Knox 2003 Trial HSC

Total marks (84)
Attempt questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) Find $\dfrac{d}{dx}\left(x\tan^{-1}2x\right)$ **2**

(b) The parametric equations of a curve are given by $x = t^2$, $y = t^3 + t$. **2**
Find the Cartesian equation of the curve (that is y in terms of x).

(c) Write down the general solution of $\sin x = \dfrac{1}{2}$. **2**

(d) The interval AB has end points A (5, 4) and B (x, y). The point P (−1, 3) divides AB internally in the ratio 2:3. Find the coordinates of B. **2**

(e) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\sin 3x}{4x}\right)$. **2**

(f) Use the table of standard integrals to find the exact value of **2**

$$\int_0^1 \frac{1}{\sqrt{4+x^2}}\ dx$$

Question 2 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) Find, correct to the nearest degree, the obtuse angle between the lines $x+y-4=0$ and $y=2x+1$. **2**

(b) Solve $\dfrac{2x+3}{x-4} \leq 1$. **3**

(c) Use the substitution $u=2-x$ to evaluate $\int_0^1 x\sqrt{2-x}\ dx$. **4**

(d) (i) Write down the domain and range of the function $y=\dfrac{\pi}{2}-\sin^{-1}\dfrac{x}{2}$. **2**

(ii) Hence sketch the function. **1**

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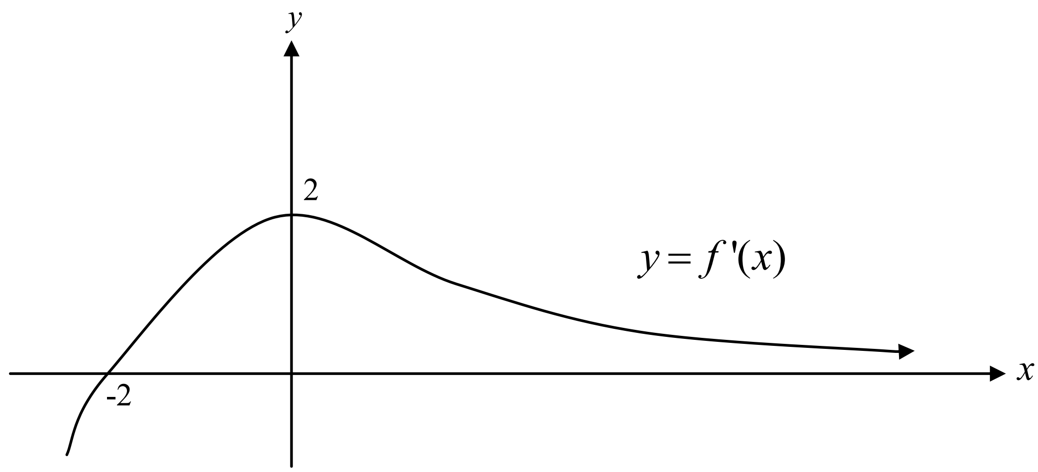
Question 3 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$ in simplest surd form, with a rational denominator. **3**

(b) **2**



The diagram above shows a sketch of the gradient function of the curve $y=f(x)$.

Copy this diagram into your writing booklet.

On the same diagram, draw a possible sketch of the function $y=f(x)$, given that $f(0)=3$ and $\lim_{x \to \infty} f(x) = 6$.

(c) Consider the point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$.

(i) Show that the equation of the normal to the parabola $x^2 = 4ay$ at the point P is given by $x + py = 2ap + ap^3$. **2**

(ii) Find the equation of the line which passes through the focus $S(0, a)$ and is perpendicular to the normal. **1**

(iii) If the line found in part (ii) meets the normal at N, find the coordinates of N. **2**

(iv) Show that the locus of N is a parabola and find its vertex. **2**

Question 4 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) Determine the exact value of $\cos\left(2\sin^{-1}\left(\frac{12}{13}\right)\right)$. **2**

(b) (i) Show that the equation $x - \tan^{-1} 3x = 0$ has a root lying between $x = 1$ and $x = 2$. **1**

(ii) By taking $x = 1.5$ as an initial approximation to the root of $x - \tan^{-1} 3x = 0$, in the interval $1 < x < 2$, use one application of Newton's method to find a second approximation to this root. **2**

(c) The velocity of a particle moving in a straight line is given by **2**

$$v = 4x + 1,$$

where x is the displacement (in metres) from a fixed point 0, and v is the velocity in metres per second.

Find the acceleration of the particle when it is 5 metres to the right of the origin.

(d) Newton's law of cooling states that the rate of cooling of a body is proportional to the excess of the temperature of a body above the surrounding room temperature. The temperature of a cup of chocolate drink satisfies an equation of the form $T = B + Ae^{kt}$ where T is the temperature of the drink, t is time in minutes, A and k are constants and B is the temperature of the surroundings.

The drink cools from 85°C to 70°C in three minutes in a room of temperature of 22°C.

(i) Find the value of A. **1**

(ii) Find the value of k, correct to 3 decimal places. **2**

(iii) Find the temperature of the cup of chocolate, to the nearest degree, after a further 9 minutes have passed. **2**

Question 5 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

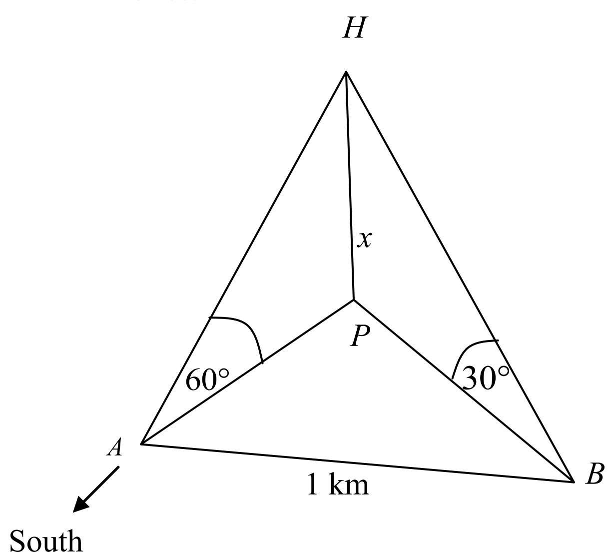
(a) Suppose $\frac{\alpha}{r}$, α and αr are the real roots of the cubic equation $2x^3 - 3x^2 - 3x + 2 = 0$.

(i) Write down the value of the sum of all three roots. **1**

(ii) Write down the value of the product of all three roots. **1**

(iii) Deduce that r can take on two real non-zero values and find them. **2**

(b) Anna (A) is standing due south of Phillip (P) who is assisting an injured bush walker. A rescue helicopter (H) is hovering directly over P and lowering a stretcher. Anna measures the angle of elevation of the helicopter to be 60° from her position. Belinda (B) is 1 kilometre due east of A and measures the angle of elevation of the helicopter to be 30°. The height of the helicopter above P is x metres.



NOT TO SCALE

(i) Write expressions for both AP and BP in terms of x. **1**

(ii) Hence or otherwise find the height of the helicopter correct to the nearest 10 m. **3**

(c) Use the Principle of Mathematical Induction to show that $9^{n+2} - 4^n$ is divisible by 5 for all positive integers n. **4**

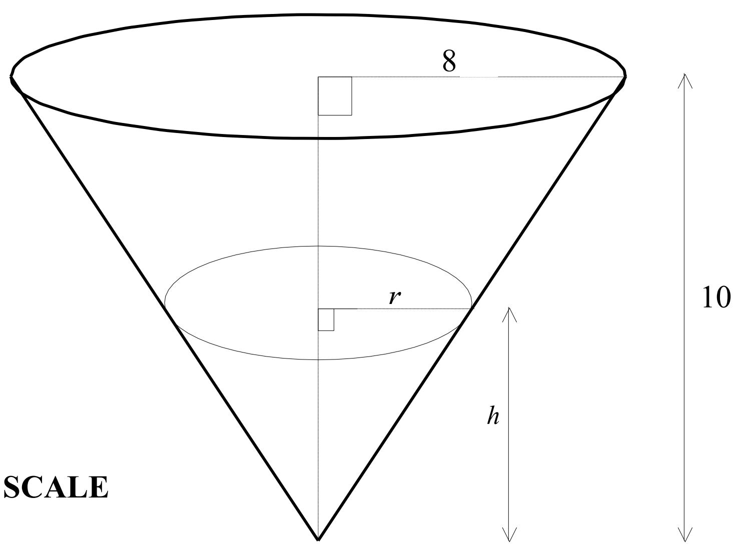
Question 6 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) (i) State the domain and range for $f(x) = 4 - \sqrt{x-1}$. **2**

(ii) Find the inverse function $f^{-1}(x)$ and state the domain and range for which it exists. **3**

(iii) Sketch the graph of $f(x) = 4 - \sqrt{x-1}$ and its inverse function $f^{-1}(x)$ on the same number plane. **2**

(b) **5**



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A bulk container for emptying grain into rail trucks is in the shape of an inverted cone with base radius 8 metres and height 10 metres. The grain is released from the apex of the cone at a constant rate of 35 m^3/s. The depth of grain in the container at any given time is *h* metres and the radius of the circle formed by the top of the grain at that same time is *r* metres.

If the grain is released continuously until the container is empty, calculate the rate at which the radius (*r*) is decreasing when the depth (*h*) is 0.65 metres.

Question 7 **(12 Marks)** Use a SEPARATE Writing Booklet.

Marks

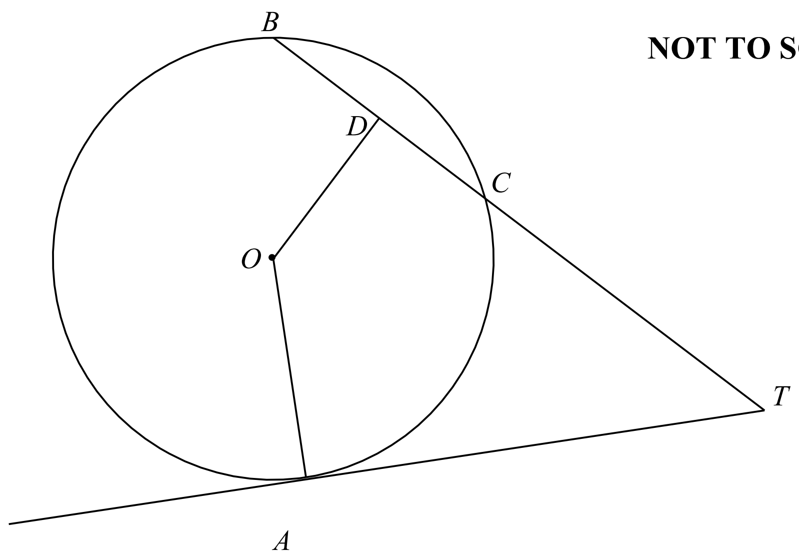
(a) By using the t – method (that is, let $t = \tan\frac{x}{2}$) solve the equation **4**

$$\cos x + \frac{1}{\sqrt{2}}\sin x = -1,$$

for x such that $0° \le x \le 360°$

(b) Find the volume of the solid formed by rotating about the x axis, the region bounded by $y = \cos 2x$, the x axis, from $x = 0$ to $x = \frac{\pi}{2}$. **4**

(c)



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In the diagram above A, B and C are three points on a circle, centre O. The tangent at A meets BC produced at T. D is the midpoint of BC.

Copy this diagram into your writing booklet.

(i) Prove that $AODT$ is a cyclic quadrilateral. **3**

(ii) Explain why $\angle AOT = \angle ADT$. **1**

End of Paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \, , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$