$ is 75°. **1**

(ii) Find the distance of buoy A from buoy B, correct to the nearest metre. **2**

(iii) Find the bearing of buoy A from buoy B, correct to the nearest degree. **3**

(b) The graph of $y = f(x)$ passes through the point (2, 3) and $f'(x) = 3x^2 - 3$. Find an expression for $f(x)$. **2**

(c) Consider the parabola $(x-1)^2 = -8(y-3)$

(i) Find the vertex and the focus of the parabola. **2**

(ii) Sketch the parabola marking the vertex and focus on it. **1**

(iii) Find the equation of another parabola with the same focal length, focus and axis of symmetry. **1**

101

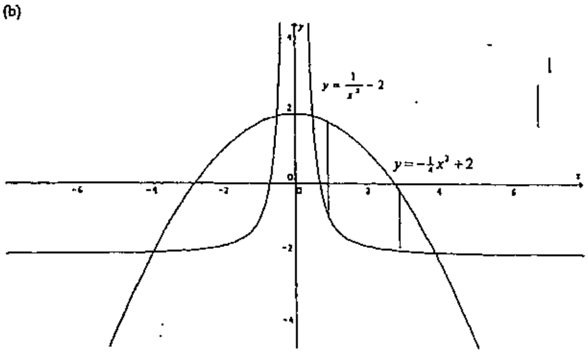
Question 5 (12 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Consider the curve given by $y = x^3 - 3x^2 + 2$.

(i) Find the two stationary points and determine their nature. **4**

(ii) Sketch the curve for $-1 \le x \le 3$. **2**

(b)



The diagram shows the graphs of $y = \frac{1}{x^2} - 2$ and $y = -\frac{1}{4}x^2 + 2$.

(i) Find the area between the curves from $x = 1$ to $x = 3$. **3**

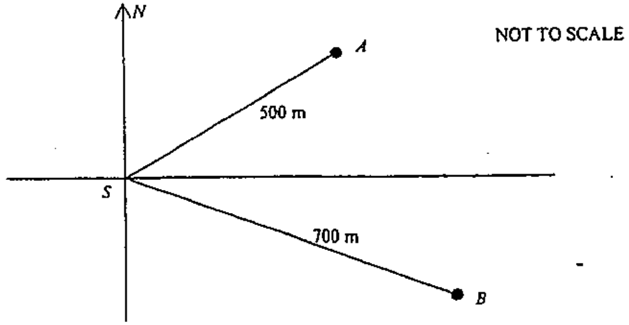
(ii) Show that the curves intersect when $x^4 - 16x^2 + 4 = 0$. **1**

(iii) Use the substitution of $u = x^2$, or otherwise, show the x coordinates of the points of intersection of the two curves are **2**

$$x = \pm\sqrt{8 \pm 2\sqrt{15}}.$$

Question 4 (12 marks). Use a SEPARATE Writing Booklet. **Marks**

(a)



From a ship at point S two buoys are observed, one at point A at a distance of 500 metres and a bearing of 043^0 T, the other at point B at a distance of 700 metres and a bearing of 118^0 T.

(i) Copy or trace the diagram into your Writing Booklet and mark on your diagram all the given information. Show $\angle ASB$ is 75^0. **1**

(ii) Find the distance of buoy A from buoy B, correct to the nearest metre. **2**

(iii) Find the bearing of buoy A from buoy B, correct to the nearest degree. **3**

(b) The graph of $y = f(x)$ passes through the point (2, 3) and $f'(x) = 3x^2 - 3$. Find an expression for $f(x)$. **2**

(c) Consider the parabola $(x-1)^2 = -8(y-3)$

(i) Find the vertex and the focus of the parabola. **2**

(ii) Sketch the parabola marking the vertex and focus on it. **1**

(iii) Find the equation of another parabola with the same focal length, focus and axis of symmetry. **1**

101

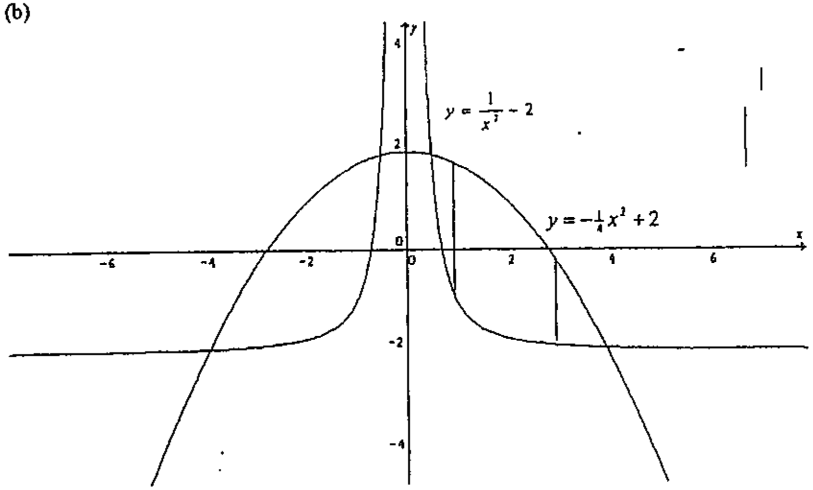
Question 5 (12 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Consider the curve given by $y = x^3 - 3x^2 + 2$.

(i) Find the two stationary points and determine their nature. **4**

(ii) Sketch the curve for $-1 \le x \le 3$. **2**

(b)



The diagram shows the graphs of $y = \frac{1}{x^2} - 2$ and $y = -\frac{1}{4}x^2 + 2$.

(i) Find the area between the curves from $x = 1$ to $x = 3$. **3**

(ii) Show that the curves intersect when $x^4 - 16x^2 + 4 = 0$. **1**

(iii) Use the substitution of $u = x^2$, or otherwise, show the x coordinates of the points of intersection of the two curves are **2**

$$x = \pm\sqrt{8 \pm 2\sqrt{15}}.$$

Question 6 (12 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Consider the sequence $T_n = 2n + 3$.

(i) Find the first 3 terms. **1**

(ii) Is this sequence an A.P. or a G.P.? Give reasons. **1**

(iii) Find the sum of the first 20 terms. **2**

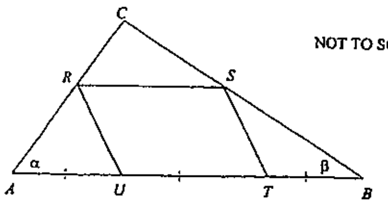
(b) A pool is being drained and the number of litres of water, L, in the pool at time t minutes is given by the equation:

$$L = 120(40 - t)^2.$$

(i) At what rate is the water draining out of the pool when $t = 6$ minutes? **2**

(ii) How long will it take for the pool to be completely empty? **1**

(c)



NOT TO SCALE

In the triangle ABC, $\angle CAB = \alpha$, $\angle CBA = \beta$, $AU = UT = TB$ and $RSTU$ is a rhombus. Copy or trace the diagram into your Writing Booklet and mark the information on it.

(i) Show that $\triangle STB$ is isosceles. **1**

(ii) Show that $\angle STU$ is double the $\angle SBT$. **1**

(iii) Hence prove that $\angle ACB$ is a right angle. **3**

Question 7 (12 marks). Use a SEPARATE Writing Booklet. **Marks**

(a) Given that $\sin\theta = \frac{3}{\sqrt{15}}$ and $\cos\theta < 0$, find the exact value of $\tan\theta$. **2**

(b) Simplify $\frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin(\pi - \theta)} \times \tan(-\theta)$. **3**

(c) The points $A(0,4)$, $B(1,4)$, $C(2,0)$, $D(3,-1)$, and $E(4,-1)$ are plotted on a number plane.
The line segments joining A to B, B to C, C to D, and D to E, define a function $g(x)$.

(i) Show clearly on a diagram the information given above. **1**

(ii) Evaluate $\int_0^4 g(x)\,dx$. **2**

(d) Consider the function $f(x) = \cos^2 x$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$\cos^2 x$				0		$\frac{3}{4}$	

(i) Copy and complete the above table using exact values. **1**

(ii) Use Simpson's rule with seven function values (from the table), to find an estimation for the volume of the solid formed when the area between the curve $y = \cos x$ and the x-axis between $x = 0$ and $x = \pi$ is revolved about the x-axis. **3**

Question 8 (12 marks). Use a SEPARATE Writing Booklet. **Marks**

(a) A particle is moving in a straight line. Its displacement from a fixed point on the line at time t seconds is given by $x = t - \ln(2t + 1)$, $t \geq 0$, where x is in metres.

(i) Find an expression for the velocity of the particle. **1**

(ii) Find an expression for the acceleration of the particle and show that the direction of the acceleration is always positive. **1**

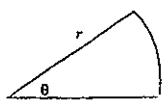
(iii) In what direction is the particle moving when $t = 0$? **1**

(iv) At what time does the particle come to rest? **1**

(v) Where does the particle come to rest? **1**

(vi) How far does the particle travel in the first two seconds of its motion? Leave answer in exact form. **2**

(b)



The sector of a circle with radius r and angle θ has a perimeter of 20 cm.

(i) Show that the area of this sector can be expressed as $A = 10r - r^2$. **1**

(ii) Find the value of θ, to the nearest degree, which will make this area a maximum. **4**

Question 9 (12 marks) Use a SEPARATE Writing Booklet. **Mark**

(a) (i) Write down an expression for the discriminant of the quadratic equation $ax^2 + 5x + a = 0$ **1**

(ii) Hence, find the values of a for which the function $f(x) = ax^2 + 5x + a$ is negative definite. **3**

(b) Solve $\log_x 4 = \log_3 \sqrt{3}$. **2**

(c) The population of solder ants, S, in a certain area increases exponentially according to $S = Ae^{kt}$, where k is a constant and t is time in weeks.

At the beginning of an observation period there were 5000 ants.

(i) Calculate the value of the constant A. **1**

(ii) If initially the population of the ants increased at a rate of 350 ants per week, calculate the value of the constant k. **2**

(iii) How many ants will there be after 8 weeks? **1**

(d) Express in simplest form the limiting sum of the geometric series **2**

$$\sin^2 x + \sin^4 x + \sin^6 x + \ldots\ \text{for } 0 < x < \frac{\pi}{2}.$$

5. a)

$\frac{d}{dx}(\sin(3x+1)) = 3\cos(3x+1)$ (1)

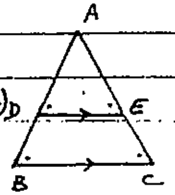
$\frac{d}{dx}(x^2 \ln x) = x^2 \times \frac{1}{x} + 2x\ln x$ (2)

$= x + 2x\ln x$.

$\frac{d}{dx}\left(\frac{e^x}{x}\right) = \frac{xe^x - e^x(1)}{x^2}$ (2)

$= \frac{e^x(x-1)}{x^2}$

) $\triangle ABC$ is isosceles



$\angle ABC = \angle ACB$ (base angles)

$\angle ADE = \angle ABC$ corresponding angles of ∥ lines DE, BC

$AED = \angle ACB$ "

$\angle ADE = \angle AED$ (2)

∴ $\triangle ADE$ is isosceles.

) ∴ $AD = AE$ also $AB = AC$.

Hence $AB - AD = AC - AE$

∴ $DB = EC$. (1)

)(i) $\int (2x+3)^5 dx = \frac{1}{2} \times \frac{1}{6}(2x+3)^6 + c$

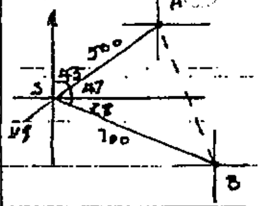
$= \frac{1}{12}(2x+3)^6 + c$. (1)

) $\int \sin\left(\frac{x}{2}\right) dx = -2\cos\left(\frac{x}{2}\right) + c$ (1)

i) $\int_0^1 \frac{dx}{3x+1} = \frac{1}{3}\ln(3x+1)\Big]_0^1$

$= \frac{1}{3}\ln(3+1) - \frac{1}{3}\ln(1)$

$= \frac{1}{3}\ln 4$ (2)



$A\hat{S}B = 47 + 28$

$= 75°$ (1)

(ii) $AB^2 = 500^2 + 700^2 - 2(500)(700)\cos 75$

$= 740\,000 - 181173.3316\ldots$

$= 558826.6684$

$AB = 747.547\ldots$

$= 748$ (nearest metre) (2)

(iii) $\frac{\sin \angle SBA}{500} = \frac{\sin 75°}{748}$

$\sin(SBA) = 0.64567\ldots$

$\angle SBA = 40.216\ldots$

$= 40°$ nearest degree

Bearing $= 270 + 28 + 40$

$= 338°T$. (3)

b) $f'(x) = 3x^2 - 3$

$f(x) = x^3 - 3x + c$ ✓

$3 = (2)^3 - 3(2) + c$

$3 = 8 - 6 + c$

$c = 1$ ✓

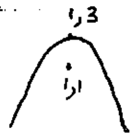
$f(x) = x^3 - 3x + 1$. (2)

c) $(x-1)^2 = -8(y-3)$.

(i) vertex $(1, 3)$ $a = 2$. (2)

focus $(1, 1)$

(ii)



(1)

(iii) new vertex $(1, -1)$ focal length 2

$(x-1)^2 = 8(y+1)$ (1)

(i) $\frac{dy}{dx} = 3x^2 - 6x$.

S.P. when $\frac{dy}{dx} = 0$ $3x(x-2) = 0$.

∴ $x = 0$ or $x = 2$

$y = 2$ $y = -2$.

S.P. $(0,2)$ $(2,-2)$

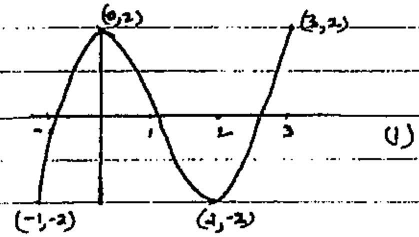
nature: $\frac{d^2y}{dx^2} = 6x - 6$.

when $x = 0$ $\frac{d^2y}{dx^2} = -6 < 0$ ∴ Concave Down.

when $x = 2$ $\frac{d^2y}{dx^2} = 12 - 6 > 0$ ∴ Concave up.

∴ $(0,2)$ max T.P. (2) $(2,-2)$ min T.P. (2).

(ii) $x = -1$ $y = -2$, $x = 3$ $y = 2$. (1)



(1)

b) $A = \int_1^3 y_T - y_B \, dx$

$= \int_1^3 -\frac{1}{4}x^2 + 2 - x^{-2} + 2 \, dx$ ✓

$= -\frac{1}{12}x^3 + 4x + \frac{1}{x}\Big]_1^3$ ✓

$= \left[-\frac{27}{12} + 12 + \frac{1}{3}\right] - \left[-\frac{1}{12} + 4 + 1\right]$

$= 10\frac{1}{12} - 5 + \frac{1}{12}$

$= 5\frac{1}{6}$ ✓

(ii) $\frac{1}{x^2} - 2 = -\frac{1}{4}x^2 + 2$

$4 - 8x^2 = -x^4 + 8x^2$ ✓

∴ $x^4 - 16x^2 + 4 = 0$

$u = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$.

$= \frac{16}{2} \pm \frac{\sqrt{16^2 - 16}}{2}$ ✓

$= 8 \pm \frac{4\sqrt{15}}{2}$

∴ $x^2 = 8 \pm 2\sqrt{15}$ ✓

$x = \pm\sqrt{8 \pm 2\sqrt{15}}$

Q6. a) $T_n = 2n + 3$.

$T_1 = 2(1) + 3 = 5$ $T_3 = 2(3) + 3 = 6 + 3 = 9$. $T_2 = 2(2) + 3 = 4 + 3 = 7$

first 3 terms. 5, 7, 9.

(ii) $T_2 - T_1 = 7 - 5 = 2$ } common

$T_3 - T_2 = 9 - 7 = 2$ } Difference

∴ A.P.

(iii) $S_{20} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{20}{2}(2 \times 5 + 19(2))$

$= 10(10 + 38)$

$= 480$

b) $L = 120(40 - t)^2$

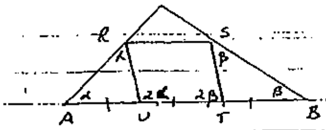
(i) $\frac{dL}{dt} = -240(40 - t)$

when $t = 6$ $\frac{dL}{dt} = -240(40 - 6)$

$= -8160$

draining out at 8160 litres per min

(ii) Pool empties in 40 minutes



rhombus has equal sides.

UT = ST = TB

∴ ΔSTB is isosceles

∠TSB = ∠TBS = β (Base angles of isosceles Δ)

STU = β + β   Exterior angle of a triangle = Sum of interior opposite angles.

= 2β

= 2∠SBT

AU = UT = RU (equal sides of rhombus)

∴ ΔAUR is isosceles.

AU = ∠ARU = α (Base angles of isosceles Δ)

RUT = α + α   ext ∠ of Δ = Σ of interior opp angles.

= 2α

= 2∠RUT

RUA = ∠STU = 2β corresponding angles of ∥ lines RU, ST

AUR + ∠RUT = 2β + 2α (Straight angle.)

= 180°

2(α+β) = 180

α + β = 90.

Δ ABC

∠ACB + α + β = 180 angle sum of triangle

∠ACB + 90 = 180.

∠ACB = 90.

∠ACB is a right angle.

$\sqrt{15}$ ∴ 90 < θ < 180.

tan θ < 0.

$\tan\theta = -\frac{3}{\sqrt{6}}$

or $= -\frac{1}{2}\sqrt{6}$.

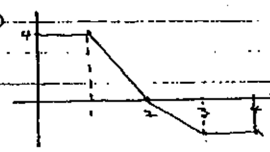
b) $\frac{\sin(\frac{\pi}{2}-\theta)}{\sin(\pi-\theta)} \times \tan(-\theta)$

$= \frac{\cos\theta}{\sin\theta} \times (-\tan\theta)$.

$= -\frac{\cos\theta}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$.

= −1.

c)



$\int_0^4 g(x)\,dx = (4\times1) + (\frac{1}{2}\times4\times1) - (\frac{1}{2}\times1\times1) - (1\times1)$

= 4 + 2 − ½ − 1

= 4½.

d)

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$\cos^2 x$	1	$\frac{3}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{3}{4}$	1
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(ii) $V = \pi\int_0^\pi \cos^2 x\,dx$

$\doteqdot \pi \times \frac{h}{3}\left[y_0 + y_6 + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$

$\doteqdot \pi \times \frac{\pi}{18}\left[1+1+4(\frac{3}{4}+0+\frac{3}{4}) + 2(\frac{1}{4}+\frac{1}{4})\right]$

$\doteqdot \frac{\pi^2}{18}\left[2+6+1\right]$

≑ 4.9348....

≑ 4.9 (1 dp) units³

---

i) $\frac{dx}{dt} = 1 - \frac{2}{2t+1}$

(ii) $\frac{d^2x}{dt^2} = \frac{4}{(2t+1)^2} > 0$ since $(2t+1)^2 > 0$.

(iii) when t = 0 $\frac{dx}{dt} = 1 - \frac{2}{1}$

= −1.

∴ moves in the negative direction when t = 0.

(iv) rest when $\frac{dx}{dt} = 0$

$1 - \frac{2}{2t+1} = 0$

2t + 1 = 2

t = ½ Second.

(v) at t = ½ $x = \frac{1}{2} - \ln(2\times\frac{1}{2}+1)$

$= \frac{1}{2} - \ln 2$.

Comes to rest at ½ − ln 2 or −0.193 (3 d.p.)

(vi) t = ½, x = ½ − ln 2; t = 0, x = 0; t = 2, x = 2 − ln 5

Distance = (ln 2 − ½) + (2 − ln 5) − (½ − ln 2)

= 1 + 2 ln 2 − ln 5 metres. (2)

or $1 - \ln\frac{5}{4}$

b) P = r + r + rθ = 20

$\theta = \frac{20-2r}{r}$.

$A = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\frac{20-2r}{r}\right)$

= r(10 − r)

A = 10r − r²

$\frac{dA}{dr}$

max when $\frac{dA}{dr} = 0$. 10 − 2r = 0

∴ r = 5.

Test. $\frac{d^2A}{dr^2} = -2 < 0$. ∴ concave down ∴ max.

$\theta = \frac{20-10}{5}$

θ = 2

$\theta = \frac{2\times180}{\pi}$

$= \frac{360}{\pi}$

= 114.59

≑ 115° (nearest degree)

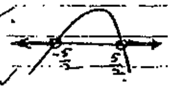
Q9. $ax^2 + 5x + a = 0$

$\Delta = 5^2 - 4(a)(a)$.

(i) $\Delta = 25 - 4a^2$

(ii) neg. def. Δ < 0 and a < 0.

$25 - 4a^2 < 0$



$a < -\frac{5}{2}$ or $a > \frac{5}{2}$

but a < 0

∴ $a < -\frac{5}{2}$.

b) $\log_x 4 = \log_3\sqrt{3}$

$= \frac{1}{2}\log_3 3$

∴ $\log_x 4 = \frac{1}{2}$

$x^{\frac{1}{2}} = 4$

x = 16.

PTO

106

---

cont.

(i) $S = Ae^{kt}$

$5000 = Ae^0$

∴ A = 5 000.

$\frac{dS}{dt} = kAe^{kt}$

$350 = k(5000)e^0$

$k = \frac{350}{5000}$

= 0.07.

$S = 5000e^{0.07\times8}$

$= 5000e^{.56}$

= ~~5362.54~~ 8753.36

= ~~5363~~ (to the nearest whole no)

8753 ants.

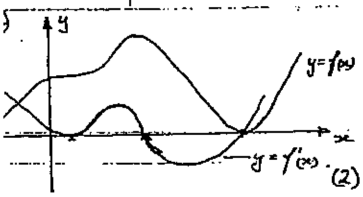
) ~~a = sin²x   r = sin²x.~~

$S_\infty = \frac{a}{1-r} = \frac{\sin^2 x}{1-\sin^2 x}$.

$= \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$.

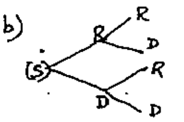
10.

)



(2)

b)



R = Rain
D = Dry

---

b) (ii) $P\binom{\text{at least once in}}{n\text{ days}} = 1 - (0.85)^n$

(iii) $1 - (0.85)^n \geq \frac{99}{100}$

$(.85)^n \leq \frac{1}{100}$

$n \ln .85 \leq \ln .01$

$n \geq \frac{\ln .01}{\ln .85}$

≥ 28.336..

∴ least no of days is 29.

c) 9% pa = .75% p month.

(i) ~~let A_n be amount owing at the end of n months.~~

$A_1 = P(1.0075) - 600$

$A_2 = A_1(1.0075) - 600$

$= P(1.0075)^2 - 600(1.0075) - 600$

$A_n = P(1.0075)^n - 600(1.0075)^{n-1} - 600(1.0075)^{n-2} \ldots - 600(1.0075) - 600$

$= P(1.0075)^n - 600(1 + 1.0075 + 1.0075^2 + \ldots + 1.0075^{n-1})$

$= P(1.0075)^n - 600\left(\frac{1.0075^n - 1}{1.0075 - 1}\right)$

$A_{120} = P(1.0075)^{120} - 600\left(\frac{1.0075^{120} - 1}{0.0075}\right)$

$\frac{1}{2}P = P(1.0075)^{120} - 600\left(\frac{1.0075^{120} - 1}{.0075}\right)$

$P(1.0075^{120} - .5) = 600\left(\frac{1.0075^{120} - 1}{.0075}\right)$

$P = \frac{600\,(1.0075^{120} - 1)}{(1.0075^{120} - .5)(.0075)}$

= 59,501.45

107

9. cont.

a)(i) $S = Ae^{kt}$

$5000 = Ae^0$

$\therefore A = 5000.$ ✓

(ii) $\frac{dS}{dt} = kAe^{kt}$

$350 = k(5000)e^0$ ✓

$k = \frac{350}{5000}$

$= 0.07.$ ✓

(iii) $S = 5000e^{0.07 \times 8}$

$= 5000e^{.56}$

$=$ ~~5362.54~~. 8753.36

$=$ ~~5363~~ (to the nearest whole no)

8753 ants.

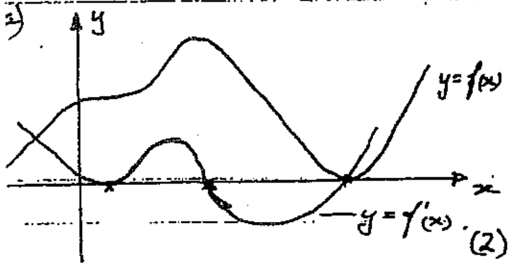
d) ~~$a = \sin^2 x$~~ $r = \sin^2 x.$

$S_\infty = \frac{a}{1-r} = \frac{\sin^2 x}{1-\sin^2 x}.$ ✓

$= \frac{\sin^2 x}{\cos^2 x} = \tan^2 x.$ ✓

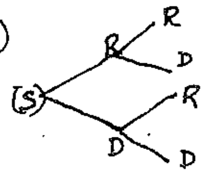
10.

a)



(2)

b)



R = Rain
D = Dry

b)(ii) $P\begin{pmatrix}\text{at least}\\ \text{once in}\\ n \text{ days}\end{pmatrix} = 1-(0.85)^n$

(iii) $1-(0.85)^n \geq \frac{99}{100}$

$(.85)^n \leq \frac{1}{100}$

$n \ln .85 \leq \ln .01$

$\therefore n \geq \frac{\ln .01}{\ln .85}$

$\geq 28.336..$

$\therefore$ least no. of days is 29.

c) 9% pa = .75% p month.

(i) ~~let A_n be amount owing at the~~ end of n months.

$A_1 = P(1.0075) - 600$

$A_2 = A_1(1.0075) - 600$

$= P(1.0075)^2 - 600(1.0075) - 600.$

$A_n = P(1.0075)^n - 600(1.0075)^{n-1} - 600(1.0075)^{n-2} \ldots - 600(1.0075) - 600$

$= P(1.0075)^n - 600(1 + 1.0075 + 1.0075^2 + \ldots + 1.0075^{n-1})$

$= P(1.0075)^n - 600\left(\frac{1.0075^n - 1}{1.0075 - 1}\right)$

$A_{120} = P(1.0075)^{120} - 600\left(\frac{1.0075^{120} - 1}{0.0075}\right)$

$\frac{1}{2}P = P(1.0075)^{120} - 600\left(\frac{1.0075^{120} - 1}{.0075}\right)$

$P(1.0075^{120} - .5) = 600\left(\frac{1.0075^{120} - 1}{.0075}\right)$

$P = \frac{600(1.0075^{120} - 1)}{(1.0075^{120} - .5)(.0075)}$

$= 59,501.45$