Set By: IM

Teachers:

DS
IM
IB (Year 12)
IB (Year 11)
RM(Year 11)
SH
LS

KNOX GRAMMAR SCHOOL
MATHEMATICS FACULTY

2005
TRIAL HSC EXAMINATION

Mathematics (Year 12)

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question

Total marks (120)

- Attempt Questions 1–10
- All questions are of equal value
- Use a **SEPARATE** Writing Booklet for each question
- Write your **Board of Studies Student Number** and **Class Teacher's Initials** on the front cover of each of your writing booklets

Board of Studies Student Number: ________________

Class Teacher's Initials: ________________

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Total marks (120)
Attempt questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) **Marks**

(a) Evaluate $\frac{3}{\log_e 9}$ correct to three significant figures. **2**

(b) Differentiate with respect to x the expression $\frac{1}{2x} + \pi$. **2**

(c) Simplify $\frac{2x}{3} - \frac{x+2}{5}$. **2**

(d) Find a primitive of $2 - \sqrt{x}$. **2**

(e) Solve $1 - 9x^2 = 0$. **2**

(f) Peter paid \$1.15 per litre of petrol. This was 12.5% above the recommended retail price. Calculate the recommended retail price per litre of petrol. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

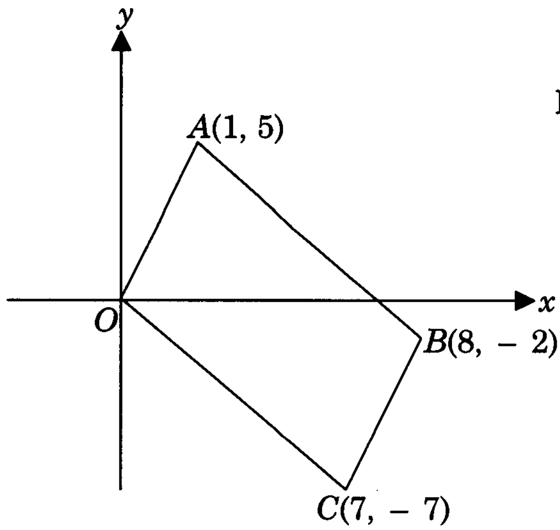


Diagram is not to scale

In the diagram, $OABC$ is a quadrilateral where the coordinates of O, A, B and C are $(0,0)$, $(1,5)$, $(8,-2)$ and $(7,-7)$ respectively.

(a) Find the midpoint of the interval joining AC. **1**

(b) Find the slope of AB. **1**

(c) Show that the equation of the line containing AB is $x + y = 6$. **1**

(d) Find the exact length of AB. **2**

(e) Show that OC is parallel to AB. **1**

(f) Explain why $OABC$ is a parallelogram. **2**

(g) Find the perpendicular distance from O to AB. Leave your answer in simplest surd form. **2**

(h) Find the area of parallelogram $OABC$. **2**

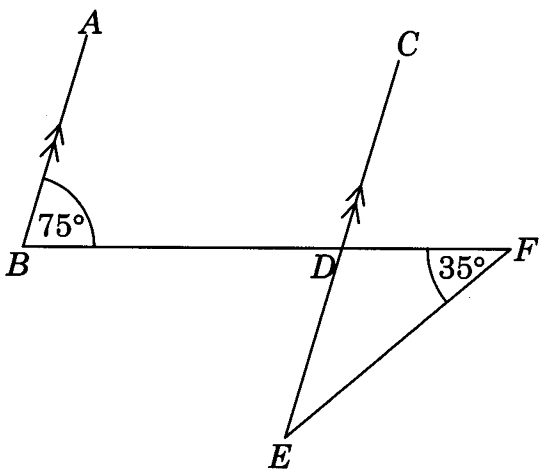
Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find:

(i) $\frac{d}{dx}\left(x^2e^{3x}\right)$. **2**

(ii) $\frac{d}{dx}\left(\frac{\sin x}{x}\right)$. **2**

(b) **Diagram is not to scale**



In the diagram, AB Π CE, $\angle ABF = 75°$ and $\angle BFE = 35°$. **2**

Find the size of $\angle DEF$, giving reasons.

(c) The graph of $y = f(x)$ passes through the point $(2, 12)$ and $f'(x) = 9x^2 + 4$. **2**

Find $f(x)$.

(d) Find $\int \frac{4x}{x^2 - 3}\,dx$. **2**

(e) Find $\int_{\frac{\pi}{6}}^{\pi} \cos 2x \; dx$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The equation of a parabola is $(x+3)^2 = -12(y-1)$.

(i) Write down the coordinates of the vertex of this parabola. **1**

(ii) State the coordinates of the focus. **1**

(iii) State the equation of the directrix. **1**

(iv) Sketch the graph of the parabola, showing the above features. **2**

(b) (i) Express $0.\dot{2}\dot{5}$ as an infinite geometric series. **1**

(ii) Hence, write $0.\dot{2}\dot{5}$ as a rational number in lowest terms. **1**

(c)

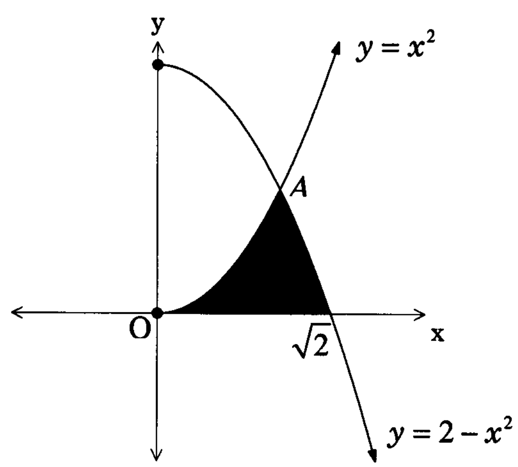


Diagram is not to scale

The diagram above shows the graphs of $y = x^2$ and $y = 2 - x^2$ intersecting at the point A. The curve $y = 2 - x^2$ crosses the x-axis at $(\sqrt{2}, 0)$. Note that $x \geq 0$.

(i) Find the x-coordinate of A. **1**

(ii) Find the area of the shaded region bounded by $y = x^2$, $y = 2 - x^2$ and the x-axis. Give your answer correct to two decimal places. **4**

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

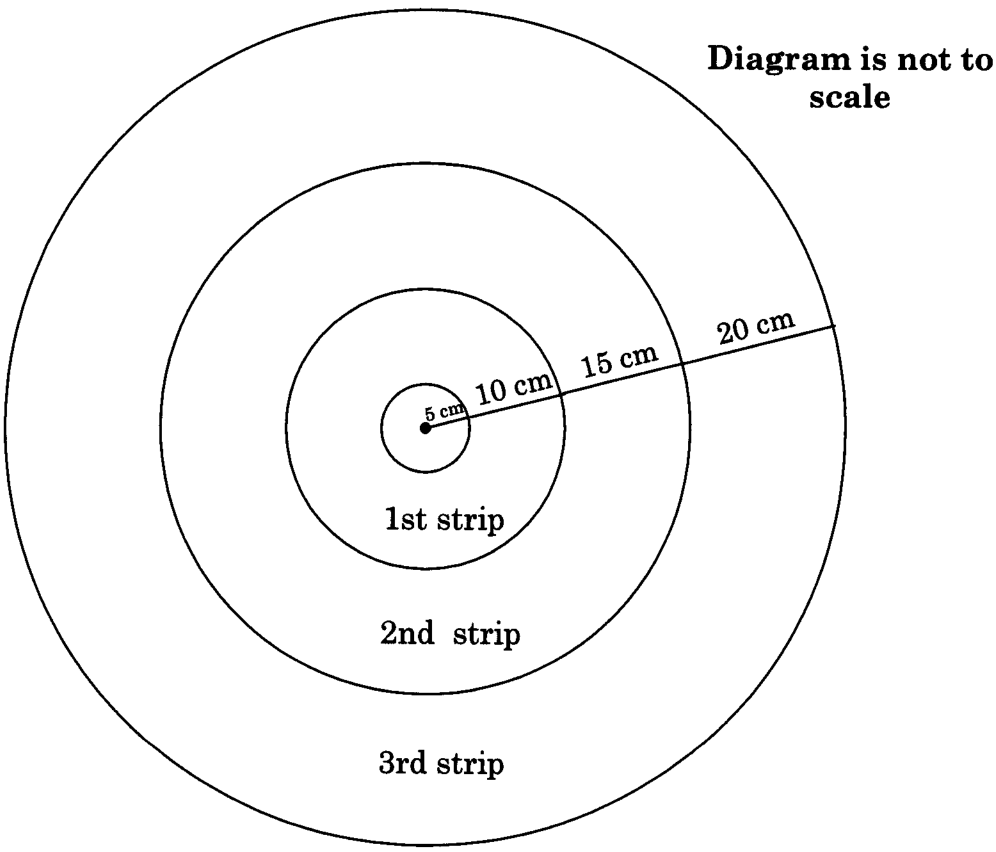
(a) Consider the function $f(x) = 2x^3 - 6x^2 - 18x + 1$.

(i) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **4**

(ii) Find the values of x for which $y = f(x)$ is increasing. **2**

(iii) For what value of x is the rate of decrease the greatest. **1**

(b)



Beginning with a circular piece of fabric of radius 5 cm, Susan sewed together circular strips of different coloured fabrics which increased in width to make a circular table cloth as shown in the diagram. The finished width of the first strip was 10 cm and of the second strip was 15 cm and so on.

(i) Show that the width of the 10^{th} strip was 55 cm. **2**

(ii) How many strips must be sewn together to complete a circular table cloth of radius 455 cm? **3**

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Consider the equation $\log_e y = x\log_e\left(\frac{1}{2}\right)$, where $y > 0$.

(i) Write down an expression for $y = f(x)$. **2**

(ii) Sketch the graph of $y = f(x)$ and label the axes appropriately. **1**

(b)

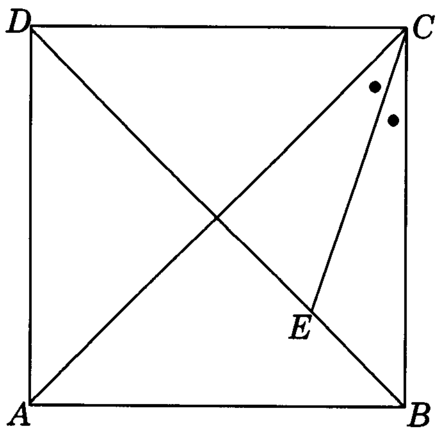


Diagram is not to scale

In the diagram, *ABCD* is a square with diagonals *AC* and *BD*.
The point *E* lies on *DB* and the interval *CE* bisects $\angle ACB$.

Copy or trace this diagram into your writing booklet.

(i) Prove, with reasons, that $\angle DCE = \angle DEC$. **2**

(ii) Hence, show that $DE = DA$. **2**

(c) An object falls into a tank filled with oil. The rate of decrease of the velocity is proportional to its velocity v centimetres per second. This statement can be expressed mathematically by the equation $\frac{dv}{dt} = -kv$, where k is a constant and t is time measured in seconds. The initial speed of the object when it enters the tank is 85 cm/s. Five seconds later, the speed of the object is 60 cm/s.

(i) Verify that $v = Ae^{-kt}$, where A is a constant, satisfies $\frac{dv}{dt} = -kv$. **1**

(ii) Find the values of A and k. **3**

(iii) Calculate the velocity of the object when $t = 8$. **1**

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)

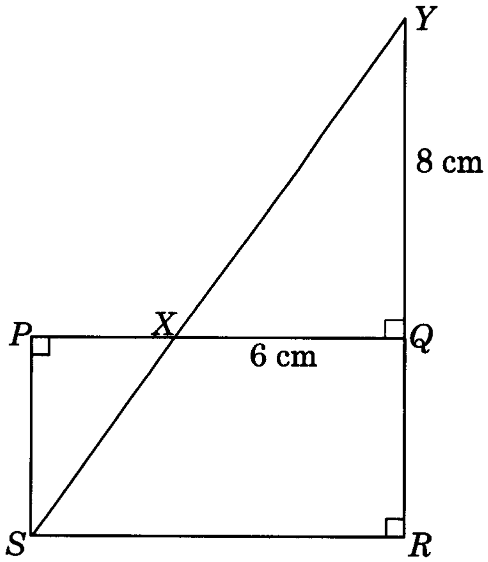


Diagram is not to scale

In the diagram, *PQRS* is a rectangle and $SR = 3PS$. The points *R*, *Q* and *Y* are collinear points. $XQ = 6$ cm and $YQ = 8$ cm.

(i) Prove that $\Delta PXS \,|||\, \Delta QXY$. **2**

(ii) Find the length of *PS*. **2**

(b) The velocity of a particle moving in a straight line is given by $v = 1 - 2\cos t$ for $0 \le t \le 2\pi$, where v is measured in metres per second and time t is measured in seconds.

(i) At what times in the interval $0 \le t \le 2\pi$, is the particle at rest? **2**

(ii) Sketch the graph of v against t for $0 \le t \le 2\pi$. **2**

(iii) What is the maximum velocity of the particle in the interval $0 \le t \le 2\pi$? **1**

(iv) Calculate the total distance travelled by the particle in the first π seconds. **3**

Question 8 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Consider the function $y = \ln(x-2)$ for $x > 2$.

(i) Sketch the graph of $y = \ln(x-2)$, showing essential features. **2**

(ii) Use Simpson's rule with three function values to evaluate: **2**

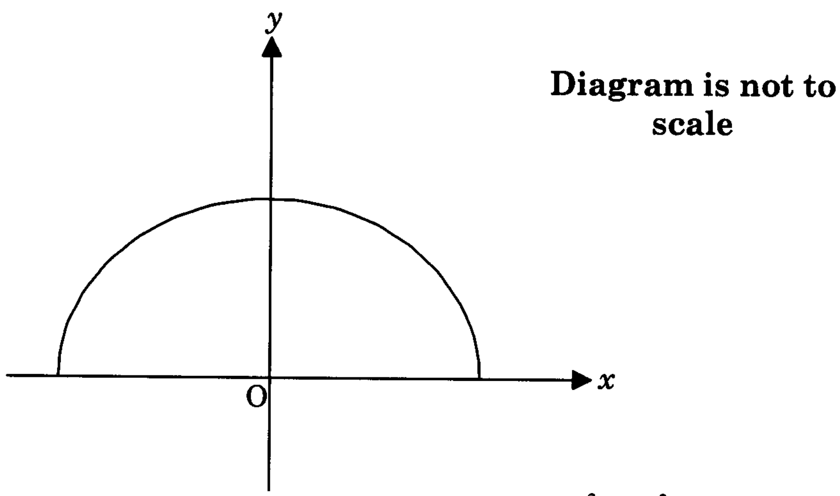
$$\int_3^5 \ln(x-2)\ dx$$

Round off your answer to one decimal place.

(b) The quadratic equation $x^2 + mx + n = 0$ has one root that is twice the other. Let one of the roots be α. **4**

Using the equations for the sum and product of the roots of $x^2 + mx + n = 0$, find the value of $\frac{m^2}{n}$.

(c)



The diagram shows part of the curve with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ which lies above the x-axis.

(i) Find the x-intercepts. **1**

(ii) Find the volume of the solid of revolution formed when this curve is rotated about the x-axis. Leave your answer in simplified form in terms of π. **3**

Question 9 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Two cards are chosen at random from the four cards shown below.

– 1	**0**	**1**	**2**

A person holds the two chosen cards in their hand.

(i) Calculate the probability that the sum of the numbers on the cards chosen is zero. 1

(ii) Find the probability that at least one of the cards in the person's hand is a 2. 1

(b) A water tank which is initially full is being emptied. The remaining quantity of water, L litres, in the tank, at any time, t minutes, after it starts to empty is given by $L = 1000(20 - t)^2$ for $0 \le t \le 20$.

(i) How much water was in the tank initially? 1

(ii) Show that $\dfrac{dL}{dt} = 2000t - 40\,000$. 1

(iii) At what time is the water flowing out of the tank at a rate of 20 000 litres per minute? 1

(iv) How long will it take to empty the tank? 1

(iv) At what rate is the tank emptying when the tank is half full? 2

(c) Consider the function $f(x) = \pi x - \cos \pi x$.

(i) Find $f''\left(\dfrac{1}{2}\right)$. 2

(ii) Show that the point $\left(\dfrac{1}{2}, \dfrac{\pi}{2}\right)$ is a point of inflexion on the graph of $y = f(x)$. 2

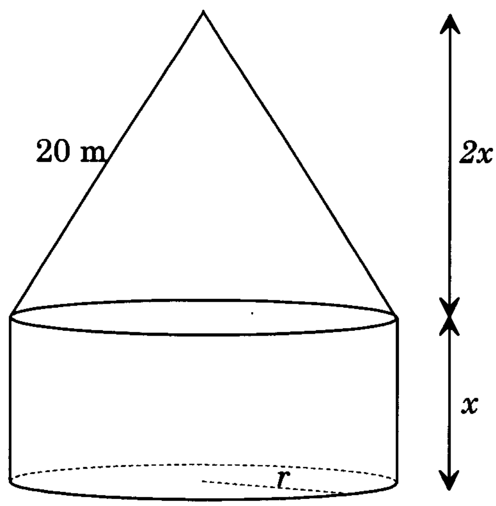
Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) An initial sum of \$2000 is invested into a retirement fund and interest is compounded at the rate of 8% per annum, every six months.

(i) If no further deposits are made into this fund, calculate how much the initial investment is worth at the end of 20 years? **2**

(ii) Now suppose that at the beginning of the second year, and the beginning of each subsequent year, a further \$500 is deposited into the fund. Determine how much the fund is worth at the end of 20 years? **4**

(b)



A grain silo has a cylindrical shaped wall and a cone shaped roof as shown in the diagram. Let the radius of the base of the silo be r metres and the height of the cylinder be x metres. The height of the cone is $2x$ metres.

(i) Show that if the length of the slant edge of the cone is 20 metres, then $r^2 = 400 - 4x^2$. **1**

(ii) Show that the volume, V, in cubic metres, of the silo is given by: **2**

$$V = \frac{20\pi}{3}\left(100x - x^3\right)$$

(iii) Find the exact height of the silo so that it holds a maximum amount of grain. **3**

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \ , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

SOLUTIONS MATHEMATICS TRIAL H.S.C 2005

QUESTION 1 (DS)

a) $\frac{3}{\log_e 9} = 1.3653588\ldots$ (1)

$= 1.37$ (3 S.F.) (1)

b) $\frac{d}{dx}\left(\frac{1}{2x} + \pi\right) = \frac{d}{dx}\left(\frac{1}{2}x^{-1} + \pi\right)$

$= -\frac{1}{2}x^{-2}$

$= \frac{-1}{2x^2}$ (2)

c) $\frac{2x}{3} - \frac{x+2}{5} = \frac{10x - 3(x+2)}{15}$

$= \frac{7x-6}{15}$ (2)

d) primitive of $2 - x^{1/2} = 2x - \frac{2}{3}x^{3/2} + C$ (2)

e) $1 - 9x^2 = 0$

$(1-3x)(1+3x) = 0$

$x = \pm\frac{1}{3}$ (2)

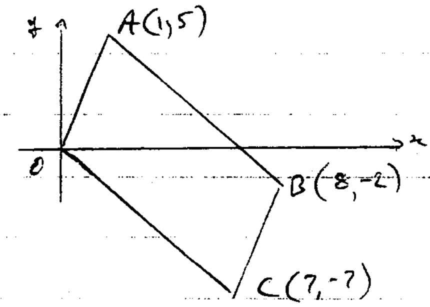
f) $112\frac{1}{2}\% = 115$

$1\% = 1.0222\ldots$

$100\% = 102.22\ldots$

Recommended retail price is 102.2 cents/litre. (2)

QUESTION 2 (SH)



a) Midpoint AC = (4, −1) (1)

b) Slope of AB = −1 (1)

c) Eqn. AB,

$y - 5 = -1(x-1)$

$y - 5 = -x + 1$

$x + y = 6$ (1)

d) length AB $= \sqrt{(8-1)^2 + (-2-5)^2}$ (1)

$= \sqrt{98}$

$= 7\sqrt{2}$ units (1)

e) Slope of OC $= \frac{-7-0}{7-0}$

$= -1$ (1)

∴ AB // AC

f) Slope OA = 5, Slope BC = 5 (1)

Since OA // BC and OC // AB (from above) then opposite sides are parallel (1) and OABC is a parallelogram.

g) ⊥ d $= \left|\frac{1\times 0 + 1\times 0 + 6}{\sqrt{1^2+1^2}}\right|$ (1) $= \frac{6}{\sqrt{2}} = 3\sqrt{2}$ units (1)

h) Area OABC $= 7\sqrt{2} \times 3\sqrt{2}$ (1)

$= 42$ units² (1)

QUESTION 3 (RM)

a) i) $\frac{d}{dx} x^2e^{3x} = e^{3x} \times 2x + x^2 \times 3e^{3x}$

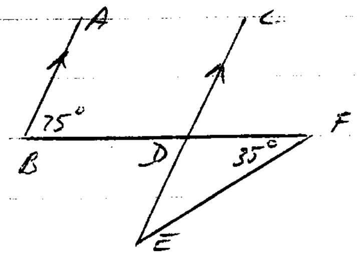
$= 2xe^{3x} + 3x^2e^{3x}$ (2)

$= xe^{3x}(2+3x)$

ii) $\frac{d}{dx} \frac{\sin x}{x} = \frac{x \times \cos x - \sin x \times 1}{x^2}$

$= \frac{x\cos x - \sin x}{x^2}$ (2)

b)



$\angle CDF = 75°$ Corresponding angles.

$\angle DEF + 35° = 70°$ Exterior angle ΔDEF

$\therefore \angle DEF = 40°$ (2)

c) $f'(x) = 9x^2 + 4$

$f(x) = 3x^3 + 4x + c$

$f(x) = 12$ when $x = 2$

$\therefore 12 = 24 + 8 + c \Rightarrow c = -20$

$\therefore f(x) = 3x^3 + 4x - 20$ (2)

d) $\int \frac{4x}{x^2-3}\,dx = 2\log_e(x^2-3) + c$ (1) (1)

e) $\int_{\pi/6}^{\pi} \cos 2x\,dx = \left[\frac{1}{2}\sin 2x\right]_{\pi/6}^{\pi}$ (1)

$= \frac{1}{2}\left(0 - \frac{\sqrt{3}}{2}\right)$

$= -\frac{\sqrt{3}}{4}$ (1)

QUESTION 4. (IB)

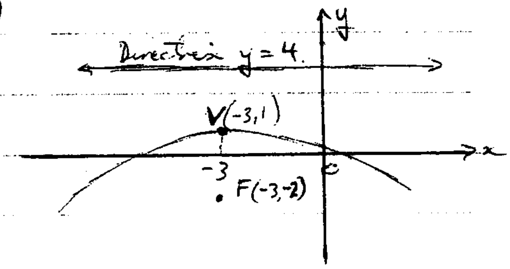
a) $(x+3)^2 = -12(y-1)$

i) Vertex $= (-3, 1)$ (1)

ii) Focus $= (-3, -2)$ (1)

iii) Directrix is $y = 4$ (1)

iv)



(2)

b) i) $0.\dot{2}\dot{5} = \frac{25}{100} + \frac{25}{10000} + \frac{25}{1000000} + \ldots$ (1)

ii) Using $S_\infty = \frac{a}{1-r}$,

$0.\dot{2}\dot{5} = \frac{25}{100} \times \frac{100}{99}$

$= \frac{25}{99}$ (1)

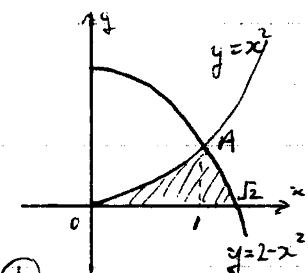
c) i) $x^2 = 2 - x^2$

$2x^2 - 2 = 0$

$x^2 - 1 = 0$

$x = \pm 1$

$\therefore$ x-coordinate of A is 1. (1)



ii) Area $= \int_0^1 x^2\,dx + \int_1^{\sqrt{2}} 2 - x^2\,dx$ (1)

$= \left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}}$ (1)

$= \left(\frac{1}{3} - 0\right) + \left[\left(2\sqrt{2} - \frac{2\sqrt{2}}{3}\right) - \left(2 - \frac{1}{3}\right)\right]$

$= \frac{4\sqrt{2}}{3} - \frac{4}{3}$ (1)

$= 0.55$ (2dp) (1)

QUESTION 5 (IM)

a) i) $f(x) = 2x^3 - 6x^2 - 18x + 1$

$f'(x) = 6x^2 - 12x - 18$

St. pts when $6x^2 - 12x - 18 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$ (1)

Stationary pts $(3, -53)$ & $(-1, 11)$ (1)

$f''(x) = 12x - 12$

at $x = 3$, $f''(3) = 24 > 0$ (1)

∴ Minimum at $(3, -53)$

at $x = -1$, $f''(-1) = -24 < 0$ (1)

∴ Maximum at $(-1, 11)$

ii) Increasing $f'(x) > 0$

i.e. $6x^2 - 12x - 18 > 0$

$(x-3)(x+1) > 0$

∴ $x < -1$ or $x > 3$ (2)

iii) Rate of decrease greatest when $f''(x) = 0$. i.e. $x = 1$ (1)

c) 10, 15, 20, . . .

i) $T_n = a + (n-1)d$

$T_{10} = 10 + 9 \times 5$ (2)

$T_{10} = 55$ ∴ Width 10^{th} strip is 55 cm

ii) Width of all strips $= 455 - 5$

$= 450$ (1)

∴ $450 = \frac{n}{2}\left[20 + (n-1)5\right]$

$900 = 15n + 5n^2$

i.e. $n^2 + 3n - 180 = 0$ (1)

$(n+15)(n-12) = 0$

∴ 12 pieces sewn together. (1)

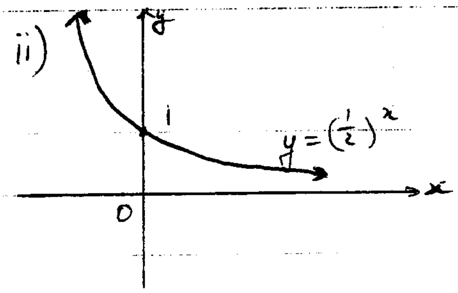
QUESTION 6 (DS)

a) $\log_e y = x \log_e\left(\frac{1}{2}\right)$

i) $\log_e y = \log_e\left(\frac{1}{2}\right)^x$

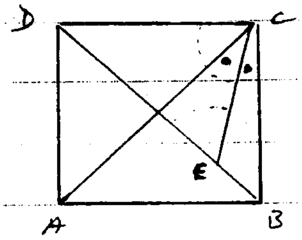
∴ $y = \left(\frac{1}{2}\right)^x$ i.e. $y = 2^{-x}$ (2)

ii)



(1)

b)



i) To prove $\angle DCE = \angle DEC$

$\angle DCA = 45°$ (Diagonals of a square bisect angles.)

Similarly $\angle DBC = 45°$

∴ $\angle DEC = 45° + \bullet$ (Exterior angle of $\triangle CBE$.)

$\angle DCE = 45° + \bullet$ (adjacent angles)

∴ $\angle DEC = \angle DCE$. (2)

ii) $\triangle DEC$ is isosceles (equal base angles.)

∴ $DC = DE$ (sides opposite equal angles are equal)

But $DC = DA$ (sides of square)

∴ $DE = DA$. (2)

c) i) $V = Ae^{-kt}$

$\frac{dV}{dt} = -kAe^{-kt}$ (1)

∴ $\frac{dV}{dt} = -kV$ $(V = Ae^{-kt})$

QUESTION 6 CONT'D.

c ii) $v = Ae^{-kt}$

$t=0, \quad V = 85$

$\therefore 85 = Ae^{0}$

$\therefore A = 85$ (1)

$V = 85e^{-kt}$

$t = 5, \quad V = 60$

$\therefore 60 = 85e^{-5k}$

$\frac{60}{85} = e^{-5k}$

$\ln\left(\frac{60}{85}\right) = -5k$

$k \doteqdot 0.0696613....$ (2)

iii) When $t = 8$

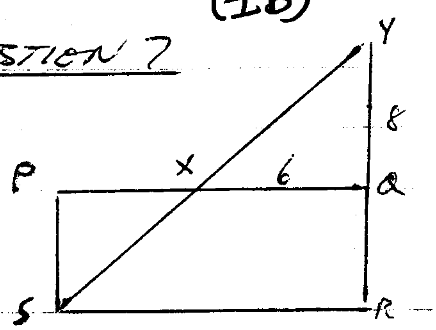
$V = 85e^{-8k}$

$V \doteqdot 48.68$ cm/s. (1)

(IB)

QUESTION 7

a)



In $\triangle PXS$ & $\triangle QXY$.

i) $\angle PXS = \angle QXY$ (Vertically opposite)

$\angle SPX = \angle XQR$ (right angles of rect. PQRS)

$= \angle YQX$ (straight angle)

$\therefore \triangle PXS \parallel\!\mid \triangle QXY$ (2 equal angles) (2)

ii) $\frac{PS}{YQ} = \frac{PX}{QX}$ corresponding sides of similar $\triangle$'s.

$\therefore \frac{x}{8} = \frac{3x-6}{6}$, $PS = x$

$6x = 24x - 48$ (2) Note: $SR = 3PS$

$18x = 48$

$\therefore x = 2\frac{2}{3}$; $PS = 2\frac{2}{3}$

b) $V = 1 - 2\cos t \qquad 0 \le t \le 2\pi$

i) At rest, $V = 0$.

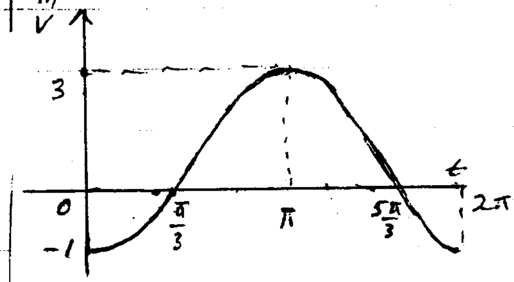
$1 - 2\cos t = 0$

$\cos t = \frac{1}{2}$

$t = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$ (2)

At rest when $t = \frac{\pi}{3}$ and $\frac{5\pi}{3}$ secs.

ii)



(2)

iii) Maximum velocity is 3 m/s (1)

iv) Distance travelled is given by

$$\int_0^{\pi/3} -(1 - 2\cos t)\,dt + \int_{\pi/3}^{\pi} 1 - 2\cos t\,dt \quad (1)$$

$$\text{distance} = \int_0^{\pi/3} 2\cos t - 1\,dt + \int_{\pi/3}^{\pi} 1 - 2\cos t\,dt$$

$$= \Big[2\sin t - t\Big]_0^{\pi/3} + \Big[t - 2\sin t\Big]_{\pi/3}^{\pi} \quad (1)$$

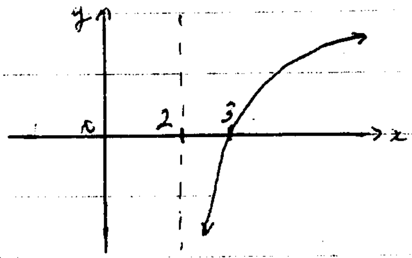
$$= \left(2\times\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right) - (0) + (\pi - 0) - \left(\frac{\pi}{3} - 2\times\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3} - \frac{\pi}{3} + \pi - \pi/3 + \sqrt{3}$$

$$= 2\sqrt{3} + \pi/3 \text{ metres.} \quad (1)$$

QUESTION 8 (IB)

a) $y = \ln(x-2) \quad x > 2$.

i) 

(2)

ii) $\int_3^5 \ln(x-2)\,dx \doteqdot \frac{h}{3}\left[y_0 + y_2 + 4y_1\right]$ S.R. 3 fn.

$= \frac{1}{3}\left[0 + \ln 3 + 4\ln 2\right]$ (1)

$= 1.3$ (1dp). (1)

b) $x^2 + mx + n = 0$

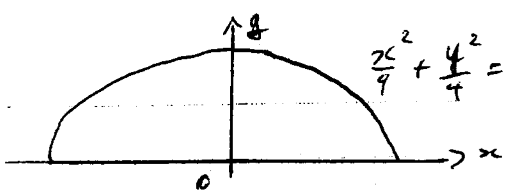
Let the roots be α and 2α

$\therefore$ Sum, $3\alpha = -m$ (1)

Product, $2\alpha^2 = n$ (1)

$\therefore \frac{m^2}{n} = \frac{9\alpha^2}{2\alpha^2}$ (1)

$= 9/2$. (1)

c) 

i) when $y=0$, $\frac{x^2}{9} + 0 = 1$

$x^2 - 9 = 0$

$x = \pm 3$ (1)

ii) $V = \pi \int_a^b y^2\,dx$

Note: $y^2 = 4 - \frac{4x^2}{9}$

$\therefore V = 2\pi \int_0^3 \left(4 - \frac{4x^2}{9}\right) dx$ (1)

$= 2\pi \left[4x - \frac{4}{27}x^3\right]_0^3$ (1)

$= 2\pi\left[(12-4) - 0\right]$

$\therefore$ Volume $= 16\pi$ units^3 (1)

QUESTION 9. (SH)

a) | -1 | | 0 | | 1 | | 2 |

i) P(sum is zero) $= \frac{1}{6}$ (1)

ii) P(one card is 2) $= \frac{1}{2}$ (1)

b) $L = 1000(20-t)^2$

i) when $t=0$, $L = 400\,000$

$\therefore$ 400 000 L of water initially. (1)

ii) $L = 1000(20-t)^2$

$\frac{dL}{dt} = 2 \times 1000(20-t)^1 \times -1$

$= -2000(20-t)$

$= -40000 + 2000t$

$= 2000t - 40000$ (1)

QUESTION 9 cont'd.

iii) $-20000 = 2000t - 40000$

$20000 = 2000t$

$t = 10$

Water is flowing out at 20 000 L/min when $t = 10$ min. (1)

iv) 20 min to empty tank (1)

v) Tank half full,

$200\,000 = 1000(20-t)^2$

$200 = (20-t)^2$

$\pm 10\sqrt{2} = 20 - t$

$\therefore t = 20 - 10\sqrt{2}$ or $(20 + 10\sqrt{2})$ $0 \le t \le 20$

$\therefore t = 20 - 10\sqrt{2}$ or 5.86 min 2dp

$\therefore \frac{dL}{dt} = 2000(20 - 10\sqrt{2}) - 40000$

(2) $= -20000\sqrt{2}$ or -28284.27

$\therefore$ tank is emptying at a rate of $20000\sqrt{2}$ L/min when half full.

c) $f(x) = \pi x - \cos \pi x$

i) $f'(x) = \pi + \pi \sin \pi x$

$f''(x) = \pi^2 \cos \pi x$

$f''(\frac{1}{2}) = 0$ (2)

ii) when $x = \frac{1}{2}$, $f(x) = \frac{\pi}{2} - \cos\frac{\pi}{2}$

$= \frac{\pi}{2}$

$\therefore (\frac{1}{2}, \frac{\pi}{2})$ lies on $f(x)$ (1)

$f''(\frac{1}{2}) = 0$

$f''(\frac{1}{2} - \epsilon) > 0$ ie $\pi^2 \cos\frac{\pi}{4} > 0$

$f''(\frac{1}{2} + \epsilon) < 0$ ie $\pi^2 \cos\frac{3\pi}{4} < 0$

Since concavity changes about $x = \frac{1}{2}$ then $(\frac{1}{2}, \frac{\pi}{2})$ is a pt of inflection. (1)

(IM).

QUESTION 10.

a) i) $A = P(1 + \frac{r}{100})^n$

$= 2000(1 + \frac{4}{100})^{40}$ (1)

$= 2000(1.04)^{40}$

$= \$9602.04$ (1)

ii)

Amount $= 500 \times 1.04^{38} + 500 \times 1.04^{36} + \dots + 500 \times 1.04^2$. (1)

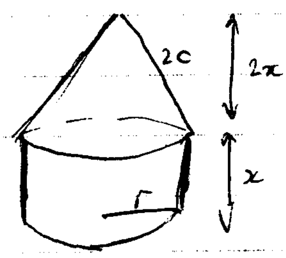
$= \dfrac{500 \times 1.04^2\left[(1.04^2)^{19} - 1\right]}{(1.04)^2 - 1}$ (1)

$= \$22790.57$ (1)

$\therefore$ Amount in account at end of 20 years is $\$9602.04 + \$22790.57$

ie. $\$32392.61$ (1)

b)



i) $r^2 + (2x)^2 = 20^2$

$r^2 + 4x^2 = 400$

$\therefore r^2 = 400 - 4x^2$ (1)

QUESTION 10 CONT'D.

ii) Volume of silo, $V = \pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \pi r^2 x + \frac{1}{3}\pi r^2 (2x)$

$= \frac{5}{3}\pi r^2 x$

$= \frac{5}{3}\pi x(400 - 4x^2)$

$= \frac{20}{3}\pi(100x - x^3)$ (1)

iii) $\frac{dV}{dx} = \frac{20}{3}\pi(100 - 3x^2)$

(1)

$\frac{d^2V}{dx^2} = \frac{20}{3}\pi(-6x)$

$= -40\pi x$

Stationary pts, $\frac{dV}{dx} = 0$

i.e. $\frac{20\pi}{3}(100 - 3x^2) = 0$

$3x^2 = 100$

$x^2 = \frac{100}{3}$

$x = \frac{10}{\sqrt{3}}$ $(x > 0)$ (1)

$\frac{d^2V}{dx^2} < 0$ for $x = \frac{10}{\sqrt{3}}$ (1)

$\therefore$ Maximum occurs when $x = \frac{10}{\sqrt{3}}$

$\therefore$ required height $= 3 \times \frac{10}{\sqrt{3}}$

$= 10\sqrt{3}$ metres

(1)