NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

HIGHER SCHOOL CERTIFICATE

Trial Examination

2015

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Write your Student Number at the top of each page
- Answer Section I- Multiple Choice on Answer Sheet provided
- Answer Section II – Free Response in a separate booklet for each question.
- Board approved calculators and templates may be used.

Section I Multiple Choice

- 10 marks
- Attempt all questions

Section II – Free Response

- 90 marks
- Each question is of equal value
- All necessary working should be shown in every question.

Weighting: 40%

<u>Section 1 Multiple Choice:</u> <u>Attempt Questions 1 – 10</u>

Answer questions on provided answer sheet.
Allow approximately 15 minutes for this section.

Q1. Which of the following is a solution to the equation $2x^4 - 5x^2 + 2 = 0$?

(A) $\frac{1}{2}$

(B) 2

(C) $-\frac{1}{2}$

(D) $\frac{1}{\sqrt{2}}$

Q2. What is the value of $\sqrt{2\pi}$, correct to 3 significant figures?

(A) 2.51

(B) 2.507

(C) 2.506

(D) 2.50

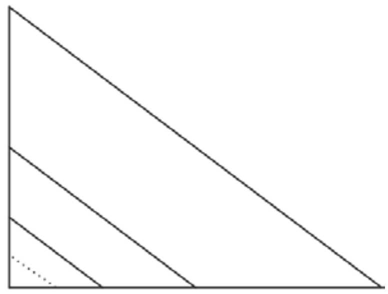
Q3. On Monday, Sophie was absent from her Year 6 class.

If a student was randomly selected from the class on Monday, the probability that the student was a boy was $\frac{5}{9}$.

If class sizes must be between 20 and 30 students, how many students are in Sophie's class when they are all present?

(A) 20
(B) 21
(C) 24
(D) 28

Q4. In the diagram right-angled triangles are drawn. Each subsequent triangle joins the midpoints of the perpendicular sides of the previous triangle. This process continues indefinitely.



NOT TO SCALE

The perpendicular sides of the largest triangle have lengths 3 and 4 units respectively.

What is the total area of all triangles?

(A) 8 units^2

(B) 10 units^2

(C) 12 units^2

(D) 16 units^2

Q5. The line $y = 5x - 1$ is tangent to the curve $y = x^2 + 3x$ at the point A.

Find the coordinates of A.

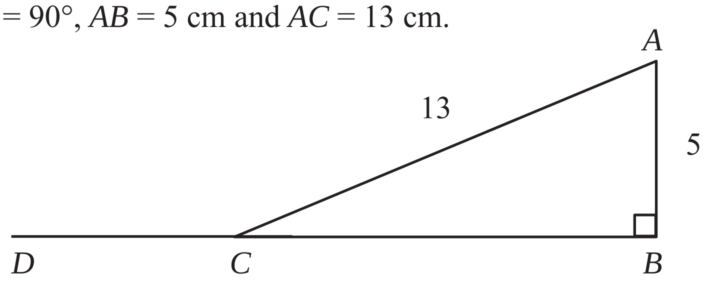
(A) $(-2, -2)$

(B) $(-1.5, -2.25)$

(C) $(1, 4)$

(D) $(0, 0)$

Q6. The diagram shows the right triangle ABC, where BC is extended to the point D.
$\angle ABC = 90°$, $AB = 5$ cm and $AC = 13$ cm.



What is the value of $\tan \angle ACD$?

(A) $-\frac{5}{12}$

(B) $\frac{12}{5}$

(C) $-\frac{12}{5}$

(D) $\frac{5}{12}$

Q7. If $\int_2^5 f(x)\ dx = 7$, what is the value of $\int_2^5 \{1 - f(x)\}\, dx$?

(A) -6

(B) -4

(C) 8

(D) 10

Q8. If $\frac{dy}{dx} = (x^2 - 9)^2(5 - x)$, which statement can be correct for the function $y = f(x)$?

(A) It has a turning point at $x = 3$.

(B) It has a minimum point at $x = 3$.

(C) It is monotonic increasing.

(D) It has a point of horizontal inflexion at $x = -3$.

Q9. The equation $x = 3\sin(nt) + 6$ has a period equal to $\frac{3\pi}{4}$. What is the value of n ?

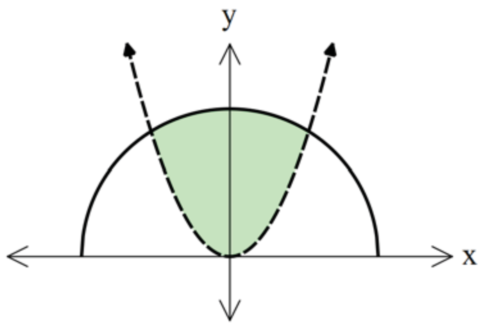
(A) 2

(B) $\frac{1}{2}$

(C) $\frac{4}{3}$

(D) $\frac{8}{3}$

Q10. The equations which satisfy the shaded region below are;



(A) $y \geq x^2$ and $y \leq \sqrt{9 - x^2}$

(B) $y \geq x^2$ and $y < \sqrt{9 - x^2}$

(C) $y > x^2$ and $y \leq \sqrt{9 - x^2}$

(D) $y > x^2$ and $y \geq \sqrt{9 - x^2}$

End of Multiple Choice

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations

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Question 11: Start A New Booklet 15 Marks

(a) Show that $\dfrac{1}{\sqrt{2}-1}-\dfrac{1}{\sqrt{2}+1}$ is a rational number **(2)**

(b) Differentiate the following with respect to x. Leave your answer in fully simplified form.

(i) $\left(x^3-1\right)\left(x^3+1\right)$ **(2)**

(ii) $\sec(x)$. **(2)**

(iii) $\ln[\ \ln(x)\ ]$ **(2)**

(c) Given the function $f(x)=(2-x)^3$, solve the equation $f\,'(x)=-3$ **(2)**

(d) Find $\displaystyle\int \frac{1}{(2x+5)^2}\,dx$. **(2)**

(e) Sketch the graph of $y=\dfrac{6}{x-1}+2$, indicating the intercepts and the asymptotes. **(3)**

Question 12 Start A New Booklet 15 Marks

(a) A parabola has focus $(1,2)$ and directrix $x = 5$.
What is the equation of the parabola? **(2)**

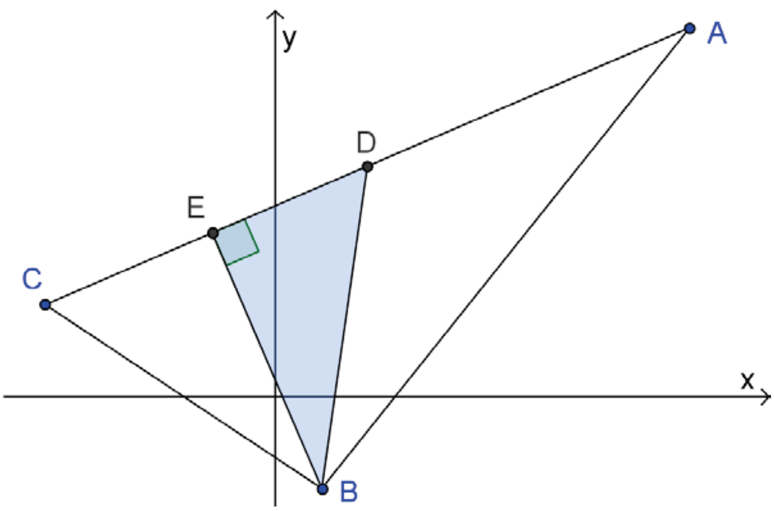
(b) (i) Differentiate $\dfrac{x^2 - 2}{x^2 + 2}$ **(2)**

(ii) Hence evaluate $\displaystyle\int_2^4 \frac{x}{\left(x^2+2\right)^2}\,dx$ **(2)**

(c) (i) Sketch $y = |x - 2|$ **(1)**

(ii) Hence, or otherwise, evaluate $\displaystyle\int_0^4 |x-2|\,dx$ **(1)**

(d) The coordinates of the points $A(9, 8)$, $B(1, -2)$ and $C(-5, 2)$ are shown in the diagram.
Points D and E lie on the line passing through A and C such that D is the midpoint of AC and $BE \perp AC$.



(i) Find the coordinates of point D. **(1)**

(ii) Show that equation of the line passing through A and C is $3x - 7y + 29 = 0$. **(2)**

(iii) Find the length of interval BE. **(1)**

(iv) Find the area of the triangle BED. Give your answer correct to one decimal place. **(3)**

Question 13 Start A New Booklet 15 Marks

(a) (i) Show that $\displaystyle\int_0^5 \frac{3}{2x+5}\,dx = \ln(3\sqrt{3})$ **(3)**

(ii) Hence use Simpson's Rule with three function values to find an approximation of $\ln(3\sqrt{3})$ **(2)**

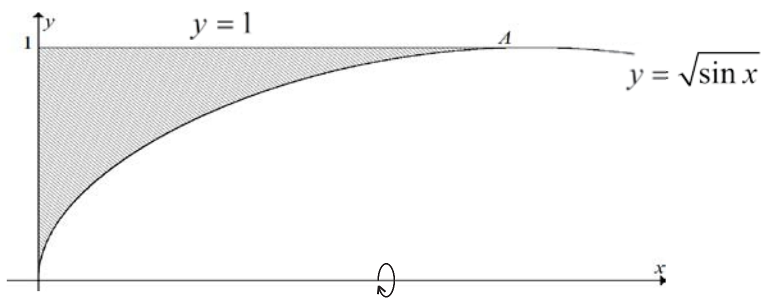
(b) Boxes in the shape of a cube have edges of length 600 mm. The boxes are stacked on top of each other in layers against a wall in a storage area. The bottom layer has 20 boxes, the next layer has 19 boxes and so on, each layer having one less box than the layer below it. The height of the wall is 10 metres.

(i) How many layers of boxes can be stored against the wall? **(1)**

(ii) How many boxes can be stored against the wall in the storage area? **(2)**

(c) Bag A contains 4 blue balls and 3 red balls. Bag B contains 2 blue balls and 3 red balls. Pierre chooses one ball at random from the 7 balls in Bag A and puts it in Bag B. He then chooses one ball at random from the 6 balls in Bag B. Find the probability he chooses a red ball from bag B? **(3)**

(d) The region bounded by the curve $y=\sqrt{\sin x}$, the y-axis and the line $y=1$ is rotated around the x-axis to form a solid.



(i) $y=\sqrt{\sin x}$ and $y = 1$ and meet at the point A.
Find the coordinates of A. **(1)**

(ii) Find the volume of the solid. (Leave your answer in exact form). **(3)**

Question 14 Start A New Booklet 15 Marks

(a) Ali borrows \$250 000 from a bank. The loan is to be repaid in 15 years. The interest rate is 6% p.a. compounded monthly. There are no repayments for the first three month.

Let A_n be the amount owing after n months and M be the monthly repayments.

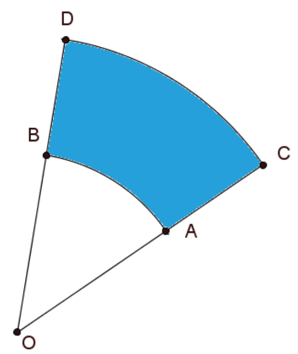
(i) Show why $A_3 = 250\,000 \times 1.005^3$ **(1)**

(ii) Show that $A_5 = 250000(1.005)^5 - M(1 + 1.005)$ **(2)**

(iii) Find the monthly repayments if the loan is to be repaid in 15 years. **(3)**

(b) Find all values of k such that $kx^2 - (k+1)x + 2 = 0$ has exactly two solutions. **(3)**

(c) In the diagram, the circular arcs AB and CD have centre O. It is given that $OD = 8$, $OB = r$ and $\angle BOA = \theta$ such that $\theta = \dfrac{2\pi r}{8}$.



(i) Show that the area of the shaded region $ABCD$ is $A = 32\theta - \dfrac{8\theta^3}{\pi^2}$. **(2)**

(ii) Find r that maximizes the area of the shaded region $ABCD$.
(Leave your answer in exact form) **(4)**

Question 15 Start A New Booklet 15 Marks

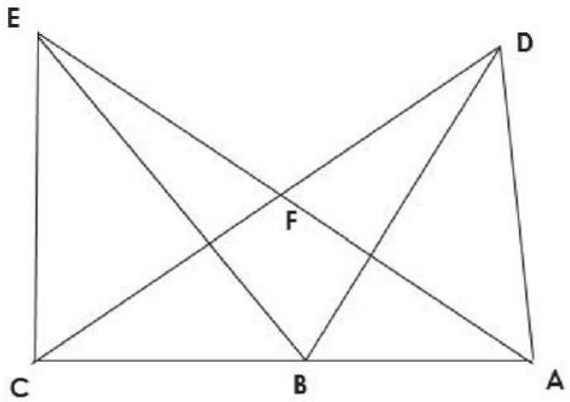
(a) Evaluate $\int_{\frac{\pi}{2}}^{\pi} \sqrt{3}\sec^2 \frac{x}{3}\, dx$. **(3)**

(b) Given the function $y = 2\cos(\pi x) + 2$

(i) Sketch the graph of the function $y = 2\cos(\pi x) + 2$ in the domain $0 \le x \le 3$. **(3)**

(ii) Find the area of the region bounded by $y = 2\cos(\pi x) + 2$ and the x-axis between $x = 0$ and $x = 2$ **(3)**

(d) In the diagram, $\angle BAE = \angle BCD$, $AD = 4$, $AF = 6$, $DF = 2$ and $EF = 3$.



NOT TO SCALE

(i) Show that triangles ADF and CEF are similar. **(2)**

(ii) Given $\Delta\, BEA \,|||\, \Delta\, BDC$, show that $BD : BE = 11 : 9$ **(2)**

(iii) Hence find BD. **(2)**

Question 16 Start A New Booklet 15 Marks

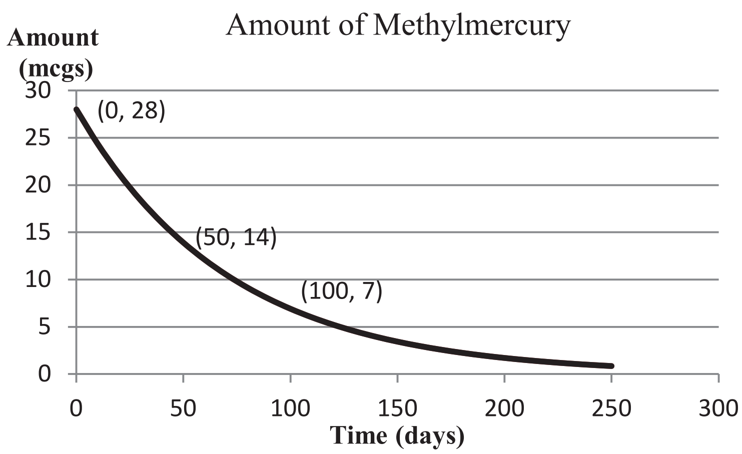
(a) A particle is moving along the x-axis. Its displacement x metres from the origin after time t seconds, is given by $x = 2e^{t} - e^{2t}$ for $0 \le t \le 2$.

(i) Find the initial velocity of the particle. **(2)**

(ii) Explain why the particle never comes to rest. **(1)**

(iii) Find the distance travelled by the particle between $t = 0$ and $t = 2$ seconds. **(2)**

(b) The pollutant methylmercury is sometimes found in fish. The graph shows the mass of methylmercury in a person's body (in micrograms, mcgs) over time (in days) after eating tuna.



(i) The graph is an exponential function of the form $m = Ae^{-kt}$.
Use the information given in the graph to find

A. the value of A. **(1)**

B. the value of k, correct to 3 decimal places. **(2)**

(ii) At what rate is the methylmercury being removed from the person's body when $t = 100$? Give your answer correct to 4 decimal places. **(2)**

(c) The velocity of a particle is measured in metres per second and is given by the piecemeal function $v(t)=\begin{cases} Ae^{kt} & 0 \le t < \frac{1}{k} \\ Ae & t \ge \frac{1}{k} \end{cases}$ where A, $k > 1$ are constants.

(i) What is the initial acceleration of the particle? **(1)**

(ii) Express the displacement x as a function of t, given that initially the particle has a displacement of $\frac{A}{k}$ and at $t=\frac{1}{k}$ the particle has a displacement of $\frac{2Ae}{k}$. **(4)**

<u>End of Examination</u>

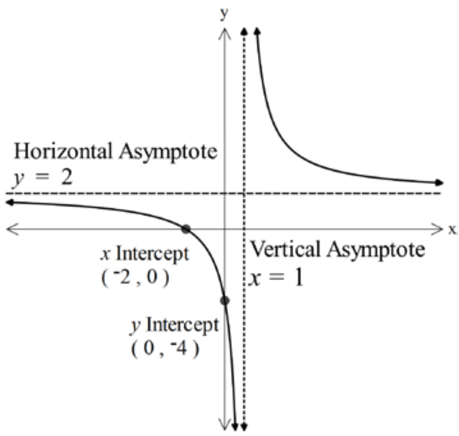
Multiple Choice

Q1	$2x^4 - 5x^2 + 2 = 0$ $(2x^2 - 1)(x^2 - 2) = 0$ $2x^2 - 1 = 0$ or $x^2 - 2 = 0$ $x^2 = \frac{1}{2}$ or $x^2 = 2$ $x = \pm\frac{1}{\sqrt{2}}$ or $x = \pm\sqrt{2}$ *D*	D
Q2	$\sqrt{2\pi}$ $= \sqrt{6{\cdot}283185307}$ $= 2{\cdot}506628275$ $= 2{\cdot}51$ (3 sig fig) *A*	A
Q3	$\mathrm{P}(\text{boy}) = \frac{5}{9} = \frac{10}{18} = \frac{15}{27}$ On Monday: no. of boys = 15 no. of girls = 12 $\therefore$ no. of students = 15 + 12 + 1 = 28	D
Q4	Area of largest triangle is $6u^2$ Both Height and base decrease by factor of $\frac{1}{2}$ each time. So area decreases by $\frac{1}{4}$ each time so … Total area: $6+6\times\frac{1}{4}+6\times(\frac{1}{4})^2+\ldots=\frac{6}{1-\frac{1}{4}}=8u^2$	A
Q5	Derivative of curve = gradient of tangent $y = x^2 + 3x$ $y' = 2x + 3$ $y = 5x - 1$ $m = 5$ $\therefore 2x + 3 = 5$ $2x = 2$ $x = 1$ $y = 5\times1-1$ $= 4$ Coords of $A = (1, 4)$	C

Q6	$13^2 = BC^2 + 5^2$ $BC = 12$ $\tan \angle ACB = \frac{5}{12}$ $\angle ACD = 180° - \angle ACB$ $\therefore \tan \angle ACD = -\frac{5}{12}$	A
Q7	$\int_2^5 \{1 - f(x)\}dx = \int_2^5 1 \ dx - \int_2^5 f(x)\, dx$ $= \left[x\right]_2^5 - 7$ $= [(5) - (2)] - 7$ $= 3 - 7$ $= -4$	B
Q8	$\frac{dy}{dx} = \left(x^2 - 9\right)(5 - x)$ for stat points $\frac{dy}{dx} = 0$ $\therefore \quad x = \pm 3$ or $\quad x = 5$ <table><tr><td>x</td><td>-4</td><td>-3</td><td>0</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>f'(x)</td><td>+</td><td>0</td><td>+</td><td>0</td><td>+</td><td>0</td><td>-</td></tr></table> Therefore *x*=-*3* horizontal inflexion point	D
Q9	$y = 3\sin(nt) + 6$ $\therefore \text{ Period} = \frac{2\pi}{n}$ $\frac{3\pi}{4} = \frac{2\pi}{n}$ $n = \frac{8}{3}$	D
Q10	Therefore greater than parabola – dotted line Less than or equal to circle – full line	C

Question 11

a	$\frac{1}{\sqrt{2}-1}-\frac{1}{\sqrt{2}+1}$ $=\frac{(\sqrt{2}+1)-(\sqrt{2}-1)}{(\sqrt{2})^2-1^2}$ $=\frac{2}{2-1}=2$	2 marks – correct solution 1 mark - correct common denominator - one correct use of a conjugate.
b-i	$y=(x^3-1)(x^3+1)$ $=x^6-1$ $\frac{dy}{dx}=6x^5$ Alternatively $\frac{dy}{dx}=3x^2(x^3+1)+3x^2(x^3-1)$ $=3x^5+3x^2+3x^5-3x^2$ $=6x^2$	2 marks – correct answer 1 mark - correct use of product rule but no simplification -incorrect expansion but correct differentiation of all resulting terms.
b-ii	$y=\sec(x)$ $=(\cos x)^{-1}$ $\frac{dy}{dx}=-\sin x\times{}^{-}1\times\cos^{-2}x$ $=\frac{\sin x}{\cos x\cos x}$ $=\sec x\tan x$	2 marks – correct answer (*nb better students will have learnt this and may not have shown all working. 1 mark - incorrect minus sign - not fully simplified.
b-iii	$y=\ln(\ln(x))$ $\frac{dy}{dx}=\frac{1}{x}\times\frac{1}{\ln x}=\frac{1}{x\ln(x)}$	2 marks – correct answer 1 mark - correct derivative for $\ln x$

c	$f(x) = (2-x)^3$ $f'(x) = {}^-3(2-x)^2$ $\therefore$ ${}^-3(2-x)^2 = {}^-3$ $(2-x)^2 = 1$ $2 - x = \pm 1$ $x = 2 \pm 1$ $x = 3$ or 1	2 marks – both solutions from correct method. 1 mark - correct *f '(x)* term
d	$\int \frac{1}{(2x+5)^2}\,dx$ $= \int (2x+5)^{-2}\,dx$ $= \frac{(2x+5)^{-1}}{2 \times {}^-1} + C$ $= -\frac{1}{2(2x+5)} + C$	2 marks – correct solution 1 mark – one error - missing minus sign
e	$y = \frac{6}{x-1} + 2$ 	3 marks – correct - asymptotes - intercepts - shape 2 mark – 2 of the above three 1 mark – 1 of the above three

Markers Comment:

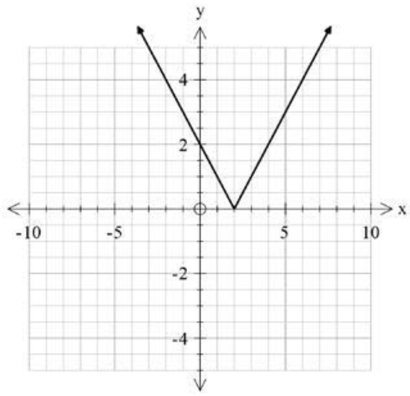
b – ii Had to get to sec$x$tan$x$ to get full marks

b – iii Recall the rule for differentiating logs

If $y = \ln[f(x)]$

then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

Question 12

a	Focus (1, 2), Directrix x=5 Standard Form $y^2 = -4ax$ Vertex (1+2, 2) = (3, 2) $(y-k)^2 = {}^-4a(x-h)$ $(y-2)^2 = {}^-4(2)(x-3)$ $(y-2)^2 = {}^-8(x-3)$	2 marks correct from correct working 1 mark correct vertex 1 mark correct standard form
b-i	$\frac{d}{dx}\frac{x^2-2}{x^2+2}$ let $u = x^2-2$ $u' = 2x$ let $v = x^2+2$ $v' = 2x$ $\therefore \frac{d}{dx}\frac{x^2-2}{x^2+2} = \frac{(x^2+2)2x-(x^2-2)2x}{(x^2+2)^2}$ $= \frac{2x^3+4x-2x^3+4x}{(x^2+2)^2}$ $= \frac{8x}{(x^2+2)^2}$	2 marks correct from correct working 1 mark correct u’ and v’
b-ii	$\int_2^4 \frac{x}{(x^2+2)^2}\,dx = \frac{1}{8}\int_2^4 \frac{8x}{(x^2+2)^2}\,dx$ $= \frac{1}{8}\left[\frac{x^2-2}{x^2+2}\right]_2^4$ $= \frac{1}{8}\left[\frac{4^2-2}{4^2+2} - \frac{2^2-2}{2^2+2}\right]$ $= \frac{1}{8}\left[\frac{14}{18} - \frac{2}{6}\right]$ $= \frac{1}{18}$	2 marks correct solution from correct working 1 mark correct integrand 1 mark correct substitution *Integrand as log: a fatal flaw*
c-i	$y = \lvert x-2\rvert$ 	1 mark correct graph (shape and intercepts)

c-ii	$\int_0^4 \lvert x-2\rvert \; dx$ $= 2 \times \frac{1}{2}\text{bh}$ $= 2 \times \frac{1}{2} \times 2 \times 2$ $= 4$	1 mark correct solution from correct working
d-i	D mid-point A(9, 8) and C (-5, 2) $D = \left(\frac{9 + {}^-5}{2}, \frac{8+2}{2}\right)$ $= (2, 5)$	1 mark correct solution from correct working
d-ii	$m_{ab} = \frac{8-2}{9 - {}^-5}$ $= \frac{3}{7}$ $y - y_1 = m(x - x_1)$ $y - 2 = \frac{3}{7}(x - {}^-5)$ $7y - 14 = 3x + 15$ $0 = 3x - 7y + 29$	2 mark correct solution from correct working 1 mark correct gradient 1 mark correct substitution into point gradient formula
d-iii	$\text{BE} = \frac{\lvert ax + by + c\rvert}{\sqrt{a^2 + b^2}}$ $3x - 7y + 29 = 0 \quad (1, 2)$ $\text{BE} = \frac{\lvert 3(1) - 7({}^-2) + 29\rvert}{\sqrt{3^2 + ({}^-7)^2}}$ $= \frac{\lvert 3 + 14 + 29\rvert}{\sqrt{9+49}}$ $= \frac{46}{\sqrt{58}}$	1 mark correct solution from correct working

d-iv	Area Δ BED $= \frac{1}{2}bh$ $= \frac{1}{2} EB \times ED$ $ED = \sqrt{BD^2 - BE^2}$ $B(1, -2)$ $D(2, 5)$ $BD^2 = (1-2)^2 + (^{-}2 - 5)^2$ $= 1 + 49$ $= 50$ $ED = \sqrt{50 - \frac{(46)^2}{58}}$ $= \sqrt{\frac{392}{29}}$ Area Δ BED $= \frac{1}{2} \times \frac{46}{\sqrt{58}} \times \sqrt{\frac{392}{29}}$ $= \frac{23}{\sqrt{58}} \times \sqrt{\frac{392}{29}}$ $= \sqrt{\frac{322}{29}}$ $= 11{\cdot}10344828$ $= 11{\cdot}1$ (1dp) $units^2$	3 marks correct solution from correct working 1 mark correct ED 1 mark correct BD

Markers Comments:

Question 13

a-i	$\int_0^5 \frac{3}{2x+5}dx = \frac{3}{2}\times\int_0^5 \frac{2}{2x+5}dx$ $=\frac{3}{2}\left[\ln(2x+5)\right]_0^5$ $=\frac{3}{2}(\ln 15-\ln 5)$ $=\frac{3}{2}\ln 3$ $=\ln 3^{\frac{3}{2}}$ $=\ln\sqrt{3^3}$ $=\ln\sqrt{3^2.3}$ $=\ln 3\sqrt{3}$ as required	3 marks correct solution 2 marks correct use of log laws to $\frac{3}{2}\ln 3$ 1 mark correct integration
a-ii	<table><tr><td>x</td><td>0</td><td>2.5</td><td>5</td></tr><tr><td>y</td><td>$\frac{3}{5}$</td><td>$\frac{3}{10}$</td><td>$\frac{1}{5}$</td></tr></table> $\int_0^5 \frac{3}{2x+5}dx \approx \frac{2.5}{3}\left(\frac{3}{5}+4(\frac{3}{10})+\frac{1}{5}\right)$ $=1.\dot{6}$ $\ln 3\sqrt{3}=1.643...$ so $\ln 3\sqrt{3}\approx 1.666...$	2 marks correct solution 1 mark correct use of Simpon's Rule
b-i	No. of layers = 10 m ÷ 600 mm = 10 000 ÷ 600 $= 16\frac{2}{3}$ 16 layers of boxes will fit in the storage area.	1 mark correct answer
b-ii	$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ $=\frac{16}{2}\left[2\times 20+(16-1)\times -1\right]$ $=200$ boxes	2 marks correct solution 1 mark correct use of formula
c	P(Red ball from Bag B) = P(RR) + P(BR) $\frac{3}{7}\times\frac{4}{6}+\frac{4}{7}\times\frac{3}{6}$ $=\frac{24}{42}=\frac{4}{7}$	3 marks correct solution 2 marks P(RR) only correctly found 1 mark correct probability tree diagram

d-i	$\sqrt{\sin x} = 1$ $\sin x = 1$ $x = \frac{\pi}{2}$ Point A is $\left(\frac{\pi}{2}, 1\right)$	1 mark correct solution
d-ii	$V = \pi\int_0^{\frac{\pi}{2}} y^2 dx$ $V = \pi\int_0^{\frac{\pi}{2}} 1^2 dx - \pi\int_0^{\frac{\pi}{2}} \left(\sqrt{\sin x}\right)^2 dx$ $= \pi\int_0^{\frac{\pi}{2}} 1\ dx - \pi\int_0^{\frac{\pi}{2}} \sin x\ dx$ $= \pi\left[x\right]_0^{\frac{\pi}{2}} + \pi\left[\cos x\right]_0^{\frac{\pi}{2}}$ $= \pi\left(\frac{\pi}{2} - 0\right) + \pi\left(0 - 1\right)$ $= \frac{\pi^2}{2} - \pi$ $= \frac{\pi^2 - 2\pi}{2}$ <u>Or</u> $V_{solid} = V_{cylinder} - V_{revolution}$ $= \pi(1)^2\frac{\pi}{2} - \int_0^{\frac{\pi}{2}} y^2 dx$ *etc*	3 marks correct solution 2 marks correct V of revolution 1 mark correct V integral

Markers Comment:

ai) Many students didn't show completely by applying all index rules

ii) some students used more than 3 function values (note process isn't possible with 4)

or calculated wrong function values from wrong x values

b) mostly well answered

c) mostly well answered, often a tree diagram will assist to cover all options

d) trig integration involves radians NOT degrees, hence point a was not (90,1). Many students correctly found the solid of revolution but didn't realise this needed to be subtracted from the cylinder formed by rotating y=1 about the x-axis.

Question 14

a-i	monthly interest $= 1 + \frac{6\%}{12} = 1.005$ $A_1 = 250000 \times 1.005$ $A_2 = A_1 \times 1.005 = 250000 \times 1.005^2$ $A_3 = 250000 \times 1.005^3$	1 mark: correct answer
a-ii	$A_4 = 250000 \times 1{\cdot}005^4 - M$ $A_5 = (A_4 \times 1{\cdot}005) - M$ $= 250000 \times 1{\cdot}005^5 - M \times 1{\cdot}005 - M$ $= 250000(1{\cdot}005)^5 - M(1 + 1{\cdot}005)$	2 marks: correct answer 1 mark: correct expression for A_4
a-iii	Since there are no repayments in the first 3 months: $A_n = 250000(1.005)^n - M(1 + 1.005 + ... + 1.005^{n-4})$ $A_n = 250000(1.005)^n - M \times 1 \times \frac{1.005^{n-3} - 1}{1.005 - 1}$ There are 180 months in 15 years. $A_{180} = 0 \Rightarrow 250000(1.005)^{180} - M \times \frac{1.005^{177} - 1}{0.005} = 0$ $M = \$2163.88$ (2 d.p.)	3 marks: correct answer 2 marks: correct equation after using sum of geometric series 1 mark: correct series for A_n
b	Require $\Delta > 0$ $\therefore [-(k+1)]^2 - 8k < 0$ $k^2 - 6k + 1 > 0$ So by the quadratic formula $\frac{6 \pm \sqrt{32}}{2}$ $= 3 \pm 2\sqrt{2}$ Since $k^2 - 6k + 1$ is concave up, then for $k^2 - 6k + 1 > 0$ we require: $k > 3 + 2\sqrt{2}$ or $k < 3 - 2\sqrt{2}$	3 marks: correct answer 2 marks: progress towards solution 1 mark: recognises the discriminant and the fact that it should be positive
c-i	Given $r = \frac{8\theta}{2\pi} = \frac{4\theta}{\pi}$ $A = \frac{1}{2}\theta(8^2 - r^2) = \frac{\theta}{2}(64 - \frac{16\theta^2}{\pi^2}) = 32\theta - \frac{8\theta^3}{\pi^2}$ using Area of Sector $= \frac{1}{2}r^2\theta$	2 marks: correct answer 1 mark: correct expression for the area of the shaded region in terms of θ and r
c-ii	$\frac{dA}{d\theta} = 32 - \frac{24\theta^2}{\pi^2}$ $\frac{dA}{d\theta} = 0 \Rightarrow \theta = +\sqrt{\frac{32\pi^2}{24}} = \frac{2\pi}{\sqrt{3}}$ $(\theta > 0)$ $\frac{d^2A}{d\theta^2} = -\frac{48\theta}{\pi^2} < 0$ for all $\theta > 0$, therefore $\frac{2\pi}{\sqrt{3}}$ maximises area Since r *and* θ are directly proportional,	

	$r=\dfrac{4\times\dfrac{2\pi}{\sqrt{3}}}{\pi}=\dfrac{8}{\sqrt{3}}$ also maximizes area. $r=4.62$ (2 d.p.) OR $A=32\theta-\dfrac{8\theta^3}{\pi^2}$ $=32\left(\dfrac{2\pi r}{8}\right)-\dfrac{8\left(\dfrac{2\pi r}{8}\right)^3}{\pi^2}$ $=8\pi r-\dfrac{\pi r^3}{8}$ $\dfrac{dA}{dr}=8\pi-\dfrac{3\pi r^2}{8}$ Stationary points occur when $\dfrac{dA}{dr}=0$ $\therefore\ 3\pi r^2=64\pi$ $r^2=\dfrac{64}{3}$ $r=\pm\dfrac{8}{\sqrt{3}}$ But $r>0$ $\therefore\ r=\dfrac{8}{\sqrt{3}}$ $\dfrac{d^2A}{dr^2}=\dfrac{-6\pi r}{8}<0$ for all radii lengths $\therefore$ concave down $\therefore$ maximum occurs at $r=\dfrac{8}{\sqrt{3}}$	4 marks: correct answer 3 marks: justification that found value is a maximum 2 marks: correct solution to $\dfrac{dA}{d\theta}=0$ or $\dfrac{dA}{dr}=0$ 1 mark: correct expression for $\dfrac{dA}{d\theta}$ or for A in terms of r

Markers Comment:

a-iii Many students did not acknowledge that there were no repayments for the first 3 months

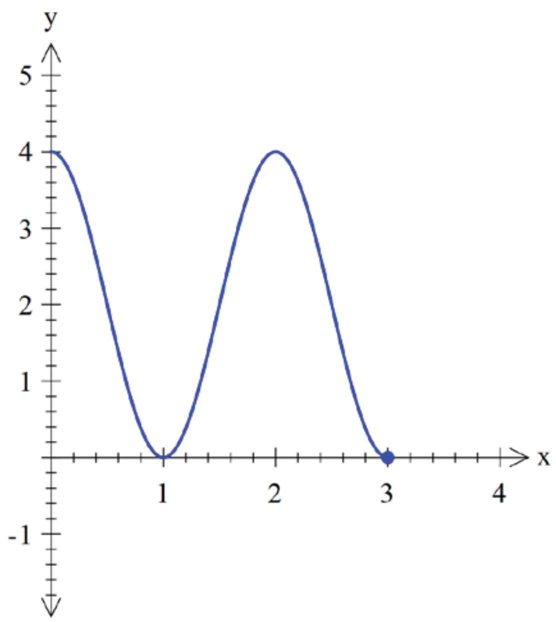
b $[-(k+1)]^2=(-k-1)(-k-1)=k^2+2k+1$ **Apply the order of operations!**

c-i You have been asked to show a statement expressed in terms of θ, not in terms of r

Therefore don't sub $\theta=\dfrac{2\pi r}{8}$ in unless it's needed.

c-ii A radius r > 0 only

Question 15

a)	$\int_{\frac{\pi}{2}}^{\pi} \sqrt{3}\sec^2\frac{x}{3}\,dx$ $=\sqrt{3}\int_{\frac{\pi}{2}}^{\pi} \sec^2\frac{x}{3}\,dx$ $=\sqrt{2}\left[3\tan\frac{x}{3}\right]_{\frac{\pi}{2}}^{\pi}$ $=3\sqrt{3}\left[\tan\frac{\pi}{3}-\tan\frac{\pi}{6}\right]$ $=3\sqrt{3}\left[\sqrt{3}-\frac{1}{\sqrt{3}}\right]$ $=6$	3 marks – correct solution 2 marks – correct approach with one error only 1 mark- correct process with two errors in calculation
b-i)		3 marks – correct graph with all features and critical points (shape, amp and period) 2 marks – correct graph with one missing feature 1 mark- one feature correct
b-ii)	$A=2\int_0^1 2\cos(\pi x)+2\,dx$ $=4\int_0^1 \cos(\pi x)+1\,dx$ $=4\left[\frac{\sin\pi x}{\pi}+x\right]_0^1$ $=4\left[\left(\frac{\sin\pi}{\pi}+1\right)-\left(\frac{\sin 0}{\pi}-0\right)\right]$ $=4[1]$ $=4\,u^2$	3 marks – correct solution 2 marks – correct approach with one error only 1 mark- correct process with two errors in calculation

c-i)	I Δ ADF and Δ CEF $\angle$ DAF = $\angle FCE$ (given) $\angle DFA = \angle$ EFC (vertically opposite) $\therefore$ Δ ADF $\parallel\!\mid$ Δ CEF (equiangular)	2 marks – correct solution 1 mark- correct similarity test with missing reason *SEE MARKER'S COMMENT
ii)	From part i) $\frac{AF}{CF}=\frac{DF}{EF}$ (correspomding sides in similar Δ) $\frac{6}{CF}=\frac{2}{3}$ CF = 9 given that Δ BDC $\parallel\!\mid$ Δ BEA $\frac{BD}{BE}=\frac{CD}{AE}$ (corres[onding sides in similar Δ) $\therefore \frac{BD}{BE}=\frac{DF+CF}{AF+EF}$ $=\frac{2+9}{6+3}$ $=\frac{11}{9}$	2 marks – correct solution showing all steps in development of the ratio from given info. 1 mark- correct value for CF **SEE MARKER'S COMMENT

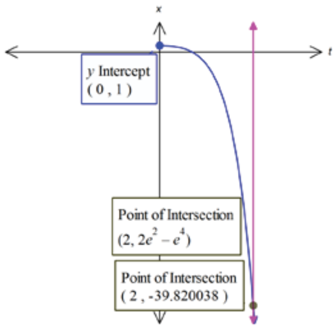
iii)	from part i) $$\frac{DF}{EF}=\frac{DA}{EC}$$ $$\frac{2}{3}=\frac{4}{EC}$$ $$EC = 6$$ from part ii $$\frac{BD}{BE}=\frac{BC}{BA}$$ $$\frac{11}{9}=\frac{BE+6}{\frac{11}{9}BE\ +9}$$ let $BE = x$ $$\frac{11}{9}=\frac{x+6}{\frac{11}{9x}+4}$$ $$x=\frac{9}{4}$$ $\therefore$ $$BD=\frac{11}{9}\times\frac{9}{4}$$ $$=\frac{11}{4}$$	2 marks – correct solution 1 mark- correct progress to answer with correct application of parts i) and ii) creating correct ratio of sides

Markers Comment:

***AA or AAA** is not a similarity test. The correct term is **equiangular**

****SHOW Qs** require all lines of working out to be clearly stated. Students who did not show the addition of sides to create the required ratio did not obtain full marks.

Question 16

a-i	$x = 2e^t - e^{2t}$ $\frac{dx}{dt} = v = 2e^t - 2e^{2t}$ At $t = 0$ $v = 2e^0 - 2e^0 = 0$ The particle is initially stationary.	2 Marks – correct differentiation and substitution. 1 mark – correct expression for *v*. **- substitution of 0 into incorrect equation**
a-ii	$e^{2t} > e^t$ for all $t > 0$ $\therefore 2e^t - 2e^{2t} < 0$ for all $t > 0$ $\therefore v \neq 0$ for all $t > 0$ therefore particle never comes to rest.	1 mark - correctly expressed answer
a-iii	 Note: Particle starts at 1 unit to the right of the origin and finishes at $(2e^2 - e^4)$ units to the left therefore distance travelled is $1 + (2e^2 - e^4)$units. (40.82)	2 marks – correct answer 1 mark – definite integral ie. displacement not distance found. **0 marks – integration of displacement function**
b-i	From graph $t = 0 \Rightarrow m = 28$ $\therefore 28 = Ae^0$ $A = 28$	1 mark **Note – simply stating** **A= 28 insufficient**
b-iI	From graph $t = 50 \Rightarrow m = 14$ $14 = 28e^{-50k}$ $\ln\left(\frac{1}{2}\right) = {}^-50k$ $k = \ln\left(\frac{1}{2}\right) \div ({}^-50)$ $= 0{\cdot}0138629$ $\approx 0{\cdot}0139$	2 mark – correct answer 1 mark – correct substitution into equation

b-iii	Mass $m = 28e^{-kt}$ $\therefore$ Rate of change is first derivative $\frac{dm}{dt} = -k \times 28e^{-kt} = -\boldsymbol{km}$ $= {}^{-}0{\cdot}0139 \times 28e^{-0{\cdot}0139 \times 100}$ $= {}^{-}0{\cdot}0970$ mcgs/day	2 marks – correct solution 1 mark – incorrect sign
c-i	$\ddot{x} = kAe^{kt}$ At $t = 0$ $\ddot{x} = kA$	1 mark
c	For $0 \le t < \frac{1}{k}$ $v = Ae^{kt}$ $x = \frac{1}{k}Ae^{kt} + C_1$ at $t = 0$ $x = \frac{A}{k}$ $\frac{A}{k} = \frac{1}{k}A \times e^{0} + C_1$ $\therefore$ $= C_1 = 0$ $\therefore$ $x = \frac{1}{k}Ae^{kt}$ For $t \ge k$ $v = Ae$ $x = Ae\,t + C_2$ at $t = \frac{1}{k}$ $x = \frac{2Ae}{k}$ $\frac{2Ae}{k} = Ae \times \frac{1}{k} + {}_2$ $\therefore$ $C_2 = \frac{Ae}{k}$ $\therefore$ $x = Ae\,t + \frac{Ae}{k}$ therefore $x(t) = \begin{cases} \frac{1}{k}Ae^{kt} & 0 \le t < \frac{1}{k} \\ Ae\,t + \frac{Ae}{k} & t \ge \frac{1}{k} \end{cases}$	4 marks 2 marks – each for correct equation including calculation of constant. Must include range for *t* for both equations 3 marks – correct but does not include range for *t*. **2 marks – correct integrals without correct calculation of the constant C** **1 mark – one correct intergral**

Markers Comment:

Generally done well – some problems with factorisation $e^{2x} \ne e^{2} \times e^{x}$