NEWCASTLE GRAMMAR SCHOOL

YEAR 12
2003
MATHEMATICS
TRIAL EXAMINATION

Time allowed – Three hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question.
  Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 10.
- Board-approved calculators may be used.
- Answer each question in a SEPARATE Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

QUESTION 1 Use a SEPARATE Writing Booklet. **Marks**

a) Calculate the value of $\dfrac{\sqrt{4\pi}}{3\cdot 6^2 - 9\cdot 8}$ correct to four significant figures **2**

b) Express $\dfrac{6}{\sqrt{3}-1}$ with a rational denominator **2**

c) Differentiate $6 - x^3$ **2**

d) Solve $\dfrac{x}{2} + \dfrac{x}{3} = 1$ **2**

e) Integrate $\dfrac{4}{x}$ **2**

f) Factorise completely $9 - 16t^2$ **2**

QUESTION 2 Use a SEPARATE Writing Booklet.

a) Differentiate:

i) $y = e^{\sin x} + \dfrac{x^4}{2}$ **3**

ii) $y = \dfrac{\log_e x}{x}$ **3**

b) Sketch the graph with the equation $y = x - x^2$ <u>showing all intercepts</u>: **2**

c) Solve $|x + 4| = 1$ **2**

d) Give the <u>exact</u> value for $\sec 210°$ **2**

QUESTION 3 Use a SEPARATE Writing Booklet. **Marks**

a) The first term of an arithmetic sequence is 6 and the common difference is 9. **3**

i) Write down the expression for the n^{th} term

ii) Which term of this sequence is 4623 ?

b) Consider the points O (0, 0), A (-1, 3) and B (11, -6)

i) Find the gradient of line AB **1**

ii) Show that the equation of AB is $3x + 4y - 9 = 0$ **2**

iii) Find the equation of line L, which passes through O and is parallel to line AB **2**

iv) The point P, (4, k), lies on line L. Find the value of k **2**

v) Calculate the perpendicular distance from P to AB **2**

QUESTION 4 Use a SEPARATE Writing Booklet. **Marks**

a) Find

i) $\int \cos 2x \, dx$ **2**

ii) $\int \frac{dx}{2x+3}$ **2**

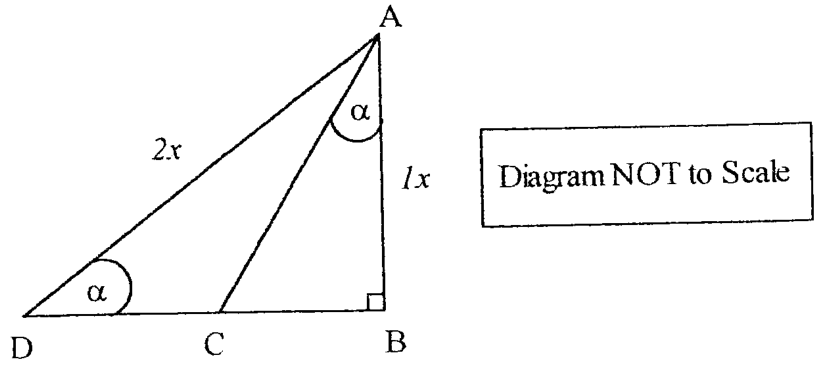
iii) $\int e^{3x} \, dx$ **2**

b) Bank X pays compound interest, compounded annually. Bank Y pays simple interest. $5000 is invested in Bank X and also in Bank Y at 9% p.a. for 6 years - at both banks. Find the difference between the compound interest and simple interest earned at each bank. **3**

c) For what values of k is $(3-k)x^2 + (3-k)x + 1$ positive definite ? **3**

QUESTION 5 Use a SEPARATE Writing Booklet. **Marks**

a) In the diagram below, AD = 2 × AB and ∠ADC = ∠BAC **5**



i) By writing an expression for sinα, show that $\alpha = 30°$

ii) Hence find the size of ∠DAC

iii) If DC = 2 cm find the length of AB

b) Solve $9^x + 6 \times 3^x - 27 = 0$ **3**

c) There are five nominees for President and Vice President of a club. Three are women and two are men. The first name, selected at random, will be the President and the second name will be the Vice President. **4**

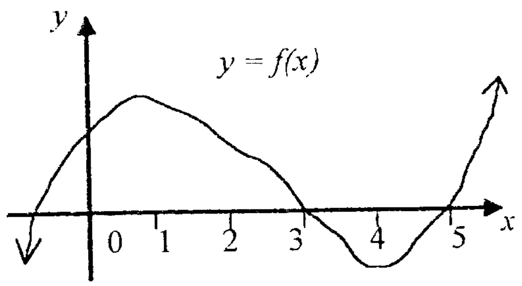
i) Draw a tree diagram to represent all possible outcomes

ii) Determine the probability that the two positions will be filled by a woman and a man, in either order.

QUESTION 6 Use a SEPARATE Writing Booklet. **Marks**

a) Given the graph of $y = f(x)$, <u>EXPLAIN</u> why **3**

$\int_0^4 f(x)dx$ is LESS than $\int_0^3 f(x)dx$



b) A sector of a circle, of radius 1 cm, has a <u>perimeter</u> of 4 cm. **4**

i) Show that the angle at the centre of the sector is 2 radians

ii) Find the area of the sector

c) Use Simpson's Rule with 5 function values (i.e. 4 strips) to find an approximation for $\int_1^5 (\log_e x)dx$ correct to 3 decimal places **5**

QUESTION 7 Use a SEPARATE Writing Booklet. **Marks**

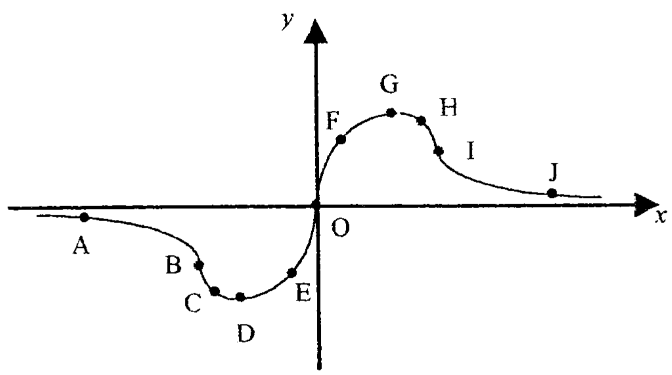
a) The rate of decay of a radioactive substance is proportional to the mass, M, present at time, t years, i.e. $\frac{dM}{dt} = -kM$ **5**

i) Show that $M = M_0 e^{-kt}$ satisfies $\frac{dM}{dt} = -kM$

ii) If the half-life of the substance is 17 600 years, find k (correct to 6 decimal places)

iii) How long will it take for $\frac{2}{3}$ (two thirds) of the substance to decay

b) For the given graph of $y = f(x)$ write down which of the labelled point(s) best demonstrate the properties below: **7**



i) $f(x) = 0$

ii) $f'(x) = 0$

iii) $f''(x) = 0$

iv) $f(x) > 0$

v) $f'(x) > 0$

vi) $f''(x) > 0$

vii) $\lim_{x \to \infty} f(x) = 0$

QUESTION 8 Use a SEPARATE Writing Booklet. **Marks**

a) i) Differentiate $y = \cos^3 x$ **5**

ii) Hence, evaluate $\int_0^{\frac{\pi}{4}} (\cos^2 x \sin x) dx$

b) Consider the parabola with the equation $x^2 - 8x = 12y - 28$ **5**

i) Show that the equation can be written as $(x-4)^2 = 12(y-1)$

ii) Find the coordinates of the vertex

iii) Find the coordinates of the focus

iv) Find the equation of the directrix

c) Find k if $\int_1^k \left(\frac{1}{x}\right) dx = 1$ **2**

QUESTION 9 Use a SEPARATE Writing Booklet.

A particle moves along the x-axis so that its displacement, x metres, after t seconds is given by $x = 3 - 2\cos t$ **12**

i) Find the initial displacement

ii) Show that the particle starts from rest

iii) When does the particle <u>next</u> come to rest ?

iv) Find the velocity when the particle passes through $x = 2$ for the second time

v) Find the particle's greatest velocity

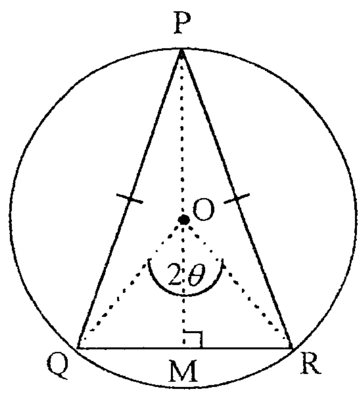
vi) Find the particle's position when it is NOT being accelerated

QUESTION 10 Use a SEPARATE Writing Booklet. **Marks**

a) i) Show that $\frac{1}{x^2-9} = \frac{1}{6}\left(\frac{1}{x-3} - \frac{1}{x+3}\right)$ **5**

ii) Hence find the exact volume generated by revolving $y = \frac{1}{\sqrt{x^2-9}}$ around the x-axis from $x=5$ to $x=6$

b) Isosceles triangle PQR is in a circle of radius 1 unit, centre O. $\angle QOR = 2\theta$ (θ is acute). PO is extended to meet QR at M such that $\angle OMR = 90°$ **7**



i) Prove that $QM = \sin\theta$ and $OM = \cos\theta$

ii) Show that the area, A, of ΔPQR is given by $A = \sin\theta(\cos\theta + 1)$

iii) Hence show that ΔPQR has a maximum area when it is equilateral

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$
$\int \frac{1}{x} dx$	$= \ln x, \quad x>0$
$\int e^{ax} dx$	$= \frac{1}{a} e^{ax}, \quad a \neq 0$
$\int \cos ax \, dx$	$= \frac{1}{a} \sin ax, \quad a \neq 0$
$\int \sin ax \, dx$	$= -\frac{1}{a} \cos ax, \quad a \neq 0$
$\int \sec^2 ax \, dx$	$= \frac{1}{a} \tan ax, \quad a \neq 0$
$\int \sec ax \tan ax \, dx$	$= \frac{1}{a} \sec ax, \quad a \neq 0$
$\int \frac{1}{a^2 + x^2} dx$	$= \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} dx$	$= \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE : $\ln x = \log_e x, \quad x > 0$

(5) continued:

(b) $9^x + 6 \times 3^x - 27 = 0$

$9^x = (3^2)^x = (3^x)^2$

$\therefore$ Letting $A = 3^x$ gives: ½

$$A^2 + 6A - 27 = 0$$
$$(A+9)(A-3) = 0$$ ½

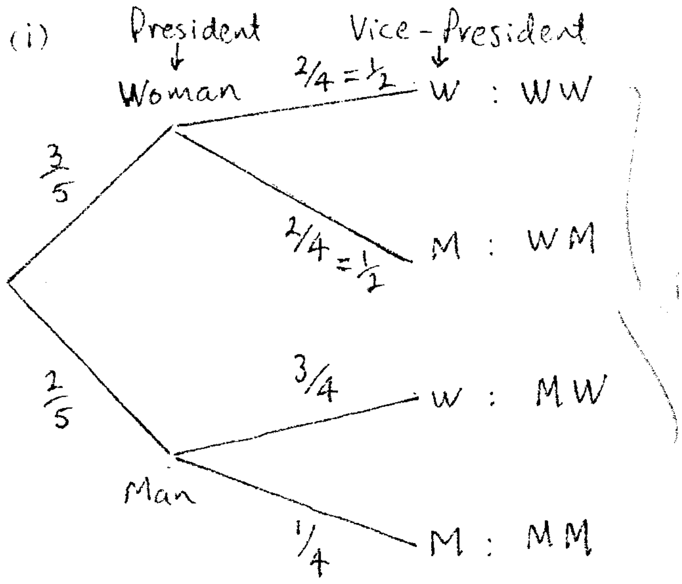
$\therefore A = -9$ or $A = 3$

i.e. $3^x = -9$ ½ $\quad 3^x = 3$ ½

no solution ½ $\quad 3^x = 3^1$ ½

$\therefore \boxed{x = 1}$

(c) (i)



Note President can not be Vice-Pres.
$\therefore$ names selected but not replaced

(ii) $P(WM \text{ or } MW)$

$= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{3}{4}$

$= \frac{3}{10} + \frac{3}{10}$ ½

$= \boxed{\frac{3}{5}}$ (or 60%) ½

(6) (a) $\int_0^3 f(x)\,dx$ is positive, say $+9$: ......(1)

$\int_3^4 f(x)\,dx$ is negative, say -2 ½

(NOTE Area from 0 to 3 > Area from 3 to 4 ½

or $|+9| > |-2|$)

Now: $\int_0^4 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^4 f(x)\,dx$

$= 9 + (-2)$

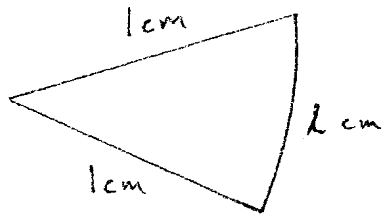
$= +7$ ..... (2) ½

$\therefore$ Comparing (1) and (2):

$7 < 9$ ½

i.e. $\int_0^4 f(x)\,dx < \int_0^3 f(x)\,dx$ (QED)

(b)



(i) $l + 1 + 1 = 4\text{cm}$ (perimeter) ½

$\therefore l + 2 = 2$

$l = 2\text{cm}$

$l = r\theta \therefore \theta = l/r$ 1

$= 2/1$

i.e. $\boxed{\theta = 2 \text{ rad.}}$ QED ½

(ii) $A = \frac{1}{2} r^2 \theta$ 1

$= \frac{1}{2} \times 1^2 \times 2$ ½

$\therefore \boxed{A = 1\,\text{cm}^2}$ ½

1)(a) on calculator:

$\sqrt{(4\pi)} \div (3.6 \times^2 - 9.8) = 1.1218\ldots$ (4 sig. fig's)

$\therefore$ Answer $= 1.122$

(b) $\frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{6(\sqrt{3}+1)}{(\sqrt{3})^2 - (1)^2}$

$= \frac{6(\sqrt{3}+1)}{3-1}$

$= \frac{{}^{3}\cancel{6}(\sqrt{3}+1)}{\cancel{2}_{1}}$

$= 3(\sqrt{3}+1)$

(c) $d/dx\,(6 - x^3) = 0 - 3x^2$

$= -3x^2$

(d) $\frac{x}{2} + \frac{x}{3} = 1$ (×6)

$\therefore 3x + 2x = 6$

$\therefore 5x = 6$

$x = 6/5$ or $1\frac{1}{5}$ or 1.2

(e) $\int 4/x \, dx = 4 \int \frac{1}{x} dx$

$= 4 \log_e x + c$

(f) $9 - 16t^2 = (3)^2 - (4t)^2$

$\{a^2 - b^2 = (a+b)(a-b)\}$

$= (3 + 4t)(3 - 4t)$

2)(a)(i) $y = e^{\sin x} + \frac{1}{2}x^4$

$\therefore \frac{dy}{dx} = \cos x \times e^{\sin x} + \frac{1}{2} \times 4x^3$

$= \cos x \cdot e^{\sin x} + 2x^3$

(ii) $y = \frac{\log_e x}{x}$ → $u = \log_e x$, $v = x$; $u' = 1/x$, $v' = 1$

$\therefore y' = \frac{vu' - uv'}{v^2}$

$= \frac{x \times \frac{1}{x} - \log_e x \times 1}{x^2}$ 1

$= \frac{1 - \log_e x}{x^2}$ ½

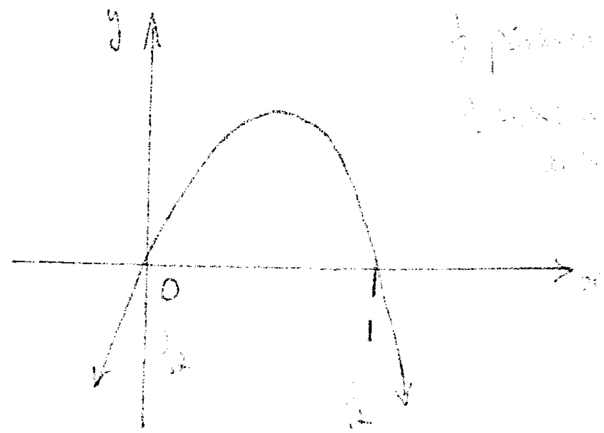
(b) $y = -x^2 + x + 0$ ← y-intercept

and for $x - x^2 = 0$

$x(1 - x) = 0$

$\therefore x = 0$ or 1 ← x-intercepts

and parabola; concave down



② continued:

(c) $|x+4| = 1$

$\therefore x+4 = 1$ ½ or $-(x+4) = 1$ ½

$\quad -4 \quad -4$

$\therefore x+4 = -1$

$\quad -4 \quad -4$

$\therefore \boxed{x = -3 \text{ or } \therefore x = -5}$ ½ ½

(d) $\sec 210° = \dfrac{1}{\cos 210°}$ ½

and for $\cos 210°$:

3rd quad : $\cos(180°+30°)$ } ½

$= -\cos 30°$

$= -\sqrt{3}/2$ ½

$\therefore \sec 210° = \dfrac{1}{-\sqrt{3}/2}$ } ½

{Answer can be checked on calculator} $= \boxed{-2/\sqrt{3}}$

③(a) Arithmetic, $a = 6$, $d = 9$.

(i) $T_n = a + (n-1)d$ ½

$= 6 + (n-1) \times 9$ } ½

$= 6 + 9n - 9$

i.e. $\boxed{T_n = 9n - 3}$ ½

(ii) Let $9n - 3 = 4623$ ½

$\quad +3 \quad +3$

$9n = 4626$ } ½

$\div 9 \quad \div 9$

$n = 514$

i.e. $\boxed{514\text{th term}}$ ½

(b)(i) Gradient$_{AB} = \dfrac{-6-3}{11-(-1)}$ $\left(\dfrac{y_2-y_1}{x_2-x_1}\right)$ } ½

$= \dfrac{-9}{12}$ } ½

$= \boxed{-3/4}$

(ii) $y - y_1 = m(x - x_1)$ ½

$\times 4 \quad \times 4$

$y - 3 = -3/4\,(x+1)$ $\times 4$ ½

$\therefore 4y - 12 = -3x - 3$ ½

$+3x + 3 \qquad +3x + 3$

$\therefore \boxed{3x + 4y - 9 = 0}$ ½ (QED!)

(or: can be done by showing – by substitution – that A and B satisfy equation). (OR 1)

(iii) // to AB $\therefore m_L = -3/4$ ½

and through origin: $y = mx + b$ ½ ($b \to 0$)

i.e. $y = mx$

$\therefore \boxed{y = -3/4\,x}$ } 1

(iv) $(4, k)$ on L $\therefore k = -3/4 \times 4$ 1

$\boxed{k = -3}$ 1

(v) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ ½

$= \dfrac{|3 \times 4 + 4 \times -3 + -9|}{\sqrt{3^2+4^2}}$ ½

$= \dfrac{|12 - 12 - 9|}{\sqrt{25}}$ ½

$= |-9|$ ½ ... $9/$... $= 1.8$ ½

4 (a)(i) $\int \cos ax \, dx = \frac{1}{a}\sin ax + c$

(from Standard Integrals Sheet)

$\therefore \int \cos 2x \, dx = \frac{1}{2}\sin 2x + c$

(ii) $\int \frac{dx}{2x+3} = \frac{1}{2}\int \frac{2}{2x+3} \, dx$

$= \frac{1}{2}\log_e(2x+3) + c$

(iii) $\int e^{3x} \, dx = \frac{1}{3}\int 3e^{3x} \, dx$

$= \frac{1}{3}e^{3x} + c$

(b) X: $A = P(1 + r/100)^n$

$= 5000\,(1 + 9/100)^6$

$= 5000 \times 1.09^6$

$= \$8385.50$ (rounded)

$\therefore$ C.I. $= 8385.50 - 5000$

$= \$3385.50$ .... (1)

Y: $SI = \frac{Prn}{100}$

$= \frac{5000 \times 9 \times 6}{100}$

$= \$2700$ ...... (2)

$\therefore$ Difference $= (1) - (2)$

$= 3385.50 - 2700$

$= \$685.50$

(c) Pos. definite if:

$a > 0$ **AND** $\Delta < 0$

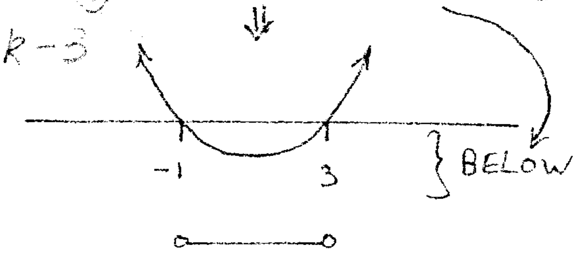
ie. $3 - k > 0$ $\quad b^2 - 4ac < 0$

$3 > k$ $\quad (3-k)^2 - 4(3-k) < 0$

or $k < 3$ ....(1) $\quad (3-k)[(3-k) - 4] < 0$

$(3-k)(-1-k) < 0$

$y = k^2 - 2k - 3$



ie $-1 < k < 3$ .....(2)

from (1) AND (2) we have:

$-1 < k < 3$

5 (a)(i) $\sin\alpha = \frac{AB}{AD}$ (from $\triangle ABD$)

$= \frac{1x}{2x}$

$= \frac{1}{2}$

$\therefore \alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$\alpha = 30^\circ$ (QED)

(ii) $\angle ADC = 30^\circ$ (above)

$\therefore \angle BAD = 60^\circ$ ($\angle$ sum of $\triangle ABD$)

$\therefore \angle DAC = 30^\circ$

(iii) $\triangle ACD$ is isosceles

$\therefore AC = DC = 2$

$\therefore$ In $\triangle ABC$:

$\cos\alpha = AB/2$

ie $\cos 30^\circ = AB/2 = \sqrt{3}/2$

$AB = \sqrt{3}$ ...

6 continued:

(c) $h = \frac{b-a}{n}$ ← 5 function values = 4 strips $\frac{1}{2}$

$= \frac{5-1}{4}$

$= 1$ (strip width) $\frac{1}{2}$

x	y ($\log_e x = \ln x$)	×	=
1	$\ln 1 = 0$	1	0
2	$\ln 2 = 0.6931$	4	2.7724
3	$\ln 3 = 1.0986$	2	2.1972
4	$\ln 4 = 1.3863$	4	5.5452
5	$\ln 5 = 1.6094$	1	1.6094
$\frac{1}{2}$	$\frac{1}{2}$ + $\frac{1}{2}$	$\frac{1}{2}$ TOTAL =	12.1242

$\therefore \int_1^5 \log_e x\, dx \doteq \frac{h}{3} \times \text{TOTAL}$ $\frac{1}{2}$

$= \frac{1}{3} \times 12.1242$ $\frac{1}{2}$

$= 4.0414$

$= \boxed{4.041}$ (3dp) $\frac{1}{2}$

(−$\frac{1}{2}$ for each "error")

(NOTE: Graph of $y = \log_e x$ above x-axis for $x = 1$ to $x = 5$

$\therefore \int_1^5 \log_e x\, dx = \text{Area}$)

7 (a) (i) $M = M_0 e^{-kt}$

$\therefore \frac{dM}{dt} = M_0 \times -ke^{-kt}$ $\frac{1}{2}$

$= -k\left(M_0 e^{-kt}\right)$ $\frac{1}{2}$

ie. $\frac{dM}{dt} = -kM$ (QED) $\frac{1}{2}$

$1\frac{1}{2}$

(ii) $M = \frac{1}{2} M_0$ at $t = 17\,600$

ie. $\frac{1}{2} M_0 = M_0 e^{-17600k}$ $\frac{1}{2}$

$\therefore \frac{1}{2} = e^{-17600k}$

$\ln \frac{1}{2} = -17600k$ $\frac{1}{2}$

$\therefore k = \ln \frac{1}{2} \div -17600$

ie. $\boxed{k = 0.000039}$ (6dp) $\frac{1}{2}$

$1\frac{1}{2}$

(iii) For $^2/_3$ decayed, $M = \frac{1}{3} M_0$ $\frac{1}{2}$

ie. $\frac{1}{3} = e^{-0.000039t}$ $\frac{1}{2}$

$\ln \frac{1}{3} = -0.000039t$

$\therefore t = \ln {}^1/_3 \div -0.000039$ $\frac{1}{2}$

$= 28169.5\ldots$

ie. $\boxed{28\,200 \text{ years}}$ $\frac{1}{2}$ (3 sig. figs)

b) (i) $f(x) = 0 \rightarrow x$-intercept(s) : $\boxed{O}$

(ii) $f'(x) = 0 \rightarrow$ st. points : $\boxed{D, G}$

(iii) $f''(x) = 0 \rightarrow$ inf. pts : $\boxed{B, O, I}$

(iv) $f(x) > 0 \rightarrow$ above x-axis : $\boxed{F, G, H, I, J}$

(v) $f'(x) > 0 \rightarrow$ increasing : $\boxed{E, O, F}$

(vi) $f''(x) > 0 \rightarrow$ concave up : $\boxed{C, D, E, J}$

(v) $\lim_{x \to \infty} f(x) = 0 \rightarrow$ approaches x-axis as x ↑ in positive direction : $\boxed{J}$

correct $\left(+\frac{1}{2}\right)$

incorrect $\left(-\frac{1}{2}\right)$

(8)(a)(i) $y = \cos^3 x$

$\therefore y = (\cos x)^3$

$\therefore \frac{dy}{dx} = 3(\cos x)^2 \times -\sin x$

$= -3\cos^2 x \sin x$ ½

(ii) $\int_0^{\frac{\pi}{4}} (\cos^2 x \sin x)\,dx$

$= -\frac{1}{3} \int_0^{\frac{\pi}{4}} (-3\cos^2 x \sin x)\,dx$ ½

$= -\frac{1}{3} \left[\cos^3 x \right]_0^{\frac{\pi}{4}}$ (from (i)) ½

$= -\frac{1}{3} \left[(\cos \pi/4)^3 - (\cos 0)^3 \right]$ ½

$= -\frac{1}{3} \left[\left(\frac{1}{\sqrt{2}}\right)^3 - (1)^3 \right]$

$= -\frac{1}{3} [0.3536 - 1]$ (4dp)

$= 0.215$ 3dp ½

(or $-\frac{1}{3}\left(\frac{1}{2\sqrt{2}} - 1\right) = \frac{1}{3}\left(1 - \frac{1}{2\sqrt{2}}\right)$)

(b)(i) $x^2 - 8x + 16 = 12y - 28 + 16$ ½

$(-8/2)^2$

$\therefore (x-4)^2 = 12y - 12$ ½ } can also be done by removing brackets and rearranging

ie. $(x-4)^2 = 12(y-1)$ ½ (QED)

Using: $\Rightarrow (x-h)^2 = 4a(y-k)$

Vertex at (h,k), focal length $= a$

$\therefore$(ii) Vertex $(4, 1)$

(iii) Concave up $\therefore$ Focus at

$(4, 1+a)$ ½

where $4a = 12$ $\therefore a = 3$ ½

$\therefore$ Focus at: $(4, 1+3)$ ½

(iv) Directrix:

$y = 1 - a$ ½

$= 1 - 3$

ie. $y = -2$ ½

(c) $\int_1^k \frac{1}{x}\,dx = 1$

$\therefore [\ln x]_1^k = 1$ ½

$\therefore \ln k - \ln 1 = 1$ ½

$\therefore \ln k - 0 = 1$ ½

$\ln k = 1$

or $\log_e k = 1$ ½

$\therefore e^1 = k$

ie. $k = e$

(9) $x = 3 - 2\cos t$

(i) at $t = 0$, $x = 3 - 2\cos 0$ ½

$= 3 - 2 \times 1$ ½

$= 3 - 2$

$\therefore x = +1\text{m}$

(ii) $x = 3 - 2\cos t$

$\therefore v = \dot{x} = 0 - 2 \times (-\sin t)$

$= 2\sin t$ ½

$\therefore$ at $t = 0$, $v = 2\sin 0$ ½

$= 2 \times 0$

$v = 0 \rightarrow$ stationary

(9) continued:

(iii) Rest $\rightarrow v = 0$ ½

$\therefore 2\sin t = 0 \quad (\div 2)$

$\sin t = 0$ ½

$\therefore t = 0, \pi, 2\pi, \ldots$ ½

↑ $\therefore$ next time

$\therefore$ $t = \pi$ seconds ($\doteqdot 3.1$ sec.) ½

(iv) When $x = 2$:

$2 = 3 - 2\cos t$ ½

$\therefore 2\cos t = 3 - 2 = 1$ ½

$\therefore \cos t = \frac{1}{2}$ (gap angle $= 60°$ or $\pi/3$)

$\therefore t = \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3}, \; \frac{\pi}{3} + 2\pi, \ldots$ ½

1st quad, 4th quad, 1st quad + 1 revolution

↑ second time ½

$\therefore t = 5\pi/3$ sec.

$\therefore v = 2\sin(5\pi/3)$

$= 2 \times -\sqrt{3}/2$ ½

$\therefore$ $v = -\sqrt{3}$ m/s ½

(v) Maximum (greatest) velocity when $\frac{dv}{dt} = 0$ (i.e. $a = \ddot{x} = 0$) ½

Here: $\ddot{x} = 2\cos t$ ½

$\therefore$ for $2\cos t = 0$

$\cos t = 0$

$t = \pi/2, \; 3\pi/2, \ldots$ ½

$\therefore v_{max} = 2\sin \pi/2$

$= 2 \times 1$

$=$ 2 m/s ½

OR $v = 2\sin t$

Amplitude $= 2$ 1

$\therefore$ $v_{max} = 2$ m/s 1

} "graphical" approach

(vi) $\ddot{x} = a = 2\cos t$

$\therefore$ for $a = 0$: $2\cos t = 0$ ½

$\cos t = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ ½

$\therefore$ at $t = \pi/2$:

$x = 3 - 2\cos \pi/2$

$= 3 - 2 \times 1$ ½

$= 3 - 2$

$x = 1$ m ½

(10) (a)(i) Show: $\frac{1}{x^2-9} = \frac{1}{6}\left(\frac{1}{x-3} - \frac{1}{x+3}\right)$

$$RHS = \frac{1}{6}\left(\frac{1 \times x+3}{x-3 \times x+3} - \frac{1 \times x-3}{x+3 \times x-3}\right)$$

$$= \frac{1}{6}\left(\frac{x+3-(x-3)}{x^2-9}\right)$$

$$= \frac{1}{6}\left(\frac{\cancel{x}+3-\cancel{x}+3}{x^2-9}\right)$$

$$= \frac{1}{\cancel{6}}\left(\frac{\cancel{6}}{x^2-9}\right)$$

$$= \frac{1}{x^2-9}$$

$$= LHS \quad (QED)$$

(ii) $$V = \pi \int_5^6 \left(\frac{1}{\sqrt{x^2-9}}\right)^2 dx$$

$$= \pi \int_5^6 \frac{1}{x^2-9}\, dx$$

$$= \pi \int_5^6 \frac{1}{6}\left(\frac{1}{x-3} - \frac{1}{x+3}\right) dx \quad (\text{from (i)})$$

$$= \pi/6 \int_5^6 \frac{1}{x-3} - \frac{1}{x+3}\, dx$$

$$= \pi/6 \left[\ln(x-3) - \ln(x+3)\right]_5^6$$

$$= \pi/6 \left[\ln\left(\frac{x-3}{x+3}\right)\right]_5^6$$

$$= \pi/6 \left[\ln\left(6-3/6+3\right) - \ln\left(\frac{5-3}{5+3}\right)\right]$$

$$= \pi/6 \left[\ln 1/3 - \ln 1/4\right]$$

$$= \pi/6 \ln\left(\frac{1/3}{1/4}\right)$$

$$= \pi/6 \ln 4/3 \text{ units}^3 \quad (\doteqdot 0.15 \text{ 2dp})$$

3.

(b)(i) $\angle MOQ = \frac{1}{2} \times \angle ROQ$

$= \theta$

In $\triangle MOQ$: $\sin\theta = \frac{QM}{QO}$ ← QO = radius = 1

ie $\sin\theta = \frac{QM}{1}$

$\therefore QM = \sin\theta$ (QED)

Similarly: $\cos\theta = \frac{OM}{QO}$

$\therefore OM = \cos\theta$ (QED)

(ii) Area $\triangle PQR = \frac{1}{2}bh$

$= \frac{1}{2} \times QR \times MP$

but: $\frac{1}{2} \times QR = QM = \sin\theta$ ....(1)

and: $MP = OM + OP$

$= \cos\theta + \text{radius}$ ← 1

$= \cos\theta + 1$ ......(2)

$\therefore$ from (1)/(2): Area $= \sin\theta(\cos\theta + 1)$ (QED)

(iii) Area_{max} when $\frac{dA}{d\theta} = 0$ $(= A')$

Using product rule: $u = \sin\theta$, $v = (\cos\theta + 1)$
$u' = \cos\theta$ $v' = -\sin\theta$

$\therefore A' = (\cos\theta + 1)\cos\theta + \sin\theta \times (-\sin\theta)$

$= \cos^2\theta + \cos\theta - \sin^2\theta$

$= \cos^2\theta + \cos\theta - (1 - \cos^2\theta)$

$= 2\cos^2\theta + \cos\theta - 1$

$\therefore$ for: $2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\therefore \cos\theta = 1/2$ or $\cos\theta = -1$

$\therefore \theta = 60°$ (or 270°: not possible)

and if $\theta = 60°$, $\angle QOR = 120°$

$\therefore \angle PQR = \angle PRQ = 60°$, $\therefore$ equilateral

AND:

θ	59°	60°	61°
A'	+0.05	0	-0.05

} easier than A'' test?

$\Rightarrow$ + 0 − $\therefore A_{max}$ {when equilateral