Northern Beaches Secondary College
Manly Selective Campus

2005
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board approved calculators and templates may be used
- All necessary working should be shown in every question
- Start each question in a new booklet

Total marks: 120

Attempt Questions 1 – 10

This paper MUST NOT be removed from the examination room

Manly Selective Campus
2005 HSC 2 Unit Trial Examination

Question 1** (Answer this question in a separate booklet)* ***12 Marks

a) Simplify $\frac{2}{x}+\frac{x-2}{5}$. *(2)*

b) Determine the value of $\frac{e^2}{5\log_e 100}$ correct to three significant places. *(2)*

c) Determine values for x such that $|2x-1|=3$. *(2)*

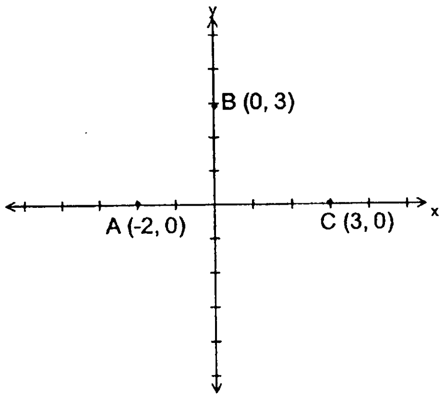
d) Car F has a fuel consumption of 12 litres/100 km and this is 12% better than Car H. What is the fuel consumption of car H? *(2)*

e) Express 15° to radians in terms of π in simplest form. *(2)*

f) Solve for x: $(3x-2)^2 = 16$ *(2)*

Question 2 ***(Answer this question in a separate booklet)*** ***12 Marks***

On a set of co-ordinate axes, A is the point (-2, 0) and B is the point (0,3).



(i) Show that the equation of the line AB is $2y = 3x + 6$. *(2)*

(ii) What is the equation of the line (call it L_2) parallel to AB through the point C (3,0). *(2)*

(iii) What is the size of the angle BAC? *(1)*

(iv) What is the shortest distance from C to the line AB? Give your answer in exact form. *(2)*

(v) Write down the equation of the circle which has centre C and AB as a tangent. *(2)*

(b) The sum of the first four terms of an arithmetic progression is 9. The first term is a and the common difference is d.

(i) Using this information, write an equation for S_4 *(1)*

(ii) The sum of the first 12 terms is 93, find the common difference between successive terms. *(2)*

Question 3** (Answer this question in a separate booklet)* — ***12 Marks

(a) Differentiate the following and simplify where possible:

i) $\cos(4x)$. *(1)*

ii) $3x\,e^x$ *(2)*

iii) $3x(x+5)^4$ *(3)*

(b) For the curve $y = 2\sin 2x$:

(i) Determine the gradient of the curve at $x = \dfrac{\pi}{6}$. *(2)*

(ii) What is the equation of the tangent to the curve at $x = \dfrac{\pi}{6}$. *(2)*

Leave your answers in exact form.

(c) Evaluate $\int_0^1 (x^2 - 1)\,dx$. *(2)*

***Question 4** (Answer this question in a separate booklet)* *12 Marks*

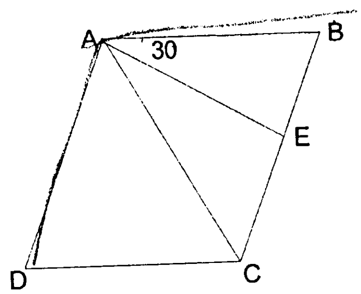
(a) The toll on the new cross-city tunnel is allowed, under the construction contract, to rise by 4% at the beginning of each quarter. At the beginning of the first quarter - on 1 July, 2005 - the toll was set at $3. The tunnel is expected to open on 4 January 2006.

(i) What will the toll be at the time of opening the tunnel (to the nearest cent)? *(1)*

(ii) What will the toll be, under this contract, on 4 January 2010? *(2)*

(b) The diagram shows a rhombus ABCD. The point E lies on BC and $\angle BAE = 30^\circ$. The interval AE bisects $\angle BAC$.

NOTE: Diagram not to scale



(i) Show that $\angle DAB = 120^\circ$. *(2)*

(ii) Find the size of $\angle ADC$ giving appropriate reasons. *(1)*

(iii) Find the size of $\angle AEB$ giving appropriate reasons. *(2)*

(c) The equation $x^2 - 6x + k = 0$ has roots α and β. If $\alpha = 2\beta$:

(i) Find the value of $\alpha + \beta$ *(1)*

(ii) Find the value of k. *(3)*

***Question 5** (Answer this question in a separate booklet)* *12 Marks*

(a) Find the following:

(i) $\int \sec^2(5x+3)\,dx$ *(1)*

(ii) $\int \frac{3\,dx}{(x+3)^2}$ *(2)*

(b) The area between the curve $y=\frac{x^2}{2}-2$ and the x and y axes is rotated about the y axis to form a bowl.

(i) Sketch the curve and indicate the defined area. *(2)*

(ii) Determine the exact volume of the bowl. *(3)*

(c) Sally is standing on the 50 m high cliff at North Curl Curl. She sees a boat out to sea heading towards the base of the cliff below her. She notes the angle of depression to the boat is 20°. Five minutes later, she notes the angle of depression to the boat is 35°.

(i) Draw a diagram with this information in it. *(1)*

(ii) How far away was the boat when Sally first saw it? *(1)*

(iii) How fast (in metres per minute) is the boat travelling towards Sally? *(2)*

Question 6** (Answer this question in a separate booklet)* — ***12 Marks

(a)

i) Show that the curve $y = \ln 2x$ crosses the x axis at $x = \frac{1}{2}$. *(1)*

ii) Draw a fully labeled sketch of $y = \ln 2x$. *(1)*

iii) Use Simpson's rule and three function values to calculate the area between the curve, the x-axis and the line $x = 2\frac{1}{2}$. Express your answer to one decimal place. *(3)*

(b) For the parabola $2y = x^2 + 6x + 11$:

(i) Express the equation in the form $(x-h)^2 = 4a(y-k)$. *(2)*

(ii) Find the focus and directrix of the parabola. *(2)*

(c) Simplify $(\text{cosec}^2 x - 1)(1 + \tan^2 x)$ *(3)*

***Question** 7 (Answer this question in a separate booklet)* *12 Marks*

(a) The pattern for a rectangular closed box is to be cut from one sheet of metal and then folded together. The volume of the box must be 9 m^3. The design requirement is that the length of one side of the base of the box must be twice the length of the other base side. Let the length of one side of the base of the box be x.

(i) Show that the surface area of the box can be expressed as $\frac{27}{x} + 4x^2$. *(3)*

(ii) What is the length of the side x which gives a minimum amount of metal to be cut from a sheet? *(4)*

(b) Two cultures of bacteria are prepared in a laboratory. They are to be used to test the effectiveness of two drugs. One culture has 1000 bacteria and Drug A reduces this number to 250 in 5 minutes. The other culture has 1250 bacteria and Drug B reduces this number to 500 in 3 minutes. Both cultures are reduced according to the model $N = N_0 e^{-kt}$ where N is the number of bacteria and t is the time since the drug was administered in minutes.

(i) Find the value for k of both cultures. *(2)*

(ii) Which drug is more effective in reducing the number of bacteria? State a brief reason to support your answer. *(1)*

(iii) How long will it take for Drug B to reduce the second culture to 10% of its original number of bacteria? *(2)*

Question 8 ***(Answer this question in a separate booklet)*** ***12 Marks***

(a) After the premiere of a new movie, 50% of the large audience claimed to have liked the movie, 30% claimed to be neutral and 20% claimed to have disliked it.

i) If 2 people from the audience are selected at random, what is the probability that one liked the movie and one disliked it? *(2)*

ii) If 3 people from the audience are selected at random, what is the probability that at least one person had disliked the movie? *(2)*

(b) When a metal disc is heated, its area A cm^2 increases at a rate given by

$$\frac{dA}{dt} = t^2 - t + 1$$

where t is the time in seconds.

(i) At what rate is the disc area increasing at the end of the third second? *(1)*

(ii) The original area of the disc was 12 cm^2. What was its area after 2 seconds? *(3)*

(c) (i) Show that $\frac{d}{dx}\left(xe^{\frac{x}{2}}\right) = e^{\frac{x}{2}} + \frac{1}{2}xe^{\frac{x}{2}}$ *(2)*

(ii) Hence find $\int xe^{\frac{x}{2}}dx$ *(2)*

Question 9 *(Answer this question in a separate booklet)* ***12 Marks***

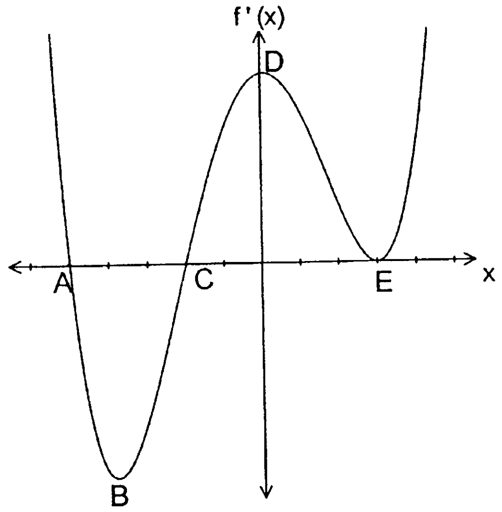
(a) Will was a Year 10 student who completed the School Certificate in 2004. He then made a decision to pay cash for the car of his dreams at the end of a three year university degree program – so in five years time.

Will decided to invest \$100 at the beginning of each month from February 2005 in a regular savings program at a fixed rate of interest of 6% p.a. payable at the end of each month.

(i) Show that by the end of March 2005, Will has $\$100(1.005) + 100(1.005)^2$ in his savings account. *(2)*

(ii) How much can Will expect to have in the Bank at the end of December 2009. *(4)*

(b) The graph of the derivative of a function f(x) is shown below:



Several points are labeled A to E.

(i) Draw a graph of the second derivative of the function f "(x). *(2)*

(ii) Draw a graph of the original function f(x) given f(0) = 0. *(4)*

In both of your graphs, indicate where the points A to E would be located approximately.

Question 10 ***(Answer this question in a separate booklet)*** *12 Marks*

(a)

(i) Show that kx, k, kx^{-1} are the first terms in a geometric series. *(1)*

(ii) For what values of x does this series have a limiting sum. *(2)*

(b) A ball falls from rest in a fluid medium with its acceleration given by

$$\ddot{x} = 10e^{\frac{-1}{3}t}\ cm/\sec^2$$

where x is the distance in centimetres below the origin at time t seconds.

(i) Find the velocity-time function for the motion of the ball. *(2)*

(ii) What is the limiting velocity of the ball. *(1)*

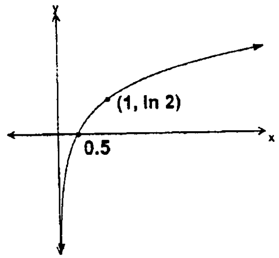
(iii) How far does the ball travel in the first 3 seconds? *(2)*

(iv) A second ball moves with the same acceleration as the first ball but it is initially forced upwards in the fluid with a speed of 10 cm/sec from a point 2 metres below the first ball at the same instant as the first ball is dropped. Do the balls ever collide and, if so, when? *(4)*

Question 1:

a)	$\frac{2}{x}+\frac{x-2}{5}=\frac{5(2)+x(x-2)}{5x}$ $=\frac{10+x^2-2x}{5x}$	**2** correct solution **1** 2 out of 3 correct for num. with correct denominator or correct numerator with incorrect denominator.
b)	$\frac{e^2}{5\log_e 100}=0\cdot 32090.....$ $=0\cdot 321$	**2** correct answer **1** solution not rounded correctly
c)	$\|2x-1\|=3$ $2x-1=3$ or $-(2x-1)=3$ $2x=4$ $2x-1=-3$ $x=2$ $2x=-2$ $x=-1$ $\therefore x=2$ or $x=-1$	**2** correct solutions **1** 1 solution correct or both solutions attempted with arithmetic mistakes in both solutions
d)	Car F 12 litres/100km $\Rightarrow$ Car H 88% of car F 88% = 12 litres/100km 100% = $\frac{12}{88}$ x 100 litres/100km Car H = $13\frac{7}{11}$ *litres/100km* $(13\cdot\dot{6}\dot{3})$	**2** correct solution (with rounding if necessary) **1** $10\frac{5}{7}$ or $10\cdot 7148..$ or $13\frac{11}{25}$ or $13\cdot 44$ *litres/100km*
e)	$15^0=\frac{15\pi}{180}$ $=\frac{\pi}{12}$	**2** correct solution **1** $\frac{15\pi}{180}$ with incorrect simplification
f)	$(3x-2)^2=16$ or $9x^2-12x+4=16$ $3x-2=\pm 4$ $9x^2-12x-12=0$ $3x=6$ or -2 $3x^2-4x-4=0$ $x=2$ or $-\frac{2}{3}$ $(3x+2)(x-2)=0$ $x=-\frac{2}{3}$ or 2	**2** both correct solutions **1** 1 solution correct or to $9x^2-12x-12=0$ if expansion method used.

Question 2:

a)	i)	$y=\log_e 2x$ at x axis $y=0$ $0=\log_e 2x$ or $y=\log_e 2(\frac{1}{2})$ $e^o=2x$ $y=\log_e 1$ $1=2x$ $y=0$ $x=\frac{1}{2}$	**1** a correct method for showing when $x=\frac{1}{2}$, $y=0$
	ii)		**1** a correct sketch showing curve crosses the x axis at $x=\frac{1}{2}$ with another point also labelled.
	iii)	$h=\frac{b-a}{n}\Rightarrow\frac{\frac{5}{2}-\frac{1}{2}}{2}=1$ x: $\frac{1}{2}$ $\frac{3}{2}$ $\frac{5}{2}$ y: $\ln 2\left(\frac{1}{2}\right)$ $\ln 2\left(\frac{3}{2}\right)$ $\ln 2\left(\frac{5}{2}\right)$ 0 $\ln 3$ $\ln 5$ $Area\approx\frac{1}{3}(0+\ln 5+4(\ln 3)\pi)$ $\approx 2\cdot 002129......$ $=2\cdot 0\,units^2$ (1dec.pl.)	**3** correct solution to 1 decimal place. **2** correct substitution into correct formula **1** correct y values <u>or</u> correct formula for this area.
b)	i)	$2y=x^2+6x+11$ $x^2+6x+\left(\frac{6}{2}\right)^2=2y-11+9$ $(x+3)^2=2y-2$ $(x+3)^2=2(y-1)$	**2** correct solution **1** some correct attempt with $(x+3)^2$

	ii)	$4a = 2$ $a = \frac{1}{2}$ $vertex = (-3, 1)$ $focus = (-3, \frac{3}{2})$ $\cot^2 x$ *or* $\sec^2 x$ $directrix: y = \frac{1}{2}$	2 both focus and directrix correct 1 correct focus or directrix with $a = \frac{1}{2}$ or correct focus and directrix for incorrect a
c)		$(\operatorname{cosec}^2 x - 1)(1 + \tan^2 x)$ $= \cot^2 x \times \sec^2 x$ $= \frac{\cos^2 x}{\sin^2 x} \times \frac{1}{\cos^2 x}$ $= \frac{1}{\sin^2 x}$ $= \operatorname{cosec}^2 x$ *or* $= \operatorname{cosec}^2 x + \operatorname{cosec}^2 x \tan^2 x - 1 - \tan^2 x$ $= \frac{1}{\sin^2 x} + \frac{1}{\sin^2 x} \times \frac{\sin^2 x}{\cos^2 x} - 1 - \frac{\sin^2 x}{\cos^2 x}$ $= \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} - 1 - \frac{\sin^2 x}{\cos^2 x}$ (*) $= \frac{\cos^2 x + \sin^2 x - \sin^2 x \cos^2 x - \sin^4 x}{\sin^2 x \cos^2 x}$ $= \frac{1 - \sin^2 x(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x}$ $= \frac{1 - \sin^2 x}{\sin^2 x \cos^2 x}$ (#) $= \frac{\cos^2 x}{\sin^2 x \cos^2 x}$ $= \frac{1}{\sin^2 x}$ $= \operatorname{cosec}^2 x$	3 a correct solution to $\frac{1}{\sin^2 x}$ 2 $\frac{\cos^2 x}{\sin^2 x}$ *or* $\frac{1}{\cos^2 x}$ with $\cot^2 x$ *and* $\sec^2 x$ or to (#) for alternate method. 1 $\cot^2 x$ *or* $\sec^2 x$ or to (*) for alternate method

Question 3:

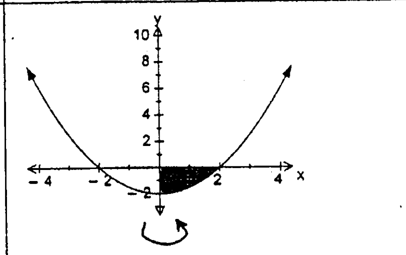
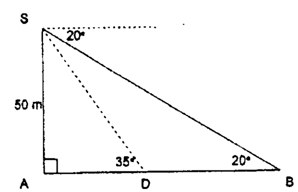
(a)i	$-4\sin 4x$	1- Answer
ii	$vu'+uv' = e^x{}_x3+3x_xe^x$. $= 3e^x + 3xe^x$ or $3e^x(1+x)$	2- Answer 1- Correct use of product rule
iii	$vu'+uv' = (x+5)^4{}_x3 + 3x_x4(x+5)^3$. $= 3(x+5)^4 + 12x(x+5)^3$. $= 3(x+5)^3[(x+5) + 4x]$ $= 3(x+5)^3[5x+5] = 15(x+5)^3[x+1]$	3- Answer fully factored 2- Correct use of product rule with answer in factored form 1- Correct use of product rule
(b)i)	$y' = 4\cos 2x$. At $x = \frac{\pi}{6}$ $y' = 4\cos\frac{\pi}{3}$ $=2$	2- Answer with correct substitution 1- Correct derivative
ii)	$y - y_1 = m(x - x_1)$ At $x = \frac{\pi}{6}$ $y = 2\sin\frac{\pi}{3} = 2\frac{\sqrt{3}}{2} = \sqrt{3}$ $y - \sqrt{3} = 2(x - \frac{\pi}{6})$ $y = 2x + \sqrt{3} - \frac{\pi}{3}$	2- Answer with appropriate working 2-Allow carry over error from i) 1- correct substitution into $y - y_1 = m(x - x_1)$ 1-One error
(c)	$\left[\frac{x^3}{3} - x\right]_0^1 = (\frac{1}{3} - 1) - 0 = \frac{-2}{3}$	2- Answer with correct integral 1- Correct integral 1- +2/3

Comments:
(a) i) Some students still make error with the sign and were again penalized in (b) i)
ii) Product Rule if used was handled well. Students need to practise factoring.
iii) Common error was $12x(x+5)^3$.
(b) i) Errors, if any, occurred at all stage, differentiating or substituting.
ii) Finding the y exact value presented some problems
(c) i) Best answered question in whole question but a few absolute valued the correct negative answer.

Question 4:

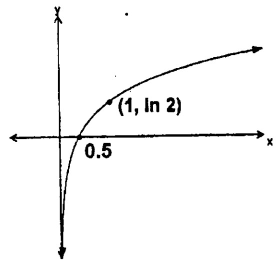
a i)	$\$3\times(1\cdot04)^2=\$3\cdot2448=\$3\cdot24$ (nearest cent)	**1 mark** for $\$3\times(1\cdot04)^2$
a ii)	Jan 2010- Jan 2006=4 years=16 quarters plus 2 quarters from July 2005 to Jan 2006 equals 18 quarters. $\$3\times(1\cdot04)^{18}=\$6\cdot077449546=\$6\cdot08$ (nearest cent)	**2 marks** for correct answer **1 mark** for rounding off error or for correct calculation with incorrect number of increments
b i)	$\angle CAE=\angle BAE=30^\circ$ (Given as bisected angles) $\angle BAC=\angle CAE+\angle BAE=30^\circ+30^\circ=60^\circ$ (Adjacent angles) $\angle DAC=60^\circ$ (The internal angles of a rhombus are bisected by its diagonals.) $\angle DAB=\angle DAC+\angle BAC=60^\circ+60^\circ=120^\circ$	**2 marks** for logical steps with correct reasons leading to correct answer. **1 mark** for proof containing illogical steps with correct reasons **or** for proof containing logical steps but incorrect reasons
b ii)	AB//DC (Opposite sides of a rhombus are parallel.) $\angle ADC=180^\circ-120^\circ=60^\circ$ (Cointerior angles in parallel lines are supplementary)	**1 mark** for correctly explained answer
b iii)	$\angle ABC=(\angle ABE)=60^\circ$ (Opposite angles of a rhombus are equal) In ΔABE, $\angle AEB=180^\circ-(\angle ABE+\angle BAE)$ (Angle sum of a triangle) $=180^\circ-(30^\circ+60^\circ)=90^\circ$	**2 marks** for logical steps with correct reasons leading to correct answer. **1 mark** for proof containing illogical steps with correct reasons or for proof containing logical steps but incorrect reasons
c i)	$\alpha+\beta=\frac{-b}{a}=6$	**1 mark** for correct answer
c ii)	Using $\alpha=2\beta$ in i) we get $2\beta+\beta=6$ $3\beta=6$ $\beta=2$ $\alpha=4$ but $\alpha\beta=\frac{c}{a}=k$ $4\times2=k$ $k=8$	**3 marks** for correct value of k. **2 marks** finding alpha and beta but not k **1 mark** for finding either alpha or beta, or for correctly stating product of roots is k

Question 5:

a i)	$\int\sec^2(5x+3)\,dx=\frac{1}{5}\tan(5x+3)+c$	**1 mark** for $\frac{1}{5}\tan(5x+3)$
a ii)	$\int\frac{3\,dx}{(x+3)^2}=3\int(x+3)^{-2}\,dx$ $=3\frac{(x+3)^{-1}}{-1\times1}+c$ $=\frac{-3}{x+3}+c$	**2 marks** for correct answer **1 mark** for neglecting to write constant of integration or for stating but incorrectly applying the rule $\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{(n+1)\times a}+c$
b i)		**2 marks** for correctly graphed parabola with x- intercepts and correctly shaded area. **1 mark** for incorrect graph of function but correct area shaded or correct graph of function with incorrect area shaded.
b ii)	$V=\pi\int_{-2}^{0}x^2\,dy$ $=\pi\int_{-2}^{0}2(2+y)\,dy$ $=2\pi\int_{-2}^{0}2+y\ dy$ $=2\pi\left[2y+\frac{y^2}{2}\right]_{-2}^{0}$ $=2\pi\left[0+0-(-4+2)\right]$ $=4\pi\ units^3$	**3 marks** for correct exact value. **2 marks** for correctly integrating to attain $2\pi\left[2y+\frac{y^2}{2}\right]_{-2}^{0}$ **1 mark** for correct volume formula $2\pi\int_{-2}^{0}2+y\ dy$
c i)		**1 mark** for correct diagram showing right angle, 20 degrees, 35 degrees and 50m
c ii)	$\tan20^\circ=\frac{50}{AB}$ $AB=\frac{50}{\tan20^\circ}=137.373871$ $=137m$ (nearest metre)	**1 mark** for correct answer

c iii)	$\tan 35^\circ = \frac{50}{AD}$ $AD = \frac{50}{\tan 35^\circ}$ $BD = AB - AD$ $= \frac{50}{\tan 20} - \frac{50}{\tan 35}$ $= 65.96647064$ $\text{speed} = \frac{\text{distance}}{\text{time}}$ $= \frac{BD}{5} = 13.19329413 = 13 \text{ metres/ min}$ (to nearest whole number)	**2 marks** for correct answer **1 mark** for correctly calculating distance BD or for incorrect BD but correct application of speed formula

Question 6:

d)	i)	$y = \log_e 2x$ *at x axis* $y = 0$ $0 = \log_e 2x$ *or* $y = \log_e 2(\frac{1}{2})$ $e^0 = 2x$ $y = \log_e 1$ $1 = 2x$ $y = 0$ $x = \frac{1}{2}$	**1** a correct method for showing when $x = \frac{1}{2}, y = 0$
	ii)		**1** a correct sketch showing curve crosses the x axis at $x = \frac{1}{2}$ with another point also labelled.
	iii)	$h = \frac{b-a}{n} \Rightarrow \frac{\frac{5}{2} - \frac{1}{2}}{2} = 1$ x: $\frac{1}{2}$ $\frac{3}{2}$ $\frac{5}{2}$ y: $\ln 2\left(\frac{1}{2}\right)$ $\ln 2\left(\frac{3}{2}\right)$ $\ln 2\left(\frac{5}{2}\right)$ 0 $\ln 3$ $\ln 5$ $Area \approx \frac{1}{3}(0 + \ln 5 + 4(\ln 3)\pi)$ $\approx 2\cdot 002129......$ $= 2\cdot 0 units^2$ (1dec.pl.)	**3** correct solution to 1 decimal place. **2** correct substitution into correct formula **1** correct y values **or** correct formula for this area.
e)	i)	$2y = x^2 + 6x + 11$ $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 2y - 11 + 9$ $(x+3)^2 = 2y - 2$ $(x+3)^2 = 2(y-1)$	**2** correct solution **1** some correct attempt with $(x+3)^2$

	ii)	$4a=2$, $a=\frac{1}{2}$ vertex $=(-3,1)$ focus $=(-3,\frac{3}{2})$ directrix : $y=\frac{1}{2}$	2 both focus and directrix correct 1 correct focus or directrix with $a=\frac{1}{2}$ or correct focus and directrix for incorrect a
f)		$(\operatorname{cosec}^2 x-1)(1+\tan^2 x)$ $=\cot^2 x\times\sec^2 x$ $=\frac{\cos^2 x}{\sin^2 x}\times\frac{1}{\cos^2 x}$ $=\frac{1}{\sin^2 x}$ $=\operatorname{cosec}^2 x$ or $=\operatorname{cosec}^2 x+\operatorname{cosec}^2 x\tan^2 x-1-\tan^2 x$ $=\frac{1}{\sin^2 x}+\frac{1}{\sin^2 x}\times\frac{\sin^2 x}{\cos^2 x}-1-\frac{\sin^2 x}{\cos^2 x}$ $=\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}-1-\frac{\sin^2 x}{\cos^2 x}$ (*) $=\frac{\cos^2 x+\sin^2 x-\sin^2 x\cos^2 x-\sin^4 x}{\sin^2 x\cos^2 x}$ $=\frac{1-\sin^2 x(\cos^2 x+\sin^2 x)}{\sin^2 x\cos^2 x}$ $=\frac{1-\sin^2 x}{\sin^2 x\cos^2 x}$ (#) $=\frac{\cos^2 x}{\sin^2 x\cos^2 x}$ $=\frac{1}{\sin^2 x}$ $=\operatorname{cosec}^2 x$	3 a correct solution to $\frac{1}{\sin^2 x}$ 2 $\frac{\cos^2 x}{\sin^2 x}$ or $\frac{1}{\cos^2 x}$ with $\cot^2 x$ and $\sec^2 x$ or to (#) for alternate method. 1 $\cot^2 x$ or $\sec^2 x$ or to (*) for alternate method

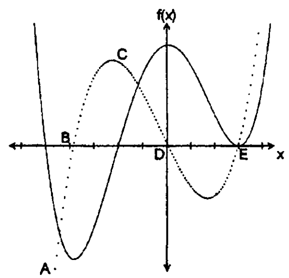
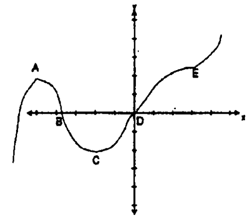
Question 7

(a)i	Let x =width and h= height Volume = $V=(x)(2x)(h)=9$ m³. $2x^2h=9$ $h=\frac{9}{2x^2}$ Surface Area = $A=2(x)(2x)+2(x)h+2(2x)h$ $=4x^2+6xh$ $=4x^2+6x\frac{9}{2x^2}=4x^2+\frac{27}{x}$	3- Result fully justified. 2- h obtained and correct substitution for A. 1- $2x^2h=9$
ii	$A=4x^2+27x^{-1}$. $\frac{dA}{dx}=8x-27x^{-2}$. and $\frac{d^2A}{dx^2}=8+54x^3$. For minimum solve $\frac{dA}{dx}=0$ and show $\frac{d^2A}{dx^2}>0$. $0=8x-\frac{27}{x^2}$ $\therefore 0=8x^3-27$ $8x^3=27$ $x^3=\frac{27}{8}$ $x=\sqrt[3]{\frac{27}{8}}=1.5$ At $x=1.5$ $\frac{d^2A}{dx^2}=8+54(1.5)^3>0$ $\therefore$ Minimum Value. $=24$	4- Correct solving for x from correct derivative and shown to be a minimum 3- Correct solving for x from correct derivative 2- Derivative correct but error in solution for x and shown to be a minimum. 1- Correct derivative 0- $x=1.5$ without justification.
(b)i)	A: $250=1000e^{-5k}$ $0.25=e^{-5k}$ $\ln(0.25)=-5k$ $k=\frac{\ln(0.25)}{-5}$ $k=0.277....$ B: $500=1250e^{-3k}$. $0.4=e^{-3k}$. $\ln(0.4)=-3k$ $k=\frac{\ln(0.4)}{-3}$ $k=0.305....$	2- Both 'k' values correct either exact or rounded to 2 or more decimal places. 1- One value of k' correct
ii)	The 'k' decay constant for B is greater than the one for A	1- Comparison of decay constants needed 1-Comparison of equal quantities over equal time for both A and B
iii)	B: $125=1250e^{-0.305t}$. $0.1=e^{-0.305t}$ $\ln(0.1)=-0.305t$ $t=\frac{\ln(0.1)}{-0.305}$ = 7.53...minutes	2- Correct answer allowing for student's value of 'k' 1- One error

Question 8:

(a)	(i)	Pr (like movie) = 0.5 Pr (dislike movie) = 0.2 Pr (1 liked, 1 disliked) = 2 × (0.5) × (0.2) = 0.2	2 marks - Correct answer 1 mark- Multiplication of correct probabilities but not multiplying by 2 or recognising need for two possible but incorrect probabilities.
	(ii)	Pr(At least one disliked the movie) = 1 – Pr (no-one disliked the movie) Hence Pr = 1 – $(0.8)^3$ = 1 – 0.512 = 0.488	2 marks - Correct answer 1 mark - Use of difference approach to recognise complementary events.
(b)	(i)	$\frac{dA}{dt}=t^2-t+1$ $=3^2-3+1$ $=7\,cm^2/\sec$	1 mark – correct answer
	(ii)	To find the area, we need to integrate: $A=\int t^2-t+1dt$ $=\frac{t^3}{3}-\frac{t^2}{2}+t+C$ $Originally\ t=0\ and\ A=12$ $12=\frac{0}{3}-\frac{0}{2}+0+C$ $C=12$ $Hence\ A=\frac{t^3}{3}-\frac{t^2}{2}+t+12$ $at\ t=2$ $A=\frac{44}{3}\ or\ 14\frac{2}{3}cm^2$	3 marks - Correct answer 2 marks - Correct approach to obtaining the constant value 1 mark - Correct integration
(c)	(i)	Using the product rule: $\frac{d}{dx}\left(xe^{\frac{x}{2}}\right)=e^{\frac{x}{2}}\times1+x\times\frac{1}{2}\times e^{\frac{x}{2}}$	2 marks - Correct use of product rule and fully demonstrated. 1 mark - Error in one term while using product rule
	(ii)	From the above: $x\times\frac{1}{2}\times e^{\frac{x}{2}}=e^{\frac{x}{2}}\times1-\frac{d}{dx}\left(xe^{\frac{x}{2}}\right)$ $\int xe^{\frac{x}{2}}=2\int e^{\frac{x}{2}}-2\frac{d}{dx}\left(xe^{\frac{x}{2}}\right)dx$ $=4e^{\frac{x}{2}}-2xe^{\frac{x}{2}}+C$	2 marks - Correct answer 1 mark - Correct rearrangement prior to integration

Question 9:

(a)	(i)	) (i) *At end of February*: $Amount=100\times\left(1+\frac{6\%}{12}\right)$ $=100\times1.005$ *At end of March* $Amount=(100\times1.005)1.005+(100\times1.005)$ $=(100\times1.005)+(100\times1.005^2)$	2 marks - Correct demonstration 1 mark - One month correctly explained. Marks not given unless explicit demonstration of relationship.
	(ii)	At the end of December 2009, there will have been 59 months of deposit and int $Amount=(100\times1.005)+(100\times1.005^2)+(100\times1.005^3)+...+(100\times1.005^{59})$ $=100\times1.005(1+1.005+1.005^2+...+1.005^{58})$ The expression in brackets is a geometric series. Hence amount is: $Amount=100\times1.005\frac{(1.005^{58}-1)}{1.005-1}$ $=\$6,877$	4 marks - Correct answer 3 marks - Correct process up to final calculation but subsequent error or all steps correct but incorrect identification of number of months. 2 marks - Demonstration of series required. 1 mark - Determination of 59 months.
(b)	(i)	 Second derivative is shown as a dotted line).	2 marks - Correct graph 1 mark - Most of the shape correct.
	(ii)		4 marks - Correct graph with labels and turning points and POI drawn 3 marks - Point E incorrect 2 marks - Points D and E incorrect 1 marks - Points A and B correct

Question 10:

(a)	(i)	To be a geometric series, need to show a common ratio. $\frac{t_2}{t_1}=\frac{t_3}{t_2}$ $\quad \frac{k}{kx}=\frac{kx^{-1}}{k}=\frac{1}{x}$	1 mark – correct answer showing correctly calculated common ratio
	(ii)	Limiting sum when $\left\|\frac{1}{x}\right\|<1$ $\frac{1}{x}<1$ *or* $-\frac{1}{x}>1$ $x>1$ $\quad -1>x$	2 marks – correct domain for x 1 mark – showing $\left\|\frac{1}{x}\right\|<1$ only
(b)	(i)	Velocity = $\int \ddot{x}.dt$ $\dot{x}=\int 10e^{\frac{-1}{3}t}.dt$ $=10\int e^{\frac{-1}{3}t}.dt$ $=10\times {}^{-}3\int \frac{^{-}1}{3}e^{\frac{-1}{3}t}.dt$ $=-30e^{\frac{-1}{3}t}+k$ *at* $t=0, \dot{x}=0$ $0=-30e^{\frac{-1}{3}\times 0}+k$ $0=-30+k$ $30=k$ *Velocity* $=-30e^{\frac{-1}{3}t}+30$	2 marks – correct answer 1 mark – correct integration lacking k in final equation.
	(ii)	Limiting velocity at $t=\infty$ as $t\to\infty$ $\dot{x}=-30e^{\frac{-1}{3}\infty}+30$ $=-30e^{-\infty}+30$ $=30\,cm/sec$	1 mark – correct answer

(iii) Travel in first 3 seconds therefore need to find distance/ time function $\dot{x} = -30e^{\frac{-1}{3}t} + 30$ $x = \int -30e^{\frac{-1}{3}t} + 30.dt$ $= 90e^{\frac{-1}{3}t} + 30t + k_2$ $At\, t = 0, d = 0$ $0 = 90 + k_2$ $-90 = k_2$ $x = 90e^{\frac{-1}{3}t} + 30t - 90$ At $t = 3$ $x = 90e^{-1} + 90 - 90$ $= 33\cdot 1cm$	2 marks – correct answer 1 mark - correct integration of velocity including k_2 term. - correct answer from an incorrect integration ie. had not included a constant term in formula for velocity. *nb. Marks not deducted for integration error followed through from earlier solution.*

(iv) Ball Two $at\ \ t = 0, v = -10$ $-10 = -30e^{\frac{-1}{3}\times 0} + k_3$ $20 = k_3$ $V_2 = -30e^{\frac{-1}{3}\times t} + 20$ $\dot{x}_{\text{Ball 2}} = -30e^{\frac{-1}{3}t} + 20$ $x_{\text{Ball 2}} = \int -30e^{\frac{-1}{3}t} + 20.dt$ $= 90e^{\frac{-1}{3}t} + 20t + k_2$ $At\, t = 0, d = 200$ $200 = 90 + k_2$ $110 = k_2$ $x_{\text{Ball 2}} = 90e^{\frac{-1}{3}t} + 20t + 110$ Balls will collide when they are at the same distance ie: $x_{\text{Ball1}} = x_{\text{Ball2}}$ $90e^{\frac{-1}{3}t} + 30t - 90 = 90e^{\frac{-1}{3}t} + 20t + 110$ $10t = 200$ $t = 20$ seconds therefore they will collide after 20 seconds.	4 marks –correct answer 3 marks – one error 2 marks – two errors 1 mark – equating formula for Distance 1 with formula for Distance 2 Possible Errors +10 instead of -10 for initial velocity 2 instead of 200 for distance Arithmetic error