Write your student exam number in the boxes								

Northern Beaches Secondary College

Manly Campus

2020 HIGHER SCHOOL CERTIFICATE

Trial Examination]

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11 – 37, show relevant mathematical reasoning and/ or calculations

---

Total marks: 100

Section I – 10 marks (pages 1-5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6-25)

- Attempt Questions 11 – 37
- Allow about 2 hour and 45 minutes for this section

MARKS	MC	Q11-18	Q19-24	Q25-29	Q30-33	Q34-37		TOTAL
STUDENT MARK								
MAXIMUM	10	17	18	16	19	20		100

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 –10

Q1. Given $\log_3 5 = 1.46$ and $\log_3 2 = 0.63$, evaluate $\log_3 100$

A. 2.09

B. 2.72

C. 3.55

D. 4.18

Q2. Given $f(x) = \log_e (x - 3)$ and $g(x) = x + 4$, the domain of $f(g(x))$ is

A. $x \geq -4$

B. $x > -1$

C. $x > 3$

D. $x \geq 3$

Q3. The sample space when a fair die is rolled is {1, 2, 3, 4, 5, 6}.
Each outcome is equally likely.

For which pair of events, A and B, are A and B independent?

A. A = {2, 3} and B = {2, 3, 5, 6}

B. A = {1, 3, 5} and B = {2, 4, 6}

C. A = {1, 2, 3} and B = {3, 4, 5}

D. A = {1, 3, 5} and B = {3, 6}

Q4. The function $f(x)$ has derivative $f'(x) = 8\sin(4x)$ and $f\left(\frac{\pi}{4}\right) = 1$

Which of the following is $f(x)$?

A $8\cos(4x) + 7$

B $-2\cos(4x) - 1$

C $2\cos(4x) + 3$

D $2\sin(4x) + 1$

Q5. Let $f(x)$ be a differentiable function such that

- $D : (-\infty, \infty)$
- $f'(-2) = 0$; and
- $f'(x) > 0$ if $x \neq -2$

At $x = -2$, the graph of $y = f(x)$ has a

A. horizontal point of inflection

B. maximum turning point

C. minimum turning point

D. vertical asymptote

Q6. The graph of $y = f(x)$ undergoes a vertical dilation by a factor of a , followed by a horizontal translation of b units to the right, and is then reflected in the x axis.

The resulting graph is

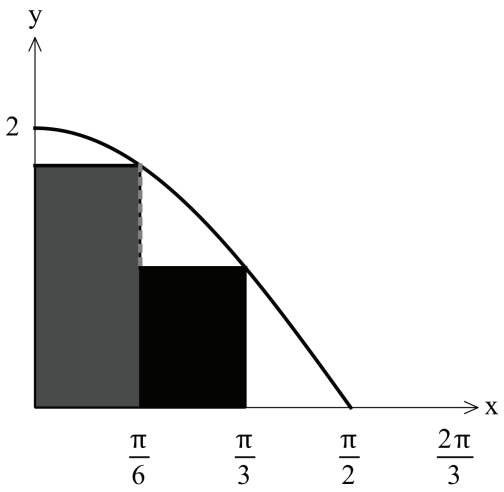
A. $y = -af(x-b)$

B. $y = af(x+b)$

C. $y = -f(ax+b)$

D. $y = f(ax-b)$

Q7. The area under the curve $y = 2\cos x$, as shown below, is approximated by two rectangles



The value of the approximation is ?

A 1

B $\frac{\pi(\sqrt{3}+1)}{6}$

C $\sqrt{3}+1$

D $2\left(\frac{\pi}{6}+\frac{\pi}{3}\right)$

Q8. Which of the following is the correct value S_n for the series given below?

$S_n = 5 + 15 + 45 + \ldots + 98\,415$

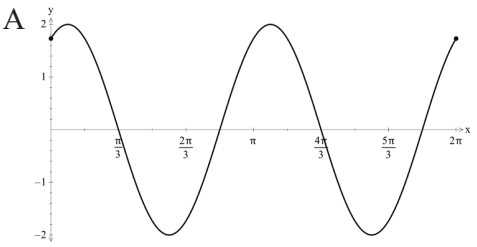
A. 125 640

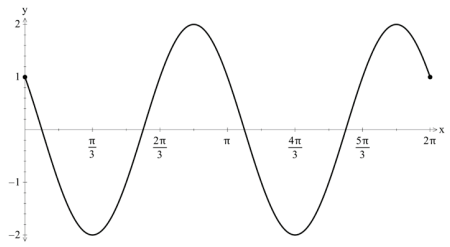
B. 147 620

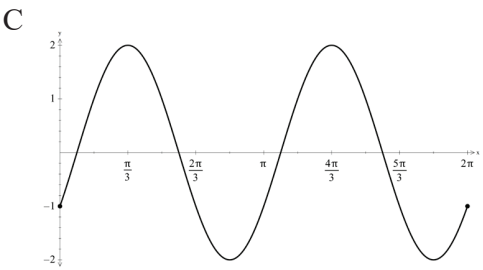
C. 155 600

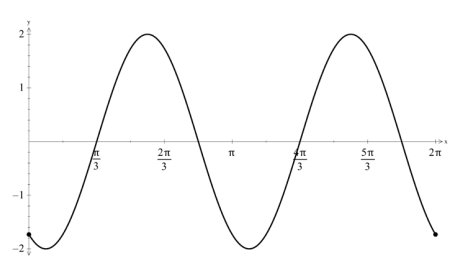
D. 162 350

Q9. The correct graph for $y = -2\sin\left(2x + \frac{\pi}{3}\right)$ for $0 \leq x \leq 2\pi$ is

A 

B 

C 

D 

Q10. The discrete random variable X has the following probability distribution.

x	0	1	2	3	6
P $(X = x)$	$\frac{1}{4}$	$\frac{9}{20}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{3}{20}$

Let μ be the mean of X.

P $(X < \mu)$ is?

A. $\frac{1}{4}$

B. $\frac{7}{10}$

C. $\frac{4}{5}$

D. $\frac{17}{20}$

End of Multiple Choice

Write your student exam number in the boxes								

Northern Beaches Secondary College

Manly Campus

Mathematics Advanced

Trial Examination

Section II – Answer Booklet

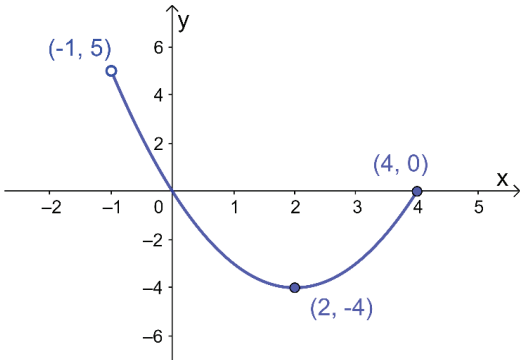
90 marks

Attempt Questions 11 – 37

Allow about 2 hour and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Additional Writing space is available at the end of the booklet.
- Your responses should include relevant mathematical reasoning and/or calculations.

Q11. The graph of a function $f(x)$ is shown below

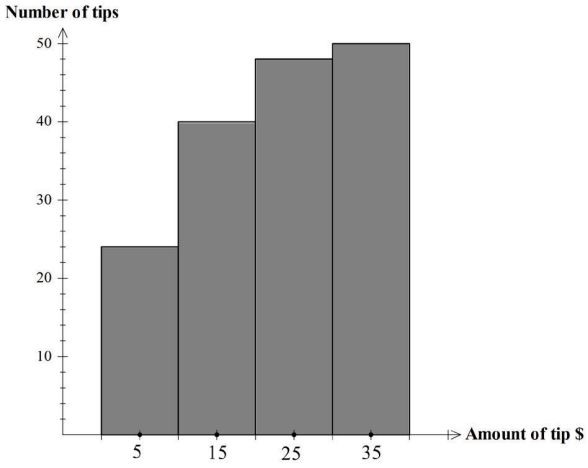


Using interval notation, state the domain and range of $f(x)$ **2**

……..

……..

Q12. Peter works in a restaurant, and each customer can leave him a tip, some extra money, for good service. On one day, Peter earns 50 tips, as shown in the cumulative histogram:



One of the tips was incorrectly listed as \$15 instead of \$25.

Giving reasons, explain what effect this would have on each of the following statistics:

i. The median: **1**

……..

……..

……..

ii. The mean: **1**

……..

……..

……..

Q13. Sarita opens an annuity account, which requires contributions to be made at the end of each period. The table below shows the future of an annuity of $1 for various interest rates per period.

Future value of an annuity of $1

Periods	**Interest rate per period**			
	0.25%	***0.75%***	***1.5%***	***3%***
2	2.0025	2.0075	2.015	2.03
6	6.0376	6.1136	6.2296	6.4684
12	12.1664	12.5076	13.0412	14.1920
24	24.7028	26.1885	28.6335	34.4265

To save for a computer for university, Sarita makes regular deposits at the end of each month into an annuity account that pays 3% p.a. The annuity account compounds interest monthly. How much will Sarita have to deposit each month if she expects the computer to cost $2800 at the end of two years? **2**

……..

……..

……..

Q14. Differentiate $e^{x \sin x}$. **2**

……..

……..

……..

……..

……..

Q15. If $f(x) = \log_e(x^2 + 1)$, calculate $f'(-1)$. **2**

……..

……..

……..

……..

……..

Q16. A shop assistant is building a display of soup cans. The top row will contain 3 cans, and each subsequent row below will have 2 more cans than the previous row above.

i. How many cans would be in the 10^{th} row? **1**

……..

……..

……..

……..

……..

……..

ii. The shop assistant has 400 cans. How many rows is the tallest display that he can build using this pattern? **2**

……..

……..

……..

……..

……..

……..

……..

……..

Q17. The completion times for the Oztown triathlon race were normally distributed with mean time 60 minutes and standard deviation 5 minutes. Using the empirical rule, find Ozzie's completion time if he finished ahead of 84% of competitors. **2**

……..

……..

……..

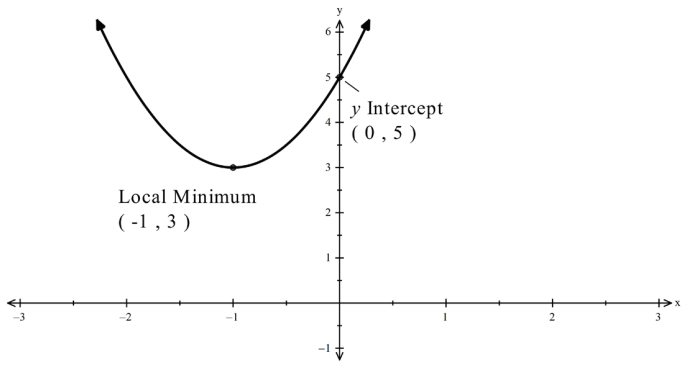
……..

……..

……..

Q18. The function $f(x) = (x - 1)^2$ is transformed and the equation of the new function is of the form $y = k\,f(x + a) + c$ where k, a and c are constants.

The graph of the new function is shown below.



What are the values of k, a and c? **2**

……..

……..

……..

……..

……..

……..

Q19. Two events, A and B, are such that $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{4}$.

If A' denotes the complement of A, calculate $P(A' \cap B)$ when

i) $P(A \cup B) = \frac{3}{4}$ **1**

……..

……..

……..

……..

ii) A are B are mutually exclusive. **1**

……..

……..

……..

……..

Q20. Given the function $g(x) = \frac{3}{2x - 1} + 1$

i. What is the equation of the vertical asymptote? **1**

……..

……..

ii. Sketch the function $g(x)$ in the space below, labelling asymptotes and intercepts. **2**

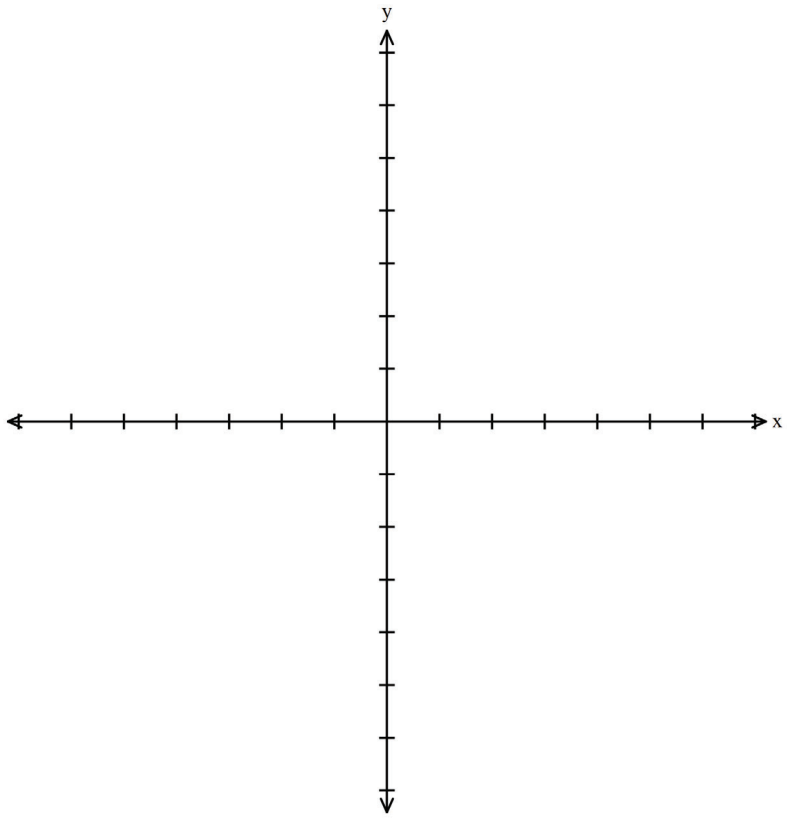
……..

……..

……..

……..

……..



iii. The function $g(x)$ was derived from applying transformations to the function $f(x) = \frac{1}{x}$.

What was the horizontal translation applied to achieve this transformation? **1**

……..

……..

……..

Q21. Find

i. $\int 24(2x-7)^5\,dx$ **1**

……..

……..

……..

……..

ii. $\int \frac{2x+2}{4x^2+8x+1}\,dx$ **2**

……..

……..

……..

……..

……..

Q22. Given the function $f(x) = x^2 - 1$ and $g(x) = \sqrt{4-x^2}$,

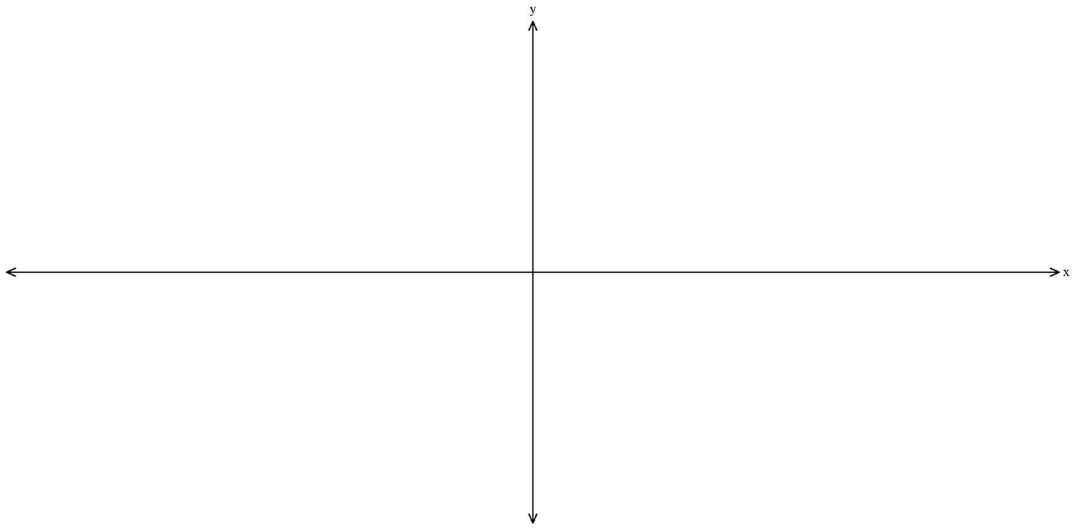
sketch $y = f(g(x))$ over its natural domain. **3**

……..

……

……..

……..



Q23. Given the series $\tan^2 x - \tan^4 x + \tan^6 x - \tan^8 x + \ldots$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

i) Determine the values of x such that the series has a limiting sum. **2**

……..

……..

……..

……..

……..

……..

ii) Write a fully simplified expression in x for the limiting sum of the given series. **1**

……..

……..

……..

……..

Q24. Solve $2\cos(3\theta) = -1$ for $0 \leq \theta \leq \pi$ **3**

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

Q25. i. Differentiate $\sin^2 x$ **1**

……..

……..

ii. Hence calculate $\int_0^{\frac{\pi}{4}} (\sin x + \cos x)^2 \, dx$, (Leave your answer in exact form.) **3**

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

Q26. A six-sided die is biased as shown in the probability distribution below.

$X = x$	1	2	3	4	5	6
$P(X = x)$	0.1	0.25	0.05	0.3	0.17	0.13

i. Find the expected value of X. **1**

……..

……..

……..

ii. Hence, using part i, calculate the variance. **2**

……..

……..

……..

……..

……..

Q27. The body weights and metabolic rate recordings for five women are shown below.

Weight (kg)	52	63	65	47	49
Metabolic rate	1671	1669	1812	1442	1607

i. Find the correlation coefficient, r. (correct to 3 decimal places) **1**

……..

……..

……..

ii. Find the coefficients of the linear regression line, $y = mx + c$, where x is the weight and y is the metabolic rate. **1**

……..

……..

……..

iii. Verify that $m = r\frac{s_y}{s_x}$, where s_x and s_y are the standard deviations of the x and y values respectively. **2**

……..

……..

……..

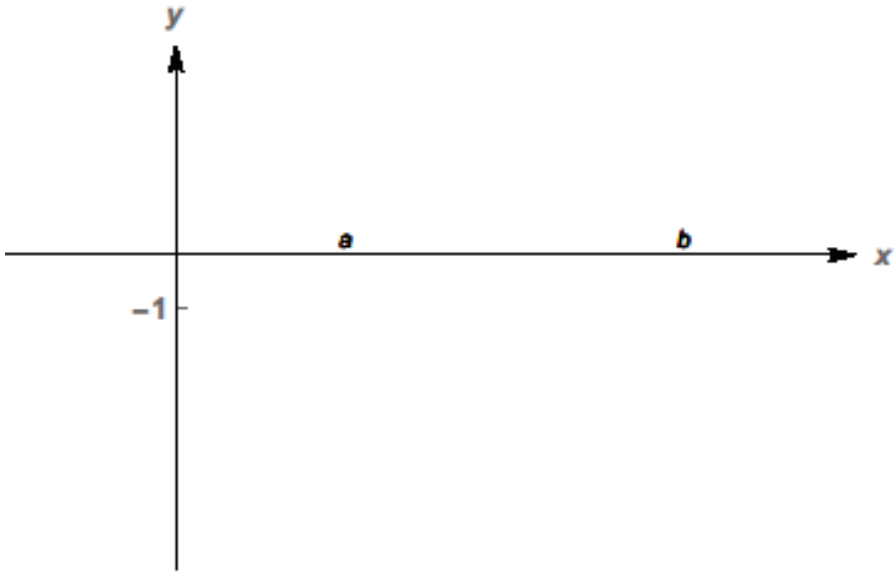
Questions 11 to 27 are worth 46 marks in total

Question 28 start on next page.

Q28. Over the domain $[a,b]$, function f satisfies the following conditions:

$f(x) < -1$, $f'(x) < 0$ and $f''(x) > 0$.

i. Sketch a possible graph of $y = f(x)$ for the domain $[a,b]$. **2**



ii. Write an expression for the minimum value of the function $g(x) = 1 - f(x)$ over the domain $[a,b]$. **1**

……..

……..

Q29. A particle is moving along the x axis. Its displacement, x metres from the origin after t seconds, is given by $x = e^t - 3e^{2t}$ for $t \geq 0$.

Explain why the particle never comes to rest. **2**

……..

……..

……..

……..

……..

……..

……..

……..

Q30. Given the function $f(x) = -x^3 + 9x^2 - 24x + 16$

i. Determine the coordinates of any stationary points and their nature. **3**

……..

……..

……..

……..

……..

……..

……..

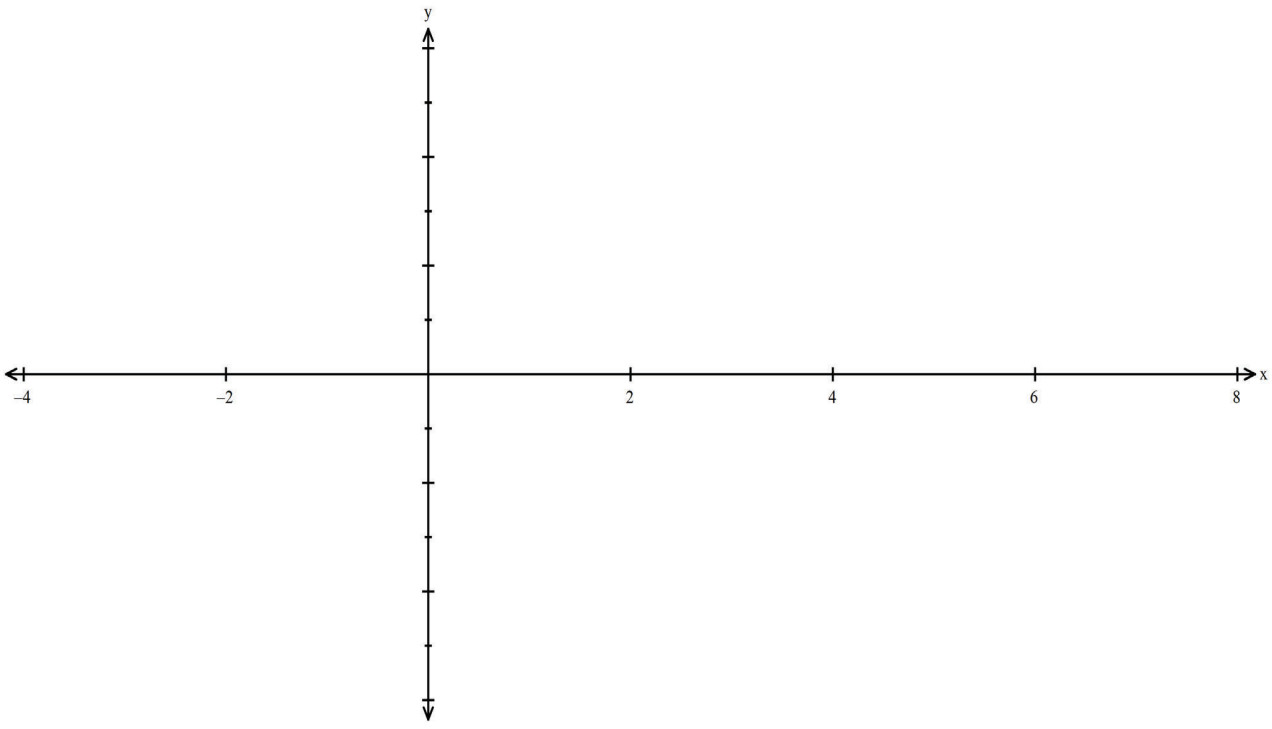
……..

……..

……..

……..

ii. Labelling the stationary points and intercepts with the coordinate axes, sketch the graph of $y = f(x)$ over the domain $[0,6]$. **3**



Question 30 continues on the next page.

iii. State the global minimum of $y = f(x)$ over the domain. $[0,6]$ **1**

……..

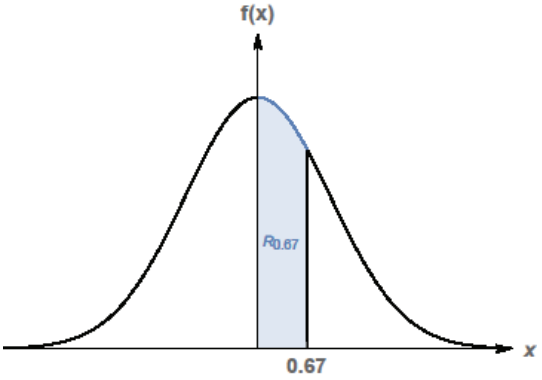
……..

……..

……..

Q31. The random variable X has the standard normal distribution, with probability density function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ over the domain $(-\infty, \infty)$.

In the diagram below, the region $R_{0.67}$ indicates the area under $y = f(x)$ over the domain [0, 0.67].



If the area of $R_{0.67}$ is 0.25, determine the range of scores that could be identified as outliers. **3**

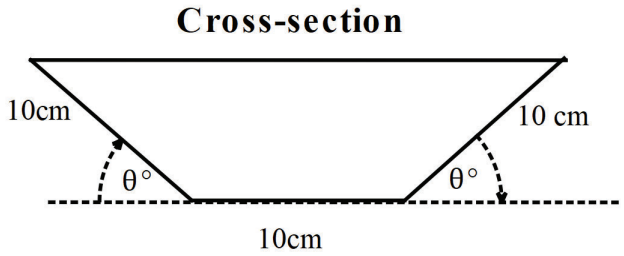
……..

……..

……..

……..

Q32. The diagram below shows a rectangular piece of metal, which is bent to form a gutter for the flow of water. The cross-section of a gutter is a trapezium.



i. Show that the cross-sectional area $A(\theta)$, as a function of θ, is given by **2**

$$A(\theta) = 100\sin\theta(1 + \cos\theta)$$

……..

……..

……..

……..

……..

……..

ii. Find the angle θ that maximizes the cross-sectional area. **3**

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

Q33. i. Prove that $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} \equiv 2\sec^2 x$ where $\sin x \neq \pm 1$ 1

……..

……..

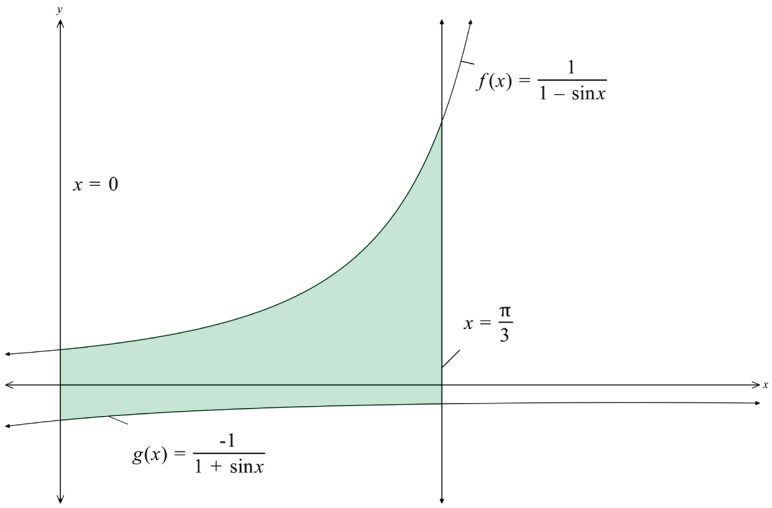
……..

……..

……..

ii. The region shown below represents the area between the curves

$f(x) = \frac{1}{1-\sin x}$ and $g(x) = \frac{-1}{1+\sin x}$, for the domain $\left[0, \frac{\pi}{3}\right]$.



Calculate the area between the 2 curves for the given domain. **3**

……..

……..

……..

……..

……..

……..

……..

……..

……..

Q34. The lizards on Goanna Island are gradually dying out. The predicted population P of lizards, t years after the first observation was made, is $P = P_0 e^{-0.01t}$, where P_0 is the initial population.

i. At what rate is the population changing 45 years later? Answer in terms of P_0. **1**

……..

……..

……..

ii. What percentage of the original population remains 45 years later, correct to the nearest 1%? **1**

……..

……..

……..

……..

iii. When will the population decline to 10% of the original population? Leave your answer in exact form. **2**

……..

……..

……..

……..

……..

……..

Q35. The monthly sales, X (in units of \$100, 000), of a business are modelled by the two probability density functions:

$$g(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{Model 1}$$

OR

$$f(x) = \begin{cases} kx^3(1 - x^2) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{Model 2}$$

i. Determine the value of k in the second model. **2**

……..

……..

……..

……..

……..

……..

ii. Show that the two models have the same median value. **3**

……..

……..

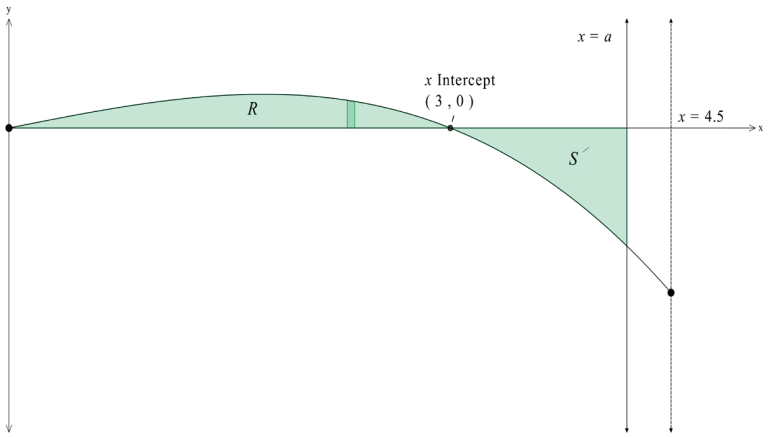
……..

……..

……..

……..

Q36. The graph below shows the cubic curve $y = 9x - x^3$, with domain $[0, 4.5]$.



The curve crosses the $x-$axis at $x = 0$ and $x = 3$.

The diagram shows two bounded regions:

R bounded by $y = f(x), y = 0, x = 0$ and $x = 3$

S bounded by $y = f(x), y = 0, x = 3$ and $x = a$

It is given that $a > 3$. Determine the value of a such that regions R and S have the same area. **3**

……..

……..

……..

……..

……..

……..

……..

……..

Q37. Sam invests $\$M$ into an account at the beginning of each month. The account earns 3% p.a. interest, compounded monthly.

i. Show that the total amount in the account at the end of the second month, A_2 is given by $A_2 = M \times (1.0025 + 1.0025^2)$ **1**

……..

……..

……..

……..

ii. Show that the amount in the account after n months, A_n, is given by $A_n = 401M(1.0025^n - 1)$ **2**

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

Question 37 continues on page 25

Question 37 continued

iii. Hence, calculate how much is needed to invest at the beginning of each month if the plan is to have \$6000 at the end of 12 months. **2**

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

iv. After the first five months of contributions, Sam increases his monthly payment to \$600. Given Sam's final contribution is less than \$600, what will be the minimum amount of this contribution (to the nearest \$5) to reach his goal of \$6000? **3**

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

……..

End of Paper.

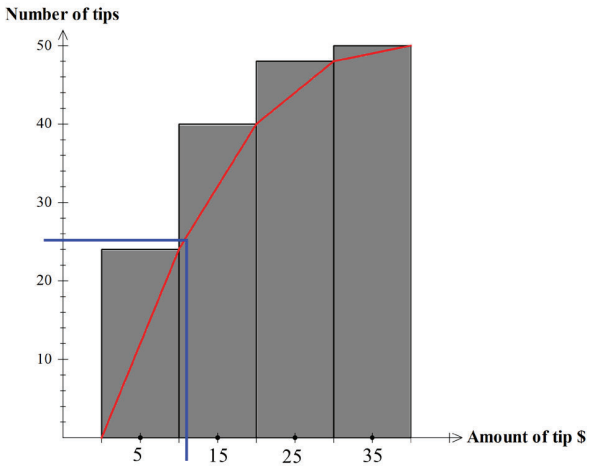
Q1-D Q2 - B Q3-D Q4-B Q5-A

Q6- A Q7- B Q8 – B Q9 – D Q10 - B

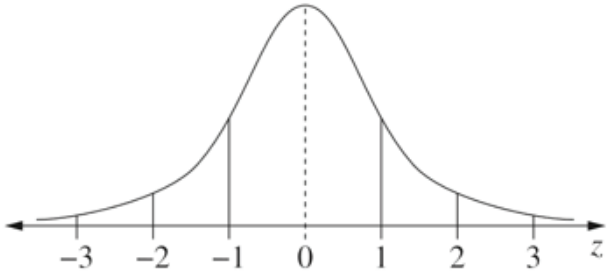
Q1	$\log_3 100$ $= 2\log_3 10$ $= 2\log_3 (2 \times 5)$ $= 2(\log_3 5 + \log_3 2)$ $= 2(1.46 \times 0.63)$ $= 4.18$	D
Q2	Domain $f(x)$ - $x > 3$ Domain $g(x)$ - all real x $g(x) = x + 4$ $f(x) = \log_e (x - 3)$ $f(g(x) = \log_e[(x + 4) - 3]$ $= \log_e(x + 1)$ Domain $x > -1$	B
Q3	A and B independent not to be confused with mutually exclusive. P(A and B) = P(A) x P(B) $A: \quad \frac{2}{6} \neq \frac{2}{6} \times \frac{4}{6}$ $B: \quad 0 \neq \frac{1}{2} \times \frac{1}{2}$ $C: \quad \frac{1}{6} \neq \frac{1}{2} \times \frac{1}{2}$ $D: \quad \frac{1}{6} = \frac{3}{6} \times \frac{2}{6}$	D

Q4	$f(x) = \int 8\sin(4x)\ dx$ $= -2\int -4\sin(4x)\ dx$ $f(x) = -2\cos(4x) + C$ $1 = -1\cos\left(4 \times \frac{\pi}{4}\right) + C$ $1 = -1 \times -1 + C$ $C = -1$ $f(x) = -2\cos(4x) - 1$	B
Q5	Single stationary point where all else is increasing therefore point of horizontal inflection	A
Q6	$y = a f(x)$ $y = af(x - b)$ $y = -af(x - b)$	A
Q7	$A = \frac{\pi}{6} \times 2\cos\frac{\pi}{6} + \frac{\pi}{6} \times 2\cos\frac{\pi}{3}$ $= \frac{2\pi}{6}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$ $= \frac{\pi(1 + \sqrt{3})}{6}$	B
Q8	$a = 5\ r = 3$ $98415 = 5 \times 3^{n-1}$ $19683 = 3^{n-1}$ $\log 19683 = (n - 1)\log 3$ $n - 1 = 9$ $n = 10$ $S_{10} = \frac{5(3^{10} - 1)}{3 - 1} = 147620$	B

Q9	$y = -2\sin\left[2\left(x + \frac{\pi}{6}\right)\right]$	D
Q10	Mean $= E(X) = 0 \times \frac{1}{4} + 1 \times \frac{9}{20} + 2 \times \frac{1}{10} + 3 \times \frac{1}{20} + 6 \times \frac{3}{20}$ $= 1.7$ $\therefore$ $P(X = 0 \text{ or } 1) = \frac{1}{4} + \frac{9}{20} = \frac{14}{20} = \frac{7}{10}$	B

		Mark
Q11	Domain is $(-1,4]$ Range is $[-4,5)$	2 marks – both correct 1 mark – one correct
Q12a	***Sample Answer:*** Through use of the OGIVE, it can be seen that the median is lower than 15 therefore the error has no effect on the median.  	1 mark – correct answer with correct explanation
Q12b	***Sample Answer:*** Since the mean calculation uses the specific value of each tip, increasing a single tip from \$15 to \$25 must increase the mean by $\frac{\$10}{50} = \$0.20$.	1 mark – correct answer with correct explanation
Q13	$3\%\ pa = 0.25\%$ pm 2 years = 24 months $\$\frac{2800}{24.7028} = \$113.347 \approx \$113.35$	2 marks – correct answer 1 mark – correct monthly interest rate.

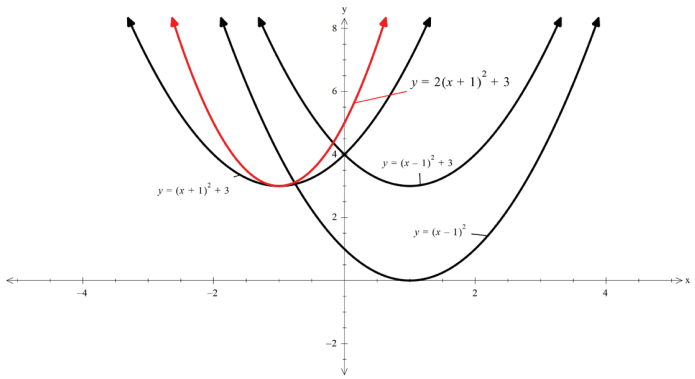
Q14	$y = e^{x\sin x}$ $f(x) = x\ \sin x$ $f'(x) = \sin x + x\cos x$ $y = e^{f(x)}$ $\frac{dy}{dx} = f'(x)\ e^{f(x)}$ $= (\sin x + x\cos x)e^{x\sin x}$	2 correct answer 1 mark - Correct derivative for index - Incorrect index derivative used correctly
Q15	$f(x) = \log_e (x^2 + 1)$ $f'(x) = \frac{2x}{x^2 + 1}$ $f'(-1) = -\frac{2}{2} = -1$	2 marks – correct answer 1 mark – correct derivative. - Correct substitution
Q16-i	Arithmetic Series $a = 3\ \ d = 2$ $T_{10} = a + (n-1)d$ $= 3 + 9 \times 2 = 21$	1 mark – correct answer
Q16-ii	$Sn = 400$ $Sn = \frac{n}{2}[2a + (n-1)d]$ $400 = \frac{n}{2}[6 + 2n - 2]$ $800 = 4n + 2n^2$ $0 = n^2 + 2n - 400$ $n = \frac{-2 \pm \sqrt{4 + 4 \times 400}}{2}$ $= (-1 \pm \sqrt{401})$ $= 19.02$ or or -21.02 ∴ 19 rows high	2 – correct answer 1 mark – correct substitution into formula.

<table>
<tr>
<td>Q17</td>
<td>
Normal distribution


approximately 68% of scores have z-scores between -1 and 1
approximately 95% of scores have z-scores between -2 and 2
approximately 99.7% of scores have z-scores between -3 and 3

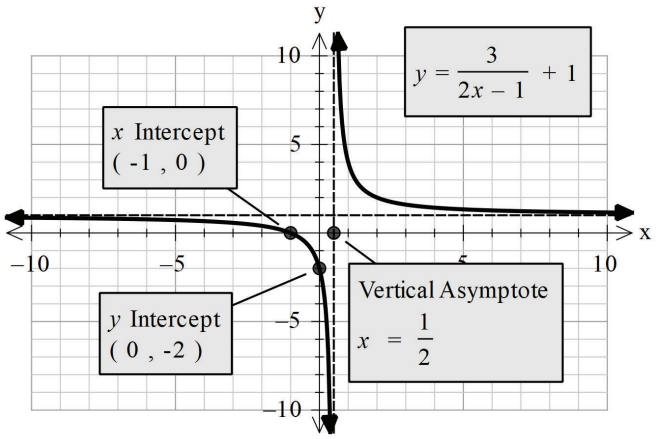
Therefore must be greater than one standard deviation less than mean to be finish ahead of 84%

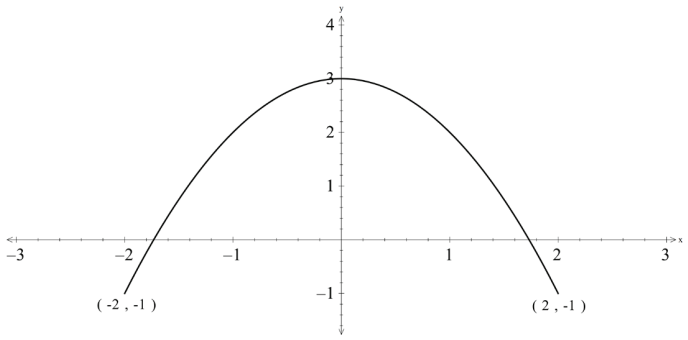
Therefore less than 55 minutes.
</td>
<td>2 – correct answer

1 mark – correctly identifies needs to be one SD from mean.</td>
</tr>
<tr>
<td>Q18</td>
<td>
Vertex moved up 3 $\therefore c = 3$ and to left 2 places
$y = k(x+1)^2 + 3$ $\quad y = f(x) = (x-1)^2$
$x = 0\ y = 5$ $\quad y = f(x+2) = \{(x+2)-1\}^2$
$5 = k \times 1^2 + 3$ $\quad y = (x+1)^2$
$k = 2$
$k = 2\ \ a = 2\ \ c = 3$


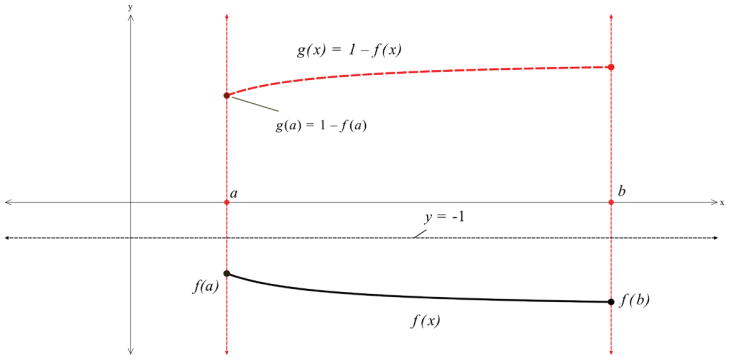
</td>
<td>2 marks – all three letters correct value

1 mark – 2 correct</td>
</tr>
<tr>
<td>Q19i</td>
<td>$P(A' \cap B) = \frac{3}{20}$</td>
<td>1 correct solution</td>
</tr>
<tr>
<td>Q19ii</td>
<td>$P(A' \cap B) = \frac{5}{20}$</td>
<td>1 correct solution</td>
</tr>
</table>

Q20i	$x = \frac{1}{2}$	1 mark correct solution
Q20ii		2 marks correct solution 1 mark some missing components
Q20iii	Right ½ unit (if follows dilation); Right 1 unit if precedes dilation,	1 mark correct solution
Q21i	$\int 24(2x-7)^5\,dx$ $= 24\int (2x-7)^5\,dx$ $= 24 \times \frac{(2x-7)^6}{2 \times 6} + C$ $= 2(2x-7)^6 + C$	1 mark correct solution
Q21ii	$\int \frac{2x+2}{4x^2+8x+1}\,dx$ let $y = 4x^2 + 8x + 1$ $\frac{dy}{dx} = 8x + 8 = 4(2x+2)$ $\frac{1}{4}\int \frac{4(2x+2)}{4x^2+8x+1}\,dx$ $= \frac{1}{4}\ln\lvert 4x^2+8x+1\rvert + C$	2 marks correct solution 1 mark incorrect multiple of correct log function

Q22		3 marks correct solution 2 marks one missing feature 1 mark two missing features
Q23i	$r = \dfrac{-\tan^4 x}{\tan^2 x}$ $= -\tan^2 x$ $\therefore \ \lvert -\tan^2 x \rvert < 1$ $-\dfrac{\pi}{4} < x < \dfrac{\pi}{4}$	2 marks correct solution 1 mark correct inequality for limiting sum
Q23ii	$S_\infty = \dfrac{\tan^2 x}{1-(-\tan^2 x)}$ $= \dfrac{\tan^2 x}{\sec^2 x}$ $= \sin^2 x$	1 mark correct solution
Q24	$2\cos(3\theta) = -1$ if $0 \leq \theta \leq \pi$ then $0 \leq 3\theta < 3\pi$ $\cos 3\theta = -\dfrac{1}{2}$ $3\theta = \cos^{-1}\left(-\dfrac{1}{2}\right)$ $3\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{8\pi}{3}$ $\theta = \dfrac{2\pi}{9}, \dfrac{4\pi}{9}, \dfrac{8\pi}{9}$	3 marks correct solution 2 marks only one error in correct process 1 mark correct ref angle and quads for solution.

Q25-i	$y = \sin^2 x$ $\frac{dy}{dx} = 2\cos x \sin x$	1 mark correct solution
Q25-ii	$\int_0^{\frac{\pi}{4}} (\sin x + \cos x)^2 \ dx$ $= \int_0^{\frac{\pi}{4}} \sin^2 x + 2\sin x \cos x + \cos^2 x \ dx$ $= \int_0^{\frac{\pi}{4}} 1 + 2\sin x \cos x \ dx$ $= \left[x + \sin^2 x \right]_0^{\frac{\pi}{4}}$ $= \left(\frac{\pi}{4} + \left(\frac{1}{\sqrt{2}} \right)^2 \right) - 0$ $= \frac{\pi}{4} + \frac{1}{2}$	3 marks correct solution 2 marks correct definite integral 1 mark correct expansion and simplified to integral $\int_0^{\frac{\pi}{4}} 1 + 2\sin x \cos x \ dx$ must get to line 3 to earn 1 mark
Q26-i	$E(x) = 1\times0.1 + 2\times0.25 + 3\times0.05 + 4\times0.3 + 5\times0.17 + 6\times0.13$ $= 3.58$	1 mark correct solution
Q26-ii	$Var(X) = E(X^2) - \mu^2$ $= 1^2\times0.1 + 2^2\times0.25 + 3^2\times0.05 + 4^2\times0.3 + 5^2\times0.17 + 6^2\times0.13 - 3.58^2$ $= 2.4636$	2 marks correct solution showing working 1 mark correct value with some relevant working
Q27-i	0.8233 **=0.823**	1 mark correct solution
Q27-ii	$m = 13.346$ $c = 903.499$	1 mark BOTH m and c values correct
Q27-iii	$\begin{bmatrix} s_y = 119.732 \\ s_x = 7.386 \end{bmatrix} \Rightarrow r \times \frac{s_y}{s_x} = 0.8233 \times 16.2096 = 13.346 = m$	2 marks correct solution, verified as shown 1 mark for finding $\frac{Sy}{Sx} = 16.21$ but not fully verified

<table>
<tr>
<td>Q28-ia</td>
<td> 
</td>
<td>2 marks for correct graph of f(x). Must be below -1 and only from a to b inclusive

1 mark if correct concavity or a decreasing graph between a and b</td>
</tr>
<tr>
<td>Q28-ii</td>
<td>Minimum => $g(a)=1-f(a)$</td>
<td>1 correct solution</td>
</tr>
<tr>
<td>Q29</td>
<td>$x = e^t - 3e^{2t}$

$\frac{dx}{t} = e^t - 6e^{2t}$

$\frac{dx}{dt} = v$
To stop $\Rightarrow \quad v = 0$

$e^t - 6e^{2t} = 0$
$e^t(1 - 6e^t) = 0$

$e^t \neq 0 \quad \Rightarrow \quad t = \ln 0$ not possible

$1 = 6e^t$
$e^t = \frac{1}{6}$
$t = \ln\left(\frac{1}{6}\right) = -1.79$

as $t \geq 0$
there are no values for $v = 0$</td>
<td>2 marks correct solution with reasons why velocity cannot =0

1 mark for correct derivative and putting = 0 and attempt to solve

0 marks just for stating exponentials can’t = 0 as question says EXPLAIN</td>
</tr>
</table>

<table>
<tr>
<td>Q30-i</td>
<td>$f(x) = -x^3 + 9x^2 - 24x + 16$

$f'(x) = -3x^2 + 18x - 24$
$= -3(x^2 - 6x + 8)$
$= -3(x - 4)(x - 2)$

$f''(x) = -3(2x - 6)$
$= -6(x - 3)$

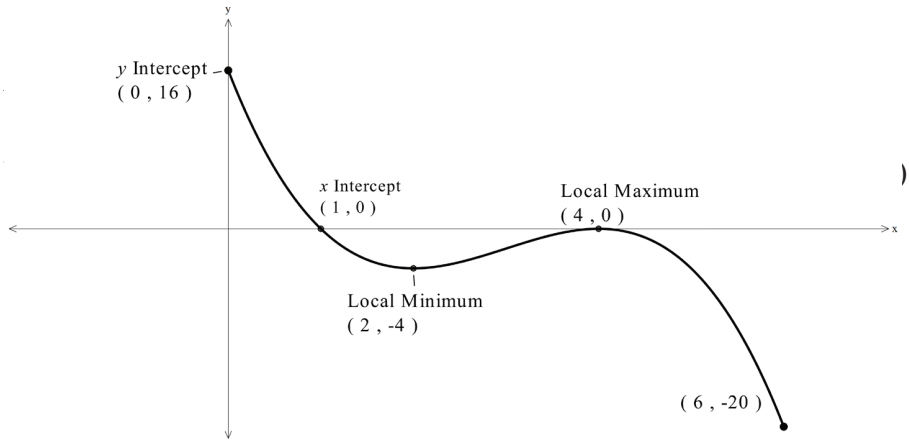
$f'(x) = 0$
at $x = 4$ and $x = 2$

at $x = 4$ $f''(x) = -6 < 0$
concave down $\therefore$ max
at $(4, 0)$

at $x = 2$ $f''(x) = 6 > 0$
concave up $\therefore$ min
at $(2, -4)$</td>
<td>3 marks

1 mark for finding the x-coordinates of the stationary points

1 mark for the finding the y-coordinates of the stationary points (Several students did this in (ii))

1 mark for correctly determining the nature of the stationary points</td>
</tr>
<tr>
<td>Q30-ii</td>
<td> 

) and</td>
<td>3 marks

1 mark for shape

1 mark for labelling the stationary points found in (i)

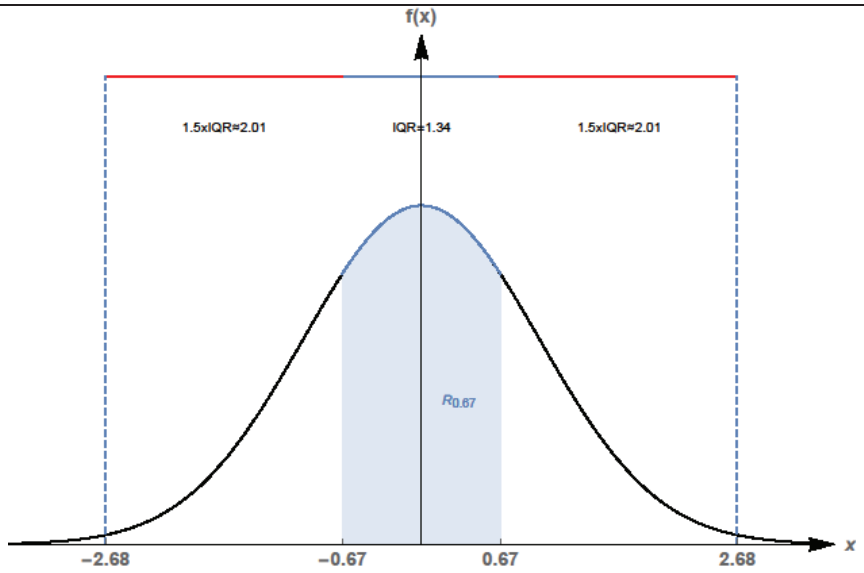
1 mark for coordinates of the endpoints

(No marks for the x-intercept at 1)</td>
</tr>
<tr>
<td>Q30-iii</td>
<td>-20
Note the global minimum / minimum value is a single number, not a point.</td>
<td>1</td>
</tr>
<tr>
<td>Q31</td>
<td>Since $R_{0.67}$ has area of 0.25, $Q_3 = 0.67$ and $Q_1 = -0.67$ by symmetry.
Hence, the outliers will be below $-2.68 = -0.67 - 1.5 \times \left(0.67 - \left(-0.67\right)\right)$ or above $2.68 = 0.67 + 1.5 \times \left(0.67 - \left(-0.67\right)\right)$</td>
<td>3 marks

1 mark for Q_3=0.67

1 mark for IQR

1 mark for using the outlier definition</td>
</tr>
</table>



Q32-i

2 marks

1 mark for some progress, e.g. a correct expression for the area of the rectangle

$$\text{Cross-sectional area} = \frac{h}{2}(a+b), \text{ where } h = 10\sin\theta, a = 10, b = 10 + 2\times 10\cos\theta$$

$$A(\theta) = \frac{10\sin\theta}{2}\left(10 + (10 + 20\cos\theta)\right)$$

$$= 5\sin\theta(20 + 20\cos\theta)$$

$$A(\theta) = 100\sin\theta(1 + \cos\theta)$$

Q32-ii

3 marks

1 mark for differentiating

1 mark for solving A' = 0

1 mark for showing a maximum

$$A'(\theta) = 100\left[\cos\theta(1+\cos\theta) + \sin\theta(-\sin\theta)\right]$$

$$= 100\left(\cos\theta + \cos^2\theta - \left(1 - \cos^2\theta\right)\right)$$

$$= 100\left(2\cos^2\theta + \cos\theta - 1\right)$$

$$= 100(\cos\theta + 1)(2\cos\theta - 1)$$

$$\therefore A'(\theta) = 0 \text{ if } \cos\theta = -1 \text{ or } \cos\theta = \frac{1}{2}$$

Since θ is acute, $\cos\theta = \frac{1}{2} \Rightarrow \theta\frac{\pi}{3}$

$$A''(\theta) = 100\left(-4\cos\theta\sin\theta - \sin\theta\right) = -100\sin\theta(4\cos\theta + 1)$$

$$A''\left(\frac{\pi}{3}\right) = -150\sqrt{3} < 0$$

Hence, the cross-sectional area is a maximum when $\theta = \frac{\pi}{3}$

Q33-i	$$\begin{aligned} LHS &= \frac{1}{1-\sin x} + \frac{1}{1+\sin x} \\ &= \frac{(1+\sin x)+(1-\sin x)}{1-\sin x^2} \\ &= \frac{2}{\cos^2 x} \\ &= 2\sec^2 x \\ &= RHS \end{aligned}$$	1 mark
Q33-ii	$$\int_0^{\frac{\pi}{3}} \left(\frac{1}{1-\sin x} - \frac{-1}{1+\sin x} \right) dx$$ $$= \int_0^{\frac{\pi}{3}} 2\sec^2 x \ dx$$ $$= 2\left[\tan x \right]_0^{\frac{\pi}{3}}$$ $$= 2\left(\tan\frac{\pi}{3} - 0 \right)$$ $$= 2\sqrt{3}$$	3 marks 1st mark for a correct expression for the area, or for some understanding of the area between two curves 2nd mark for simplifying using (i)
Q34-i	$$\frac{dP}{dt} = kP$$ at $t = 45$ $$P = P_0 e^{-0.45}$$ $$\frac{dP}{dt} = -0.1P_0 e^{-0.45}$$	1 mark for correct solution
Q34-ii	at $t = 45$ $$P = P_0 e^{-0.45}$$ $$\frac{P_0 e^{-0.45}}{P_0} \times 100 = 63.76 \cong 64\%$$	1 mark for correct solution

Q34-iii	$P = P_0\ e^{-0.01t}$ 10% $\frac{P_0}{10} = P_0\ e^{-0.01t}$ $0.1 = e^{-0.01t}$ $\ln(0.1) = -0.01t$ $t = \frac{-\ln 10}{-0.01}$ $= 100 \ln 10$	2 marks for correct working and solution 1 mark for writing the equation, then not making any progress.
Q35-i	$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow k\int_0^1 \left(x^3 - x^5\right)dx = 1$ Since $\int_0^1 \left(x^3 - x^5\right)dx = \left[\frac{x^4}{4} - \frac{x^6}{6}\right]_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ then $\frac{k}{12} = 1 \Rightarrow k = 12$	2 marks for working and solution 1 mark for integrating.
Q35-ii	Let $x = M$ be the median of model g. Then: $\frac{1}{2} = \int_0^M g(x)\,dx = \int_0^M 2x\,dx = \left[x^2\right]_0^M = M^2$, i.e. $M^2 = \frac{1}{2}$ Consider $\int_0^M f(x)\,dx = \int_0^M 12\left(x^3 - x^5\right)dx = \left[3x^4 - 2x^6\right]_0^M = 3M^4 - 2M^6$ Since $M^2 = \frac{1}{2}$, $\int_0^M f(x)\,dx = 3\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 = \frac{1}{2}$ Hence $x = M$ is also the median of model f, i.e. the two models have the same median.	3 marks for correct solution 2 marks for finding M^2 and integrating model 2, but did not sub M. 1 mark for only finding M^2
Q36	$\int_0^3 9x - x^3\ dx + \int_3^a 9x - x^3\ dx = 0$ $\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3 + \left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_3^a = 0$ $\left(\frac{81}{2} - \frac{81}{4}\right) + \left(\frac{9a^2}{2} - \frac{a^4}{4}\right) - \left(\frac{81}{2} - \frac{81}{4}\right) = 0$ $\frac{9a^2}{2} = \frac{a^4}{4}$ $18a^2 = a^4\ ,\ a \neq 0$ $a^2 = 18$ $a = 3\sqrt{2}$ as $a > 0$	3 marks for correct solution 2 marks for integrating with different limits and then equating. However did not make progress. 1 mark for integrating with different limits.

	Alternatively $\int_0^a 9x - x^3 \ dx = 0$ $\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^a = 0$ $18a^2 - a^4 = 0$ $a^2(18 - a^2) = 0$ $a = 0$ or $a = \pm 3\sqrt{2}$ but $a > 0$ therefore $a = 3\sqrt{2}$	
Q37 – i	$A_1 = M \times \left(1 + \frac{3}{1200}\right) = M \times 1.0025$ $A_2 = A_1 \times 1.0025 + M \times 1.0025$ $= M \times 1.0025^2 + M \times 1.0025$ $= M(1.0025 + 1.0025^2)$	1 mark for solution. Must show A_1 being substituted into A_2.
Q37-ii	$A_n = M(1.0025 + 1.0025^2 + \ldots 1.0025^n)$ $= M \times 1.0025(1.0025 + 1.0025^2 + \ldots 1.0025^{n-1})$ $= M \times 1.0025\left(\frac{1.0025^n - 1}{1.0025 - 1}\right)$ $= 401M(1.0025^n - 1)$	2 marks for correct working and solution. Must show the geometric progression. 1 mark for showing geometric progression. 1 mark for only using the formula without showing the geometric progression.

Q37-iii	$S_n = 6000$ $6000 = 401M(1.0025^{12} - 1)$ $M = \dfrac{6000}{401 \times (1.0025^{12} - 1)}$ $= \$491.93$	2 marks for correct solution 1 mark for equation however did not progress and solve.

Q37-iv	**Amount saved after 5 months** $S_n = 401 \times 491.93(1.0025^5 - 1)$ $= \$2478.16$ Amount left to save = $\$6000 - 2478.16 = \$3521.84$ **Time required for \$600 payment to reach goal of \$6000.** $S_n = 3521.84$ $3521.84 = 401 \times 600(1.0025^n - 1)$ $1.0025^n = \frac{3521.84}{401 \times 600} + 1 = 1.01463$ $n = \frac{\ln(1.01463)}{\ln 1.0025} = 5.81$ therefore 5 months $< n <$ 6 months **5 months total \$600** $S_n = = 401 \times 600(1.0025^5 - 1) = \$3022.575$ **5 months compound interest for \$2478.16** $S_5 = 2478.16 \times (1.0025)^5 = \$2509.29$ Combined total after 10 months = 2509.29 +3022.58 = 5531.87 Last month payment = \$6000 - \$5531.87=\$468.13 = \$470 (nearest \$5)	3 marks – correct answer . 2 marks - Compound interest amount of \$2509.29 - Amount = \$3022.57 - Have found 2 (incorrect) subtotals by considering compound interest and subtracted their 2 subtotals from \$6000 1 mark – - $n = 5.81$