VIRILE AGITUR

Number: ____________________

Teacher: ____________________

2014
Trial Higher School Certificate Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen only
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I: Pages 1-3

10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II: Pages 4-11

90 marks
- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section

Teachers:
Mr Sedgman
Mr Johansen*
Ms Tran
Mr Vuletich
Mr Ockenden
Mr Salmon

Write your Board of Studies Student Number, your name and your teacher's name on the front cover of each writing booklet

This paper MUST NOT be removed from the examination room

Number of Students in Course: 127

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 What is 4.09784 correct to three significant figures?

(A) 4.09

(B) 4.10

(C) 4.097

(D) 4.098

2 The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β.

What is the value of $\alpha\beta + (\alpha + \beta)$?

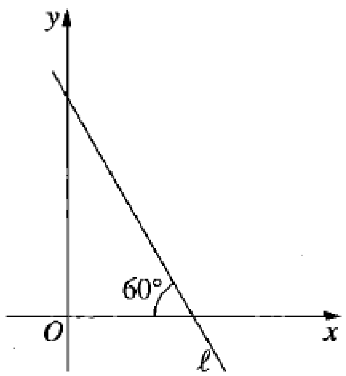
(A) 4

(B) 2

(C) -4

(D) -2

3 The diagram shows the line ℓ.



What is the slope of the line ℓ?

(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) $\dfrac{1}{\sqrt{3}}$

(D) $-\dfrac{1}{\sqrt{3}}$

4 What is the derivative of $\frac{x}{\cos x}$?

(A) $\frac{\cos x + x\sin x}{\cos^2 x}$

(B) $\frac{\cos x - x\sin x}{\cos^2 x}$

(C) $\frac{x\sin x - \cos x}{\cos^2 x}$

(D) $\frac{-x\sin x - \cos x}{\cos^2 x}$

5 What is the sum of the first ten terms of the series $96 - 48 + 24 - 12 + ...$?

(A) 63.9375

(B) 191.8125

(C) –32.736

(D) 98.208

6 Which of the following statements is INCORRECT?

(A) $\log a^n = n\log a$

(B) $\log ab = \log a + \log b$

(C) $\log(a-b) = \frac{\log a}{\log b}$

(D) $\log e = 1$

7 The curve $y = ax^2 - 6x + 3$ has a stationary point at $x = 1$. What is the value of a?

(A) 2

(B) –1

(C) 3

(D) –3

8 What is the value of $\int_1^4 \frac{1}{3x}\,dx$?

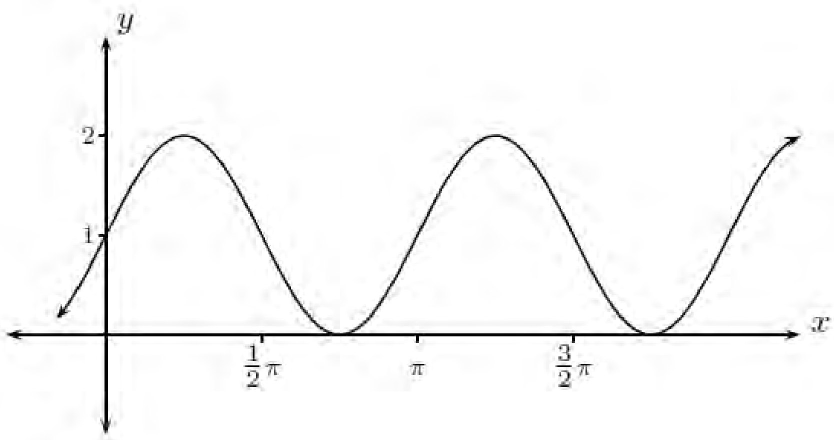
(A) $\frac{1}{3}\ln 3$

(B) $\frac{1}{3}\ln 4$

(C) $\ln 9$

(D) $\ln 12$

9



The equation of the graph sketched above could be:

(A) $y = 1 + \sin 2x$
(B) $y = 1 - \sin 2x$
(C) $y = 1 + 2\sin 2x$
(D) $y = 1 - 2\sin x$

10 What is the range of the function $y = |x| - x$?

(A) All real y

(B) $y \geq 0$

(C) $y \leq 0$

(D) $y = 0$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Factorise $2x^2 - 7x + 3$. **2**

(b) Solve $|3x - 1| < 2$. **2**

(c) Find the equation of the tangent to the curve $y = x^2$ at the point where $x = 3$. **2**

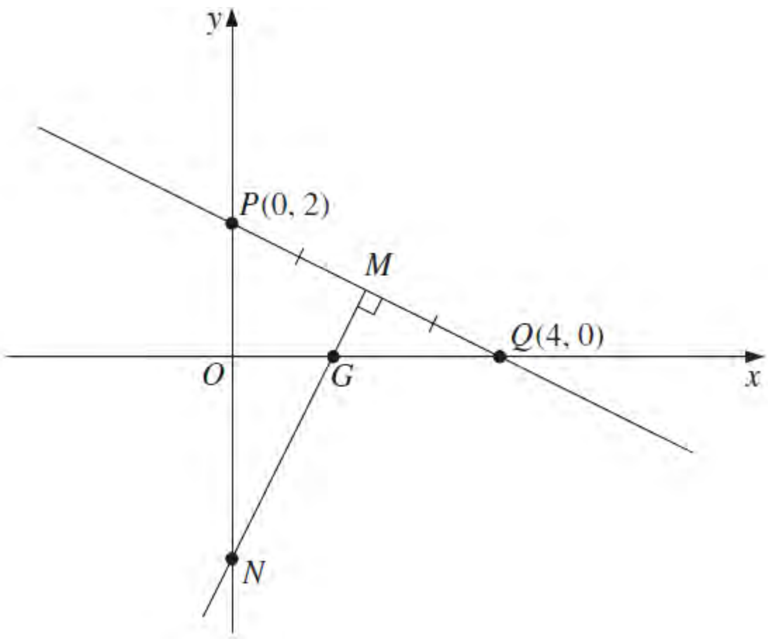
(d) Differentiate $\left(3 + e^{2x}\right)^5$. **2**

(e) Find the coordinates of the focus of the parabola $x^2 = 16(y - 2)$. **2**

(f) The area of a sector of a circle of radius 6 cm is 50 cm^2. **2**
Find the length of the arc of the sector.

(g) Find $\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2}\, dx$. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet.



(a) The diagram shows the points $P(0,2)$ and $Q(4,0)$. The point M is the midpoint of PQ. The line MN is perpendicular to PQ and meets the x axis at G and the y axis at N.

(i) Show that the gradient of PQ is $-\frac{1}{2}$. **1**

(ii) Find the coordinates of M. **2**

(iii) Find the equation of the line MN. **2**

(iv) Show that N has coordinates $(0,-3)$. **1**

(v) Find the distance NQ. **1**

(vi) Find the equation of the circle with centre N and radius NQ. **2**

(vii) Hence show that the circle in part (vi) passes through the point P. **1**

(viii) The point R lies in the first quadrant, and $PNQR$ is a rhombus. Find the coordinates of R. **2**

(b) The gradient of a curve is given by $\frac{dy}{dx} = \frac{2x}{x^2 + e}$. The curve passes through the point $(0,2)$. What is the equation of the curve? **3**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) For an arithmetic progression, the fifth term is 16 and the eleventh term is 40.

(i) Find the first term and the common difference. **3**

(ii) How many terms in the sequence must be added to reach a sum of 312? **2**

(b) Solve the following equation for x: **2**

$$e^{2x} + 3e^{x} - 10 = 0.$$

(c) Find the exact value of $\cos\theta$ given that $\tan\theta = 7$ and $\sin\theta < 0$. **2**

(d) Let $f(x) = (x+2)(x^2+4)$.

(i) Show that the graph $y = f(x)$ has no stationary points. **2**

(ii) Find the values of x for which the graph $y = f(x)$ is concave down, and the values for which it is concave up. **2**

(iii) Sketch the graph $y = f(x)$, indicating the values of the x and y intercepts. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Simplify $1-\sin^2\theta$. **1**

(ii) Prove the identity $\tan\theta\left(1-\sin^2\theta\right)=\sin\theta\cos\theta$. **2**

(b) A particle is moving on the x axis with displacement x metres after t seconds given by the function

$$x=2t^2-25t+50$$

(i) What was the initial position of the particle? **1**

(ii) What was the initial velocity of the particle? **1**

(iii) At what times was the particle at the origin? **2**

(iv) At what time was the particle instantaneously at rest? **1**

(v) How far did the particle travel between its visits to the origin? **2**

(c) Henry borrows \$200 000 which is to be repaid in equal monthly instalments. The interest rate is 7.2 % per annum reducible, calculated monthly.

It can be shown that the amount, $\$A_n$, owing after the nth month is given by the formula

$$A_n=200\,000\,r^n-M\left(1+r+r^2+\ldots+r^{n-1}\right),$$

where $r=1.006$ and $\$M$ is the monthly repayment. (Do NOT show this.)

(i) The minimum monthly repayment is the amount required to repay the loan in 300 instalments. **3**
Find the minimum monthly repayment.

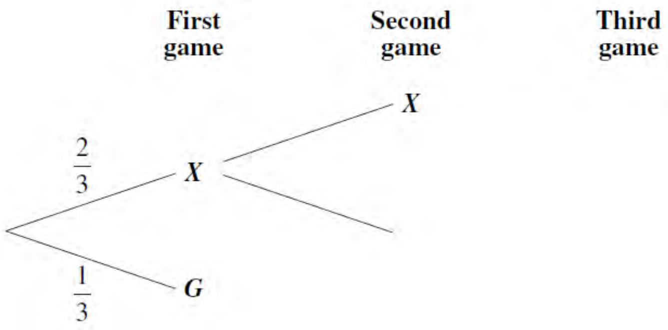
(ii) Henry decides to make repayments of \$2800 each month from the start of the loan. **2**
How many months will it take for Henry to repay the loan?

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) Show that $3x^2 + 4x + 5$ is positive definite. **2**

(b) Xena and George compete in a series of games. The series finishes when one player has won two games. In any game, the probability that Xena wins is $\frac{2}{3}$ and the probability that George wins is $\frac{1}{3}$.

Part of the tree diagram for the series of games is shown.



(i) Copy and complete the tree diagram showing the possible outcomes. **1**

(ii) What is the probability that George wins the series? **2**

(iii) What is the probability that three games are played in the series? **2**

(c) The rate of elimination $\frac{dQ}{dt}$ of a drug by the kidneys is given by the equation

$$\frac{dQ}{dt} = -kQ$$

where k is a constant and Q is the quantity of drug present in the blood. In this question, t is measured in minutes and Q in milligrams.

(i) Show that $Q = Q_0 e^{-kt}$ satisfies the equation $\frac{dQ}{dt} = -kQ$. **1**

(ii) The initial quantity of the drug present was mesasured to be 100 mg and at time $t = 20$ minutes, the quantity was 74 mg. Find the values of Q_0 and k. **2**

Give k correct to five decimal places and Q_0 to the nearest mg.

(iii) What is the initial rate of elimination of the drug? Give your answer correct to one decimal place? **1**

(iv) How long is it until only half the original quantity of drug remains? Give your answer correct to the nearest minute? **2**

Question 15 continues on page 9

Question 15 (continued)

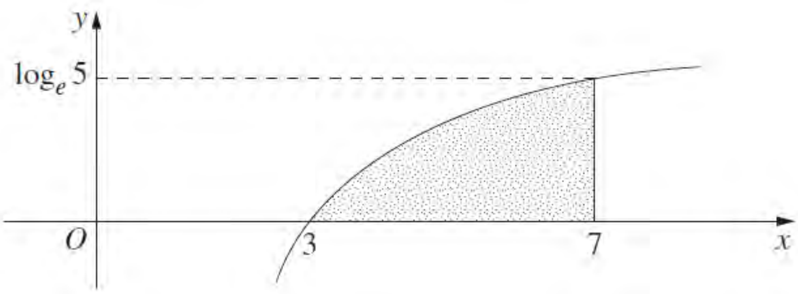
(d) Use Simpson's rule with three function values to find an approximation to the value of $\int_{0.5}^{1.5} (\log_e x)^3 \, dx$. **2**

Give your answer correct to three decimal places.

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

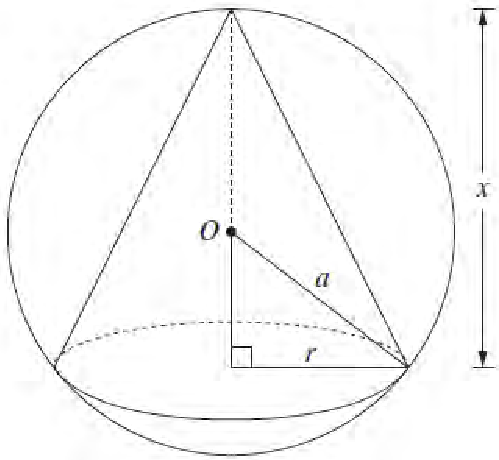
(a) **5**



In the diagram, the shaded region is bounded by $y = \log_e(x-2)$, the x axis and the line $x = 7$.

Find the exact value of the area of the shaded region.

(b)



A cone is inscribed in a sphere of radius a, centred at O. The height of the cone is x and the radius of the base is r, as shown in the diagram.

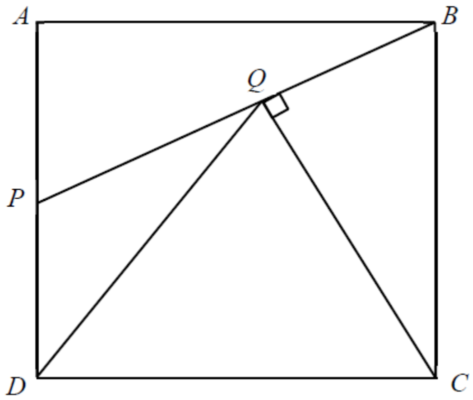
(i) Show that the volume, V, of the cone is given by $V = \frac{1}{3}\pi\left(2ax^2 - x^3\right)$. **2**

(ii) Find the value of x for which the volume of the cone is a maximum. **3**

You must give reasons why your value of x gives the maximum volume.

Question 16 continues on page 11

(c)



NOT TO SCALE

$ABCD$ is a square of side length 2 units. P is the midpoint of AD.

CQ is drawn perpendicular to PB and $\angle APB = x^\circ$.

(i) Prove that $\angle APB = \angle QBC$. **1**

(ii) Hence, or otherwise, show that $QC = \dfrac{4}{\sqrt{5}}$ units. **2**

(iii) Show that $QD = CD$. **2**

End of paper

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x\,, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Section I		5. $96 - 48 + 24 - 12 + \ldots$	
1. 4.09784 $= 4.10 \rightarrow$ (B)		$a = 96,\ r = -\frac{1}{2},\ S_{10} = ?$ $S_{10} = \dfrac{96\left[\left(-\frac{1}{2}\right)^{10} - 1\right]}{-\frac{1}{2} - 1}$ $= 63.9375 \rightarrow$ (A)	
2. $\alpha + \beta = -3$ $\alpha\beta = -1$ $\therefore\ \alpha\beta + (\alpha + \beta) = -1 - 3$ $= -4 \rightarrow$ (C)		6. $\log(a - b) \neq \dfrac{\log a}{\log b} \rightarrow$ (C)	
3. $m_L = -\tan 60^\circ$ $= -\sqrt{3} \rightarrow$ (B)		7. $\dfrac{dy}{dx} = 2ax - 6$ $= 0$ when $2ax - 6 = 0$ $x = 1 \rightarrow 2a(1) - 6 = 0$ $a = 3 \rightarrow$ (C)	
4. $\dfrac{d}{dx}\left(\dfrac{x}{\cos x}\right)$ $= \dfrac{(\cos x)(1) - (x)(-\sin x)}{\cos^2 x}$ $= \dfrac{\cos x + x \sin x}{\cos^2 x} \rightarrow$ (A)		8. $\displaystyle\int_1^4 \frac{1}{3x}\,dx$ $= \frac{1}{3}\Big[\ln x\Big]_1^4$ $= \frac{1}{3}\left[\ln 4 - \cancel{\ln 1}^{\,0}\right]$ $= \frac{1}{3}\ln 4 \rightarrow$ (B)	

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Section I cont'd 9. $y = 1 + \sin 2x$ $\left.\begin{aligned}\text{Period} &= \frac{2\pi}{2} \\ &= \pi \\ \text{Amplitude} &= 1\end{aligned}\right\} \rightarrow$ (A) 10. $x \geqslant 0,\ y = x - x$ $= 0 \rightarrow R: y = 0$ $x < 0,\ y = -x - x$ $= -2x \rightarrow R: y > 0$ $\therefore$ Range of function is $y \geqslant 0 \rightarrow$ (B)			

Suggested Solution (s)	Comments
Section II	
Q11:	
a) $2x^2 - 7x + 3$ $= (2x-1)(x-3)$	
b) $\lvert 3x - 1 \rvert < 2$ $-2 < 3x - 1 < 2$ $-1 < 3x < 3$ $-\frac{1}{3} < x < 1$	
c) $x = 3,\ y = 9$ $\frac{dy}{dx} = 2x$ At $x = 3$, $\frac{dy}{dx} = 6$ $= m_{\text{tangent}}$ $\therefore\ y - 9 = 6(x-3)$ $y - 9 = 6x - 18$ $0 = 6x - y - 9$	
d) Let $y = (3 + e^{2x})^5$ $\frac{dy}{dx} = 5(3+e^{2x})^4 \cdot 2e^{2x}$ $= 10e^{2x}(3+e^{2x})^4$	
e) $x^2 = 16(y-2)$ $4a = 16$ $a = 4$ $\therefore$ Focus is $(0, 6)$	
f) $50 = \frac{1}{2}(6)^2\theta$ $\theta = \frac{50}{18}$ $= \frac{25}{9}$ $\therefore\ l = 6\left(\frac{25}{9}\right)$ $= \frac{50}{3}$ or $16\frac{2}{3}$ cm	
g) $\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2}\, dx = \left[2 \tan \frac{x}{2}\right]_0^{\frac{\pi}{2}}$ $= 2\left[\tan\frac{\pi}{4} - 0\right]$ $= 2(1-0)$ $= 2$	

3/

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q12: a) i) $m_{PQ} = \frac{0-2}{4-0}$ ✓ $= \frac{-1}{2}$ ii. $M_{PQ} = \left(\frac{0+4}{2}, \frac{2+0}{2}\right)$ ✓ $= (2,1)$ ✓ iii. $m_{MN} = 2$ $(MN \perp PQ)$ ✓ $y-1 = 2(x-2)$ $y-1 = 2x-4$ $0 = 2x-y-3$ ✓ iv. y int $(x=0)$ $\rightarrow 0 = 2(0)-y-3$ $\therefore y=-3$ ✓ ie N is $(0,-3)$ v) $N(0,-3)$ $Q(4,0)$ $d_{NQ} = \sqrt{(4)^2+(3)^2}$ $= \sqrt{25}$ $= 5$ units ✓		vi. Centre $(0,-3)$, $r=5$ $(x-0)^2+(y+3)^2 = 5^2$ ✓ $x^2+(y+3)^2 = 25$ ✓ or $x^2+y^2+6y-16=0$ vii. $(0,2) \rightarrow x^2+(y+3)^2=25$ $0^2+(2+3)^2=25$ $(5)^2=25$ ✓ $\therefore$ Circle passes thru P viii. If $PNQR$ is a rhombus, $M(2,1)$ is the midpoint of RN. $\therefore R(x,y)$ $N(0,-3) \rightarrow M(2,1)$ $\therefore \frac{x+0}{2}=2$ and $\frac{y-3}{2}=1$ $x=4$ ✓, $y=5$ ✓ $\rightarrow R(4,5)$ [or similar] b) $y = \int \frac{2x}{x^2+e}\,dx$	

$= \ln(x^2+e) + C$ ✓

4/

$\overset{x\ y}{(0,2)} \rightarrow 2 = \cancel{\ln e^1} + C \rightarrow C=1$ ✓

$\therefore y = \ln(x^2+e) + 1$ ✓

Y12 Mathematics (HSC ASSESSMENT TASK 4: TRIAL HSC EXAMINATION) - Term 3 2014

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q13 :

a) i. $T_5 = a + 4d = 16$ (1) ✓

$T_{11} = a + 10d = 40$ (2)

(2) − (1) → $6d = 24$

$d = 4$ ✓

$d = 4 \rightarrow a + 16 = 16$

$a = T_1$

$= 0$ ✓

ii. $S_n = 312$

$312 = \frac{n}{2}\left[2(0) + (n-1)4\right]$

$312 = \frac{n}{2}\left[4n - 4\right]$

$312 = 2n^2 - 2n$ ✓

$0 = n^2 - n - 156 = 0$

$0 = (n-13)(n+12)$

$n = 13$ only $(n > 0)$ ✓

ie $S_{13} = 312$

b) Let $u = e^x$

$u^2 + 3u - 10 = 0$

$(u+5)(u-2) = 0$

$u = -5$ or $u = 2$ ✓

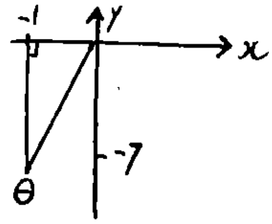
$\therefore e^x = -5$ ✓ $e^x = 2 \rightarrow x = \ln 2$ only

$\downarrow$

NO SOLN $(e^x > 0) \rightarrow$ REASON required ✓

5

c)



$r = \sqrt{1^2 + 7^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$ ✓

$\therefore \cos\theta = \frac{-1}{5\sqrt{2}}$ or $\frac{\sqrt{2}}{10}$ ✓

$\left(\frac{-1}{\sqrt{50}} \text{ okay}\right)$

d) i. $y = (x+2)(x^2+4)$

$= x^3 + 2x^2 + 4x + 8$

$\frac{dy}{dx} = 3x^2 + 4x + 4$ ✓

$\Delta = (4)^2 - 4(3)(4)$

$= -32 < 0$ ✓

$\therefore$ No real roots for $\frac{dy}{dx} = 0$

$\therefore$ No stationary points

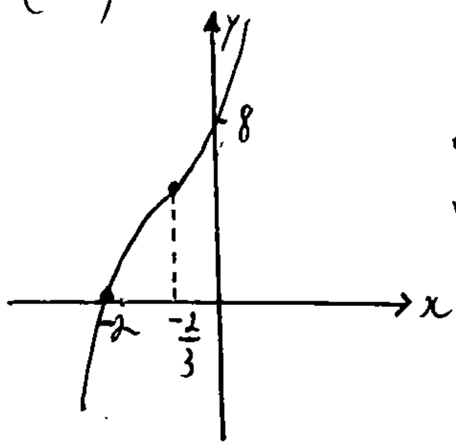
ii. $\frac{d^2y}{dx^2} = 6x + 4$

Concave down $\rightarrow 6x + 4 < 0$

$6x < -4$

$x < -\frac{2}{3}$ ✓

Similarly concave up $\rightarrow x > -\frac{2}{3}$ ✓

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q13 cont'd: x int $\rightarrow$ $0 = (x+2)(x^2+4)$ $(y=0)$ $\quad x = -2$ only y int $\rightarrow$ $y = 8$ $(x=0)$ 	✓ shape ✓ intercepts		

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q14:

a) i) $1-\sin^2\theta = \cos^2\theta$ ✓

ii) $LHS = \tan\theta\,(1-\sin^2\theta)$

$= \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cos^{\cancel{2}}\theta$ ✓

$= \sin\theta\cos\theta$ ✓

$= RHS$

b) i. $t=0$, $x = 50$ m ✓

ii. $v = 4t-25$

$t=0$, $v = -25$ m/s ✓

iii. $x=0 \rightarrow 2t^2-25t+50=0$

$(2t-5)(t-10)=0$

$t = 2\frac{1}{2}$ ✓ or $t=10$ ✓

iv. $v=0 \rightarrow 4t-25=0$

$t = 6.25$ ✓

v.

t	2.5	6.25	10
x	0	$-28\frac{1}{8}$	0

✓ ✓

$\therefore$ Distance $= 2(28.125)$

$= 56.25$ m ✓

c) i. Loan repaid $\rightarrow A_{300} = 0$, $M = ?$

$0 = 200\,000(1.006)^{300} - M(1+1.006+\dots+1.006^{299})$

GP where $a=1$, $r=1.006$, $n=300$

$0 = 200\,000(1.006)^{300} - M\left[\frac{1.006^{300}-1}{0.006}\right]$ ✓

$M\left[\frac{1.006^{300}-1}{0.006}\right] = 200\,000(1.006)^{300}$

$M = \frac{200\,000(1.006)^{300} \times 0.006}{1.006^{300}-1}$ ✓

$= \$1439.18$ ✓

ii. $A_n = 0$, $M = 2800$, $n = ?$

$0 = 200\,000(1.006)^n - 2800\left[\frac{1.006^n-1}{0.006}\right]$

$0 = 200\,000(1.006)^n(0.006) - 2800[1.006^n-1]$

$0 = 1200(1.006)^n - 2800(1.006)^n + 2800$

$1600(1.006)^n = 2800$

$(1.006)^n = \frac{7}{4}$ ✓

$n\ln(1.006) = \ln\frac{7}{4}$

PTO

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q14 cont'd: c) ii. $n = \dfrac{\ln\left(\frac{7}{4}\right)}{\ln 1{\cdot}006}$ $= 93{\cdot}5488\ldots$ $\therefore$ the loan will be repaid in 94 months	✓		

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q11 cont'd: c) i. $\angle APB = x$ $\angle APB = \angle QBC$ (alternate angles, $AD \parallel BC$) $\rightarrow$ ABCD is a square or $x + \angle ABP + 90° = 180°$ (Angle sum of ΔAPB and $\angle A = 90°$ $\downarrow$ ABCD is a square) $\therefore \angle ABP = 90° - x$ $\angle QBC = 90° - \angle ABP$ ($\angle B = 90°$) $= 90° - (90° - x)$ $= x$ $\therefore \angle ABP = \angle QBC = x$ ii. $PB = \sqrt{AP^2 + AB^2}$ $= \sqrt{2^2 + 1^2}$, given $AP = \frac{1}{2}AB$ $= \sqrt{5}$ $\sin x = \frac{AB}{PQ} = \frac{2}{\sqrt{5}}$ (1) $\cos x = \frac{AP}{PB} = \frac{1}{\sqrt{5}}$ (2) In ΔQBC, $\sin x = \frac{QC}{BC} = \frac{2}{\sqrt{5}}$ from (1)	✓ ✓	$\therefore \frac{QC}{2} = \frac{2}{\sqrt{5}}$ ✓ $QC = \frac{4}{\sqrt{5}}$ (3) or by similar Δ's ABP, BCQ (PROOF) iii. $\angle QCB = 90 - x$ (angle sum of ΔQBC) $\therefore \angle QCD = x$ ($\angle C = 90°$) $QD^2 = QC^2 + CD^2 - 2(QC)(CD)\cos x$ $= \frac{16}{5} + 4 - 2\left(\frac{4}{\sqrt{5}}\right)(2)\left(\frac{1}{\sqrt{5}}\right)$ ✓ from (2) + (3) $= \frac{16}{5} + 4 - \frac{16}{5}$ $= 4$ $\therefore QD = 2$ ✓ and $QD = CD = 2$	

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q15 cont'd: iii. $t=0,\ Q=Q_0=100$ $\frac{dQ}{dt}=-kQ_0$ $=-(0.01506)\times 100$ $=-1.506$ ✓ $=-1.5$ mg/min iv. $Q=50,\ Q_0=100,\ t=?$ $50=100e^{-kt}$ $e^{-kt}=\frac{1}{2}$ ✓ $-kt=\ln\frac{1}{2}$ $t=\frac{\ln\frac{1}{2}}{-0.01506}$ $=46.0257092$ ✓ $\doteqdot 46$ min		d) x: 0.5, 1, 1.5 y: $(\ln 0.5)^3$, $(\ln 1)^3$, $(\ln 1.5)^3$ $\int_{0.5}^{1.5}(\ln x)^3\,dx \doteqdot \frac{0.5}{3}\left[(\ln 0.5)^3+0+(\ln 1.5)^3\right]$ ✓ $=-0.0443942$ ✓ $=-0.044$	

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q16:

a) $y = \log_e(x-2)$

$x - 2 = e^y$

$x = e^y + 2$

$\text{Area} = (7 \times \ln 5) - \int_0^{\ln 5} (e^y + 2)\,dy$

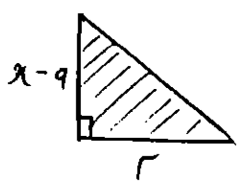
$= 7\ln 5 - \left[e^y + 2y \right]_0^{\ln 5}$

$= 7\ln 5 - \left[\left(e^{\ln 5} + 2\ln 5\right) - \left(e^0 + 0\right) \right]$

$= 7\ln 5 - 5 - 2\ln 5 + 1$

$= (5\ln 5 - 4)$ units^2

b)



$a^2 = (x-a)^2 + r^2$

$\cancel{a^2} = x^2 - 2ax + \cancel{a^2} + r^2$

$\therefore\ r^2 = 2ax - x^2$ (1)

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 x$

$= \frac{1}{3}\pi (2ax - x^2)x$ from (1)

$= \frac{1}{3}\pi (2ax^2 - x^3)$

ii. $V = \frac{1}{3}\pi(2ax^2 - x^3)$

$\frac{dV}{dx} = \frac{1}{3}\pi(4ax - 3x^2)$

$= \frac{4\pi}{3}ax - \pi x^2$

$= 0$ when $\frac{4\pi}{3}ax - \pi x^2 = 0$

i.e. $\pi x\left(\frac{4}{3}a - x\right) = 0$

$x = 0$ or $\frac{4a}{3}$

$\frac{d^2V}{dx^2} = \frac{4\pi a}{3} - 2\pi x$

$\therefore\ x = \frac{4a}{3}$ only as $x > 0$

$\frac{d^2V}{dx^2} = \frac{4\pi a}{3} - 2\pi\left(\frac{4a}{3}\right)$

$= -\frac{4\pi a}{3} < 0$

$\therefore$ Cone has a maximum volume when $x = \frac{4\pi a}{3}$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q15:

a) $\Delta = (4)^2 - 4(3)(5)$

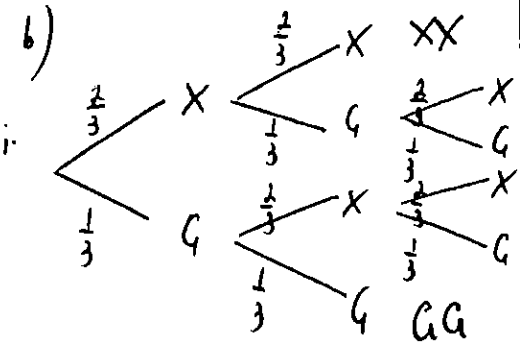
$= -44 < 0$

and coefficient of x^2 is positive $(a > 0)$

then graph of $3x^2 + 4x + 5$ lies entirely above x-axis

ie positive definite.

b) i.



ii) $P(\text{Georgia wins})$

$= P(XGG) + P(GXG) + P(GG)$

$= 2\left(\frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{1}{3}\right)$

$= 2\left(\frac{2}{27}\right) + \frac{1}{9}$

$= \frac{7}{27}$

iii. $P(3 \text{ games played})$

$= 1 - P(XX) - P(GG)$

$= 1 - \left(\frac{2}{3} \times \frac{2}{3}\right) - \left(\frac{1}{3} \times \frac{1}{3}\right)$

$= \frac{4}{9}$

c) i. $Q = Q_0 e^{-kt}$

$\frac{dQ}{dt} = Q_0 \cdot -k e^{-kt}$

$= -k \cdot Q_0 e^{-kt}$

$= -kQ \rightarrow$ Hence shown

ii. $t = 0$, $Q = 100$

$\therefore 100 = Q_0 \cdot e^{\cancel{-k(0)}}$ 1

$\therefore Q_0 = 100$ mg

$t = 20$, $Q = 74$

$74 = 100 e^{-20k}$

$0.74 = e^{-20k}$

$-20k = \ln 0.74$

$k = \frac{\ln 0.74}{-20}$

$= 0.01505525$

$= 0.01506$

(must have 0.01506)