Manly Trial 2U 2001

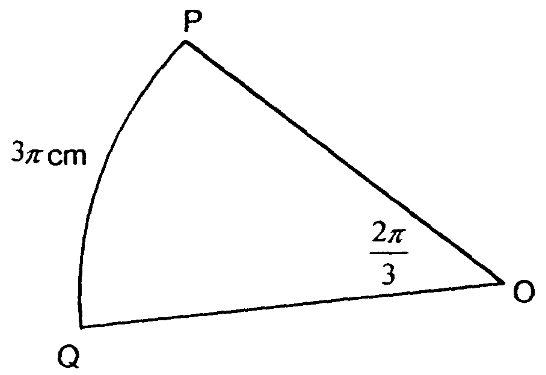
Question 1 **Marks**

(a) Evaluate correct to 3 significant figures : $\frac{\pi}{e^3}$. 2

(b) Solve $7-3x<5$ and graph your solution set on a number line. 2

(c) Express $\frac{1}{\sqrt{3}-2}-\frac{1}{\sqrt{3}+2}$ in its simplest form. 3

(d) 2



Not to Scale

In the diagram above, PQ is an arc of a circle, centre O. The length of the arc is 3π centimetres and angle POQ is $\frac{2\pi}{3}$ radians.
Find the radius of the circle.

(e) The length of the line joining the points A $(a,-2)$ and B $(3,-7)$ is $5\sqrt{2}$ units 3
Find all possible values of a.

Question 2

(a) Differentiate:

(i) $(2x^3-5)^7$ 2

(ii) x^2e^{3x} 2

(iii) $\frac{\sin x}{x}$. 2

Q 2(cont)

(b) On the same number plane draw the graphs of: **3**

$$y = \sqrt{4 - x^2} \text{ and } y = |x|$$

Shade in on your graph, the region where $y \le \sqrt{4 - x^2}$ and $y \ge |x|$ hold simultaneously

(c) The curve $y = ax^3 + bx$ passes through the point $(1,7)$. The tangent at this point is parallel to the line $y = 2x - 6$. **3**
Find the values of a and b.

Question 3

(a) Find primitives of:

(i) $\sec^2 7x$ **1**

(ii) $\dfrac{x^2}{x^3 + 3}$ **2**

(b) (i) On the same diagram draw graphs of the functions $y = x^2$ and $y = 5 - 4x$ showing all intercepts with the x and y- axes. **2**

(ii) Show that the graphs intersect at $x = 1$ and $x = -5$. **1**

(iii) Hence find the exact area bounded by the two functions. **2**

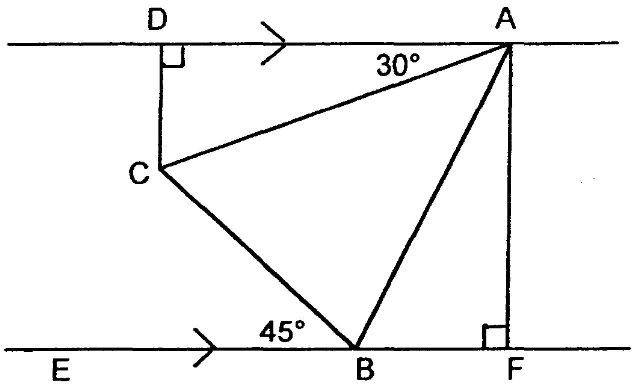
(c) Evaluate :

(i) $\displaystyle\int_{-2}^{-1} \left(\frac{1}{x^2}\right) dx$ **2**

(ii) $\displaystyle\int_0^{\frac{\pi}{3}} \cos(2x + \pi)\, dx$ **2**

Question 4 *Start a new page* **Marks**

(a)



Not to Scale

In the diagram above AC = AB and DA is parallel to EF. Angle DAC = 30° and angle CBE = 45°. CD is perpendicular to DA and AF is perpendicular to EF.

Copy or trace the diagram onto your working paper.

(i) Find the size of angle ACB, giving reasons. **2**

(ii) Hence find the size of angle CAB. **1**

(iii) Prove that $\Delta ACD \equiv \Delta ABF$. **2**

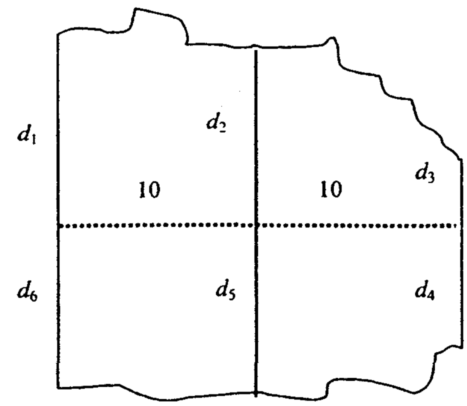
(b) A set of packing crates has been designed each in the shape of a rectangular prism. When empty, each crate packs inside the next sized crate. The largest crate is 2 metres long by 2metres wide by 1 metre high. The crate inside this one is 1 metre by 1 metre by 0.5 metres. Each succeeding crate has dimensions which are half those of the preceding one.

(i) Write down the dimensions of the third largest crate. **1**

(ii) Calculate the maximum possible total volume for the complete set. **2**

Question 4 **Marks**

(c) The diagram shows the face of a 20m wide vertical cliff. The distances d_1 - d_6 are given in the table.



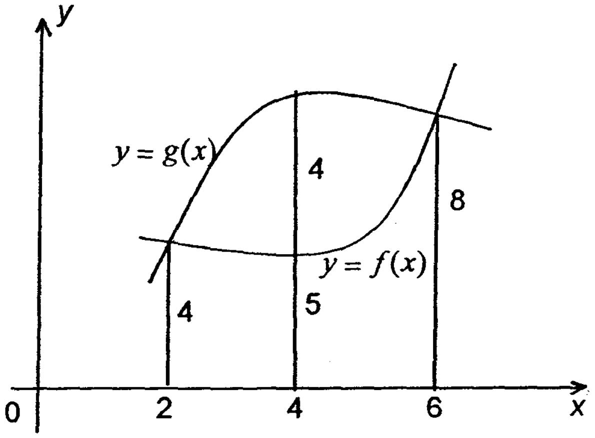
d_1	d_2	d_3	d_4	d_5	d_6
15	14	5.4	8.8	15	14.4

(i) Find an estimate for the area of the cliff face using the trapezoidal rule. Give your answer correct to the nearest square metre. **2**

(ii) Is the estimate greater than or less than the actual area of the cliff? Justify your answer. **2**

Question 5 *Start a new page* **Marks**

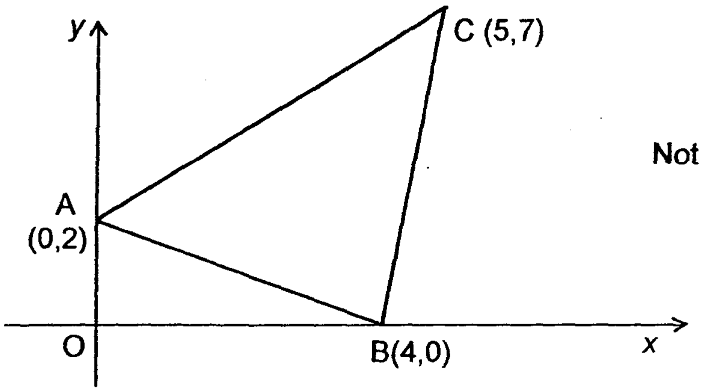
(a) 3



Not to Scale

In the diagram above, the graphs of the functions $y = f(x)$ and $y = g(x)$ are shown. Using Simpson's Rule, find an approximate value for the area enclosed by the two curves.

(b)



Not to Scale

The diagram shows the points A (0,2), B (4,0) and C (5,7).
Copy the diagram onto your worksheet.

(i) Find the coordinates of M, the midpoint of AB. 1

(ii) Show that the gradient of AB is $-\frac{1}{2}$ 1

(iii) Find the equation of the perpendicular bisector of AB. 2

(iv) Show that the perpendicular bisector of AB passes through C. 1

(v) What type of triangle is ABC? (Give a reason for your answer) 1

(c) Solve : $2^{2x} - 15(2^x) - 16 = 0$ 3

Question 6 *Start a new page* **Marks**

(a) Consider the curve $y = x^3 + 4x^2 - 3x$.

(i) Show that the gradient of the curve at $x = 1$ is 8. 1

(ii) Hence find the equation of the normal at $x = 1$ 2

(b) Janice lives in Springwood and is starting a new job in Parramatta. She needs to catch the train to get to work. Her new boss says that she cannot be late on the first two days of her new job or she will lose it.
The probability that her train arrives on time is 0.95.

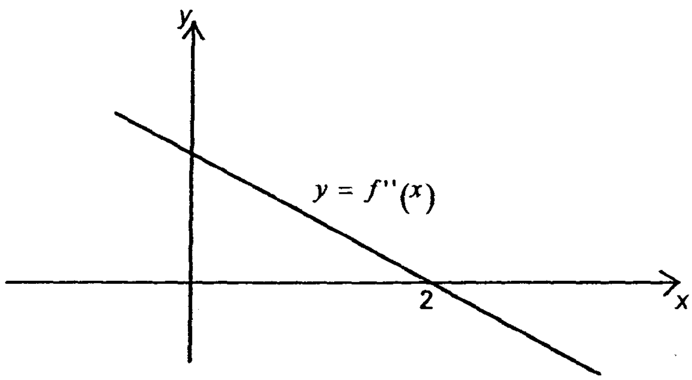
(i) What is the probability that Janice's train is late on the first day? 1

(ii) What is the probability of the train being late on the first two days? 1

(iii) What is the probability of Janice keeping her job? 1

(iv) What is the probability that Janice arrives late on exactly one of the first three days of her new job? 2

(c) 2



The diagram above shows the graph of the $y = f''(x)$, the second derivative of the function $y = f(x)$. Given that $f(2) = 0$ and $f'(1) = 0$, draw a possible sketch of the function $y = f(x)$.

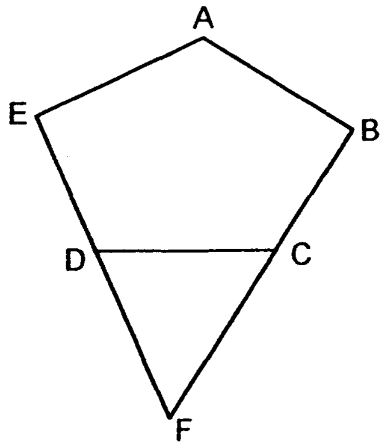
(d) Consider the sequence : $a,\ 3a-1,\ 5a-2, \ldots$

(i) Find the twentieth term 1

(ii) Find the sum of the first twenty terms. 1

Question 7 *Start a new page* **Marks**

(a)



Not to Scale

ABCDE is a regular pentagon. BC and ED are produced to meet at F.

Copy or trace the diagram onto your working paper.

(i) Show that the size of each internal angle in the pentagon is $108°$ **1**

(ii) Show that triangle FCD is isosceles. **1**

(iii) Prove that triangle FCD is similar to triangle FBE. **2**

(iv) If the sides of the pentagon are each 5 centimetres and BE = 8 centimetres, determine the length of CF.. **2**

(b) For the curve represented by the equation $y = x^3 + 3x^2 - 1$

(i) Find $\frac{dy}{dx}$. **1**

(ii) Find all stationary points and determine their nature. **3**

(iii) Sketch the curve in the domain $-3 \le x \le 2$, showing the above information. **2**

Question 8 *Start a new page* **Marks**

(a) A particle is moving along the x-axis. The distance of the particle, x metres, from the origin O is given by the equation $x = 6t + e^{-4t}$, where t is the time in seconds.

(i) What is the position of the particle when $t = \frac{1}{2}$? **1**

(ii) Write down an expression for the velocity of the particle and find the initial velocity. **2**

(iii) Show that the initial acceleration of the particle is 16 cm/sec^2. **2**

(iv) Explain why the particle will never come to rest. **1**

(b) Given that $\cos\alpha = -\frac{5}{\sqrt{29}}$ and $\tan\alpha < 0$, find the value of cosec α **2**

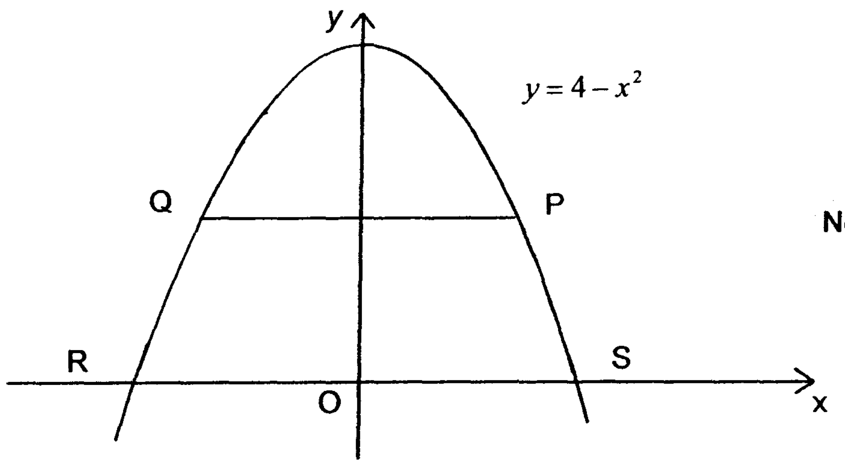
(c) (i) Sketch the graph of $y = \cos 2x$, for $0 \le x \le \pi$ **1**

(iii) Find all values of x for $0 \le x \le \pi$, such that $2\cos 2x = 1$ **2**

(iii) By drawing a straight line on your graph, illustrate these solutions. **1**

Question 9 *Start a new page* **Marks**

(a)



Not to Scale

The parabola $y = 4 - x^2$ cuts the x-axis at R and S. The point P (x,y) lies on the parabola in the first quadrant. Q also lies on the parabola such that PQ is parallel to the x-axis.

(i) Write down the coordinates of R and S. **1**

(ii) Show that the area of trapezium PQRS is given by : **2**

$$A = (2 + x)(4 - x^2)$$

(iii) Hence find the value of x which gives a maximum value of A, justifying your answer **3**

(b) The size of the population, P, of a colony of whiteants after t days is given by the equation $P = 3000e^{kt}$

(i) What was the initial size of the colony? **1**

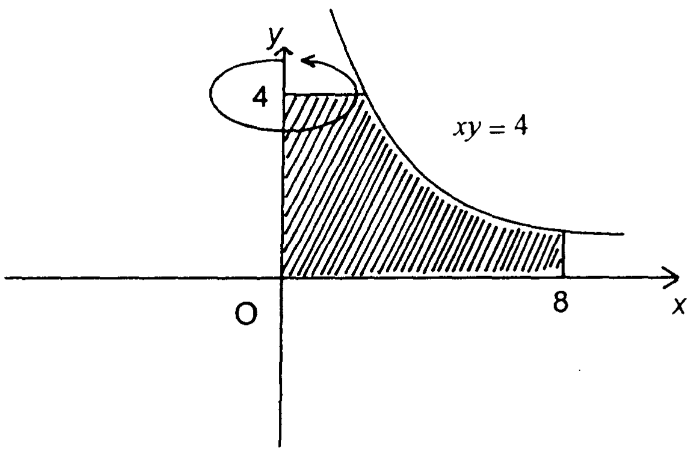
(ii) If there are 4000 whiteants in the colony after 1 day, find the value of k correct to 2 decimal places. **2**

(iii) What is the size of the colony after 2 days? **2**

(iv) When will the colony quadruple in size? (Answer to the nearest day) **1**

Question 10 *Start a new page* **Marks**

(a)



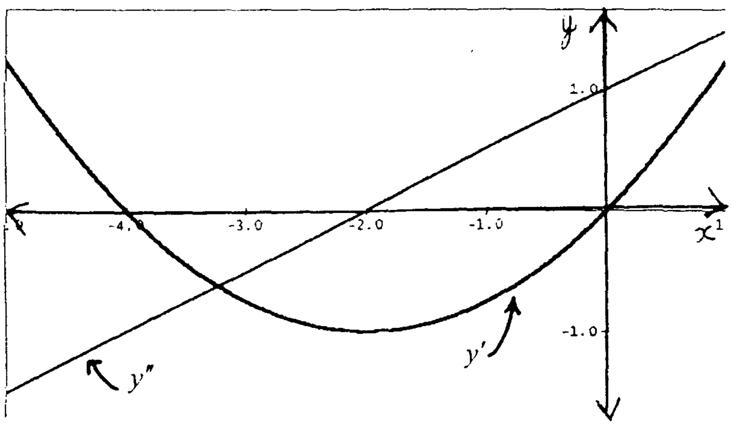
The area enclosed by the curve $xy = 4$, the x and y-axes and the lines $y = 4$ and $x = 8$, is rotated about the y-axis.

(i) Show that the volume of the solid of revolution obtained is given by; **2**

$$V_y = 32\pi + \pi\int_{\frac{1}{2}}^{4} \frac{16}{y^2}\,dy$$

(ii) Hence find the volume of the solid. **2**

(b) The graph shows y' and y'' for a function $y = f(x)$. **3**



Sketch the graph of $y = f(x)$ clearly showing the x values of any turning points and points of inflexion.

Question 10 (cont'd) **Marks**

(c) Michael has decided to invest in a superannuation fund. He calculates that he will need \$1 000 000 if he is to retire in 20 years time and maintain his present lifestyle. The superannuation fund pays 12% per annum interest on his investments.

(i) Michael invests \$P at the beginning of each year. Show that at the end of the first year his investment is worth \$P(1.12). 1

(ii) Show that at the end of the third year the value of his investment is given by the expression $\$P(1.12)(1.12^2 + 1.12 + 1)$. 2

(iii) Find a similar expression for the value of his investment after 20 years and hence calculate the value of P needed to realise the total of \$1 000 000 required for his retirement. 3

End of Paper