Student Number

Knox Grammar School

2008

Trial Higher School Certificate Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time - 3 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt Questions 1 – 10
- Answer each question in a separate writing booklet
- All questions are of equal value

Subject Teachers

Mr A. Johansen
Mr J. Harnwell
Mr I. Mulray
Miss L. Schultz
Miss F. Yamamer

This paper MUST NOT be removed from the examination room

Number of Students in Course: 76

Number of Writing Booklets Per Student (Four Page) 10

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Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet.

		Marks
(a)	Evaluate $\dfrac{0.1}{\sqrt{e+1}}$ correct to two significant figures.	2
(b)	Factorise $2x^2-4x+2$ completely.	2
(c)	Write down the primitive function of $\dfrac{3}{x}+5$.	2
(d)	Solve $\dfrac{x}{4}=3-\dfrac{x-2}{3}$, leaving your answer as an improper fraction.	2
(e)	If $a+\sqrt{b}=4(7+\sqrt{5})$ find a and b if they are both integers.	2
(f)	Sketch the graph of $y=\lvert 4-x \rvert$	2

Question 2 (12 marks) Use a SEPARATE writing booklet

		Marks
(a)	Draw a neat sketch of a number plane and plot the points $A(-4, 0)$, $B(4, 0)$ and $C(0, 8)$ on it.	1
(b)	Find the gradient of AC and show that the equation of AC is $2x-y+8=0$.	2
(c)	Find the perpendicular distance of AC from $Z(0, 3)$.	2
(d)	If X is the midpoint of AC, and Y is the midpoint of BC, find the coordinates of X and Y.	1
(e)	Show that XZ is perpendicular to AC.	2
(f)	Show that the lengths $AZ=BZ=CZ=5$ units.	2
(g)	Find the equation of the circle passing through A, B and C.	2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate the following with respect to x.

(i) $y=\dfrac{\sin x}{x}$ **1**

(ii) $y=\left(x^2+3\right)^6$ **1**

(iii) $y=x\ln x$ **1**

(b) Find an expression for each of the following integrals.

(i) $\displaystyle\int \frac{8}{x^2}\,dx$ **1**

(ii) $\displaystyle\int \sec^2 \pi x\,dx$ **1**

(c) Evaluate $\displaystyle\int_0^1 e^{2x}-e^{-x}\,dx$ **3**

(d) The gradient function of a curve is given by $\dfrac{dy}{dx}=6x^2-4$. The curve passes through the point (1, 8). Determine the equation of the curve. **2**

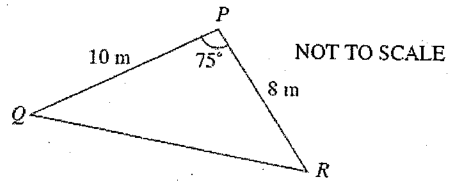
(e) The exterior angle of a regular polygon is $\dfrac{\pi}{10}$ radians.

(i) What is the size of each interior angle in radians? **1**

(ii) How many sides does this regular polygon have? **1**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



(i) Determine the length of QR, correct to 2 decimal places. **2**

(ii) What is the area of triangle PQR? Answer correct to 2 decimal places. **1**

(b) A function $f(x)$ is defined as $f(x)=x^4-8x^2$.

(i) Locate all stationary points and any points of inflexion. Distinguish between them. **4**

(ii) Determine the coordinates of the points where $y=f(x)$ crosses the x-axis. **2**

(iii) On a half-page diagram, sketch the function $y=f(x)$. Clearly label the stationary points, points of inflexion and intercepts with the x-axis. **2**

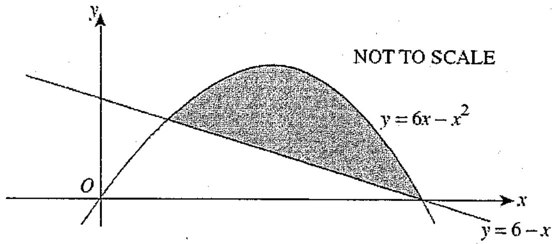
(iv) What is the maximum value of $f(x)$ in the interval $-2\le x\le 3$? **1**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Simplify $\log_b a^m \div \log_m a$ as a single expression in a logarithm of base b. 2

(b) The roots of the quadratic equation $2x^2 + kx + D = 0$ are α and β. $\alpha\beta = -5$ and $\alpha + \beta = 3$. Determine the values of k and D. 2

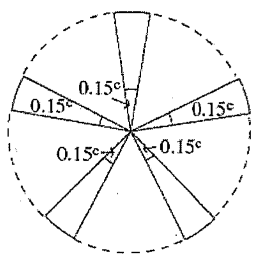
(c) The diagram shows the graph of $y = 6x - x^2$ and $y = 6 - x$.



(i) Use simultaneous equations to show that $y = 6x - x^2$ and $y = 6 - x$ intersect at (1, 5) and (6, 0). 1

(ii) Use calculus to determine the size of the shaded area. 3

(d)



The five blades on a windmill are identical sectors of the same circle. The angle of each blade at the centre of the circle is 0.15^c and the radius is 1.2 metres.

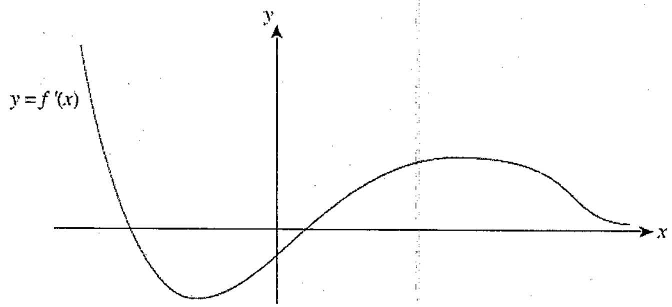
(i) All the edges on each of the blades are to be covered by a protective metal strip. Calculate the total length of metal strip required to protect the edges of all five blades. 2

(ii) The front and back surface of each blade is to be painted with a metal protector. A 100 mL container of the metal protector covers 400 cm^2. Calculate the quantity of metal protector required. 2

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\sin\theta = \dfrac{-\sqrt{3}}{2}$ for $0 \le \theta \le 2\pi$. 2

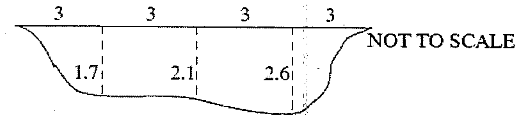
(b)



The diagram shows the gradient function $y = f'(x)$. Copy or trace the diagram into your answer booklet.

The curve $y = f(x)$ passes through the origin. Sketch the function $y = f(x)$ on the same set of axes. Clearly indicate any turning points, points of inflexion, and the behaviour of the graph for very large positive and negative values of x. 3

(c)

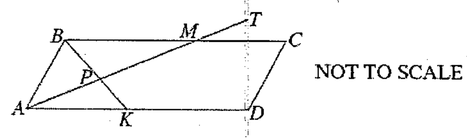


The diagram shows the cross-section of a 12-metre-wide pond. The depths are taken every 3 metres.

(i) Use Simpson's rule with five function values to find an approximate value for the area of the cross-section. 2

(ii) The pond is 25 metres long. Calculate the approximate quantity of water in the pond. Express the volume in cubic metres. 1

(d)



$ABCD$ is a parallelogram. Line AT bisects $\angle BAD$ and cuts BC at M. Line BK bisects $\angle ABC$. AT and BK intersect at P.

Copy the diagram onto your answer page and prove that

(i) $\angle BPA = 90°$. 2

(ii) $AB = BM$. 2

Question 7 (12 marks) Use a SEPARATE writing booklet. **Mark**

(a) Solve the equation $e^{2x} - 28e^{x} + 27 = 0$. Leave your answer in exact form. **3**

(b) An ambulance is delivering a patient to the hospital who is unconscious from a drug overdose. The medical staff don't know how much of the drug the unconscious patient has taken.

The rate of change of the concentration of the drug (C) in the blood is proportional to the concentration, i.e. $\frac{dC}{dt} = kC$.

(i) Prove that $C = C_0 e^{kt}$ is a solution to $\frac{dC}{dt} = kC$. **1**

(ii) Three hours after the patient took the overdose, the blood concentration of the drug was 2.45 mg/L. Half an hour later the concentration was 1.84 mg/L. Determine the initial concentration of the drug in the patient's blood. Give your answer correct to two decimal places. **3**

(iii) If the medical staff don't give the patient any further medication, when will the drug concentration fall below the critical value of 0.5 mg/L? **1**

(c) Two particles moving in a straight line are initially at the origin. The velocity of one particle is $\frac{2}{\pi}$ m/s and the velocity of the other particle at time t seconds is given by

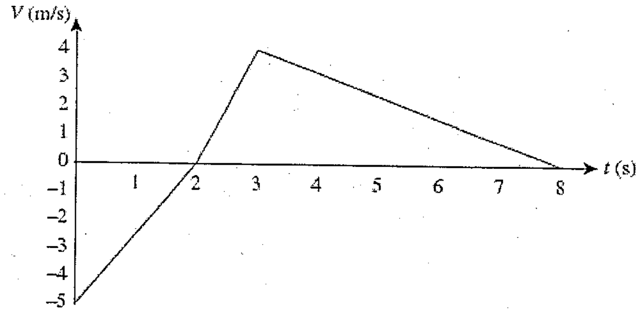
$v = -2\cos t$ m/s.

(i) Determine equations that give the displacements, x_1 and x_2 metres, of the particles from the origin at time t seconds. **2**

(ii) Hence, or otherwise, show that the particles will never meet again. **2**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



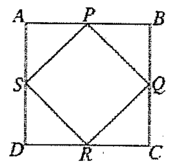
The graph shows the velocity of a particle moving in a straight line for 8 seconds.

(i) When does the particle change direction? **1**

(ii) Determine the total distance covered by the particle during the 8 seconds. **2**

(iii) What is the particle's position relative to its starting position when $t = 8$ seconds? **1**

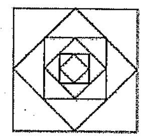
(b)



$ABCD$ is a square with sides 16 cm long. P, Q, R and S are the midpoints of the sides of the square $ABCD$. P, Q, R and S are joined to make another square.

(i) Show that $PS = 8\sqrt{2}$ and that the area of $PQRS$ is 128 cm^2. **2**

A 'squares within squares' pattern is produced by joining midpoints of the sides of successive squares.



(ii) $ABCD$ is the first square and $PQRS$ is the second square. What is the area of the 10th square? **2**

(iii) Which square has a perimeter of $\sqrt{2}$ cm? **2**

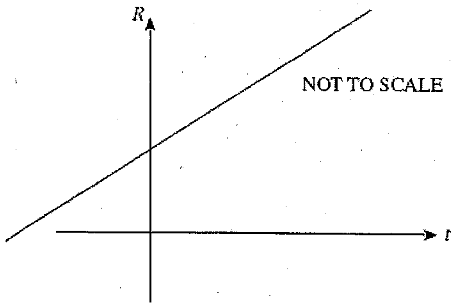
(iv) Imagine the pattern can be repeated infinitely. What is the relationship between the sum of the areas of all the squares and the original square $ABCD$? Use a calculation to justify your answer. **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) During a famine in Europe in the 19th century people in a small rural city ate an increasing quantity of potatoes each month as other food became increasingly scarce.

The rate at which potatoes were eaten (R) was given by $R = 15 + 2t$ tonnes per month, where t is the time in months after the beginning of the famine.

(i)



The diagram shows the graph of $R = 15 + 2t$. Copy the graph onto your answer page and show on the graph the region representing the total quantity of potatoes eaten in the city in 12 months. **1**

(ii) Calculate the total amount of potatoes that were eaten in the city during the 12-month famine. **3**

(b) Beth and Cathy are best friends who work in the same office. Each year on January 1, they each receive a cash bonus of \$5000. They received their first bonuses in 1997. Every year Beth invests her \$5000 in superannuation at 9% p.a. compounding interest. Each year Cathy spends her bonus on an overseas trip.

(i) Show that the expression $5000(1.09^{10} + 1.09^{9} + 1.09^{8} + \ldots + 1.09)$ represents the amount in Beth's superannuation account on January 1, 2007, immediately before her 2007 bonus was added to the account. **2**

(ii) Show that Beth had almost \$88 000 in her superannuation account on January 1, 2007, after her 2007 bonus was credited to her account. **2**

Cathy decides that on January 1, 2007, she will start saving for her retirement, which will occur in 20 years' time. She would like to have the same amount that Beth will have in 20 years' time from saving her annual \$5000 bonus. Cathy's account also pays 9% p.a. compounding interest.

(iii) How much will Cathy need to save each year to have the same total amount as Beth will have in 20 years' time (i.e. including the amounts Beth invested in the first 10 years)? **3**

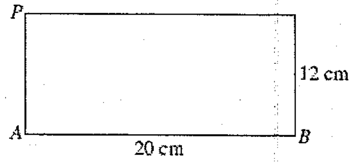
(iv) How much more will Cathy have to invest over the 20 years than Beth will have invested over the 30 years? **1**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

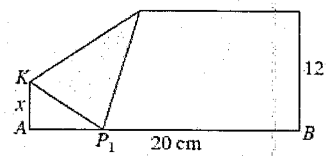
(a) (i) Shade the region bounded by $y \le 4 - x^2$, $x \ge 0$ and $y \ge 0$. **2**

(ii) Find the volume of the solid of revolution formed when the region defined in part (i) above is rotated about the x-axis. **4**

(b)



I have a rectangular sheet of paper 12 cm high by 20 cm long. I take the vertex labelled P and place it on the side AB. P now lies on top of P_1.



At the bottom left of the rectangle there is a small triangle AKP_1. Let the length of KA be x cm.

(i) Explain why KP_1 is $(12 - x)$ cm long. **1**

(ii) Show that the area of ΔAKP_1 is given by $A = x\sqrt{36 - 6x}$. **2**

(iii) Hence show that when x is one-third the length of PA the area of ΔAKP_1 is a maximum. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

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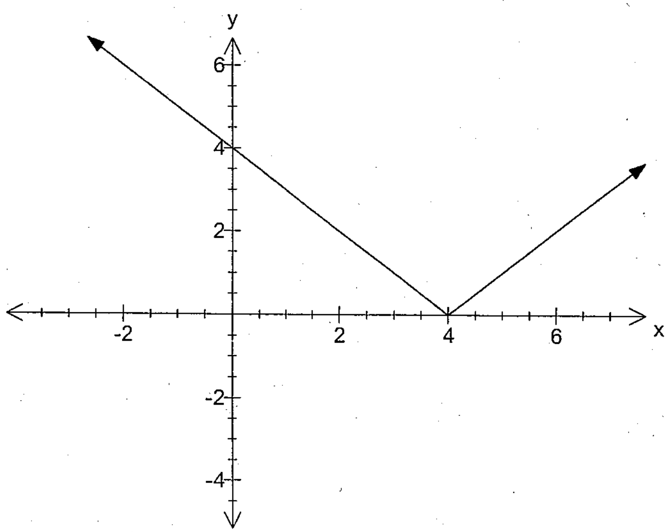
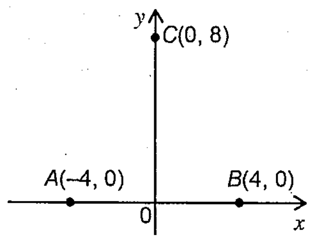
Marking guidelines

Question 1

Criteria	Marks
(a) $\dfrac{0.1}{\sqrt{e+1}}=0.0518\ldots$ $=0.052$ (correct to 2 significant figures)	1 value 1 rounding 2 full answer
(b) $2x^2-4x+2=2(x^2-2x+1)$ $=2(x-1)^2$	1 common factor 2 full
(c) $\int\left(\dfrac{3}{x}+5\right)dx=3\ln x+5x+c$	1 for each part 2 full
(d) $\dfrac{x}{4}=3-\dfrac{x-2}{3}$ $3x=36-4(x-2)$ $3x=36-4x+8$ $7x=44$ $x=\dfrac{44}{7}$	1 for 2nd line 2 for +8 3 full
(f) 	1 for y = 4-x 2 for complete graph

Question 2

Criteria	Marks
(a) 	1
(b) $m=\dfrac{8-0}{0-(-4)}=2$ C is the y-intercept, so $b=8$ $y=2x+8$ $2x-y+8=0$	1 for mostly correct 2 full answer
(c) $d=\dfrac{\lvert 2x-y+8\rvert}{\sqrt{2^2+(-1)^2}}$ $=\dfrac{\lvert 2\times0-3+8\rvert}{\sqrt{5}}$ $=\dfrac{5}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\sqrt{5}$	1 for mostly correct 2 full answer
(d) $X\left(\dfrac{-4+0}{2},\dfrac{0+8}{2}\right)$ and $Y\left(\dfrac{4+0}{2},\dfrac{0+8}{2}\right)$ $X(-2,4)$ and $Y(2,4)$	1
(e) $m_{XZ}=\dfrac{3-4}{0-(-2)}=\dfrac{-1}{2}$ $m_{XZ}\times m_{AC}=2\times\dfrac{-1}{2}=-1$, so XZ is perpendicular to AC.	1 for gradient 1 for test 2 full answer
(f) $d_{AZ}=\sqrt{(0--4)^2+(3-0)^2}=5$ $\quad d_{BZ}=\sqrt{(0-4)^2+(3-0)^2}=5$ $d_{CZ}=\sqrt{(0-0)^2+(3-8)^2}=5$	1 for mostly correct 2 full answer
(g) From part (f), Z is the centre of the circle with radius 5 passing through A, B and C. $x^2+(y-3)^2=25$	1 for circle equation 2 full answer

Question 3

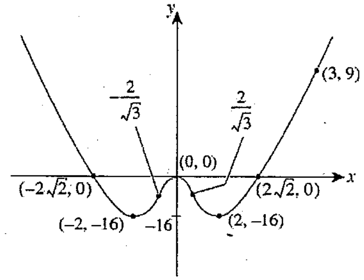
Criteria	Marks
(a) (i) $\frac{dy}{dx}=\frac{x\cos x-\sin x}{x^2}$	1
(ii) $\frac{dy}{dx}=12x(x^2+6)^5$	1
(iii) $\frac{dy}{dx}=1+\ln x$	1
(b)	
(i) $\int\frac{8}{x^2}dx=-\frac{8}{x}+C$	1
(ii) $\int\sec^2\pi x\,dx=\frac{1}{\pi}\tan\pi x+C$	1
(c) $\int_0^1 e^{2x}-e^{-x}dx=\left[\frac{1}{2}e^{2x}+e^{-x}\right]_0^1$	1 for each primitive
$=\left(\frac{1}{2}e^2+e^{-1}\right)-\left(\frac{1}{2}e^0+e^0\right)$	
$=\frac{1}{2}e^2+e^{-1}-\frac{3}{2}$	3 for correct answer.
(d) $y=2x^3-4x+C$ at $x=1, y=8 \therefore C=2$ $\therefore y=6x^2-4x+2$	1 for primitive 2 marks correct equation.
(e) (i) $\frac{9\pi}{10}$	1
(ii) $2\pi\div\frac{\pi}{10}=20$ sides	1

Question 4

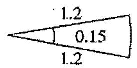
	Sample answer	Syllabus outcomes and marking guide
(a)	(i) $x^2=10^2+8^2-2\times10\times8\cos75$ $\therefore x=11.07$ to 2 decimal places	P4 • Gives the correct answer .... 2 • Makes the correct substitution into the correct formula.................... 1
	(ii) area $=\frac{1}{2}\times10\times8\times\sin75=38.64\text{ m}^2$	• Gives the correct answer 1
(b)	(i) $f(x)=x^4-8x^2$ $f'(x)=4x^3-16x$ stationary points occur when $f'(x)=0$ i.e. $4x^3-16x=0$ $4x(x^2-4)=0$ $\therefore$ stationary points occur at $(0,0)$, $(2,-16)$ and $(-2,-16)$ Testing $f''(x)=12x^2-16$ At $(0,0)$, $f''(x)=-16<0$. $\therefore$ maximum turning point. At $x=\pm2$, $f''(x)=32>0$. $\therefore$ minimum turning points. For inflexions, $f''(x)=0$ and a change in concavity occurs. $\therefore x=\pm\sqrt{\frac{16}{12}}=\pm\frac{2}{\sqrt{3}}$	H6, H9 • Gives the correct solutions........... 4 • Locates stationary points and determines nature or equivalent progress......... 3 • Locates stationary points or equivalent progress .... 2 • Correctly identifies the x-values of a cubic derivative 1

x	-2	$-\frac{2}{\sqrt{3}}$	0	$\frac{2}{\sqrt{3}}$	2
$f''(x)$	>0	0	<0	0	>0

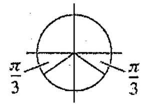
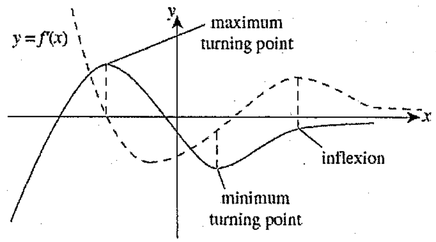
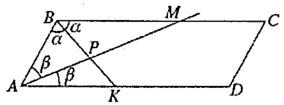
Therefore, points of inflexion are at $x=\pm\frac{2}{\sqrt{3}}$.

	Sample answer	Syllabus outcomes and marking guide
	(ii) crosses x-axis at $x^4-8x^2=0$ $x^2(x^2-8)=0$ $x=0$ or $\pm2\sqrt{2}$	P4, H9 • Gives the correct answers .... 2 • Gives one correct answer, or attempts to solve $x^4-8x^2=0$ 1
	(iii) 	H6, H9 • Gives a correct sketch showing all features (or correct from previous answer) (point (3, 9) not required) .... 2 • Gives any quartic-shaped sketch (or correct from previous answer)...... 1
	(iv) maximum value in $-2\le x\le3$ is 9	P4, H9 • Gives correct answer (or correct from previous answer) 1

Question 5

	Sample answer	Syllabus outcomes and marking guide
(a)	$\log_b a^m \div \log_m a$ $= m\log_b a \div \dfrac{\log_b a}{\log_b m}$ $= m\log_b a \times \dfrac{\log_b m}{\log_b a}$ $= m\log_b m$	H3, H9 • Gives the correct answer .... 2 • Uses the change of base law 1
(b)	$\alpha+\beta = -\dfrac{b}{a}$ $\alpha\beta = \dfrac{c}{a}$ $3 = -\dfrac{k}{2}$ $-5 = \dfrac{D}{2}$ $k = -6$ $D = -10$	P4 • Gives the correct answers .... 2 • Gives the correct answer for either D or k. 1
(c) (i)	$6 - x = 6x - x^2$ $x^2 - 7x + 6 = 0$ $(x-1)(x-6) = 0$ $\therefore x = 1$ or 6 $y = 6-1, 6-6$ Therefore, the points are (1, 5) and (6, 0).	P4, H9 • Gives the correct answers 1
(ii)	$\int_1^6 [6x - x^2 - (6-x)]dx$ $= \int_1^6 (7x - x^2 - 6)dx$ $= \left[\frac{7}{2}x^2 - \frac{1}{3}x^3 - 6x\right]_1^6$ $= 7\times 18 - \frac{1}{3}\times 6\times 36 - 36 - \left(\frac{7}{2} - \frac{1}{3} - 6\right)$ $= 20\frac{5}{6}$ square units	H8 • Gives the correct solution .... 3 • Makes significant progress .... 2 • Gives the correct expression for area or equivalent merit 1
(d) (i)	 length for 1 blade = $2\times 1.2 + 1.2\times 0.15 = 2.58$ m length for 5 blades = 12.9 m	H4 • Gives the correct answer .... 2 • Gives the correct length for one blade OR • Gives 0.18×5 as the length for the five arcs 1
(ii)	area $= 10\times\frac{1}{2}\times(120)^2\times 0.15$ $= 10\,800$ cm^2 quantity $= 10\,800 \div 400\times 100$ mL $= 2.7$ L	H4 • Gives the correct quantity .... 2 • Gives the correct area or equivalent merit 1

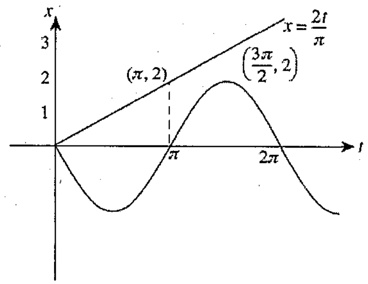
Question 6

	Sample answer	Syllabus outcomes and marking guide
(a)	$\sin\theta = \dfrac{-\sqrt{3}}{2}$  $\theta = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{4\pi}{3}, \frac{5\pi}{3}$	H5 • Gives the correct solutions........... 2 • Gives one correct solution 1
(b)		H7 • Gives a correct sketch which shows all features.......................... 3 • Gives a sketch which shows two correct features.......................... 2 • Gives a sketch showing one correct feature 1
(c) (i)	$A = \frac{h}{3}\{(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\}$ $= \frac{3}{3}\{0+0+4(1.7+2.6)+2\times 2.1\}$ $= 21.4$ m^2	H8, H9 • Gives the correct answer .... 2 • Gives a correct evaluation for h or equivalent merit.................... 1
(ii)	$V = Ah = 21.4\times 25 = 535$ m^3	H8 • Gives the correct answer 1
(d) (i)	 Let $\angle BAP = \beta$, $\angle ABP = \alpha$ $\therefore \angle MBP = \alpha$ $\angle PAK = \beta$ (given BK and PA bisect $\angle ABM$ and $\angle BAK$ respectively) Now $2\alpha + 2\beta = 180°$ (co-int angles $BM \parallel AK$ $ABCD$ parm) $\therefore \alpha+\beta = 90°$ $\therefore \angle BPA = 90°$ (angles in a triangle add to 180°)	P2, P4 • Gives the correct proof............... 2 • Makes some progress................ 1
(ii)	In ΔBAP and ΔBPM, $\angle BPA = \angle BPM = 90°$ (proved in (i)) BP is common $\angle ABP = \angle MBP$ (given PB bisects ΔABC) $\therefore \Delta ABP \equiv MPB$ (AAS) $\therefore AB = BM$ (corresponding sides in congruent triangles)	P2, P4 • Gives the correct proof............... 2 • Makes some progress (e.g. establishes congruency without reasons).......... 1

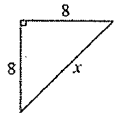
Question 7

	Sample answer	Syllabus outcomes and marking guide
(a)	Let $k = e^x$ $k^2 - 28k + 27 = 0$ $(k-27)(k-1) = 0$ $\therefore k = 27$ or 1 Hence, $e^x = 27$ or $e^x = 1$ $x = \log_e 27$ or $x = 0$	H3 • Gives the correct solutions .... 3 • Reduces equation to quadratic, and correctly factorises and solves for k ..... 2 • Reduces equation to a quadratic or equivalent merit 1
(b) (i)	$C = C_0 e^{kt}$ $\frac{dC}{dt} = k \times C_0 e^{kt}$ $= kC$ as required	H3 • Gives the correct proof 1
(ii)	$t = 3$ $C = 2.45$ $t = 3.5$ $C = 1.84$ $2.45 = C_0 e^{3k} \Rightarrow C_0 = \frac{2.45}{e^{3k}}$ $1.84 = C_0 e^{3.5k} \Rightarrow C_0 = \frac{1.84}{e^{3.5k}}$ $\frac{2.45}{e^{3k}} = \frac{1.84}{e^{3k} \times e^{0.5k}}$ $e^{0.5k} = \frac{1.84}{2.45}$ $0.5k = \log_e \frac{1.84}{2.45}$ $k = 2\log_e \frac{1.84}{2.45}$ $= -0.5726$ $\therefore C_0 = \frac{2.45}{e^{3 \times -0.2487}}$ $= 13.65$ mg/L	H3, H4 • Gives the correct solution (ignore rounding) .... 3 • Makes significant progress .... 2 • Establishes two values for C_0 or equivalent merit 1
(iii)	$t = ?$ $C = 0.5$ $0.5 = 13.65 \times e^{-0.5726t}$ $0.03663 = e^{-0.5726t}$ $\therefore t = \log_e 0.09677 \div -0.5726$ $= 5.78$ hours $\therefore$ after 5.78 hours	H3, H4 • Gives the correct answer 1

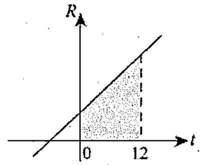
Question 7 **(Continued)**

	Sample answer	Syllabus outcomes and marking guide
(c) (i)	$t = 0, x = 0$ $v_1 = \frac{2}{\pi}$ $v_2 = -2\cos t$ $\therefore x_1 = \frac{2t}{\pi} + C_1$ $x_2 = -2\sin t + C_2$ when $t = 0, x = 0 \Rightarrow C_1 = 0$ when $t = 0, x = 0 \Rightarrow C_2 = 0$ $\therefore x_1 = \frac{2t}{\pi}$ $\therefore x_2 = -2\sin t$	H4, H5 • Gives the correct answers .... 2 • Gives one correct answer 1
(ii)	 The graphs don't intersect again. $x = \frac{2t}{\pi}$ has a value greater than 2 for $x > \pi$, and the maximum value of $x = -2\sin t$ is 2.	H2, H4, H5 • Gives the correct justification and explanation .... 2 • Draws graphs, with no justification or explanation 1

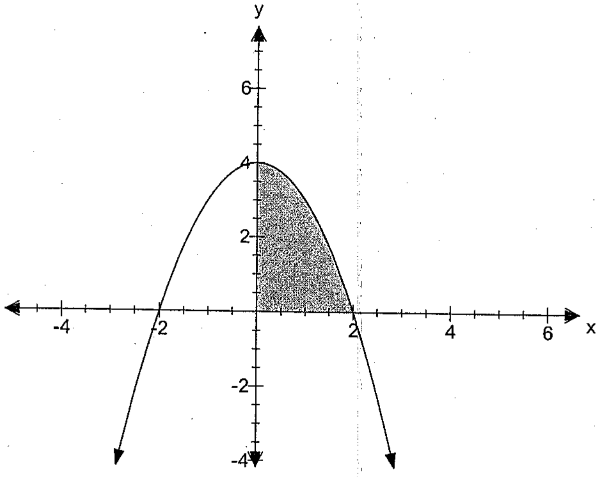
Question 8

		Sample answer	Syllabus outcomes and marking guide
(a)	(i)	At $t = 2$, because $v = 0$.	H4, H5 • Gives the correct answer 1
	(ii)	Total distance covered equals the area under the v–t graph. Therefore, the distance covered $= \frac{1}{2} \times 5 \times 2 + \frac{1}{2} \times 6 \times 4$ $= 5 + 12$ $= 17$ m	H4, H5, H8 • Gives the correct distance .... 2 • Correctly calculates one area or equivalent merit 1
	(iii)	7 metres on the positive side of the starting position.	H4, H5 • Gives the correct answer 1
(b)	(i)	 $x^2 = 64 + 64$ $= 64 \times 2$ $x = 8\sqrt{2}$ area of $PQRS = (8\sqrt{2})^2$ $= 128$ cm^2	H4, H5 • Gives the correct proof and the correct area .... 2 • Gives one correct proof 1
	(ii)	The areas are 256, 128, ... This is a geometric sequence: $a = 256$, $r = \frac{1}{2}$. $T_{10} = ar^9$ $= 256 \times \left(\frac{1}{2}\right)^9$ $= \frac{1}{2}$ cm^2	H5 • Gives the correct answer .... 2 • Identifies the correct $r = \frac{1}{2}$ 1
	(iii)	The perimeters are 64, $32\sqrt{2}$, 32 ... $T_n = \sqrt{2} = 64 \times \left(\frac{1}{\sqrt{2}}\right)^{n-1}$ $(\sqrt{2})^n = 64$ $2^{\frac{1}{2}n} = 2^6$ $n = 12$	H5 • Gives the correct answer .... 2 • Determines a correct equation, or solves an incorrect, non-trivial exponential equation for n 1
	(iv)	areas $= 256, 128 ..., r = \frac{1}{2}$ $S_\infty = \frac{a}{1 - \frac{1}{2}}$ $= 2a$ The sum of the areas of all the squares is twice the area of the original square.	H5 • Gives the correct answer .... 2 • Uses limiting sum 1

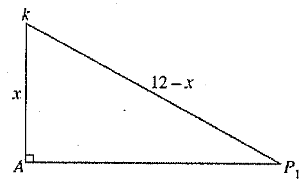
Question 9

		Sample answer	Syllabus outcomes and marking guide
(a)	(i)		P4, H4, H9 • Gives the correct answer (12 must be shown) 1
	(ii)	$\int_0^{12} (15 + 2t)dt = [15t + t^2]_0^{12}$ $\frac{15 + 39}{2} \times 12$ $= 15 \times 12 + 12^2$ $= 324$ tonnes OR area $= \frac{15 + 39}{2} \times 12$ $= 324$ tonnes	H4, H8, H9 • Gives the correct answer .... 3 • Makes significant progress .... 2 • Makes limited progress 1
(b)	(i)	Let a_n = amount in the account at the end of n years immediately before the next addition. $a_1 = 5000(1.09)$ $a_2 = [5000(1.09) + 5000](1.09)$ $= 5000(1.09)^2 + 5000(1.09)$ $= 5000[(1.09)^2 + 1.09]$ $a_3 = [5000\{(1.09)^2 + 1.09\} + 5000](1.09)$ $= 5000[1.09^3 + 1.09^2 + 1.09]$ $a_n = 5000[1.09^n + 1.09^{n-1} ... 1.09]$ $\therefore a_{10} = 5000[1.09^{10} + 1.09^9 ... 1.09]$	H5, H9 • Gives the correct demonstration .... 2 • Makes some progress 1
	(ii)	$A_{10} + 5000$ $= 5000 + 5000 \times \frac{1.09(1.09^{10} - 1)}{1.09 - 1}$ $= \$87\,801.46$ Beth has almost \$88 000 in her account.	H5, H9 • Gives the answer \$87 801.46 .... 2 • Uses the sum of geometric series 1
	(iii)	In 20 more years, Beth will have $5000 \times \frac{1.09(1.09^{30} - 1)}{1.09 - 1} = \$742\,876.09$ Cathy $742\,876.09 = A \times \frac{1.09(1.09^{20} - 1)}{1.09 - 1}$ $= A \times 55.7645$ $\therefore A = 13\,321.66$ Cathy will need to invest \$13 321.66 each year.	H5, H9 • Gives the answer \$13 321.66 .... 3 • Makes significant progress .... 2 • Makes some progress (e.g. determines \$742 876.09) 1
	(iv)	$\$13\,321.66 \times 20 - \$5000 \times 30 = \$116\,433.19$ Cathy will have to invest \$116 433.19 more than Beth.	• Gives the correct answer (accept correct from previous answer) 1

Question 10

Sample Answer	Marking Guide
(i)  	2 for correct region 1 for parabola
(ii) $V=\pi\int_0^2 y^2\,dx$ where $y=4-x^2$ so $y^2=16-8x^2+x^4$ $=\pi\int_0^2 16-8x^2+x^4dx$ $=\pi\left[16x-\frac{8}{3}x^3+\frac{1}{5}x^5\right]_0^2$ $=\frac{256\pi}{15}$	4 for correct answer 3 for correct primitive prior to evaluation 2 for correct integral with integrand in terms of *x*. 1 for formula with correct limits of integration.

Question 10 (Continued)

<table>
<tr><th>Sample answer</th><th>Syllabus outcomes and marking guide</th></tr>
<tr><td>(b) (i) $kA+kP_1=$ length of AP
$kA+kP_1=12$
$kP_1=12-kA$
$=12-x$</td><td>H2
• Gives the correct explanation. . . . . 1</td></tr>
<tr><td>(ii)


$(AP_1)^2=(12-x)^2-x^2$
$=4(36-6x)$
$AP_1=2\sqrt{36-6x}$
area $=\frac{1}{2}\times Ak\times AP_1$
$=\frac{1}{2}x\times 2\sqrt{36-6x}$
$=x\sqrt{36-6x}$</td><td>H2, H4
• Gives the correct demonstration 2

• Makes progress (e.g. shows $(AP_1)^2=4(36-6x)$) . . . . 1</td></tr>
<tr><td>(iii) For a maximum $\frac{dA}{dx}=0$
$\frac{dA}{dx}=\sqrt{36-6x}+\frac{1}{2}\times x(36-6x)^{-\frac{1}{2}}\times -6=0$
$\sqrt{36-6x}-\frac{3x}{\sqrt{36-6x}}=0$
$36-6x-3x=0$
$9x=36$
$x=4$
test in the first derivative
<table>
<tr><td>x</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y'</td><td>$\frac{9}{\sqrt{18}}$
↗</td><td>0
—</td><td>$\frac{-9}{\sqrt{6}}$
↖</td></tr>
</table>
Therefore it is a maximum.
When $x=4$, which is $\frac{1}{3}$ of AP.
The area of the triangle is a maximum.</td><td>H4, H5, H9
• Gives the correct proof. 3

• Makes significant progress 2

• Makes some progress, e.g. equates the correct expression for $\frac{dA}{dx}$ to 0 . . . . 1</td></tr>
</table>