STUDENT NAME……………………………. **STUDENT NUMBER**…………………

MLC School

YEAR 12 MATHEMATICS 2 unit

Trial HSC Examination

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided and should be used
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I

Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II

Pages 7 – 16

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.
Use the multiple-choice answer sheet for Questions 1 – 10.

1. What is 532 124 999 written in scientific notation correct to 3 significant figures?

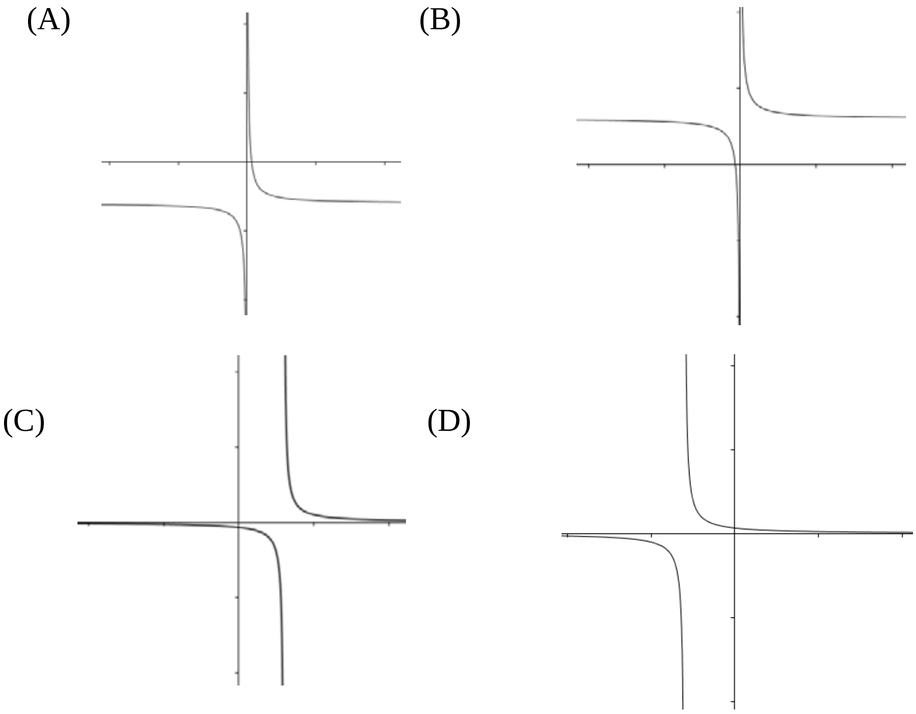
(A) 5.32×10^8

(B) 5.321

(C) 532 100 000

(D) 5 000

2. Which of the following is the correct graph of $y = \frac{1}{x-3}$



3. Fully simplify the algebraic fraction $\frac{4x^2-16}{x-2}$.

(A) $2x - 8$

(B) $4(x + 2)$

(C) $4x$

(D) $\frac{2x-8}{x}$

4. What is the derivative of $(3x^2 - 1)^3$?

(A) $3(3x^2)^2$

(B) $3(6x - 1)^2$

(C) $3(6x - 1)^2$

(D) $18x(3x^2 - 1)^2$

5. The quadratic function $3x^2 - 5x + 2$ has roots α and β.

Which of the following statements is false?

(A) $3\alpha\beta = 2$

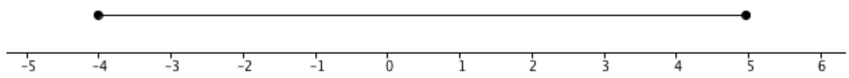
(B) $\alpha + \beta = \frac{5}{3}$

(C) $\alpha^2\beta + \alpha\beta^2 = \frac{10}{9}$

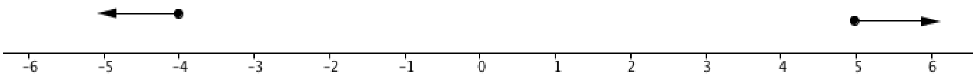
(D) $(\alpha\beta)^2 = \frac{4}{3}$

6. Which of the following best represents the solutions to the inequality $|2x - 1| < 9$

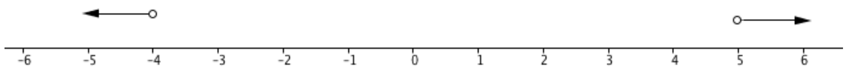
(A)



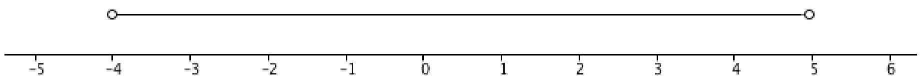
(B)



(C)



(D)



7. State the domain and range of $y = \frac{1}{\sqrt{2x-8}}$.

(A) Domain: $x = 4$ Range: $y < 0$

(B) Domain: $x > 4$ Range: $y > 0$

(C) Domain: $x \neq 0$ Range: $y \neq 0$

(D) Domain: all real x Range: all real y

8. Which equation represents the line perpendicular to $y = 5x + 6$, at the point $(1,11)$.

(A) $y = 5x - y + 6$

(B) $x + 5y - 56 = 0$

(C) $x - 5y + 10 = 0$

(D) $y = -5x - 54$

9. A parabola has focus $S(5,0)$ and directrix $x = 1$.

What is the equation of the parabola?

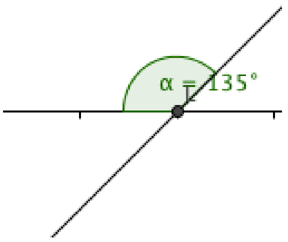
(A) $y^2 = 8(x - 3)$

(B) $y^2 = -8(x - 3)$

(C) $x^2 = 8(y - 3)$

(D) $x^2 = -8(y - 3)$

10. The diagram shows the line l at the point where it crosses the x −axis.



What is the gradient of line l.

(A) $\frac{1}{\sqrt{3}}$

(B) $\sqrt{3}$

(C) $\frac{1}{2}$

(D) 1

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2hours and 45 minutes for this section.

Answer each question in a NEW WRITING BOOKLET. Write your name on each booklet.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW WRITING BOOKLET.

(a) Rationalise the denominator of $\frac{1}{3-\sqrt{5}}$. **2**

(b) Solve $3x^2 + 10x + 4 = 0$. **3**

Give your answer as an exact surd in its simplest form.

(c) Fully factorise $9x^2 - 36$. **2**

(d) Differentiate $2x^2e^x$. **2**

(e) Differentiate $(1 - x^2)\sin x$. **2**

(f) Find the following indefinite integral $\int \frac{x^2}{x^3-2}\,dx$. **2**

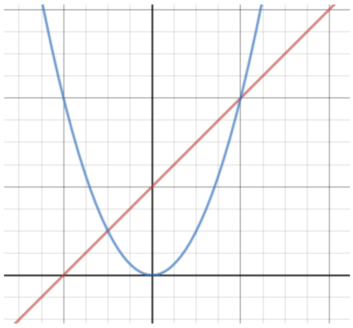
(g) Evaluate $\int_0^{\pi/2} \sin 2x\,dx$. **2**

End of Question11

Question 12 (15 marks) Use a NEW WRITING BOOKLET.

(a) Find the values of x for which the function $y = x - x^2$ is decreasing. **2**

(b) The diagram below shows the graphs of $f(x) = x^2$ and $g(x) = x + 2$

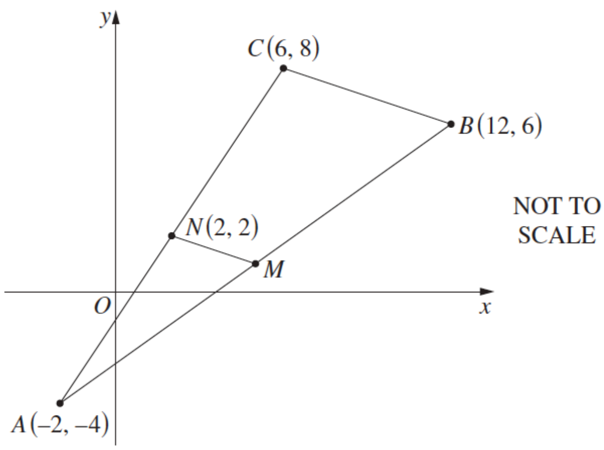


(i) Show that $f(x)$ and $g(x)$ intersect when $x = -1$ and $x = 2$. **1**

(ii) Copy the diagram into your answer booklet. Clearly label the intersections you just found and draw the line $x = -2$. **3**
Find the total area enclosed between the curves $f(x)$ and $g(x)$, from $x = -2$ to $x = 2$.

(c) In the diagram below $A(-2, -4)$, $B(12,6)$ and $C(6,8)$ form a triangle.

The point $N(2,2)$ is the midpoint of AC. The point M is the midpoint of AB.



(i) Find the coordinates of M. **1**

(ii) Find the distance AB. **1**

(iii) Prove that ΔABC is similar to ΔANM. **3**

(d) The first four terms of a sequence are 3, 6, 9, 12.

(i) Show that this is an arithmetic sequence. **1**

(ii) Hence, or otherwise, find the sum of the terms of this sequence with value between 100 and 200. **3**

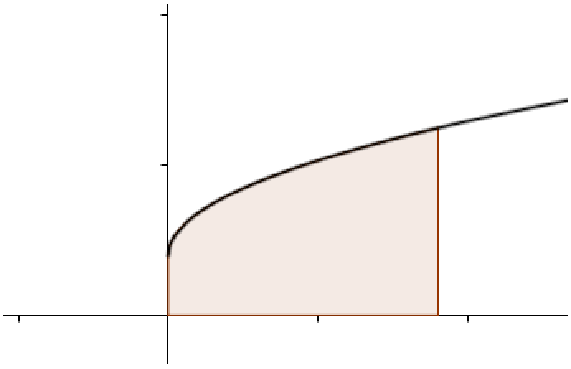
End of Question 12

Question 13 (15 marks) Use a NEW WRITING BOOKLET.

(a) Find all solutions of $2\cos\theta = 1$, for $0 \le \theta \le 2\pi$. **2**

(b) Sketch the graph of $y = \sin\frac{1}{2}x$ in the domain $0 \le x \le 4\pi$. **2**

(c) The region bounded by the curve $y = 2 + \sqrt{2x}$ and the x-axis between $x = 0$ and $x = 9$ is rotated about the x-axis to form a solid. **3**



Find the volume of the solid generated. Give your answer exactly in terms of π.

(d) (i) Solve the equation $\sin x = \cos x$ for $0 \le x \le 2\pi$. **2**

(ii) On the same diagram, sketch the graphs of the curves $y = \cos x$ and $y = \sin x$ for $0 \le x \le 2\pi$. Clearly label all intersections. **2**

(e) Consider the series $\ln 2, \ln 10, \ln 50 \ldots$

(i) Express $\ln 50$ in the form $a\ln 2 + b\ln 5$. **1**

(ii) Show that this is an arithmetic sequence and express the common difference as a logarithm in exact terms. **2**

(iii) Write an expression for the n^{th} term of the sequence. **1**

End of Question 13

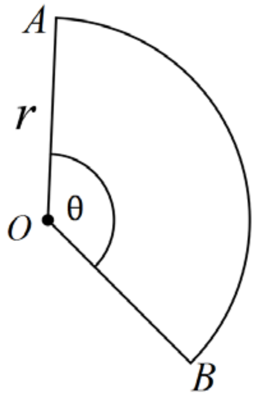
Question 14 (15 marks) Use a NEW WRITING BOOKLET.

(a) (i) Differentiate $3 + \sin 2x$. **1**

(ii) Hence, or otherwise, find $\int \frac{\cos 2x}{3+\sin 2x}\,dx$. **2**

(b) The sector OAB shown below has area $\frac{25\pi}{6}$ square units. Arc AB has length $\frac{5\pi}{3}$ units. **4**

Find the length of the radius, r, of the circle.



(c) The curve $y = e^{3x}$ has gradient 2 at the point P.

Show that the coordinates of P are $\left(\frac{1}{3}\ln\left(\frac{2}{3}\right), \frac{2}{3}\right)$. **3**

(d) A petrol tank is accidentally punctured and the fuel begins flowing out. The volume, V, of petrol left in the tank reduces at a rate of $\frac{dV}{dt} = -1000 + 20t$ where t is measured in minutes since the engine was punctured and the 30,000 litre tank was full.

(i) When does the fuel stop flowing? **1**

(ii) Explain why $\frac{dV}{dt}$ is negative until this time. **1**

(iii) Use integration to find the volume of fuel in the tank at any time, t. **2**

(iv) How much fuel has flowed out of the tank by the time it stops draining? **2**

End of Question 14

Question 15 (15 marks) Use a NEW WRITING BOOKLET.

(a) Given the cubic $y = ax^3 + bx + c$, where a, b and c are positive constants

(i) Show that there is a point of inflection at $x = 0$. **1**

(ii) Show that the graph has no maximum or minimum points. **2**

(b) Use Simpson's Rule with 5 function values to approximate $\int_1^9 (\ln x)^2\,dx$. Answer correct to 3 significant figures. **4**

(c) A cylindrical container closed at both ends is to be made from thin sheet metal. The container is to have a radius r cm and a height of h cm such that its volume is 2000π cm^3. (Such a cylinder closed at both ends has surface area S and volume V given by the formulae $S = 2\pi r^2 + 2\pi rh$ and $V = \pi r^2 h$.)

(i) Find an expression for h in terms of r. **2**

(ii) Show that the total surface area of metal to be used is $\left(2\pi r^2 + \frac{4000\pi}{r}\right)$ cm^3. **2**

(iii) Hence, find the minimum area of sheet metal required to make the container. Answer to 4 significant figures. **4**

End of Question 15

Question 16 (15 marks) Use a NEW WRITING BOOKLET.

(a) The gradient function of a curve is given by $f'(x) = 3x^2 - 4x + 2$. **2**

The curve passes through the point $(1,1)$.

Find the equation of the curve.

(b) Cane toads were first introduced to Australia from Hawaii in 1935 by sugar farmers in an attempt to control cane beetles which were damaging sugar crops. They have since grown out of control and are considered a feral species detrimental to the natural ecology of Australia. Their population is monitored and controlled.

Approximately 100 cane toads were brought to Australia from Hawaii in 1935. The number of toads has grown at a rate proportional to the population, so $\frac{dP}{dt} = kP$, where k is a positive constant and P is the population.

75 years later, in 2010, there were approximately 200 million cane toads in Australia.

(i) Show that $P = 100e^{kt}$ can be used to model the population of cane toads. **1**

(ii) Find the value of k. Answer correct to 2 decimal places. **2**

(iii) How many cane toads are there in 2016? Answer correct to the nearest hundred million. **2**

(iv) Explain what will happen to the rate at which the cane toad population increases over time. **1**

(c) Ellen and Crist start jobs at the beginning of the same year. Ellen's salary is higher than Crist's. Both Ellen's and Crist's employers pay into their superannuation funds at the beginning of each month.

Ellen's employer deposits \$550 per month into her superannuation fund which earns interest at 0.5% per month. Crist's employer deposits \$M per month into her superannuation fund which earns 0.6% per month.

(i) Derive an expression for E_n where E_n represents the *total amount* of Ellen's superannuation after *n* months. Hence show that the amount in Ellen's superannuation fund after *n* months is given by: $E_n = \$110\ 550(1.005^n - 1)$. **3**

(ii) Find the amount of *interest* that Ellen's superannuation earned in the first year. **2**

(iii) Ellen and Crist work for 25 years. Find a formula for, C_n, the total amount in Crist's superannuation after n months. Use this to determine, to the nearest dollar, \$M such that Crist's superannuation matches Ellen's superannuation after 25 years. **2**

End of Question 16

Q11

a) $\dfrac{1}{3-\sqrt{3}} \times \dfrac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \dfrac{3+\sqrt{3}}{9-3}$

$= \dfrac{3+\sqrt{3}}{6}$

b) $3x^2 + 10x + 4 = 0$ $a=3$ $b=10$ $c=4$

$x = \dfrac{-10 \pm \sqrt{100-48}}{6}$

$x = \dfrac{-10+\sqrt{52}}{6}$ OR $x = \dfrac{-10-\sqrt{52}}{6}$

$x = \dfrac{-10+2\sqrt{13}}{6}$

$x = \dfrac{-5+\sqrt{13}}{3}$ OR $x = \dfrac{-5-\sqrt{13}}{3}$

c) $9x^2 - 36 = 9(x^2-4)$
$= 9(x-2)(x+2)$

d) $u = 2x^2$ $v = e^x$
$u' = 4x$ $v' = e^x$

$\frac{d}{dx} = 2x^2e^x + 4xe^x = 2xe^x(x+2)$

e) $\frac{d}{dx}(1-x^2)\sin x$ $\quad u = 1-x^2 \quad v = \sin x$

$u' = -2x \quad v' = \cos x$

$= (1-x^2)\cos x \mp 2x\sin x$

f) $\int \frac{x^2}{x^3-2}\,dx$ $\quad f(x) = x^3-2$

$f'(x) = 3x^2$

$= \frac{1}{3}\int \frac{3x^2}{x^3-2}\,dx$

$= \frac{1}{3}\ln|x^3-2| + C$

g) $\int_0^{\pi/2} \sin 2x\,dx$

$= \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/2}$

$= \left[-\frac{1}{2}\cos\pi - -\frac{1}{2}\cos(0)\right]$

$= \left[\frac{1}{2} + \frac{1}{2}\right]$

$= 1$

Q12

a) decreasing when $y' < 0$

$y' = 1 - 2x$

$1 - 2x < 0$

$2x > 1$

$x > 1/2$

b) i) at intersection, $f(x) = g(x)$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$ OR $x = 2$

ii) Area $= \int_{-2}^{-1} x^2 - (x+2)\,dx + \int_{-1}^{2} x + 2 - x^2\,dx$

$= \int_{-2}^{-1} x^2 - x - 2\,dx + \int_{-1}^{2} x + 2 - x^2\,dx$

$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-2}^{-1} + \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$= 11/6 + 9/2$

Area $= \frac{19}{3}$ units^2

c)

i) $M\left(\frac{-2+12}{2}, \frac{-4+6}{2}\right)$

$\therefore M(5,1)$

ii) $AB = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{(-2-12)^2+(-4-6)^2}$

$= \sqrt{296}$

$= \sqrt{4}\sqrt{74}$

$= 2\sqrt{74}$ units

iii) Consider ΔABC and ΔANM

$\angle NAM = \angle CAB$ (common)

$\frac{AN}{AC} = \frac{1}{2}$ (N is the midpoint of AC)

$\frac{AM}{AB} = \frac{1}{2}$ (M is the midpoint of AB)

$\Rightarrow \Delta ABC \sim \Delta ANM$ (SAS, two corresponding sides have the same ratio and their included angles are equal) d)

i) $T_3 - T_2 = 9 - 6 = 3$

$T_2 - T_1 = 6 - 3 = 3$

$\Rightarrow$ AP with common difference 3.

ii) $T_n = 3 + (n-1)3$

$T_n = 3n$

$T_{34} = 102 \qquad T_{66} = 198$

sum between these values.

$S_{33} = \frac{33}{2}(102 + 198)$

$= 4950.$

Q13

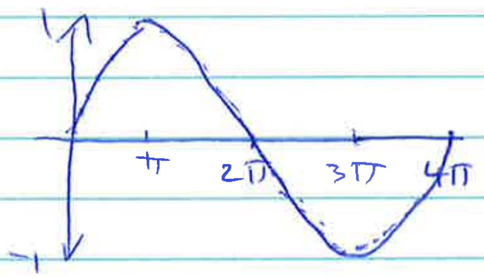
a) $2\cos\theta = 1$ $\qquad 0 \leq \theta \leq 2\pi$

$\cos\theta = 1/2$

$\cos^{-1}(1/2) = \theta$

$\theta = \pi/3$ or $5\pi/3$ using exact ratios.

b)



$\sin \frac{1}{2} x.$

c) $V = \pi \int y^2 \, dx$

$y = 2 + \sqrt{2}\sqrt{x}$

$y^2 = 4 + 2\sqrt{2}\sqrt{x} + 2\sqrt{2}\sqrt{x} + 2x$

$y^2 = 4 + 4\sqrt{2}\sqrt{x} + 2x$

$$V = \pi \int_0^9 4 + 2x + 4\sqrt{2}x^{1/2} \, dx$$

$$= \pi \left[4x + x^2 + \frac{8\sqrt{2}}{3} x^{3/2} \right]_0^9$$

$$= \pi \left[\left(36 + 81 + \frac{8\sqrt{2}}{3} \times 27 \right) - (0) \right]$$

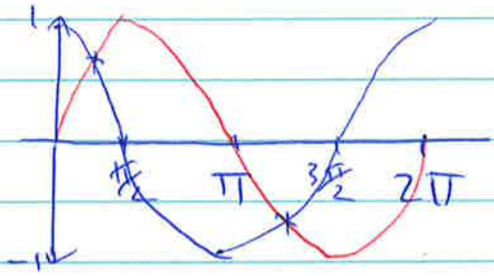
$$= \pi \left[117 + 72\sqrt{2} \right] cm^3$$

d) i) $\sin x = \cos x \qquad 0 \le x \le 2\pi$

$\tan x = 1$ using exact ratios

$x = \pi/4$ OR $5\pi/4$

ii)



$\sin x$

$\cos x$

e) i) $\ln 50 = \ln 2 \times 25$

$= \ln 2 + \ln 25$

$= \ln 2 + 2\ln 5$

ii) $T_2 - T_1 = \ln 2 + \ln 5 - \ln 2 = \underline{\ln 5}$

$T_3 - T_2 = \ln 2 + 2\ln 5 - \ln 2 - \ln 5 = \underline{\ln 5}$

$d = \ln 5 \qquad a = \ln 2$

iii) $T_n = \ln 2 + (n-1)\ln 5$

Q14

a) i) $\frac{d}{dx}\, 3+\sin 2x = 2\cos 2x$

ii) $\int \frac{\cos 2x}{3+\sin 2x}\,dx = \frac{1}{2}\int \frac{2\cos 2x}{3+\sin 2x}\,dx = \ln|3+\sin 2x| + C$

b) $\text{Area} = \frac{25}{6}\pi u^2 \quad A = \frac{1}{2}r^2\theta \quad \Rightarrow \frac{1}{2}r^2\theta = \frac{25}{6}\pi$

$\text{Arc length} = \frac{5}{3}\pi u \quad l = r\theta \quad \Rightarrow r\theta = \frac{5}{3}\pi$

So $\frac{1}{2}r\left(\frac{5}{3}\pi\right) = \frac{25}{6}\pi$

$\frac{5r\cancel{\pi}}{\cancel{6}} = \frac{25}{\cancel{6}}\cancel{\pi}$

$r = 5$

c) $y = e^{3x}$

$y' = 3e^{3x}$

$2 = 3e^{3x}$

$\frac{2}{3} = e^{3x}$ so $y = e^{3\times\frac{1}{3}\ln\frac{2}{3}}$

$\ln\frac{2}{3} = 3x$ $\quad y = \frac{2}{3}$

$\frac{1}{3}\ln\frac{2}{3} = x$ $\quad P\left(\frac{1}{3}\ln\frac{2}{3}, \frac{2}{3}\right)$

d) Stops wh

i) $\frac{dV}{dt}=0$

$0=-1000+20t$

$1000=20t$

$50=t$ after 50 minutes.

ii) The volume of water is decreasing over time as water leaves the tank.

iii) $\frac{dV}{dt}=-1000+20t$

$V=-1000t+10t^2+C$

at $t=0$, $V=30,000$

$30,000=C$

$V=-1000t+10t^2+30,000$.

iv) at $t=50$

$V=-1000\times 50+10\times 50\times 50+30,000$

$V=5,000$

Hence $30,000-5,000=25,000$ l left the tank.

Q15

a) i) $y' = 3ax^2 + b$

$y'' = 6ax$

at inflection

$0 = 6ax$

$\underline{0 = x}$

ii) at max/min

$0 = 3ax^2 + b$

$-\frac{b}{3a} = x^2$

$\frac{b}{3a}$ is +ve, $-\frac{b}{3a}$ is negative, cannot square root a negative.

b)

x	1	3	5	7	9
$f(x)$	0	$(\ln 3)^2$	$(\ln 5)^2$	$(\ln 7)^2$	$(\ln 9)^2$

$\approx \frac{2}{3}\left\{0 + 4\times(\ln 3)^2 + 2\times(\ln 5)^2 + 4\times(\ln 7)^2 + (\ln 9)^2\right\}$

≈ 19.988

≈ 20 3.S.F. ~~$\cancel{\#}$~~ OR multiple applications

c) i) $2000\pi = \pi r^2 h$

$$\frac{2000\pi}{\pi r^2} = h$$

$$\frac{2000}{r^2} = h$$

ii) $S = 2\pi r^2 + 2\pi r \times \frac{2000\pi}{\pi r^2}$

$$S = 2\pi r^2 + \frac{4000\pi}{r}$$

iii) $S' = 4\pi r - \frac{4000\pi}{r^2}$

at min

$$0 = 4\pi\left(r - \frac{1000}{r^2}\right)$$

$$0 = r - 1000r^{-2}$$

$$\frac{1000}{r^2} = r$$

$$1000 = r^3$$

$$10 = r$$

at $r = 10$

$$S = 2\pi \times 100 + \frac{400\cancel{0}}{1\cancel{0}}\pi$$

$$S = 200\pi + 400\pi$$

$$S = 600\pi$$

$$S = 1880\,cm^2$$

Q16

a) $f'(x) = 3x^2 - 4x + 2$

$f(x) = x^3 - 2x^2 + 2x + c$

at $(1,1)$

$1 = 1^3 - 2(1)^2 + 2(1) + c$

$1 = 1 - 2 + 2 + c$

$0 = c$

$f(x) = x^3 - 2x^2 + 2x$

b) i) $P = 100e^{kt}$

$\frac{dP}{dt} = k100e^{kt}$

$\frac{dP}{dt} = kP$ as required

ii) at $t = 75$

$200\text{million} = 100e^{75k}$

$2\text{million} = e^{75k}$

$\ln 2\text{million} = 75k$

$k = 0.19$

iii) in 2016 $k = 81$

$P = 100e^{75 \times 81}$

$P \approx 500,000,000$

iv) The rate of growth of the population will continue to increase.

c) i) $E_1 = 550(1\cdot005)$

$E_2 = 550(1\cdot005)^2 + 550(1\cdot005)$

$\vdots$

Geometric series with $r = 1\cdot005$ and $a = 550(1\cdot005)$

$$E_n = \frac{550(1\cdot005)(1\cdot005^n - 1)}{0\cdot005}$$

$$E_n = 110550\,(1\cdot005^n - 1)$$

ii) Paid in 6600. But fund is worth

$E_{12} = 110550(1\cdot005^{12} - 1)$

$= \$6818\cdot48$

Interest earned $= \$218\cdot48$

iii) $C_n = \dfrac{1\cdot006M}{0\cdot006}(1\cdot006^n - 1)$ 25 years $= 12 \times 25 = 300$ months

$E = 383052\cdot41$

$$383052\cdot41 = \frac{1\cdot006M}{0\cdot006}(1\cdot006^{300} - 1)$$

$M = \$455\cdot36.$