



2002
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (120)

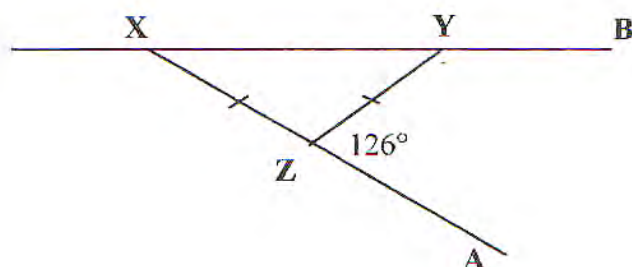
- Attempt Questions 1-10
- All questions are of equal value

QUESTION 1

- a) Find the exact value of $\log_2 128$ (1)
- b) Expand and simplify $5a^2 + 7ab - a(2a + b)$ (1)
- c) Factorise $49x^2 - y^2$ (1)
- d) Evaluate e^4 correct to 3 significant figures. (1)
- e) Find the exact value of $\tan \frac{5\pi}{6}$ (2)
- f) i) Express $4\sqrt{27} - \sqrt{243}$ in simplest surd form. (2)
- ii) Determine the value of x given $4\sqrt{27} - \sqrt{243} = \sqrt{x}$ (1)
- g) Solve $|3x - 2| \geq 7$ and graph the solution on a number line. (3)

QUESTION 2*Start a new page*

- a) Differentiate:
- i) $x^5 - 2x^3$ (1)
- ii) $(x + 5)^4$ (1)
- iii) $3 \sin \frac{2x}{5}$ (2)
- iv) xe^{-x} (2)
- b) For the parabola $(x - 5)^2 = -8(y - 2)$ determine:
- i) the focal length (1)
- ii) the co-ordinates of the vertex (1)
- iii) the co-ordinates of the focus (1)
- iv) the equation of the directrix. (1)
- c) In the figure below triangle XYZ is isosceles and $\angle AZY$ is 126° . Determine the size of $\angle ZYB$ giving reasons. (2)



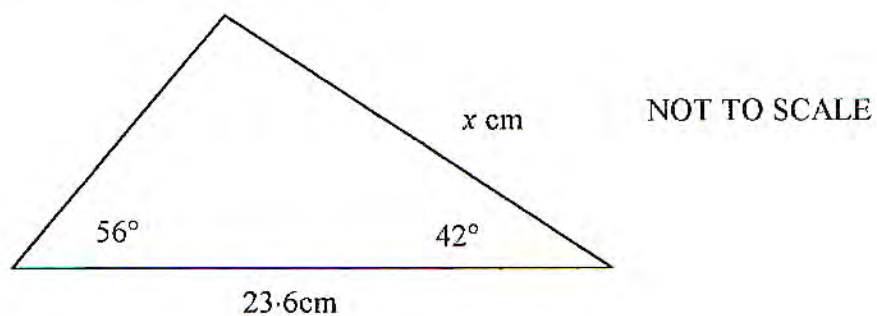
QUESTION 3 *Start a new page*

A (2,5), B (-2, -1) and C (1, -5) are the vertices of triangle ABC.

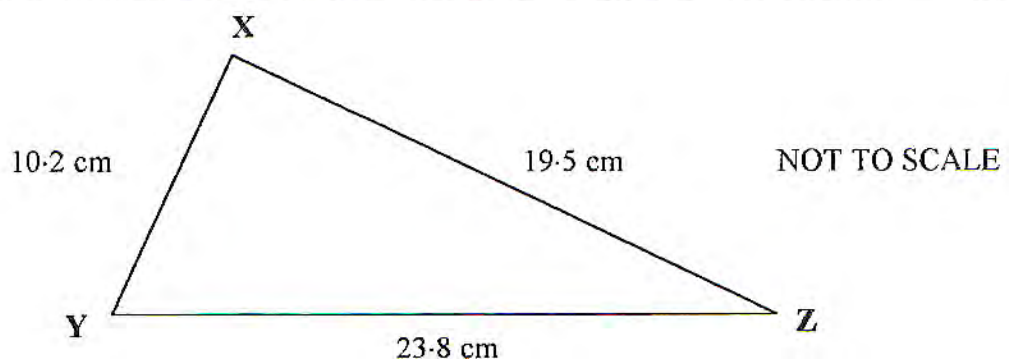
- a) Show this information clearly on a diagram (1)
- b) Determine the exact length of AB in simplest form. (2)
- c) Show the gradient of AB is $\frac{3}{2}$. (1)
- d) Find the equation of AB in general form. (2)
- e) Calculate the exact distance from C to side AB. (2)
- f) Find the exact area of triangle ABC. (2)
- g) Determine the co-ordinates of D such that ABCD is a parallelogram. (1)
- h) Calculate the angle (nearest minute) that side AB is inclined to the positive direction of the x -axis. (1)

QUESTION 4 *Start a new page*

- a) Find the value of x correct to 3 significant figures. (3)



- b) Without solving, state the largest angle in triangle XYZ, justifying your answer. (1)



- c) Consider the arithmetic series $9 + 13 + 17 + \dots$
- i) Determine the first term and the common difference. (1)
 - ii) Find the sum of the first 20 terms. (1)
 - iii) Which term is the first term greater than 1000 and what is its value? (2)
- d) 8 black and 5 yellow discs are placed in a bag and 12 black and 10 yellow discs in a second bag. A disc is drawn from each bag. What is the probability of:
- i) a black disc being drawn from the first bag? (1)
 - ii) a yellow disc being drawn from the second bag? (1)
 - iii) a black and a yellow disc being drawn? (2)

QUESTION 5 *Start a new page*

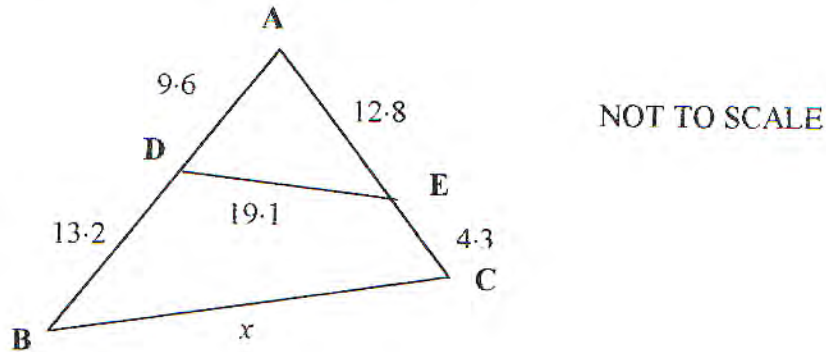
- a) Find a primitive of:
- i) $3x^4 - e^{-2x}$ (2)
 - ii) $\frac{6x}{x^2 - 1}$ (2)
 - iii) $\sqrt{5x + 2}$ (2)
- b) Find the equation of the normal to the curve $y = e^{2x}$ at the point where $x = 0$ in general form. (3)
- c) Solve in the domain, $0 \leq \alpha \leq 2\pi$,
- $$2 \cos^2 \alpha + 3 \cos \alpha + 1 = 0 \quad (3)$$

QUESTION 6 *Start a new page*

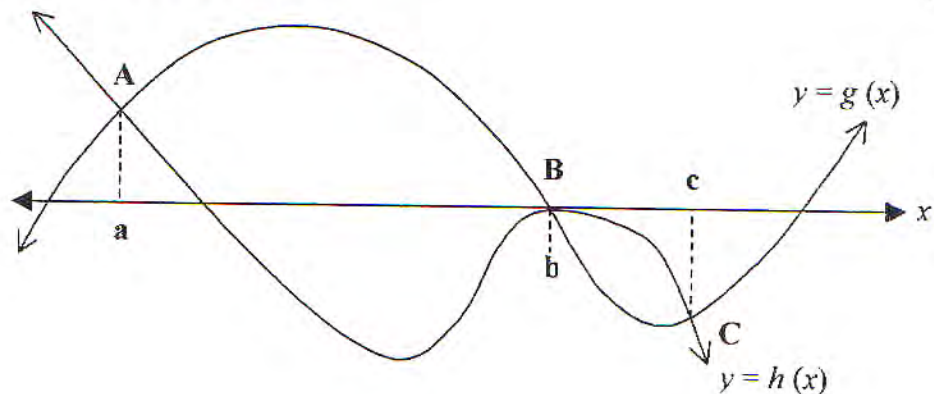
- a) A function is defined by the rule $f(x) = x^3 - 3x^2 - 9x + 6$.
- i) Find the co-ordinates of any stationary points and determine their nature. (3)
 - ii) Find any point(s) of inflexion. (2)
 - iii) Sketch the curve in the domain $-3 \leq x \leq 5$ showing clearly important features. (2)
- b) i) Sketch $y = 3 \cos 2x$ in the domain $0 \leq x \leq \pi$. (1)
- ii) Without solving explain why $\int_0^\pi 3 \cos 2x \, dx = 0$. (1)
 - iii) Determine the area bounded by the curve $y = 3 \cos 2x$, the x -axis and values from $x = 0$ to $x = \pi$. (3)

QUESTION 7 Start a new page

- a) Prove triangle ADE and triangle ACB are similar and determine the value of x (correct to 1 decimal place). Units are mm. (2)

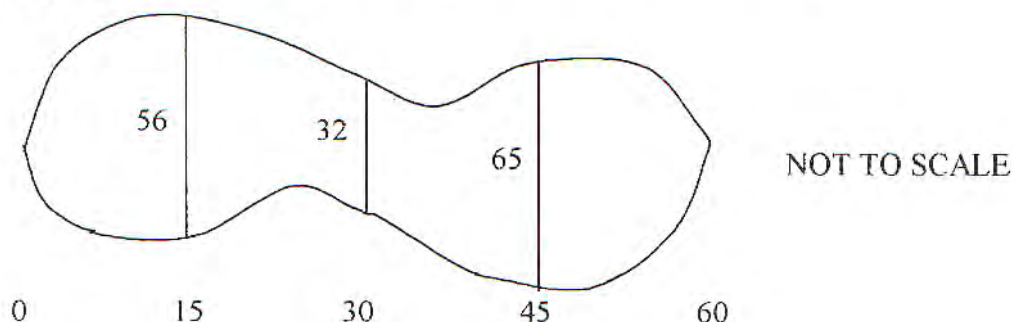


- b) The volume of water in a tank is given by $V = 200 - 2t + \frac{t^2}{200}$ litres, where t is in minutes.
- i) How much water is in the tank after 15 minutes? (1)
 - ii) At what rate is water flowing out of the tank after 25 minutes? (2)
 - iii) How long will it take to empty the tank? (1)
- c) A ball drops from a height of 20 metres and bounces with each successive bounce only $\frac{4}{5}$ the height of the previous bounce.
- i) How far will the ball rebound after it has hit the ground for the 3rd time? (1)
 - ii) Through what distance will the ball eventually travel? (3)
- d) The diagram below shows 2 curves $y = g(x)$ and $y = h(x)$ which intersect at points A, B and C. Write down an expression which could be used to evaluate the area between the 2 curves. (2)



QUESTION 8 *Start a new page*

- a) The diagram shows the widths of a garden bed in a park at 15m intervals. Use Simpson's Rule to estimate the surface area of the garden bed. (3)
Units are metres.



- b) i) Find the derivative of $y = \ln(\ln x)$. (1)
ii) Hence or otherwise evaluate $\int_e^{e^4} \frac{2dx}{x \ln x}$ (2)

- c) A block of ice is melting at a rate proportional to its mass, M kg, left in the block after time t minutes, i.e. $\frac{dM}{dt} = -kM$
After 25 minutes, 15% of the block has melted.
- i) Show that $M = M_0 e^{-kt}$ is a solution of the equation $\frac{dM}{dt} = -kM$ (1)
ii) Determine the value of k . (2)
iii) How long, to the nearest minute, will it take for 70 % of the block of ice to have melted? (2)

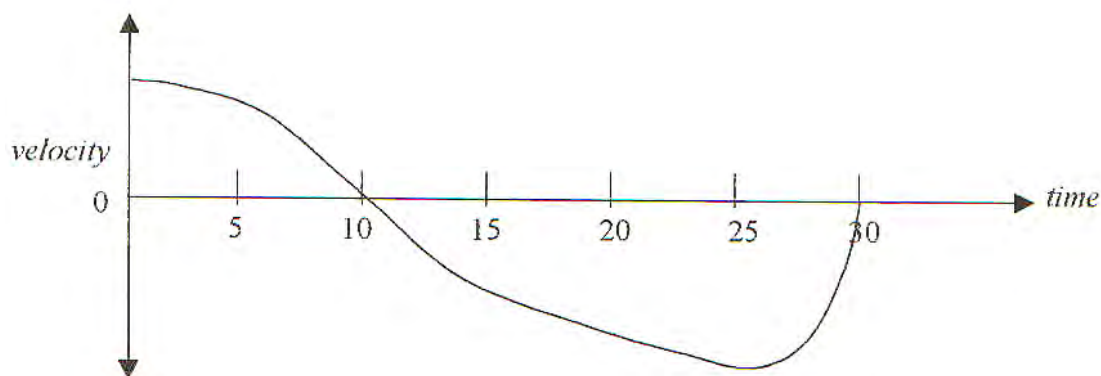
- d) Evaluate $\lim_{x \rightarrow 0} \frac{\sin \frac{3x}{2}}{6x}$ (1)

QUESTION 9 *Start a new page*

- a) The Smith's borrow \$150 000 from Friendly Bank at 9% p.a. charged monthly for 25 years. Friendly Bank has a special deal to help customers, **no repayments are required for the first 9 months.**
- i) Show that the amount owing after 1 month is 150 000 (1.0075) (1)
ii) Show an expression for the amount owing after 10 months assuming the monthly repayment is \$ M . (1)
iii) Derive a similar expression for the amount owing after n months. (1)
iv) Calculate (to nearest cent) the amount of each monthly payment. (2)

b) The diagram shows a velocity-time graph for particle P for its first 30 seconds of motion.

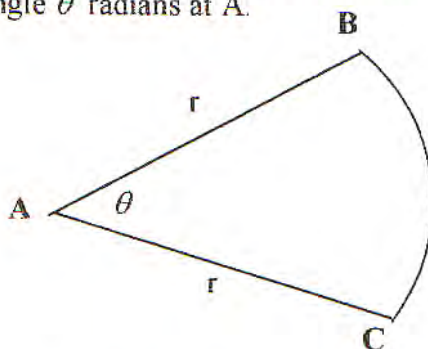
- i) When is P moving forwards? (1)
- ii) When is the acceleration positive? (1)
- iii) When is P furthest from its starting position in a positive direction? (1)
- iv) When is it furthest in the negative direction? (1)
- v) About when does it return to its starting position? (1)
- vi) Assuming the particle starts at the origin, sketch graphs of its displacement and its acceleration. (2)



QUESTION 10 *Start a new page*

- a) i) Shade the region between the curve $y = \log_e x$ and the x -axis from $x = 1$ to $x = 2$. (1)
- ii) This region is rotated about the y -axis. Determine the exact volume. (4)

b) In the figure AB and AC are radii of length r metres of a circle centre A. Arc BC subtends an angle θ radians at A.



- i) If the perimeter of the sector is 8 metres, show that the area of the sector is given by $A = \frac{32\theta}{(\theta + 2)^2}$ (3)
- ii) Hence determine the maximum area of the sector. (4)