Set By: EH

Teachers:

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VIRILE AGITUR

KNOX GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

2004
TRIAL HSC EXAMINATION

Mathematics

(Year 12)

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page of this paper
- All working should be shown in every question

Total marks (120)

- Attempt Questions 1–10
- All questions are of equal value
- Use a **SEPARATE** Writing Booklet for each question
- Write your **Student Number** and **Teacher's Initials** on the front cover of each writing booklet

NAME:______________________________ TEACHER:__________________________

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Total marks (120)
Attempt questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the value of x if: $x^3 = \dfrac{12.167 \times 42.875}{125}$ **1**

(b) A laptop is purchased for \$2300. Calculate the value of the laptop at the end of 3 years if it depreciates by 12% each year. **2**

(c) Determine the value of x which satisfies the simultaneous equations: **2**

$$\begin{cases} 3x - 2y = 5 \\ 7x - 2y = 17 \end{cases}$$

(d) If $\dfrac{5}{\sqrt{5}-2} = a + b\sqrt{5}$, find the value of a and b. **2**

(e) Solve for x if: $5 - 6x > 8$ **2**

(f) α and β are the roots of the equation $2x^2 - x - 5 = 0$.

Without solving the equation, evaluate:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha - 2)(\beta - 2)$ **1**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Sketch the graph of the function $f(x)=\begin{cases} x^2 & \text{for } -1 \le x < 1 \\ \dfrac{1}{x} & \text{for } x \ge 1 \end{cases}$ **2**

(ii) State the range of $f(x)$. **1**

(b) Solve $|2x+3| > 5$. **2**

(c) Find the values of k for which the quadratic equation $x^2 - kx + 9 = 0$ has no real roots. **3**

(d) Find a primitive function of:

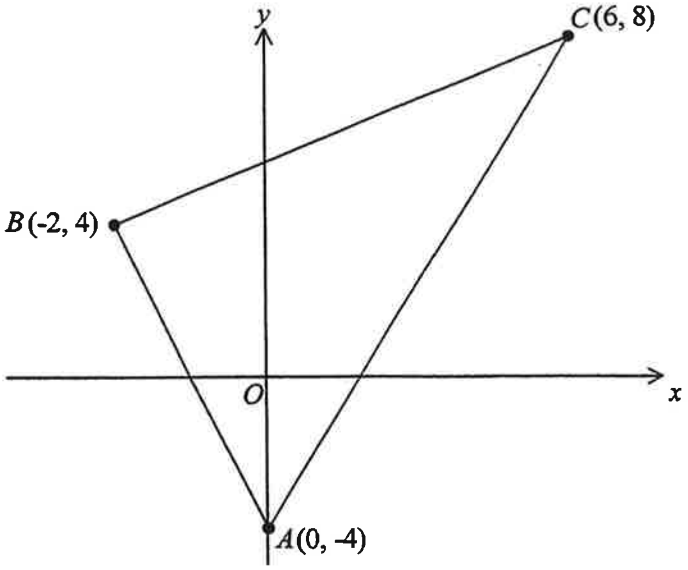
(i) $\dfrac{\sqrt{x}}{x}$ **1**

(ii) $\sin\left(\dfrac{x}{5}\right)$ **1**

(iii) $\dfrac{1}{2}e^{1-4x}$ **2**

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



NOT TO SCALE

In the diagram, the coordinates of the points A, B and C are respectively $(0, -4)$, $(-2, 4)$, and $(6, 8)$.

Copy this diagram into your writing booklet and label all the given information.

(i) Find in the form $ax + by + c = 0$ the equation of the line containing BC. **1**

(ii) If D is the midpoint of BC write down the coordinates of D. **1**

(iii) The perpendicular bisector of BC intersects the x axis at the point E. **3**

Find the coordinates of E.

(iv) Show that the perpendicular distance from A to the line BC is $\frac{18}{\sqrt{5}}$ units. **2**

(v) Hence, show that the ratio of the area of ΔABC to the area of ΔBEC is $6:5$. **2**

(b) Find the equation of the normal to the curve $y = (2x - 3)^4$ at the point where $x = 1$. Leave your answer in the general form. **3**

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

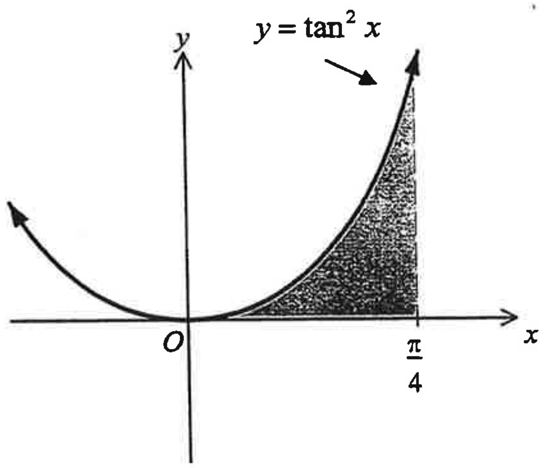
(a) Find the number of terms in the arithmetic series $180 + 165 + 150 + \ldots$ if the last term is -30. **2**

(b) Use the *product rule* to find: $\frac{d}{dx}(x \log_e 2x)$ **2**

(c) (i) By expressing $\sec x$ and $\tan x$ in terms of $\sin x$ and $\cos x$, show that: **1**

$$\sec^2 x - \tan^2 x = 1$$

(ii)



The diagram above shows the region bounded by $y = \tan^2 x$, $y = 0$ from $x = 0$ to $x = \frac{\pi}{4}$. Find the exact area of the shaded region. You may use the result of part (i). **2**

(d) A tree is 50 m high when first observed. In the first week of observation it grows vertically by 120 cm, and in each succeeding week the vertical growth in height is 70% of the previous week's vertical growth. **2**

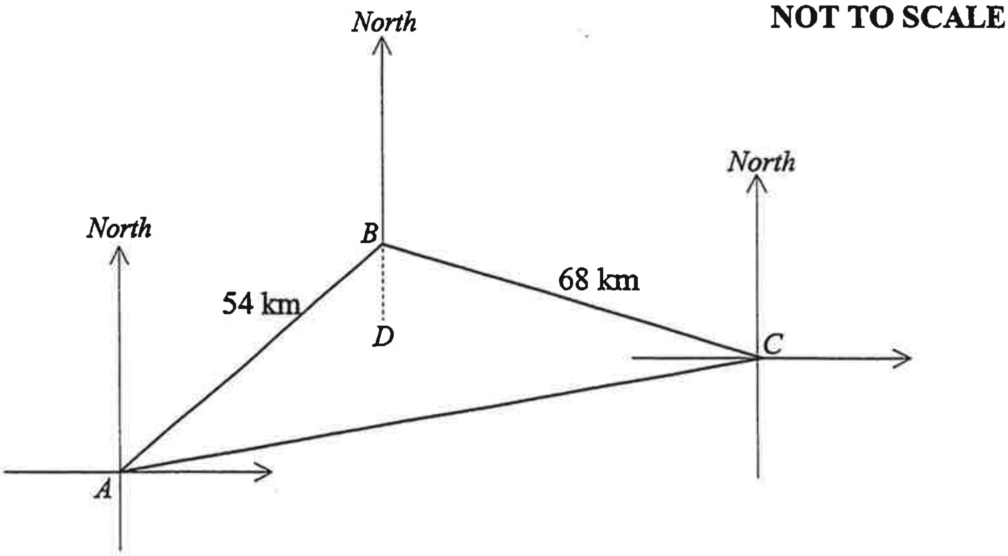
If this pattern of vertical growth continues, what will be the tree's eventual height?

(e) (i) Show that $f(x) = x\sqrt{4 - x^2}$ is an odd function in x. **2**

(ii) Hence, without using integration, state the value of $\int_{-1}^{1} f(x)\ dx$. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



A helicopter flies 54 km from A to B, on a bearing of 055°T. The helicopter then proceeds to C distant 68 km on a bearing of 105°T.

Copy this diagram into your writing booklet and label all the given information.

(i) Explain why $\angle ABC = 130°$. **2**

(ii) Use the cosine rule in ΔABC to find the distance (to 0.1 km) from C to A. **2**

(iii) Use the sine rule in ΔABC to calculate the size of $\angle BAC$ (to the nearest degree). **2**

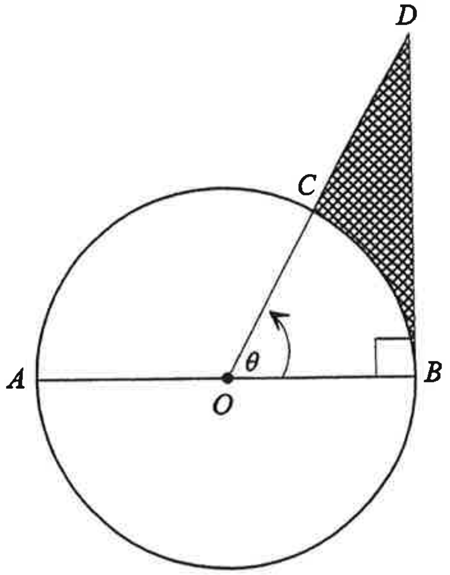
(iv) Hence, find the bearing of C from A (to the nearest degree). **1**

(b) (i) Find the equation of the locus of the point $P(x, y)$ which moves so that $(PA)^2 + (PB)^2 = (AB)^2$, where $A = (0, 4)$ and $B = (-2, 4)$. **3**

(ii) Sketch the locus of P. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



NOT TO SCALE

In the diagram, AB is a diameter of a circle, centre at O. Radius OC is extended to meet the tangent BD at D. $\angle OBD = \frac{\pi}{2}$ radians. The measures of arc AC and arc CB are in the ratio $2:1$ and AB is 32 cm. Let $\angle COB = \theta$.

(i) Find the size of θ in radians. **1**

(ii) Calculate the exact value of the shaded area which is contained by BD, arc BC and CD. **3**

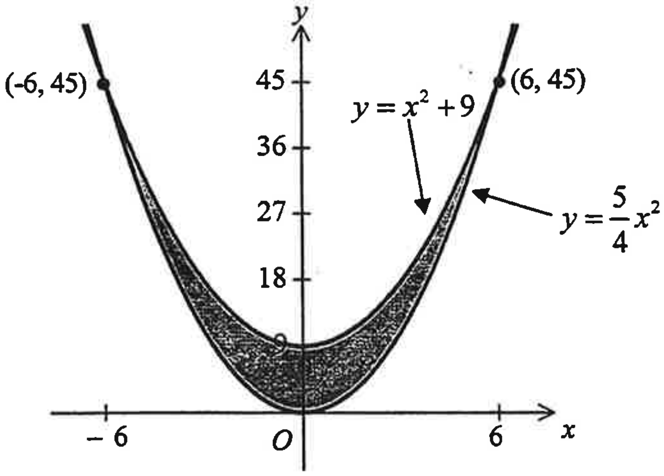
(b) Sketch the graph of the parabola represented by the equation $y^2 = 12(x-4)$. **4**

Show on your diagram, the directrix and the coordinates of the focus and vertex.

Question 6 is continued on the next page ...

Question 6 (continued)

(c)



The area bounded by the curves $y = \frac{5}{4}x^2$ and $y = x^2 + 9$ is revolved about the y axis to obtain a bowl. The graphs intersect at $(-6, 45)$ and $(6, 45)$ as shown. **4**

Find the volume of the material used in making the bowl.

(Note that the volume of the material used is determined by revolving the shaded area as shown in the diagram above about the y axis)

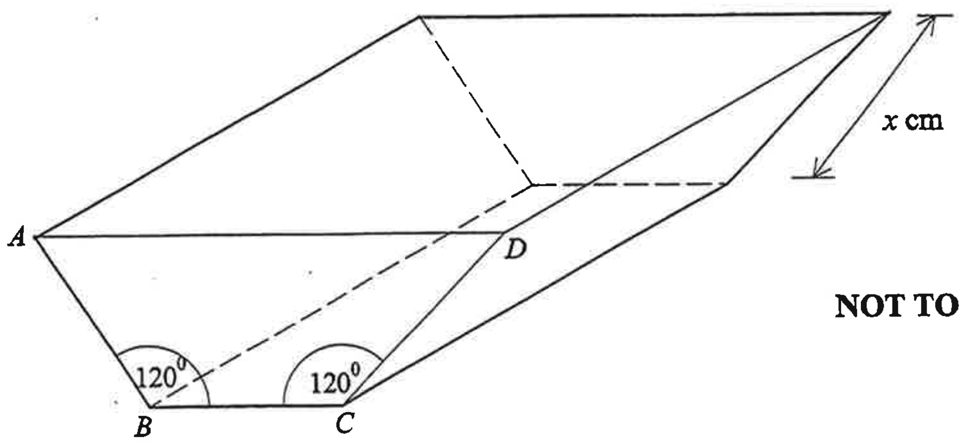
Question 7 commences on the next page ...

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve the equation $2\cos^2 x = 2\sin x\cos x$ for $0 \le x \le \pi$. **3**

(b) Simplify the expression: $\log_e\left(e^{x+y}\right)^2$ **2**

(c)



NOT TO SCALE

In the diagram, the two sides and the base of a trough are to be made by bending a long rectangular piece of tin 36 cm wide (that is where $AB + BC + CD = 36$ cm).

The cross-section of the trough is a trapezium with sides making angles of 120° with the base as shown.

(i) If a side of the trough is x cm, show that the width of the base BC is $(36 - 2x)$ cm and the top AD is $(36 - x)$ cm. **3**

(ii) Show that the cross-sectional area A, of the trough, in terms of x, is given by: **2**

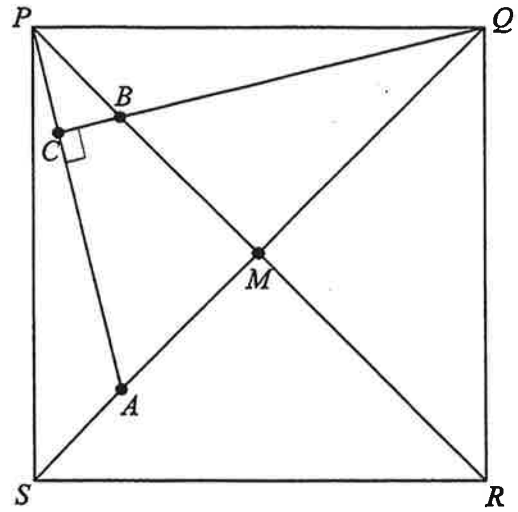
$$A = \frac{3\sqrt{3}}{4}\left(24x - x^2\right)$$

(iii) In order to maximise the cross-sectional area of $ABCD$, what should be the length of the side CD? **2**

(Testing for maxima is not required)

Question 8 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



NOT TO SCALE

In the diagram, $SRQP$ is a square. A is any point on diagonal SQ and $QC \perp PA$.

B and M lie on diagonal PR as shown. Let $\angle PQC = x°$ and $\angle SPA = y°$.

Copy this diagram into your writing booklet and label all the given information.

(i) Explain why $\angle CPB = (45 - y)°$. **1**

(ii) Hence, show that $x = y$. **2**

(iii) Prove that $\Delta PBQ \equiv \Delta SAP$. **2**

(b) A panel beater borrows \$90 000 to purchase new machinery.

The interest is calculated monthly at the rate of 2% per month, and is compounded each month. The panel beater intends to repay the loan with interest in two equal annual instalments of $\$M$ at the end of the first and second years.

(i) How much does the panel beater owe at the end of the first month? **1**

(ii) Write an expression involving M for the total amount owed by the panel beater after 12 months (that is, just after the first instalment of $\$M$ has been paid). **1**

(iii) Find an expression for the amount owed at the end of the second year and hence deduce that $M = \dfrac{90\,000 \times (1.02)^{24}}{(1.02)^{12} + 1}$. **2**

Question 8 continues on the next page ...

(c) The depth of water in a harbour is changing at a rate given by the equation: **3**

$$\frac{dh}{dt} = 0.8\cos\left(\frac{t}{2}\right),$$

where h is the depth of water (in metres), t hours after observation started.

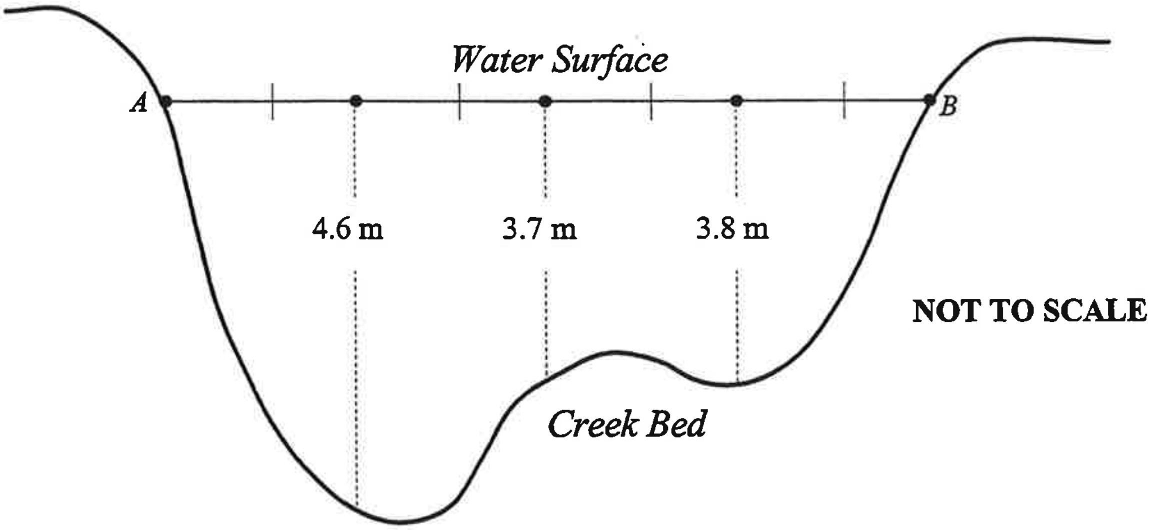
If the water was 3 m deep at the start, find the depth of water 3 hours later.

(Give your answer to one decimal place)

Question 9 commences on the next page ...

Question 9 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



The diagram above shows a vertical cross-section of a creek.

The horizontal distance from A to B is 64 metres.

(i) By using Simpson's rule with five function values, find an approximation for the area of this cross-section of the creek. **2**

(ii) Assume that a 120 metre length of this creek has approximately the same cross-section as above. Estimate the volume of water in this section of the creek. **1**

(b) Find the value of k (where $k > 0$) such that: $\int_1^k \frac{t}{4t^2-1}\,dt = \frac{1}{8}\log_e 5$ **3**

(c) The velocity of a particle is given by $\dot{x} = 3\sin 2t$ for $0 \le t \le 2\pi$, where v is measured in metres per second and t is measured in seconds.

(i) At what times is the particle at rest during the first π seconds? **2**

(ii) Write down an expression for the acceleration of the particle in terms of t. **1**

(iii) What is the maximum velocity of the particle during this period? **1**

(iv) Sketch the graph of v as a function of t for $0 \le t \le 2\pi$. **2**

Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) In a certain strain of plant the probability that a seed will produce a red flower is $\frac{1}{4}$.

(i) Write down the probability that a seed does not produce a red flower. **1**

(ii) Calculate the probability that no red flower is produced, if three seeds are planted. **1**

(iii) What is the probability that at least one red flower is produced if three seeds are planted? **1**

(iv) If n seeds are planted, write down the probability in terms of n, of obtaining at least one red flower? **1**

(v) Determine the least number of seeds that must be planted in order to ensure that the probability of obtaining at least one red flower exceeds 0.9. **2**

(b) After the nuclear accident at Chernobyl, there were increased amounts of strontium-90 found in many parts of Europe. Strontium-90 is a radioactive substance with a half-life of 26 years.

The rate of decline in strontium-90 is proportional to the amount of strontium-90 left (S) after t years, that is:

$$\frac{dS}{dt} = -kS,$$

where k is a constant.

(i) Verify that $S = S_o e^{-kt}$ is a solution to the above equation, where S_o is a constant. **1**

(ii) Given that $S = \frac{1}{2} S_o$ when $t = 26$ years, show that $k = \frac{\ln 2}{26}$. **2**

(iii) What fraction of the fallout will still be present after 10 years? **1**

(iv) How long will it be (in approximate years) until 85% of the strontium-90 is decomposed? **2**

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \,, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

EH: Q1 & [illegible]; [illegible]: Q2, [illegible]; FT: Q3, 8; Student Number YEAR 12 [illegible] 2U MATHEMATICS TRIAL.

QUESTION 1:

LT: Q4, 7;
GH: Q5, 6
RD: Q9, 10 (ACCELERATED GROUP).

COMMENTS:

(a) $x = \sqrt[3]{\dfrac{12.167 \times 42.875}{125}}$

$= 1.61$

✓ must have the correct answer.

(b) Value $= 2300\,(0.88)^3$

$= \$1567.39$ (nearest cent)

✓ for appropriate working

✓ final answer. (nearest $ is OK)

(c) $3x - 2y = 5$ ①

$7x - 2y = 17$ ②

① − ② $-4x = -12$

$\therefore x = 3$

✓ for appropriate working.

✓ answer.

[award 2 if answer only]

(d) $\dfrac{5}{\sqrt{5}-2} = \dfrac{5}{\sqrt{5}-2} \times \dfrac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \dfrac{5\sqrt{5}+10}{5-4}$

$= 10 + 5\sqrt{5}$

$\therefore a = 10, \quad b = 5$

✓ for working.

✓ must have both values — need not explicitly separate a & b.

(e) $5 - 6x > 8$

$-6x > 3$

$\therefore x < -\frac{1}{2}$

✓ method.

✓ correct solution.

(f) (i) $\alpha + \beta = \frac{1}{2}$

(ii) $\alpha\beta = -5/2$

(iii) $(\alpha-2)(\beta-2) = \alpha\beta - 2(\alpha+\beta) + 4$

$= -5/2 - 2(\frac{1}{2}) + 4$

$= \ldots$

✓ correct answer

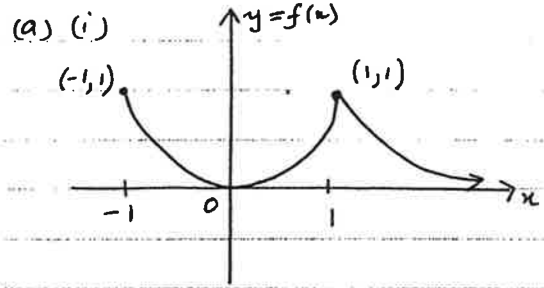
✓ correct answer.

✓ correct answer.

QUESTION 2:

COMMENTS

(a) (i)



✓✓ correctly draws each function in the specified domain.

(ii) Range $= 0 \le y \le 1$.

✓ correct answer.

(b) $|2x + 3| > 5$

either $2x + 3 > 5$ or $2x + 3 < -5$

$\therefore x > 1$ or $x < -4$

✓ method

✓ correct answer (both inequalities are required).

(c) Want $\Delta < 0$ (i.e. $b^2 - 4ac < 0$)

$(-k)^2 - 4(1)(9) < 0$

$k^2 - 36 < 0$

$(k-6)(k+6) < 0$

$\therefore \{k : |k| < 6\}$

✓ correct LHS

✓ correct inequality

✓ correct answer. $\{k > 6$ or $k < -6\}$ is OK.

(d) (i) $\displaystyle\int \frac{\sqrt{x}}{x}\,dx = \int x^{-1/2}\,dx$

$= 2x^{1/2} + c$

$(= 2\sqrt{x} + c)$

✓ do not penalise if 'c' is not present.

(ii) $\displaystyle\int \sin\left(\frac{x}{5}\right) dx = -5\cos\left(\frac{x}{5}\right) + c$

✓

(iii) $\displaystyle\int \frac{1}{2} e^{1-4x}\,dx = -\frac{1}{8} e^{1-4x} + c$

✓ correct coeff.

✓ correct (e^{1-4x}).

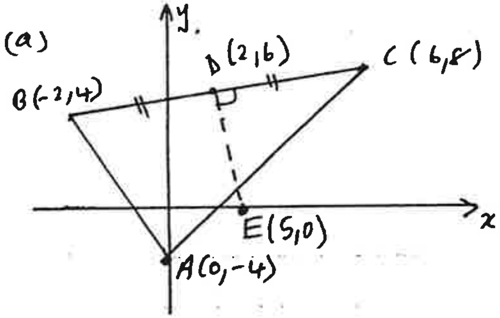
(or ② for correct answer).

Student Number _ _ _ _ _ _ _ _ _

QUESTION 3:

COMMENTS

(a)



(i) $m_{BC} = \frac{8-4}{6+2}$

$= \frac{1}{2}$ ✓

$\therefore$ using $y - y_1 = m(x - x_1)$

Equation of BC is: $y - 8 = \frac{1}{2}(x-6)$

$2y - 16 = x - 6$

$\therefore x - 2y + 10 = 0$ ①

(ii) $D = \left(\frac{6-2}{2}, \frac{8+4}{2}\right)$

$= (2, 6)$ ✓

(iii) Equation of DE is of the form

$2x + y + k = 0$ since $DE \perp BC$

sub $D(2,6)$ to find k.

$\therefore 4 + \overset{(10)}{y} + k = 0$

$\therefore k = -10$

$\therefore$ Equation of DE is: $2x + y - 10 = 0$

The x-intercept found by subst $y=0$

$\therefore 2x = 10$

ie $x = 5$

$\therefore E(5,0)$. ✓

Comments: There are several other approaches award marks appropriately.

✓✓ deriving or stating the correct equation of ⊥ bisector.

Student Number _ _ _ _ _ _ _ _ _

QUESTION 3 (ctd)

COMMENTS.

(iv) Equation of BC: $x - 2y + 10 = 0$

using $A(0,-4)$ & $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$

$\therefore d = \frac{|(1)(0) + (-2)(-4) + 10|}{\sqrt{1^2 + (-2)^2}}$ ✓✓

$= \frac{|18|}{\sqrt{5}}$

$= \frac{18}{\sqrt{5}}$

It is not necessary that a student actually writes down what is to be shown to gain the marks.

(v) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BEC} = \frac{\frac{1}{2} \times BC \times \frac{18}{\sqrt{5}}}{\frac{1}{2} \times BC \times \sqrt{36+9}}$ ✓ correct expressions

$= \frac{18}{\sqrt{5}} \times \frac{1}{3\sqrt{5}}$ ✓ simplifying.

$= \frac{6}{5}$

$= 6 : 5$ as required.

(b) $y = (2x-3)^4$

$\frac{dy}{dx} = 4(2x-3)^3(2)$

$= 8(2x-3)^3$ ✓ correct gradient function

at/when $x = 1$, $y = (-1)^4 = 1 \Rightarrow (1,1)$

$y' = 8(-1)^3 = -8$ ✓ correct slope of normal.

$\therefore$ slope of normal is $\frac{1}{8}$.

$\therefore$ Equation of normal is:

$y - 1 = \frac{1}{8}(x-1)$

$8y - 8 = x - 1$

ie $x - 8y + 7 = 0$. (General form) ✓ (must be in the general form).

Student Number

QUESTION 4:

(a) Let $a = 180$

$d = -15$

& $T_n = -30$ $(T_n = a+(n-1)d)$

$\therefore -30 = 180 - 15(n-1)$

$14 = n-1$

$\therefore n = 15.$

$\therefore$ The number of terms is 15.

(b) $\frac{d}{dx} x(\ln 2x) = (\ln 2x)(1) + x\left(\frac{2}{2x}\right)$

$= \ln 2x + 1$

(N.B $x>0$)

(c) (i) Let $LHS = \sec^2 x - \tan^2 x$

$= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$

$= \frac{1-\sin^2 x}{\cos^2 x}$

$= \frac{\cos^2 x}{\cos^2 x}$

$= 1$

hence $\tan^2 x = \sec^2 x - 1.$

(ii) $A = \int_0^{\pi/4} \tan^2 x \, dx$

from (i) $A = \int_0^{\pi/4} \sec^2 x - 1 \, dx$

$= [\tan x - x]_0^{\pi/4}$

$= \tan \pi/4 - \pi/4$

$= (1 - \pi/4)$ square units.

COMMENTS:

✓ correct common diff.

✓

✓✓

(or 2 for correct simplified answer).

✓

✓ correct change of identity

✓ correct simplified answer

Student Number

QUESTION 4 (ctd):

(d) Let T_1 = initial height of tree.

$= 50m$

Let T_2 = height of tree after 1st increase

$= (50 + 1.2)$

T_3 = height of tree after 2nd increment

$= 50 + 1.2 + 1.2\times0.7$

similarly,

$T_4 = 50 + 1.2 + 1.2\times0.7 + 1.2\times0.7^2$

$\therefore T_n = 50 + (1.2 + 1.2\times0.7 + 1.2\times0.7^2 + \dots)$

This is an infinite geometric series with $a = 1.2$, $r = 0.7$ $(|r|<1)$

$\therefore S_\infty = \frac{a}{1-r}$

$= \frac{1.2}{0.3}$

$= 4$

$\therefore$ The eventual height $= 50+4$

$= 54$ m.

(e)(i) $f(x) = x\sqrt{4-x^2}$

$\therefore f(-x) = (-x)\sqrt{4-(-x)^2}$

$= -(x\sqrt{4-x^2})$

$= -f(x)$

(ii) $\int_{-1}^{1} f(x)\,dx = 0.$

COMMENTS:

✓

✓

✓ This is essential

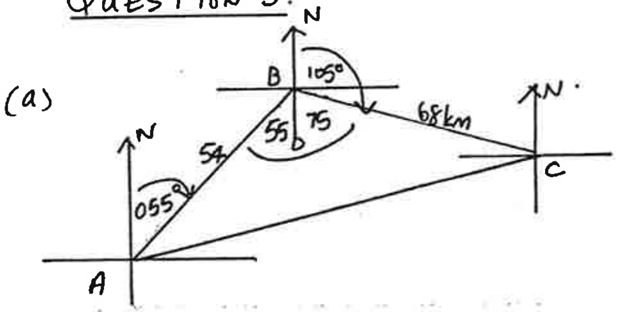
✓ conclusion.

✓ no explanation needed.

QUESTION 5:

COMMENTS

(a)



(i) $\angle ABD = 55°$ (alt ∠'s: AN||DN) ✓

$\angle DBC = 180 - 105$

$= 75°$ (supp. ∠ & adj) ✓

} may be implied or drawn on diagram.

$\therefore \angle ABC = 130°$.

(ii) $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos A\hat{B}C$

$= 54^2 + 68^2 - 2(54)(68)\cos 130°$

$\therefore AC = 110 \cdot 7$ km (nearest 0.1 km)

✓ correct use of cosine rule (may use: $\cos\theta = \frac{a^2+b^2-c^2}{2ab}$)

✓ (correctly rounded answer).

(iii) $\frac{\sin B\hat{A}C}{68} = \frac{\sin A\hat{B}C}{110 \cdot 7}$ ← from (ii)

✓ need to see correct subst of lengths matched with correct 'opp' ∠'s.

$\therefore \sin B\hat{A}C = \frac{68 \times \sin 130°}{110 \cdot 7}$

$\therefore \angle BAC = 28°$ (nearest degree) ✓ (need not round off to level required)

(iv) The bearing of C from A = 55 + 28

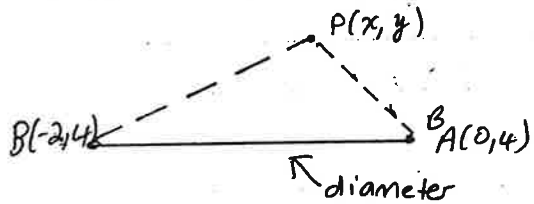
= 83°. ✓ correct answer.

QUESTION 5 ctd:

COMMENTS

Given: $PA^2 + PB^2 = AB^2$

(b) Noting for this to be true then $\angle APB = 90°$.



{ centre = (-1, 4) ✓
{ r = 1 ✓

$\therefore$ locus in cartesian form is:

$(x+1)^2 + (y-4)^2 = 1$. ✓

There are several ways to do this problem, for example actually find the lengths PA^2 & PB^2 & equate to AB^2; This suggest Pythagoras' result. ie. Use $(x-0)^2 + (y-4)^2 + (x+2)^2 + (y-4)^2 = (0+2)^2 + (4-4)^2$ etc.

* { or $m_{PA} \times m_{PB} = -1$

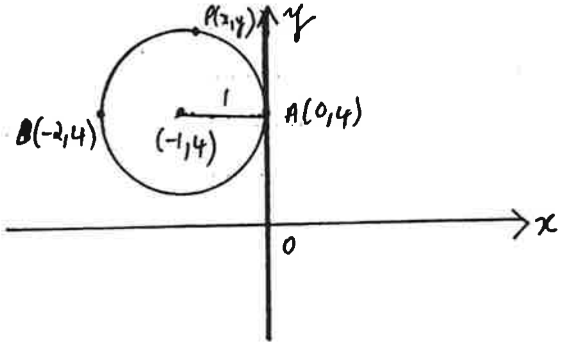
ie $\left(\frac{y-4}{x+2}\right)\left(\frac{y-4}{x}\right) = -1$

ie $y^2 - 8y + 16 = -(x^2 + 2x)$

ie $x^2 + 2x + y^2 - 8y = -16$

$x^2 + 2x + \left(\frac{2}{2}\right)^2 + y^2 - 8y + \left(\frac{-8}{2}\right)^2 = -16 + 1 + 16$ } *

ie: $(x+1)^2 + (y-4)^2 = 1$



✓✓

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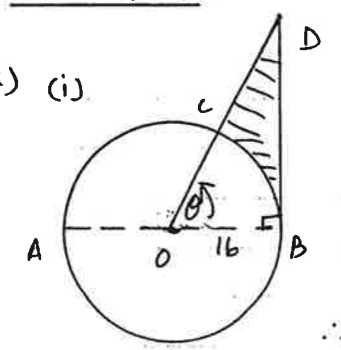
Question 6:

Comments

(a) (i)



arc AC : arc CB

= 2 : 1

∴ ∠AOC : ∠COB

= 2 : 1

∴ $\theta = \frac{1}{3} \times \pi$

$= \pi/3$ radians ✓

(ii) Shaded area = Area of ΔBOD − Area of sector COB

now: $\frac{BD}{OB} = \tan\theta$ $(OB = \frac{1}{2} \times AB)$

∴ $BD = 16 \times \tan \pi/3$

$= 16\sqrt{3}$ cm   ✓ find BD.

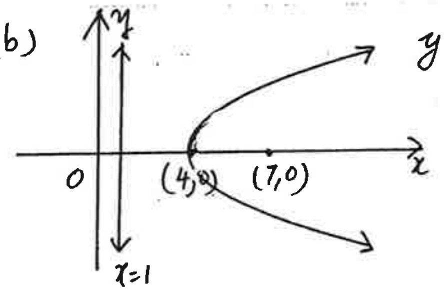
∴ Shaded area $= \frac{1}{2}(OB)(DB) - \frac{1}{2}(OB)^2\left(\frac{\pi}{3}\right)$ ✓ method.

$= \frac{1}{2} \times 16 \times 16\sqrt{3} - \frac{1}{2}(16)^2\left(\frac{\pi}{3}\right)$

$= \left(\frac{1}{2} \cdot 16^2\right)\left(\sqrt{3} - \frac{\pi}{3}\right)$

$= 128\left(\sqrt{3} - \frac{\pi}{3}\right)$ square units.   ✓ simplified answer is OK.

(b)



$y^2 = 4(3)(x-4)$

∴ $a = 3$

$V(4,0)$

✓ sketch

✓ focus with coords

✓ directrix with equation

✓ Vertex with coords.

Question 6(ctd):

Comments

(c) Required Volume (V) = that formed by revolving the shaded area about $x=0$.

Given: $y = \frac{5x^2}{4}$ ∴ $x^2 = \frac{4y}{5}$

Also: $y = x^2 + 9$ ∴ $x^2 = y - 9$

} ✓ changing the subject; can be implied in an integral.

∴ $V = \pi\int_0^{45} \frac{4y}{5}\,dy - \pi\int_9^{45} (y-9)\,dy$ ✓ correct expression leading to a correct solution.

$= \pi\left[\int_0^{45} \frac{4y}{5}\,dy - \int_9^{45} y - 9\,dy\right]$

$= \pi\left[\left[\frac{4y^2}{2(5)}\right]_0^{45} - \left[\frac{y^2}{2} - 9y\right]_9^{45}\right]$ ✓ correct integration of expression.

$= \pi\left(\frac{2y^2}{5}\Big|_0^{45} - \left(\frac{y^2}{2} - 9y\right)\Big|_9^{45}\right)$

$= \pi\left(\frac{2 \times 45^2}{5} - \left(\frac{45^2}{2} - 9(45) - \frac{9^2}{2} + 81\right)\right)$

$= \pi\left(810 - \frac{2025}{2} + 405 + \frac{81}{2} - 81\right)$

$= 162\pi$ cubic units.   ✓ correct answer.

QUESTION 7:

COMMENTS

(a) $2\cos^2 x = 2\sin x \cos x$

ie. $2\cos^2 x - 2\sin x \cos x = 0$

ie $\cos x(\cos x - \sin x) = 0$ — ✓ method.

$\therefore$ either $\cos x = 0$ or $\cos x = \sin x$

ie $\cos x = 0$ $\tan x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ $x = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots$

in the domain: $0 \le x \le \pi$

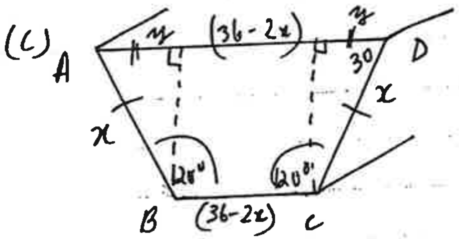
$x = \frac{\pi}{4}, \frac{\pi}{2}$ only. — ✓✓ each correct answer.

(Students that omit $\cos x = 0$ can only get a maximum 2/3 marks).

(b) $\log_e (e^{x+y})^2 = 2\log_e e^{x+y}$ ✓

$= 2(x+y)$ ✓

(c)



NB: $AB + BC + CD = 36\text{ cm}$.

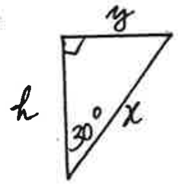
Since $AB = CD = x$

then $x + BC + x = 36$

$\therefore BC = (36 - 2x)\text{ cm}$. — ✓ making use of $AB+BC+CD=36$.

QUESTION 7 ctd:

COMMENTS



$\frac{y}{x} = \sin 30°$

$\therefore y = x\sin 30°$

$= \frac{x}{2}$ — ✓ Finding y in terms of x.

$\therefore AD = y + y + (36 - 2x)$

$= \frac{x}{2} + \frac{x}{2} + 36 - 2x$

$= (36 - x)\text{ cm}$ — ✓ calculation leading to desired answer.

(ii) Area of trapezium ABCD $= \frac{h}{2}(AD + BC)$

but $\frac{h}{x} = \cos 30°$ (see diagram above)

$\therefore h = \frac{x\sqrt{3}}{2}$ — ✓ getting h

$\therefore \text{Area} = \frac{1}{2}\left(\frac{x\sqrt{3}}{2}\right)\left[(36 - x) + (36 - 2x)\right]$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\left[x(72 - 3x)\right]$

$= \frac{3\sqrt{3}}{4}x(24 - x)$

$\therefore A = \frac{3\sqrt{3}}{4}(24x - x^2)$ as required. — ✓ correct use of area of trapezium formula or similar leading to desired result.

(iii) Max. area when $A' = 0$ — ✓ statement

ie when $24 - 2x = 0$

$24 = 2x$

$\therefore x = 12\text{ cm}$. — ✓ answer.

(Note: A is only valid for $0 < x < 18$).

(No test is required as A is a Quadratic)

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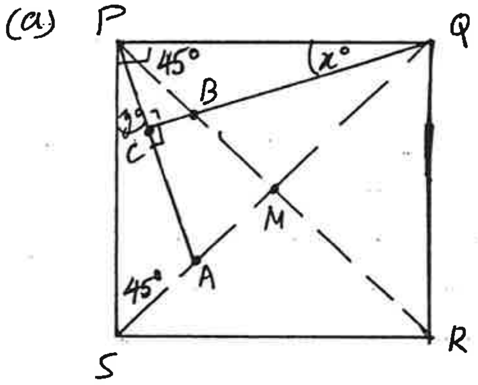
QUESTION 8:

COMMENTS

(a)



(i)

PQRS is a square & PR is a diagonal ∴ $\angle SPM = 45°$ (diagonals of a square bisect the angles through which they pass). ✓

since $\angle SPM = \angle SPA + \angle CPB$

∴ $\angle CPB = (45 - y)°$

(ii) $\angle QPC = (90 - x)°$
(complementary ∠ in ΔQPC) ✓

but $\angle SPA + \angle CPQ = 90°$ ✓

ie $y + (90 - x) = 90$

∴ $y - x = 90$

∴ $x = y$

∴ $\angle PQC = \angle SPA$.

(iii) RTP: $\Delta PBQ \equiv \Delta SAP$

QUESTION 8 ctd:

COMMENTS

In Δ's PBQ, SAP

1. $\angle PQC = \angle SPA$ (proven in (ii))
2. $PQ = PS$ (sides of square =) ✓
3. $\angle QPB = \angle ASP = 45°$ (diagonals of a square bisect the angles through which they pass). ✓

These two are required deductions with reasons.

∴ $\Delta PBQ \equiv \Delta SAP$ (A.S.A test for congruency).

(b) \$90,000 $\quad (A = P(1+r)^n)$

$r = 0.02$.

(i) Amount $= 90000(1.02)^1$
$= \$91,800$ ✓

(ii) Let A_n = amount owing at end of month n

∴ $A_{12} = 90000(1.02)^{12} - M$ ✓

(since only a one payment made in 12 months time).

(iii) $A_{24} = [90000(1.02)^{12} - M](1.02)^{12} - M$ ✓

but $A_{24} = 0$ (since loan is repaid then)

∴ $0 = 90000(1.02)^{24} - M(1 + 1.02^{12})$ ✓ must make reference to $A_{24} = 0$ or equivalent.

∴ $M = \dfrac{90000(1.02)^{24}}{1 + 1.02^{12}}$

(as required)

QUESTION 8 ctd:

COMMENT

(c) Given $\frac{dh}{dt} = 0.8\cos\left(\frac{t}{2}\right)$

(N.B. use of radians is essential here).

$\therefore h = \int 0.8\cos\left(\frac{t}{2}\right)\,dt$

ie $h = (0.8)(2)\sin\left(\frac{t}{2}\right) + c$ ✓

when $t = 0$, $h = 3$

$\therefore 3 = 1.6\sin\left(\frac{0}{2}\right) + c$ ✓

$\therefore c = 3$

ie $h = 1.6\sin\left(\frac{t}{2}\right) + 3$

$\therefore$ when $t = 3$,

$h = 1.6\sin\left(\frac{3}{2}\right) + 3$

$= 4.6\text{ m}$ ✓

QUESTION 9:

COMMENTS.

A student may use $A \doteqdot \frac{b-a}{6}\left\{f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right\}$ award marks accordingly.

(a) (i) Let
$y_0 = 0$
$y_1 = 4.6$
$y_2 = 3.7$
$y_3 = 3.8$
$y_4 = 0$
} from the diagram

Also, $h = 16$.

$\therefore A \doteqdot \frac{h}{3}\left\{y_0 + y_4 + 4(y_1 + y_3) + 2y_2\right\}$

$= \frac{16}{3}\left\{0 + 0 + 4(4.6 + 3.8) + 2(3.7)\right\}$ ✓

$= \frac{16}{3} \times 41$

$= 218\frac{2}{3}\text{ m}^2$ ✓

(ii) Volume $= 218\frac{2}{3} \times 120$

$= 26{,}240\text{ m}^3$ ✓

(b) $\int_1^k \frac{t}{4t^2 - 1}\,dt = \frac{1}{8}\log_e 5$

ie $\frac{1}{8}\left[\ln(4t^2 - 1)\right]_1^k = \frac{1}{8}(\log_e 5)$ ✓

$\therefore \ln(4k^2 - 1) - \ln(3) = \ln 5$

ie $\ln\left(\frac{4k^2 - 1}{3}\right) = \ln 5$ ✓

$\therefore \frac{4k^2 - 1}{3} = 5$

ie $4k^2 - 1 = 15$

$4k^2 = 16$

$k^2 = 4$

$\therefore k = 2 \quad (k > 0)$ ✓

QUESTION 9 ctd:

COMMENTS: This entire question can be answered by making reference to graphs for $\ddot{x}/x$.

(c) $\dot{x} = 3\sin 2t$, $0 \le t \le 2\pi$.

(i) $\dot{x} = 0$

ie $3\sin 2t = 0$ $\left\{\begin{matrix} a=3 \\ p=\pi \end{matrix}\right\}$

$\sin 2t = 0$

$\therefore 2t = n\pi$ (n is an integer) ✓

$t = \frac{n\pi}{2}$

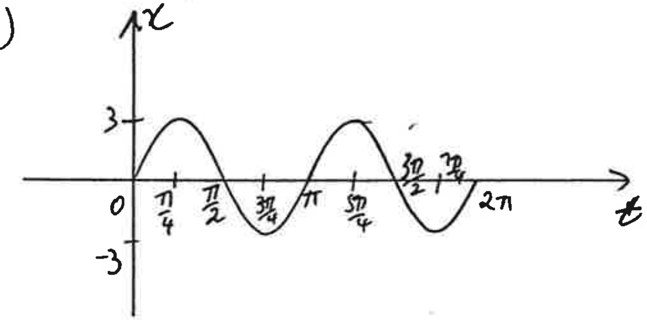
in $0 \le t \le 2\pi$, $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ ✓

but $0 \le t \le \pi$, $\therefore t = 0, \frac{\pi}{2}, \pi$. ✓ (or correct answers without working & reference to $\ddot{x}/x$ graphs → ②).

(ii) $\ddot{x} = 6\cos 2t$ ✓

(iii) $\dot{x}_{max} = 3$ m/s.

(iv)



✓✓ sine curve showing accurately x-intercepts & max/min.

QUESTION 10:

COMMENT

(a) $P(\text{Red flower}) = \frac{1}{4}$

(i) $P(\text{no red flower}) = 3/4$ ✓

(ii) $P(\text{no. red flower produced from 3 seeds}) = \left(\frac{3}{4}\right)^3$ ✓

$= \frac{27}{64}$

(iii) $P(\text{at least one red seed in/from 3 seeds}) = 1 - P(\text{no red from 3 seeds})$ ✓

$= 1 - \left(\frac{3}{4}\right)^3$ ← leave in this form.

$\left(= \frac{37}{64}\right)$

(iv) $P(\text{at least one red from } n \text{ seeds}) = 1 - \left(\frac{3}{4}\right)^n$ ✓

(v) We want $1 - \left(\frac{3}{4}\right)^n > 0.9$ ✓

ie $0.1 > \left(\frac{3}{4}\right)^n$

ie $\left(\frac{3}{4}\right)^n < 0.1$

$n \ln(3/4) < \ln 0.1$

ie $n > \frac{\ln(0.1)}{\ln(3/4)}$

$> 8.0039\ldots$

We should take $n = 9$ (92.5% approx) however $n = 8$ (89.9% is much closer). ✓

Either is OK.

QUESTION 10 ctd:

COMMENTS:

(b) $\frac{dS}{dt} = -kS$

(i) $S = S_0 e^{-kt}$ — ①

$\frac{dS}{dt} = S_0[-k e^{-kt}]$

$= -k[S_0 e^{-kt}]$

$= -kS$ from ①.

∴ $S = S_0 e^{-kt}$ satisfies $\frac{dS}{dt}$.

✓ must see reference to derivative on RHS.

(ii) $\frac{1}{2}S_0 = S_0 e^{-kt}$, $t = 26$

∴ $\frac{1}{2} = e^{-26k}$ ✓

∴ $-26k = \ln \frac{1}{2}$

ie $k = \frac{\ln \frac{1}{2}}{-26}$ ✓

$= \frac{\ln 2}{26}$

(iii) N.B: In 26 years, the fraction of fallout left is $\frac{S_{26}}{S_0}$ → new / old (initial).

ie $\frac{\frac{1}{2}S_0}{S_0} = \frac{1}{2}$ ∴ after t years fraction left e^{-kt}

In 10 years:

∴ $\frac{S_{10}}{S_0} = e^{-\left(\frac{\ln 2}{26}\right)t}$

when $t = 10$, $\frac{S_{10}}{S_0} = e^{-\left(\frac{10}{26}\ln 2\right)}$

ie $\frac{S_{10}}{S_0} = e^{\ln 2^{\left(-\frac{10}{26}\right)}}$

$= 2^{\left(-10/26\right)}$ ✓ exact answer.

$\doteqdot 0.76598\ldots$

∴ about 77% of the strontium-90 will remain.

(or expressed as a % is OK).

(iv) Let $S = 0.15 S_0$

∴ $0.15 S_0 = S_0 e^{-kt}$ ✓

$0.15 = e^{-\frac{\ln 2}{26}t}$

∴ $t = \frac{\ln 0.15}{-\frac{\ln 2}{26}}$

$\doteqdot 71$ years (from calc). ✓

(a max. of one mark /2 if $0.85 S_0$ is used).