NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

HIGHER SCHOOL CERTIFICATE

Trial Examination

2016

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3hours
- Write using black or blue pen
- Write your Student Number at the top of each page
- Answer Section I- Multiple Choice on Answer Sheet provided
- Answer Section II – Free Response in a separate booklet for each question.
- Board approved calculators and templates may be used.
- Reference sheet provided.

Section I Multiple Choice

- 10 marks
- Attempt all questions

Section II – Free Response

- 90 marks – Questions 11-16
- Each question is of equal value
- All necessary working should be shown in every question.

Weighting: 40%

<u>Section 1 Multiple Choice:</u> <u>Attempt Questions 1 – 10</u>
Answer questions on provided answer sheet.
Allow approximately 15 minutes for this section.

Q1 The domain of the function $f(x)=\sqrt{x^2-9}$ is given by

A $x \geq 3$

B $x \leq -3$

C $-3 \leq x \leq 3$

D $x \leq -3, x \geq 3$

Q2 For $x > 1$, which one of the following expressions is the limiting sum of the given infinite series?

$$1-\frac{1}{x}+\frac{1}{x^2}-\frac{1}{x^3}+\frac{1}{x^4}-\frac{1}{x^5}+\ldots$$

A $\dfrac{x}{1+x}$

B $\dfrac{x}{1-x}$

C $\dfrac{1+x}{x}$

D $\dfrac{1-x}{x}$

Q3 If $y = 10^x$, which of the following is true?

A $x = \sqrt[10]{y}$

B $x = \ln y$

C $\dfrac{dy}{dx} = 10^x$

D $\dfrac{dy}{dx} = (\ln 10) \times 10^x$

Q4 When a cyclist rides a bicycle along a straight road, the height h cm of the pedal above the ground at time t is given by $h = 30 + 20\sin(\pi t)$.
What is the closest the pedal comes to the ground?

A 10 cm

B 20 cm

C 30 cm

D 50 cm

Q5 What is the equation of the parabola with directrix $y = 3$ and focus (0,-3)?

A $x^2 = -12y$

B $x^2 = -12(y-3)$

C $x^2 = -24y$

D $x^2 = -24(y-3)$

Q6 A line L is perpendicular to the line $2x - 5y - 8 = 0$.

The gradient of the line L is:

A $\frac{2}{5}$

B $-\frac{2}{5}$

C 2

D $-\frac{5}{2}$

Q7 The derivative of a function is $y = f(x)$ is given by $f'(x) = x^3 - 8$.
Here are two statements about $f(x)$

(1) $f(x)$ is increasing at $x = 1$
(2) $f(x)$ is stationary at $x = 2$

Which of the following is true?

A. Neither statement is correct

B. Only statement (i) is correct

C. Only statement (ii) is correct

D. Both statements are correct

Q8 Find the value of $\int_1^3 \frac{1}{x^3}\, dx$

A $\frac{4}{9}$

B $\frac{5}{3}$

C $\frac{2}{9}$

D $\frac{7}{16}$

Q9 What is the primitive of $\frac{1}{\sqrt{2x-5}}$?

A $\frac{(2x-5)^{-\frac{3}{2}}}{3} + C$

B $\sqrt{2x-5} + C$

C $-\sqrt{2x-5} + C$

D $\frac{3}{(2x-5)^{\frac{3}{2}}} + C$

Q10 Timothy plays a video game three times. The probability that he wins at least once is $\frac{37}{64}$. What is the probability that Timothy wins exactly one game only?

A $\frac{27}{64}$

B $\frac{3}{4}$

C $\frac{1}{64}$

D $\frac{1}{4}$

<u>End of Multiple Choice</u>

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations

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Question 11: Start A New Booklet **15 Marks**

a. Rationalise the denominator for $\dfrac{6}{1-\sqrt{3}}$ **1**

b. State the values(s) of x where the function $f(x) = \dfrac{x+3}{x-3}$ is discontinuous. **1**

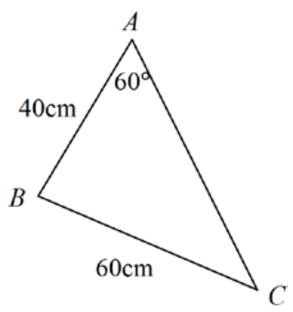
c. Find $\displaystyle\int \frac{4x+6}{x^2+3x}\,dx$. **2**

d. (i) Find the equation of the line passing through the point $(0,-3)$ that is parallel to the line $2x+3y=6$. Give your answer in general form. **2**

(ii) Find the perpendicular distance between the parallel lines. **1**

e. If $\log_b m = 0{\cdot}2$ and $\log_b n = 0{\cdot}3$, evaluate $(\log_b mn)^2$ **2**

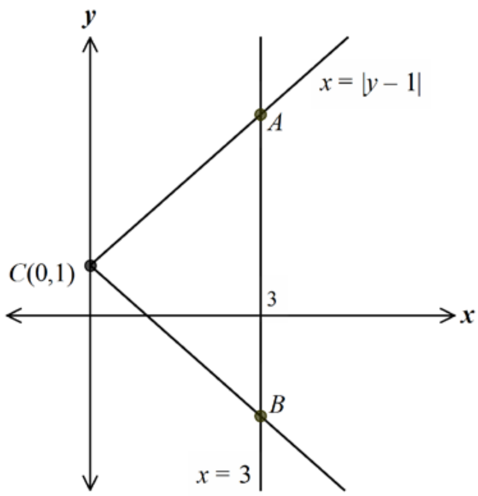
f. Determine the length of AC to the nearest cm **2**



Question 11 continues on the next page.

Question 11 continued

g. The diagram shows triangle ABC formed from the intersection of $x=|y-1|$ and $x=3$.



(i) Find the y ordinates of the points A and B. **2**

(ii) Using $x=|y-1|$ and $x = 3$, write down two inequalities for the region that defines the area of triangle ABC **2**

End of Question 11

Question 12 Start A New Booklet 15 Marks

a. Determine the equation of the tangent to the curve $y = (2x^3 + 1)^4$ at the point $(-1, 1)$. Express your answer in general form. **2**

b. (i) Sketch the function $f(x) = \dfrac{1}{x-2}$. Your diagram should be **<u>at least</u>** one third of a page. Include asymptotes and intercepts on the coordinate axes in your diagram. **2**

(ii) Show clearly on your sketch from part (i) the region where $\dfrac{1}{x-2} \geq 1$ **1**

c. A particle is moving in a straight line and its velocity v m/s at time t seconds is given by $v = 2 - 4\cos(2t)$, where $t \geq 0$. Initially the particle is at the origin.

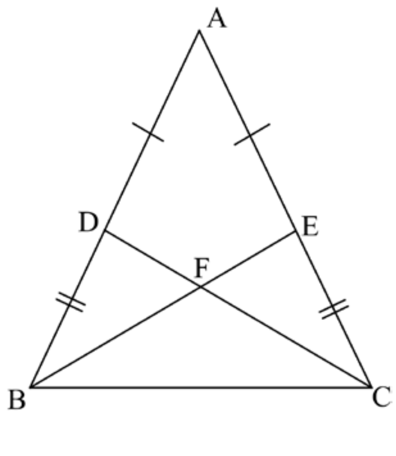
(i) Show that the particle is at rest at $t = \dfrac{5\pi}{6}$. **1**

(ii) Find the maximum acceleration of the particle. **2**

(iii) Express the displacement x, as a function of t. **2**

d. The diagram shows a triangle ABC. The points D and E lie on the sides AB and AC respectively. DC intersects EB at F; AD= AE and BD = CE.

Copy the diagram into your answer booklet. Your diagram should be **<u>at least</u>** one third of a page.



(i) Prove that $\Delta ADC \equiv \Delta AEB$ **2**

(ii) Prove that ΔBFC is isosceles. **2**

(iii) Explain why ADFE is a kite. **1**

End of Question 12

Question 13 Start A New Booklet 15 Marks

a. A parabola is defined by the equation $8y = x^2 - 12x$.

(i) Find the focal length of the parabola. **2**

(ii) Find the y coordinate of the focus of the parabola. **1**

b. Consider the curve $f(x) = 12x^2 - x^4$

(i) Find all stationary points and determine their nature. **3**

(ii) Find any points of inflexion. **2**

(iii) Sketch the curve for $-3 \le x \le 3$ showing the stationary points, points of inflexion and the x and y intercepts. **3**

c. The charge of a battery after t hours is given by the formula $C = C_0 e^{-kt}$, where C_0 and k are constants. It takes 4 hours for the charge of the battery to decrease to half of its initial charge.

(i) Show that $k = 0{\cdot}25 \ln 2$. **2**

(ii) How long does it take for the battery to reduce to 10% of its initial charge? Give your answer to the nearest minute. **2**

End of Question 13

Question 14 Start A New Booklet 15 Marks

a. Evaluate $\int_1^7 \frac{x^2 - 7}{x^2}\, dx$ **3**

b. Determine the exact value of $\sin\theta$ given $\tan\theta = \frac{1}{2}$ and $180° \leq \theta \leq 360°$ **2**

c. (i) The fifth term of an arithmetic series is 3 times the second term. **1**
Show that $d = 2a$, where a is the first term and d is the common difference.

(iii) Hence, or otherwise, find the third term of the arithmetic series **2**

if the sum of the first six terms is 144.

d. An excavation site has been flooded due to recent wet weather. The water is pumped out so that the building can commence. The rate $\frac{dV}{dt}$ at which the water is being pumped out in thousands of litres per hour is given by $5 - \frac{1}{1 + 2t}$ where $t \geq 0$.

(i) Find the initial rate at which the water is being pumped out of the excavation site. **1**

(ii) Calculate the total amount of water pumped out during the first 2 hours.
Give your answer to the nearest litre. **3**

e. The curve $y = \frac{1}{\sqrt{1 + x^2}}$ is rotated about the x – axis from x = -1 to x = 1.

Use Simpson's rule with 5 function values to estimate this volume to two decimal places. **3**

End of Question 14

Question 15 Start A New Booklet 15 Marks

a. Determine the exact value of $\int_0^{\log_e 3} e^{\frac{x}{2}}\,dx$ **2**

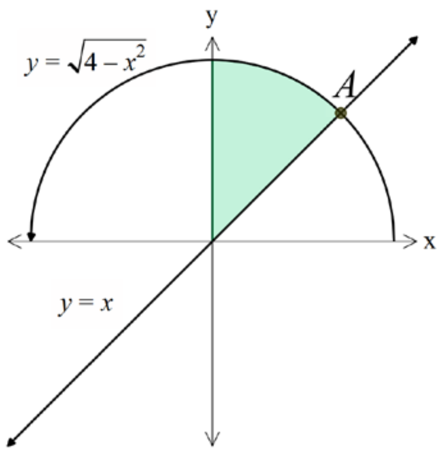
b. There are two student council groups at a particular school. Council A has 5 boys and 4 girls as its members while Council B has 3 boys and 7 girls as members.

(i) Determine the probability that if the two students are chosen from Council A, they are both the same sex. **2**

(ii) To choose the two school captains, one student is selected from each council group. Find the probability that a boy and a girl are selected. **2**

c. The diagram below shows the shaded region between the functions $y = \sqrt{4 - x^2}$ and $y = x$

.



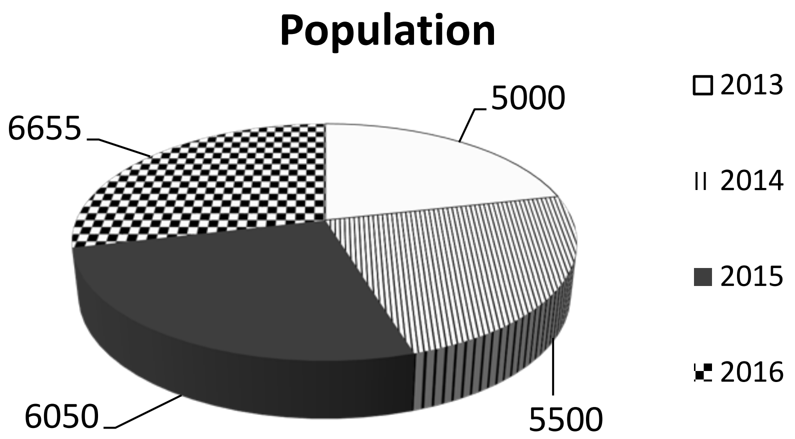
(i) Determine the coordinates of the point of intersection *A*. **1**

(ii) The shaded region is rotated around the *y*-axis. Determine the exact volume of the resulting solid of revolution **4**

Question 15 continues on next page.

Question 15 continued:

d. The population of a town for the years 2013 to 2016 is shown in the pie chart below.



The population increases each year forming a geometric sequence

5000, 5500, 6050, 6655………. .

(i) Calculate the population of the town in 2033, to the nearest whole number. **2**

(ii) A new pie chart was created showing the population of the town for the years 2013 to 2033. What percentage of the pie chart would represent the population in 2033? Give your answer correct to one decimal place. **2**

End of Question 15

Question 16 Start A New Booklet 15 Marks

a. Find the values of A and B for which the expressions $x^2 + x$ and $A(x-1)(x+1) + B(x+1)(x+2)$ are equal for more than two values of x **2**

b. i) Show that $\frac{d}{dx}(\cos^3 3x) = -9\sin 3x\cos^2 3x$ **1**

ii) Hence, or otherwise find $\int (\sin 3x - \sin^3 3x)\, dx$ **3**

c. Zac borrows \$200 000 from a bank. The loan is to be repaid in equal monthly repayments of \$$M$, at the end of each month, over 25 years. Reducible interest is charged at 6% per annum, calculated monthly.

Let \$ A_n be the amount owing after the nth repayment.

(i) Derive an expression for A_2, in terms of M, for the amount Zac owes after the second payment**.** **2**

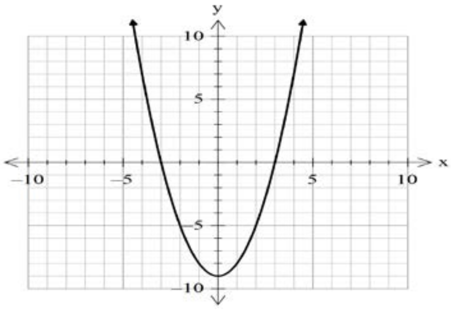
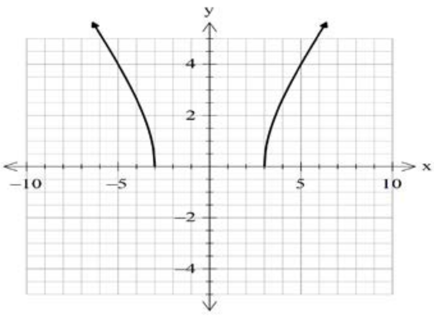
(ii) Write an expression for A_n, in terms of M and n, and hence calculate how much Zac pays each month. Give your answer correct to the nearest cent. **2**

(iii) Just after he makes the 120th payment, Zac starts making monthly payments of \$2 500. In how many more months will he pay off the loan? **3**

(iv) How much money will Zac save a result of changing his repayment amount? **2**

<u>End of Examination</u>

Multiple Choice

Q1	$x^2 - 9 \geq 0$ $(x-3)(x+3) \geq 0$  $\therefore x \geq 3$ and $x \leq -3$ OR 	D
Q2	$s_\infty = \dfrac{a}{1-r}$ $a = 1$ $r = -\dfrac{1}{x}$ $s_\infty = \dfrac{1}{1-\left(-\dfrac{1}{x}\right)}$ $= \dfrac{x}{x+1}$	A
Q3	$D: y = a^x \Rightarrow \dfrac{dy}{dx} = a^x \ln(x)$ $y = 10^x \Rightarrow \dfrac{dy}{dx} = 10^x \ln(x)$	D
Q4	$A: -20 \leq 20\sin(\pi t) \leq 20 \Rightarrow -20+30 \leq 30 + 20\sin(\pi t) \leq 20+30$ $\Rightarrow 10 \leq h \leq 50$ $\therefore$ minimum value is 10 cm	A

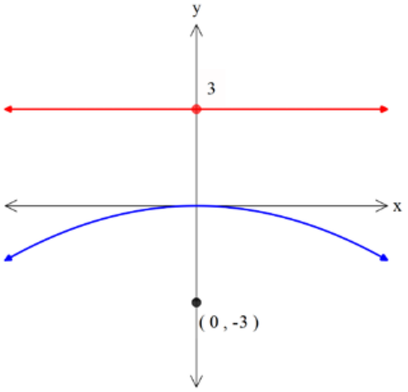
$13^2 = BC^2 + 5^2$

$BC = 12$

$\tan \angle ACB = \dfrac{5}{12}$

$\angle ACD = 180° - \angle ACB$

$\therefore \tan \angle ACD = -\dfrac{5}{12}$

Q5	focus: (0, -3), directrix: $y = 3$  The vertex is at the origin and a = 3 The parabola is concave down so the equation is $x^2 = -4ay$ $x^2 = -12y$	A
Q6	$2x - 5y - 8 = 0$ $5y = 2x - 8$ $y = \frac{2}{5}x - \frac{8}{5}$ $m = \frac{2}{5}$ The lines are perpendicular $\therefore$ Gradient of line L = $-1 \div \frac{2}{5} = -\frac{5}{2}$	D
Q7	At $x = 1$ $\quad f'(x) = 1 - 8 < 0$ therefore decreasing At $x = 2$ $\quad f'(x) = 8 - 8 = 0$ therefore stationary point	C

Q8	$\int_1^3 \frac{1}{x^3}\,dx$ $= \int_1^3 x^{-3}\ dx$ $= \left[\frac{x^{-2}}{-2}\right]_1^3$ $= -\frac{1}{2}\left[\frac{1}{x^2}\right]_1^3$ $= -\frac{1}{2}\left(\frac{1}{9}-1\right)$ $= -\frac{1}{2} \times -\frac{8}{9}$ $= \frac{4}{9}$	A
Q9	$f(x) = (2x-5)^{-\frac{1}{2}}$ $F(x) = \frac{(2x-5)^{\frac{1}{2}}}{\frac{1}{2}\times 2} + C$ $= \sqrt{2x-5} + C$	B
Q10	$P(\text{LLL}) = 1 - \frac{37}{64}$ $= \frac{27}{64}$ $P(\text{L}) = \frac{3}{4}$ $\text{P(exactly one } win) = 3 \times \left[\left(\frac{1}{4}\right) \times \left(\frac{3}{4}\right)^2\right]$ $= \frac{27}{64}$	A

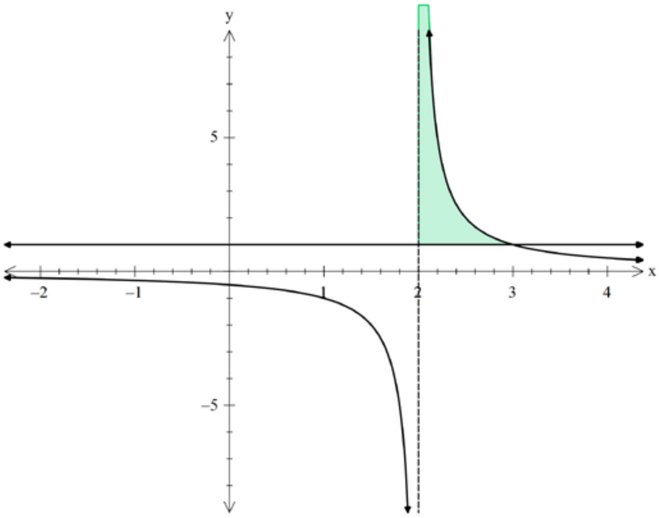
Question 11

11a	$\dfrac{6}{1-\sqrt{3}} = \dfrac{6}{1-\sqrt{3}} \times \dfrac{1+\sqrt{3}}{1+\sqrt{3}}$ $= \dfrac{6(1+\sqrt{3})}{1-3}$ $= {}^-3(1+\sqrt{3})$	1 mark correct solution from correct working – mark awarded at $\dfrac{6(1+\sqrt{3})}{1-3}$
11b	given $f(x) = \dfrac{x+3}{x-3}$ as $x - 3 \neq 0$ $\therefore$ curve is discontinuous at $x = 3$	1 mark correct solution from correct working – if students indicated understanding through $x \neq 3$ also accepted.
11c	$\displaystyle\int \frac{4x+6}{x^2+3x} = \int \frac{2(2x+3)}{x^2+3x}\,dx$ $\displaystyle = 2\int \frac{2x+3}{x^2+3x}\,dx$ $= 2(\ln(x^2+3x)) + C$	2 marks correct solution from correct working. 1 mark correct equivalent integral.
11di	From $2x + 3y = 6$ $y = \dfrac{6}{3} - \dfrac{2x}{3}$ $\therefore \quad m = -\dfrac{2}{3}$ $y - y_1 = m(x - x_1)$ $y - {}^-3 = -\dfrac{2}{3}(x-0)$ $y + 3 = -\dfrac{2x}{3}$ $\dfrac{2x}{3} + y + 3 = 0$ $2x + 3y + 9 = 0$	2 marks correct solution from correct working. 1 mark correct gradient.
11dii	$d = \dfrac{\lvert ax_1 + by_1 + c\rvert}{\sqrt{a^2+b^2}}$ $= \dfrac{\lvert 2(0) + 3({}^-3) - 6\rvert}{\sqrt{2^2+3^2}}$ $= \dfrac{15}{\sqrt{13}}$ $= \dfrac{15\sqrt{13}}{13}$	1 mark correct exact solution from correct working Nb – other solutions accepted $\dfrac{\lvert{}^-15\rvert}{\sqrt{13}}$ or $4 \cdot 16$

11e	$(\log_b mn)^2 = (\log_b m + \log_b n)^2$ $= (0{\cdot}2 + 0{\cdot}3)^2$ $= 0{\cdot}25$	2 marks correct solution from correct working. 1 mark correct expansion of log expression.
11f	let $AC = x$ $60^2 = 40^2 + x^2 - 2(40)(x)\cos 60°$ $60^2 = 40^2 + x^2 - 80x\left(\frac{1}{2}\right)$ $3600 = 1600 + x^2 - 40x$ $x^2 - 40x - 2000 = 0$ $x = \frac{-(^-40)\pm\sqrt{40^2 - 4(1)(^-2000)}}{2(1)}$ $= \frac{40\pm\sqrt{1600 + 8000}}{2}$ $= \frac{40\pm\sqrt{9800}}{2}$ $= \frac{40\pm 97{\cdot}979797}{2}$ $= 68{\cdot}9877\ldots$ as $x > 0$ $x = 69$	2 marks correct solution from correct working. I mark correct substitution into correct formula.
11gi	$\|y - 1\| = 3$ $+(y - 1) = 3$ or $-(y - 1) = 3$ $y - 1 = 3$ $\quad$ $y - 1 = {}^-3$ $y = 4$ $\quad$ $y = {}^-2$ $\therefore$ coordinates $A(3, 4)$ $B(3, -2)$ $\therefore$ y ordinates A $y = 4$ $\quad$ B $y = {}^-2$	2 marks correct solution from correct working
11gii	$x \leq 3$ and $x \geq \|y - 1\|$	2 marks correct solution from correct working

Markers Comment:

Question 12

12 a)	$y=(2x^3+1)^4 \Rightarrow \dfrac{dy}{dx}= 4(2x^3+1)^3 \times 6x^2 = 24x^2(2x^3+1)^4$ At $x=-1$ $m=24(-1)^2(2(-1)^2+1)^3 = -24$ $y-y_1=m(x-x_1) \Rightarrow y-1=-24(x--1)$ $y-1=-24x-24 \Rightarrow 24x+y+23=0$	2 Marks- correct solution. 1 Mark- correct expression f dy/dx or correct equation from incorrect gradient.
b i), ii)		2 Marks – correct solution. 1 Mark – correct shape of graph and correct intercept or asymptote.
c i)	$t=\dfrac{5\pi}{6} \rightarrow v=2-4\cos(2t) \Rightarrow v=2-4\cos\left(2\times\dfrac{5\pi}{6}\right)$ $v=2-4\cos\left(\dfrac{5\pi}{3}\right) \Rightarrow v=2-4\times\dfrac{1}{2}=0$	1 Mark- correct answer
c ii)	$a=\dfrac{dv}{dt}=-4\times -2\sin(2t)=8\sin(2t)$ Max value = amplitude=8	2 Marks- correct solution. 1 Mark- correct expression for dv/dt.
c iii)	$x=\int(2-4\cos(2t))dt = 2t-4\times\dfrac{1}{2}\times\sin(2t)+C$ $x=2t-2\sin(2t)+C$ $t=0$ and $x=0 \rightarrow C=0$ $x=2t-2\sin(2t)$	2 Marks- correct solution. 1 Mark- correct primitive function but no evaluation constant.
d i)	In ΔADC ΔAEB: 1. $\angle$ A is common 2.AD = AE (given) 3.AD + DB = AE + EC (*given*) $\Rightarrow$ AB = AC $\Delta ADC = \Delta AEB$ (*SAS*)	2 Marks- correct solution. 1 Mark- correct identificat and justification of two relevant facts.
d ii)	AB = AC $\Rightarrow$ ΔABC is isosceles $\Rightarrow \angle B = \angle C$ $\angle ABE = \angle ACD$ (corr $\angle$'s in congruent Δ 's) $\angle B - \angle ABE = \angle C - \angle ACD \Rightarrow \angle EBC = \angle DCB$ ΔBFC is isosceles (2 eq $\angle$'s)	2 Marks – correct solution 1 Mark – identification of equal angles or equal side in ΔABE and ΔACD

d iii)	AD = AE (given) and DC = EB (corr sides in cong Δ 's) FC = FB (= sides opp base $\angle$'s isos Δ BFC) DC – FC = EB – FB $\Rightarrow$ DF = EF $ADFE$ is a kite (2 pairs adjacent = sides)	1 Mark- correct, justified explanation.

Markers Comments:

Markers Comment:

12 a) Most students responded well to this part of the question. Those students who failed to gain 2 marks for this part made minor errors such as incorrectly applying the chain rule, incorrect substitution or transposition of terms.

12 b) The majority of students correctly identified the function as a hyperbola and were able to sketch the graph though some responses failed to show the y intercept as required.

The main error was in part ii) where the region required was not correctly identified due to a failure to test values in the inequality.

12 c i) Mostly well done but too many students spent too much time solving the equation formed by setting v equal to 0 rather than simply substituting t=0 into the equation for velocity.

12 c ii) The majority of students were able to find the correct expression for the acceleration but many then failed to realise that the maximum value is given by the amplitude of 8sin(2t).

12 c iii) This part of the question was reasonably well done. The most common errors were the incorrect sign and/or coefficient of sin(2t) as well as failing to include and evaluate the primitive function constant.

12 d i) Most students were able to provide a reasonable response to part i) but there were many students whose setting out and provision of reasons was poor.

12 d ii) This part of the question was not as well done as part i) with many students providing lengthy responses which, even when correct, were not necessary.

12 d iii) Many students were unable to correctly respond to this part of the question. Lengthy responses were common; even when correct these responses indicated that too much time was spent to gain only 1 mark. The most common error was to assume that a kite is determined by one pair of equal side and one pair of equal and opposite angles.

Question 13

<table>
<tr><td>ai</td><td>$$8y = x^2 - 12x$$ $$8y + 36 = x^2 - 12x + 36$$ $$8(y + 4\cdot5) = (x - 6)^2$$ $$4(2)(y + 4\cdot5) = (x - 6)^2$$ $$\therefore \quad a = 2$$</td><td>2 marks

1 mark for correct focal length

1 mark for completing the square correctly or for stating 4a=8</td></tr>
<tr><td>aii</td><td>Vertex: (6, -4.5)
Focus: (6, -4.5+2)
= (6, -2.5)
y-coordinate of the focus is -2.5</td><td>1 mark for finding y-coordinate of focus based on focal length found in part ai</td></tr>
<tr><td>bi</td><td>$$f(x) = 12x^2 - x^4$$ $$f'(x) = 24x - 4x^3$$ stationary points when f'(x)=0 $$24x - 4x^3 = 0$$ $$4x(6 - x^2) = 0$$ $$\therefore \quad x = 0$$ or $$6 - x^2 = 0$$ $$x^2 = 6$$ $$x = \pm\sqrt{6}$$ $$f(0) = 0$$ $$f(\pm\sqrt{6}) = 36$$ $\therefore$ the stationary points are $(0, 0)$, $(\sqrt{6}, 36)$ and $(-\sqrt{6}, 36)$

Determine their nature either by using the second derivative: $$f''(x) = 24 - 12x^2$$ $$f''(0) = 24 > 0 \therefore \text{ concave up, minimum at } (0, 0)$$ $$f''(\pm\sqrt{6}) = -48 \therefore \text{ concave down, maxima at } (\pm\sqrt{6}, 36)$$ OR by testing the first derivative in the neighbourhood of each point:

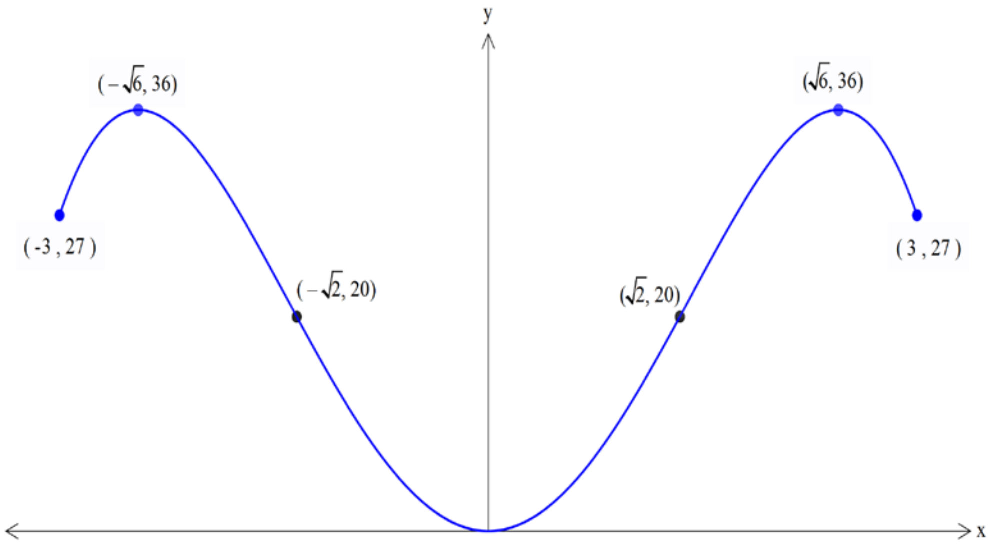
<table>
<tr><td>x</td><td>-2.5</td><td>$-\sqrt{6}$</td><td>-2.4</td><td>-0.1</td><td>0</td><td>0.1</td><td>2.4</td><td>$\sqrt{6}$</td><td>2.5</td></tr>
<tr><td>y’</td><td>2.5</td><td>0</td><td>-2.304</td><td>-2.396</td><td>0</td><td>2.396</td><td>2.304</td><td>0</td><td>-2.5</td></tr>
<tr><td>sign</td><td>+</td><td>0</td><td>-</td><td>-</td><td>0</td><td>+</td><td>+</td><td>0</td><td>-</td></tr>
</table>

Therefore, minimum at $(0, 0)$, maxima at $(\pm\sqrt{6}, 36)$</td><td>3 marks

1 mark for finding the x-values of the stationary points

1 mark for finding y-values of the stationary points

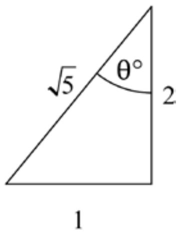
1 mark for correctly determining the nature</td></tr>
</table>

bii	$f''(x) = 24 - 12x^2$ possible points of inflexion when $f''(x) = 0$ $24 - 12x^2 = 0$ $12x^2 = 24$ $x^2 = 2$ $x = \pm\sqrt{2}$ Check concavity either side of these x-values $f''(1{\cdot}4) = 0{\cdot}48 > 0$ $f''(1{\cdot}5) = -3 < 0$ Therefore there is a change of concavity either side of $x = \sqrt{2}$, which means this is a point of inflexion. Similarly, after testing, there is a point of inflexion at $x = -\sqrt{2}$ $f(\pm\sqrt{2}) = 20$ Hence the points of inflexion are $(\pm\sqrt{2}, 20)$	2 marks 1 mark for finding the *x*-values of the points of inflexion 1 mark for the finding the *y*-values of the points of inflexion
biii	 Note that there are *x*-intercepts at $x = \pm\sqrt{12}$ which are not to be included, since the sketch is for $-3 \le x \le 3$	3 marks for correct shape and labelling stationary points, points of inflexion and endpoints
ci		2 marks for fully demonstrated solution 1 mark for obtaining a

	$C = C_0 e^{-kt}$ when $t = 0$, $C = C_0$, so C_0 is the intitial charge when $t = 4$, $C = \frac{1}{2} C_0$ $\frac{1}{2} C_0 = C_0 e^{-4k}$ $\frac{1}{2} = e^{-4k}$ $\ln\left(\frac{1}{2}\right) = -4k$ $\ln 2^{-1} = -4k$ $-\ln 2 = -4k$ $k = 0{\cdot}25 \ln 2$	correct equation in terms of *k*
cii	Find t when $C = 0{\cdot}1C_0$ $0{\cdot}1C_0 = C_0 e^{-kt}$ $0{\cdot}1 = e^{-kt}$ $\ln 0{\cdot}1 = -kt$ $t = \frac{\ln 0{\cdot}1}{-k}$ $t = 13°17'15''{\cdot}76...$ It takes 13 hours and 17 minutes for the battery to reduce to 10% of its initial charge	2 marks for fully demonstrated solution 1 mark for a correct equation involving *t*

Markers Comment:

Question 14

a	$\int_1^7 \frac{x^2-7}{x^2}\ dx$ $= \int_1^7 \frac{x^2}{x^2} - \frac{7}{x^2}\, dx$ $= \int_1^7 1 - 7x^{-2}\ dx$ $= \left[x - 7 \times \frac{x^{-1}}{-1} \right]_1^7$ $= \left[x + \frac{7}{x} \right]_1^7$ $= (7+1) - (1+7)$ $= 8 - 8 = 0$	3 marks – correct solution 2 marks – correct integrand but incorrect final answer. 1 – correct substitution into an incorrect integral form (not if integral is result of $\int_1^7 1 - x^2\ dx \times \int_1^7 \frac{1}{x^2}\, dx$ or similar product.
b	$\tan\theta = \frac{1}{2} \therefore$ Quad 3 $\therefore \sin\theta = -\frac{1}{\sqrt{5}}$ as $180° \le \theta° \le 360°$ 	2 marks – correct solution. 1 mark – either correct magnitude or correct sign.
c-i	$T_5 = 3 \times T_2$ $\therefore\ a + 4d = 3(a + d)$ $4d - 3d = 3a - a$ $d = 2a$	1 mark – correctly demonstrated
c-ii	$S_6 = \frac{n}{2}[2a + (n-1)d]$ $144 = \frac{6}{2}[2a + 5(2a)]$ $48 = 12a$ $a = 4$ $\therefore\ d = 8$ $T_3 = 4 + 2 \times 8 = 20$	2 marks – correct answer 1 mark – correct *a* value.
d-i	$\frac{dV}{dt} = 5 - \frac{1}{1+2t}$ at $t = 0$ $\frac{dV}{dt} = (5-1)$ thousand litres per hour $= 4000$ litres per hr	1 mark – correct solution including per thousand litres

<table>
<tr>
<td>d-ii</td>
<td>

$$\int_0^2 5-\frac{1}{1+2t}\,dt$$
$$=\left[5t-\frac{1}{2}\ln(1+2t)\right]_0^2$$
$$=\left(10-\frac{1}{2}\ln 5\right)-(0+\ln 1)$$
$$=\left(10-\frac{1}{2}\ln 5\right)\text{ kilolitres}$$
$$=9195\text{ litres}$$

</td>
<td>3 marks – correct solution.

2 marks – incorrect rounding to nearest litre.

1 mark – correct integral</td>
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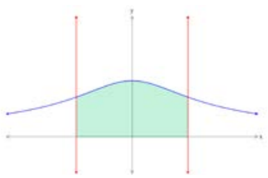

x	-1	-0.5	0	0.5	1
y	$\frac{1}{\sqrt{1+1}}=\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{1\cdot 25}}$	1	$\frac{1}{\sqrt{1\cdot 25}}$	$\frac{1}{\sqrt{2}}$
y^2	$\frac{1}{2}$	0.8	1	0.8	$\frac{1}{2}$
	× 1	× 4	× 2	× 4	× 1

$$Vol=\pi\times\frac{h}{3}\left[y_0+4(y_1+y_3)+2\times y_2+y_4\right]$$
$$=\frac{\pi}{6}\times\left(\frac{1}{2}+4\times 0\cdot 8+2\times 1+4\times 0\cdot 8+\frac{1}{2}\right)$$
$$=\frac{\pi}{6}\times\frac{47}{5}=4\cdot 92\,u^2$$

</td>
<td>3 marks – correct solution

2 marks – missing term eg π

1 mark – finding area not volume.</td>
</tr>
</table>

Markers Comment:

(a) – simplification of $\frac{x^2-7}{x^2}=1-\frac{7}{x^2}=1-7x^{-2}$ **was major problem for this question.**

(d) – numerous students failed to read “thousand litres per hour”and/or to “the nearest litre”

(e) - common mistake was to not recognise this was a Volume problem involving π and y^2

Question 15

a)	$\int_0^{\ln 3} e^{\frac{x}{2}}\,dx$ $=\left[2e^{\frac{x}{2}}\right]_0^{\ln 3}$ $2\left(e^{\frac{1}{2}\ln 3} - e^{\frac{0}{2}}\right)$ $=2(\sqrt{3}-1)$	2 marks- correct solution 1 mark- correct integration
b) i)	$P(\text{BB})$ or $P(\text{GG})$ $=\left(\frac{5}{9}\times\frac{4}{8}\right)+\left(\frac{4}{9}\times\frac{3}{8}\right)$ $=\frac{20}{72}+\frac{12}{72}$ $=\frac{4}{9}$	2 marks- correct solution 1 mark- partial correct product or sum
b)ii)	$P(B_AG_B)+P(G_AB_B)$ $=\left(\frac{5}{9}\times\frac{7}{10}\right)+\left(\frac{4}{9}\times\frac{3}{10}\right)$ $=\frac{47}{90}$	2 marks- correct solution 1 mark- partial correct product or sum
c) i)	Pt int: $\sqrt{4-x^2}=x$ $4-x^2=x^2$ $2x^2-4=0$ $x^2-2=0$ $x=\pm\sqrt{2}$ $\therefore$ $x=\sqrt{2}$ since quad 1 and $y=\sqrt{2}$ $(\sqrt{2},\sqrt{2})$	1 mark- correct solution

c) ii)	$V = \text{cone} + \pi\int_{\sqrt{2}}^{2} x^2\ dy$ $= \frac{1}{3}\pi(\sqrt{2})^2\sqrt{2} + \pi\int_{\sqrt{2}}^{2} 4 - y^2\ dy$ $= \frac{2\sqrt{2}\pi}{3} + \pi\left[4y - \frac{1}{3}y^3\right]_{\sqrt{2}}^{2}$ $= \frac{2\sqrt{2}\pi}{3} + \pi\left[4(2) - \frac{1}{3}(2)^3 - 4(\sqrt{2}) - \frac{1}{3}(\sqrt{2})^3\right]$ $= \frac{2\sqrt{2}\pi}{3} + \pi\left[\frac{24}{3} - \frac{8}{3} - \frac{12\sqrt{2}}{3} + \frac{2\sqrt{2}}{3}\right]$ $= \frac{16\pi - 8\sqrt{2}\pi}{3}$	4 marks - correct solution 3 marks – one error ONLY in solution of correct integrand and bounds 2 marks – only two errors in correct process without simplification of Question 1 mark – correct integration and evaluation of a simplified Question OR 1 mark – identifying summation of two volumes
d) i)	$a = 5000\ r = \frac{11}{10}$ $T_n = 5000\left(\frac{11}{10}\right)^{20}$ $= 33637$	2 marks- correct solution 1 mark- partial correct a and r
d) ii)	$S_n = \dfrac{5000\left(\left(\frac{11}{10}\right)^{21} - 1\right)}{\frac{11}{10} - 1}$ $= 320012$ $\therefore \% = \frac{33637}{320012} \times 100\%$ $= 10{\cdot}5\%$	2 marks- correct solution 1 mark- correct Sn or calc of %

Markers Comment:

a) small portion of students giving decimal approx. or leaving in *ln* form
b) some students using incorrect probabilities for council A
c) loss of mark for not evaluating y-coordinate

d) poorly done with most students integrating and not realising the formation of a cone , and many using wrong sum of volumes or incorrectly subtracting volumes and a small proportion rotating about x-axis

e) most common error involved the incorrect use of 20 terms

Question 16

a	$x^2+x=A(x-1)(x+1)+B(x+1)(x+2)$ $let\ \ x=1$ $1+1=A(1-1)(1+1)+B(1+1)(1+2)$ $2=6B$ $B=\frac{1}{3}$ $let\ \ x=-2$ $4-2=A(-2-1)(-2+1)+B(-2+1)(-2+2)$ $2=3A$ $A=\frac{2}{3}$	1 mark for each value
b (i)	$\frac{d\left(\cos^3 3x\right)}{dx}$ $=3\cos^2 3x\times -3\sin 3x$ using chain rule $=-9\sin 3x\cos^3 3x$	1 mark for solution
(ii)	$\int\left(\sin 3x-\sin^3 3x\right)\,dx$ $=\int \sin 3x\left(1-\sin^2 3x\right)dx$ $=\int \sin 3x\cos^2 3x\ \ dx$ $=-\frac{1}{9}\int -9\sin 3x\cos^2 3x\ \ dx$ $=-\frac{1}{9}\cos^3 3x+C$	3 marks for correct working and solution 2 marks for using trig identity and setting up the integral, however incorrect solution 1 mark for using trig identity, but incorrect integral and incorrect solution
c (i)	$A_1=200000\left(1.005\right)-M$ $A_2=A_1(1.005)-M$ $=200000(1.005)^2-M\left(1.005+1\right)$	2 marks for A_1 and A_2 1 mark for writing just A_1 or just A_2
(ii)		2 marks for writing A_n or the equation and the correct solution 1 mark for writing

	$A_n = 200000(1.005)^n - M\left(1+1.005+1.005^2+.........+1.005^{n-1}\right) = 0$ $200000(1.005)^{300} - M\dfrac{1.005^{300}-1}{0.005} = 0$ $M = \$1288.60$	equation 1 mark for writing just A_n
(iii)	$A_{120} = 200000(1.005)^{120} - 1288.60\times\dfrac{1.005^{120}-1}{0.005}$ $= \$152704.42$ $A_n = 152704.42(1.005)^n - 2500\times\dfrac{1.005^n-1}{0.005} = 0$ $n = 73.07$ Final payment would be in the 74th month.	3 marks for correct working and solution 2 marks for writing amount owing and setting up equation 1 mark for writing amount owing or 74 months.
(iv)	$Total\ \ paid = \$1288.60\times 300$ $= \$386580$ $Total\ \ paid\ \ with\ change = \$1288.60\times 120 + 2500\times 74$ $= \$339632$ $Money\ Saved = \$386580 - \$339632$ $= \$46948$	2 marks for correct working and correct solution (either 73 or 74 months used.) 1 mark for just the solution without working 1 mark for both total paid

Markers Comment: