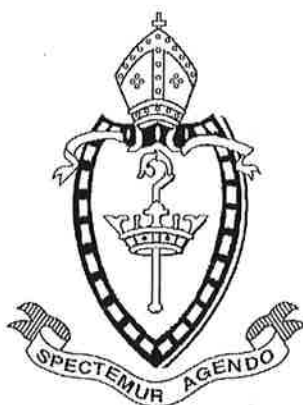


# NEWCASTLE GRAMMAR SCHOOL



## YEAR 12 2005 MATHEMATICS TRIAL EXAMINATION

*Time allowed – Three hours  
(Plus 5 minutes reading time)*

### DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 8.
- Board-approved calculators may be used.
- Answer each question in a SEPARATE Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

**QUESTION 1** Use a SEPARATE Writing Booklet.**Marks**

- a) Calculate the value of  $\sqrt[3]{2^{1.9} + 7}$  correct to four significant figures. 2
- b) Factorise completely  $6x^3 - 48$  2
- c) Express  $300^\circ$  in radians, in terms of  $\pi$  2
- d) State the domain of the function with the equation  $f(x) = \sqrt{3 - x}$  2
- e) Differentiate  $\frac{4}{x}$  2
- f) Simplify completely  $\log_a a^2 - \log_a \frac{1}{a}$  2

**QUESTION 2** Use a SEPARATE Writing Booklet.

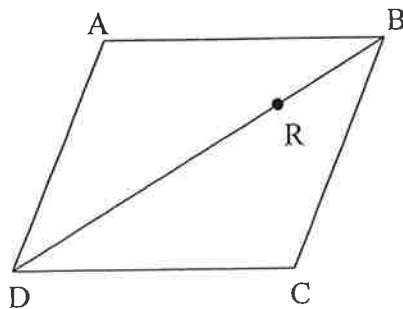
- a) Differentiate:
- i)  $y = x^5 e^x$  3
- ii)  $y = \sqrt{\sin x}$  3
- b) The points  $A(-2,3)$ ,  $B(3,8)$  and  $C(10,9)$  are three of the vertices of parallelogram  $ABCD$ .
- i) Show that the coordinates of  $D$  are  $(5,4)$ . 2
- ii) Find the coordinates of the point  $K$  where the diagonals meet. 2
- iii) Prove that this parallelogram is a rhombus. 2

**QUESTION 3** Use a SEPARATE Writing Booklet.**Marks**

- a) The probability that Mary-Anne catches no fish in any one day is 0.6. Find the probability that when Mary-Anne goes fishing for one week that she has at least one day where she does catch fish, answer correct to 2 decimal places. 2
- b) Find:
- i)  $\int \sqrt{2x+1} dx$  3
- ii)  $\int \frac{2x}{4x^2+1} dx$  3
- c) Solve  $2 \sin^2 \theta + \sin \theta = 0$  for  $0^\circ \leq \theta \leq 360^\circ$  4

**QUESTION 4** Use a SEPARATE Writing Booklet.

- a) For the function with the equation  $y = 3 \sin 2x$ :
- i) state the period of the function 1
- ii) state the amplitude of the function 1
- iii) sketch the graph of the function for  $0 \leq x \leq 2\pi$  2
- b) Find the values of  $k$  for which  $x^2 - 2kx + 1 = 0$  has real roots 4
- c) ABCD is a rhombus. R is any point on diagonal BD.



- i) Prove that triangles ARD and CRD are congruent. 3
- ii) Hence, show that  $AR=RC$ . 1

**QUESTION 5** Use a SEPARATE Writing Booklet.

Marks

- a) For the parabola with the equation  $y = x^2 - 4$  find the
- i) coordinates of the focus and the equation of the directrix. **4**
  - ii) exact volume generated when the area in the fourth quadrant, between the curve and the axes, is rotated around the  $y$ -axis. **3**
- b) Show, using calculus, that the graph with the equation  $y = x^3(x - 4)$  has a horizontal inflection point at  $x = 0$ . **5**

**QUESTION 6** Use a SEPARATE Writing Booklet.

- a) If  $\alpha$  and  $\beta$  are the roots of the equation  $3x^2 - 2x + 1 = 0$  find the value of  $\alpha^2\beta + \alpha\beta^2$  **3**
- b) The area of an isosceles triangle, in which the two equal sides each have a length of 6 cm, is  $10 \text{ cm}^2$ . Calculate the angle between the two equal sides, correct to the nearest minute. **4**
- c) i) Use the trapezoidal rule, with 4 sub-intervals (strips) to estimate the area between the curve  $y = e^x$  and the  $x$ -axis, from  $x = -1$  to  $x = 1$ , correct to 3 decimal places. **3**
- ii) Calculate the percentage error in this estimated area. **2**

## QUESTION 7 Use a SEPARATE Writing Booklet.

Marks

- a) A loan of \$50 000 is taken out by a small business. The loan plus interest and charges are to be repaid at the end of each month in equal monthly instalments, \$M, over 8 years. Interest of 12% p.a. on the balance owing at the start of each month is added to the account at the end of each month. Additionally, at the end of each month a management charge of \$15 is added to the account. Let  $A_n$  be the amount owing after  $n$  months.

i) Show that  $A_1 = 50000 \times 1.01 - (M - 15)$  2

ii) Show that  $A_2 = 50000 \times 1.01^2 - (M - 15)(1 + 1.01)$  2

iii) Find the amount of each monthly instalment, \$M. 4

- b) A large tank of liquid chemical which contains  $L$  litres of chemical is being drained. The amount of chemical in the tank over time,  $t$  minutes, is given by:

$$L = 120(40 - t)^2$$

i) At what rate is the water draining out of the tank after 6 minutes? 2

ii) How long will it take for the tank to be completely empty? 2

## QUESTION 8 Use a SEPARATE Writing Booklet.

- a) The number of bees,  $B$ , in a hive after  $t$  days is given by  $B = 1200e^{kt}$ , where  $k$  is a constant.

i) Show that  $B = 1200e^{kt}$  satisfies  $\frac{dB}{dt} = kB$  2

ii) If there are 4 200 bees after 6 days, find the population after a further 10 days. 4

b) i) For  $y = \frac{\log_e \sqrt{x}}{x}$  show that  $\frac{dy}{dx} = \frac{1}{2} \left( \frac{1 - \log_e x}{x^2} \right)$  3

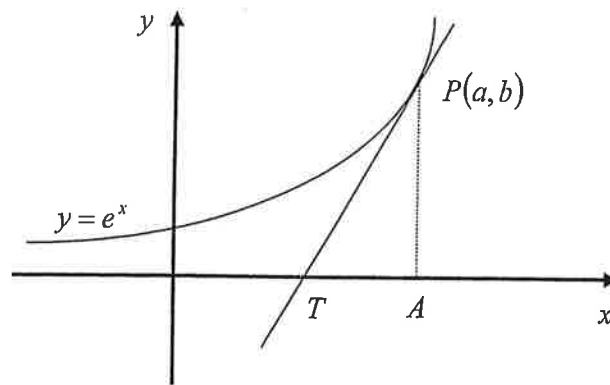
ii) Hence, evaluate  $\int_1^e \frac{1 - \log_e x}{x^2} dx$ , answer in exact form. 3

QUESTION 9 Use a SEPARATE Writing Booklet.

Marks

a) Solve  $2 \log_e x = \log_e (x + 6)$  3

b) The point  $P(a, b)$  lies on the curve with the equation  $y = e^x$ .



i) Show that the tangent at  $P$  has the equation 3

$$y - e^a = e^a(x - a)$$

ii) Hence, find the coordinates of  $T$ , the point where the tangent meets the  $x$ -axis. Give your answer in terms of  $a$ . 2

iii)  $A$  is the foot of the perpendicular from  $P$  to the  $x$ -axis. Show that the length of the interval  $TA$  is constant, for any point  $P$ . 2

c) Find  $\int \tan x \, dx$  2

## QUESTION 10 Use a SEPARATE Writing Booklet.

Marks

Two particles A and B move along the  $x$ -axis, both starting when  $t = 0$ . The displacement of particle A is given by  $x = 4t + 21 - t^2$ , where  $x$  is measured in metres and  $t$  is in seconds. The displacement of particle B is given by  $x = 2t(t - 7)$ .

- |      |  |   |
|------|--|---|
| i)   | Find when and where particle A is stationary.  | 2 |
| ii)  | On the same diagram, sketch each particle's displacement graph, showing <u>only</u> the $t$ -intercepts.     | 3 |
| iii) | Show that the particles meet <u>only</u> after 7 seconds.  | 1 |
| iv)  | Show that the distance, $D$ , between the two particles at any time, $t$ , is given by $D = 18t + 21 - 3t^2$ | 1 |
| v)   | During the first 7 seconds, when are the particles furthest apart?   | 3 |
| vi)  | Find the time when both particles have the same velocity.  | 2 |

1)  $\sqrt[3]{2^{1.9} + 7}$  : on calculator :-  
 a)

$$\sqrt[3]{(2^{1.9} + 7)} = 2.20577 \dots$$

4 sig. fig.

=  $\boxed{2.206}$

b)  $6x^3 - 48 = 6(x^3 - 8)$   
 $= 6(x^3 - 2^3)$

using:  $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$\therefore \boxed{6(x-2)(x^2 + 2x + 4)}$

c)  $\frac{300^\circ}{1} \times \frac{\pi}{180} = \frac{5\pi}{3} \text{ rad}$

d) For domain: we need :-

$3 - x \geq 0$

i.e.  $\boxed{3 \geq x \text{ or } x \leq 3}$

e)  $\frac{d}{dx} \left( \frac{4}{x} \right) = 4 \times \frac{d}{dx} \left( \frac{1}{x} \right)$   
 $= 4 \times \log_e x$   
 $= \boxed{4 \log_e x \text{ (or } 4 \ln x)}$

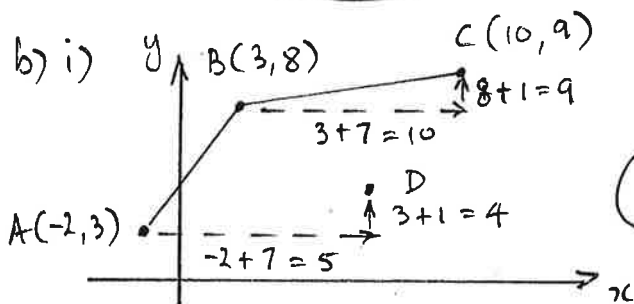
f)  $\log_a a^2 - \log_a \frac{1}{a} = \log_a \left( \frac{a^2}{1/a} \right)$   
 $= \log_a (a^2 \div a^{-1})$   
 $= \log_a (a^3)$   
 $= \underline{\underline{3}}$

2)  $y = x^5 e^x$        $u = x^5$      $v = e^x$   
 a)  $\therefore u' = 5x^4$      $\therefore v' = e^x$

$y' = vu' + uv'$   
 $= e^x \times 5x^4 + x^5 \times e^x$       ③  
 $= \boxed{x^4 e^x (5 + x)}$

b)  $y = \sqrt{\sin x}$   
 $= (\sin x)^{\frac{1}{2}}$       ③

$\therefore \frac{dy}{dx} = \frac{1}{2} (\sin x)^{-\frac{1}{2}} \times \cos x$   
 $= \frac{\cos x}{2\sqrt{\sin x}}$



$\therefore D(5, 4)$  (Q.E.D)

ii) Diagonals of parallelogram BISECT  
 $\therefore$  find midpoint of AC (or BD)

$\therefore K = \left( \frac{-2+10}{2}, \frac{3+9}{2} \right)$       ②  
 $= \boxed{(4, 6)}$

iii) Parallelogram = rhombus  
 if adjacent pair of sides equal  
 $\therefore$  show  $AB = BC$       ②  
 $AB = \sqrt{(3+2)^2 + (8-3)^2} = \sqrt{50}$  units  
 $BC = \sqrt{(10-3)^2 + (9-8)^2} = \sqrt{50}$  units  
 i.e.  $AB = BC \therefore$  rhombus (Q.E.D)



③ a) P(at least 1 day catch fish)

$$= 1 - P(\text{no fish})$$

$$= 1 - (0.6)^7 \leftarrow \text{for 7 days}$$

$$= 0.972 \dots$$

$$= \boxed{0.97}$$

b) i)  $\int \sqrt{2x+1} dx$

$$= \int (2x+1)^{\frac{1}{2}} dx$$

$$= \frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + C$$

using:  $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$

$$= \frac{(2x+1)^{\frac{3}{2}}}{3} + C$$

or  $\frac{1}{3} \sqrt{(2x+1)^3} + C$

ii)  $\int \frac{2x}{4x^2+1} dx$

$$= \frac{1}{4} \int \frac{8x}{4x^2+1} dx$$

$$= \boxed{\frac{1}{4} \log_e(4x^2+1) + C}$$

{ using:  $\int \frac{f'(x)}{f(x)} dx = \log_e(f(x)) + C$  }

(2)

c)  $2 \sin^2 \theta + \sin \theta = 0$

$$\therefore \sin \theta (2 \sin \theta + 1) = 0$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$\therefore \sin \theta = 0 \quad \text{or} \quad 2 \sin \theta + 1 = 0$$

boundary angle  
or

use sine graph  
 $\pi$ -intercepts

$$\therefore \sin \theta = -\frac{1}{2}$$

$$\text{Gap } L = 30^\circ$$

$\sin \theta < 0 \therefore$  3rd/4th quadrants

$$\therefore \theta = 0^\circ, 180^\circ \text{ or } 360^\circ$$

$$\therefore \theta = 180+30^\circ, 360-30^\circ$$

$$\therefore \theta = 210^\circ, 330^\circ$$

$$\therefore \boxed{\theta = 0^\circ, 180^\circ, 210^\circ, 330^\circ, 360^\circ}$$

④ a)  $y = 3 \sin 2x$

i) period:  $P = \frac{2\pi}{n} = \frac{2\pi}{2}$

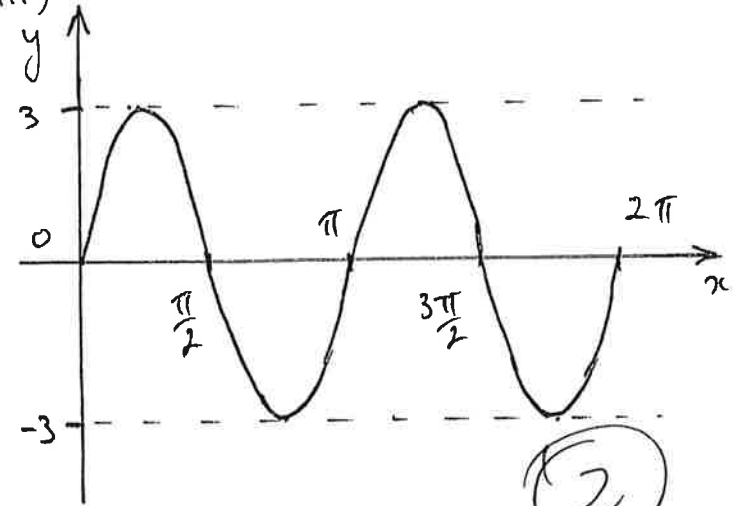
$$\therefore \boxed{P = \pi}$$

①

ii) amplitude =  $\boxed{3}$

①

iii)



②

④ b) For real roots we need:

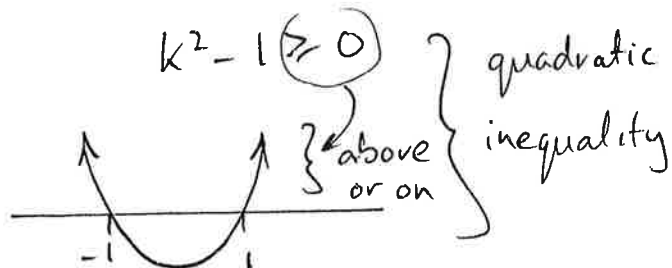
$$\Delta \geq 0$$

ie.  $b^2 - 4ac \geq 0$

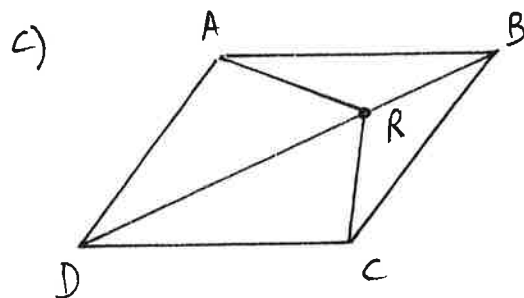
$$\therefore (-2k)^2 - 4 \times 1 \times 1 \geq 0$$

$$\therefore 4k^2 - 4 \geq 0 \quad (\div 4) \quad \textcircled{4}$$

$$k^2 - 1 \geq 0$$



$$\therefore k \leq -1 \text{ or } k \geq 1$$



i)  $AD = CD$  (sides of rhombus) S  
 $\angle ADR = \angle CDR$  (L's bisected by diagonal) A

$DR = DR$  (common) S

$\therefore \triangle ARD \cong \triangle CRD$  (SAS)

ii)  $AR = RC$

corresponding sides in cong.  $\Delta$ 's

③

①

⑤ a)  $y = x^2 - 4$  } down 4 \*

i)  $\therefore x^2 = y + 4$

or:  $(x-0)^2 = 1(y+4)$

ie.  $(x-0)^2 = 4 \times \frac{1}{4} (y - (-4))$

FORMULA:  $(x-h)^2 = 4a(y-k)$

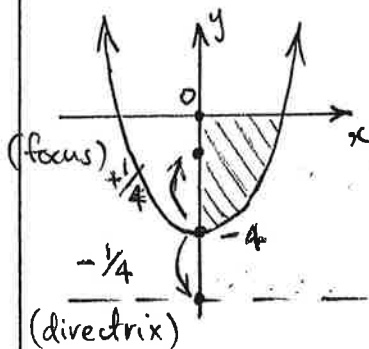
Vertex =  $(h, k)$

Focal length =  $a$

$\therefore$  Vertex at  $(0, -4)$  (\* as above)

Focal length =  $\frac{1}{4}$

$\therefore$  Sketch:



Focus is:

$$(0, -3\frac{3}{4})$$

Directrix is:-

$$y = -4\frac{1}{4}$$

ii)  $V = \pi \int_a^b x^2 dy$  } revolving (around) y-axis

$\therefore V = \pi \int_{-4}^0 (y+4) dy$  } of shaded area in diagram above

$$= \pi \left[ \frac{y^2}{2} + 4y \right]_{-4}^0$$

$$= \pi \left[ \left( \frac{0^2}{2} + 4 \times 0 \right) - \left( \frac{(-4)^2}{2} + 4 \times (-4) \right) \right]$$

$$= \pi [ (0) - 8 + 16 ]$$

$$= 8\pi \text{ units}^3$$

(5) b) Horizontal  $\Rightarrow \frac{dy}{dx} = 0 \dots (1)$

Inflection  $\Rightarrow \frac{d^2y}{dx^2} = 0 \dots (2)$   
AND sign changes

(1)  $y = x^3(x-4)$   
 $= x^4 - 4x^3$

$\therefore \frac{dy}{dx} = 4x^3 - 12x^2$   
 $= x^2(4x - 12)$

Letting:  $x^2(4x - 12) = 0$   
 $\downarrow \qquad \qquad \downarrow$   
 $x^2 = 0 \qquad \text{not required}$   
 $\therefore x = 0$

$\therefore$  St. pt. at  $x = 0$   
(i.e. horizontal)

(2)  $\frac{d^2y}{dx^2} = 12x^2 - 24x$   
 $= 12x(x - 2)$

Letting:  $12x(x - 2) = 0$   
 $\downarrow$   
 $12x = 0$   
 $\therefore x = 0$

$\therefore$  POSSIBLE inf. pt. at  $x = 0$ .

AND

$x$	-1	0	1
$\frac{d^2y}{dx^2}$	+	0	-

at  $x = -1 : \frac{d^2y}{dx^2} = 12(-1)^2 - 24(-1) = +36$

$x = 1 : \frac{d^2y}{dx^2} = 12(1)^2 - 24(1) = -12$

in sign change: (then inf at  $x = 0$ )

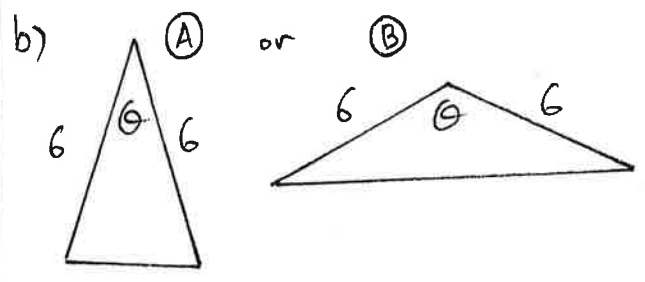
(6) a)  $3x^2 - 2x + 1 = 0$

$\therefore \alpha + \beta = -b/a$   
 $= -(-2)/3$   
 $= 2/3$

$\alpha\beta = c/a$   
 $= 1/3$

(3)

and  $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$   
 $= \frac{1}{3} \times \frac{2}{3}$   
 $= \frac{2}{9}$



Area =  $\frac{1}{2} ab \sin C$

$\therefore \frac{1}{2} \times 6 \times 6 \times \sin \theta = 10$

$18 \times \sin \theta = 10$

$\sin \theta = 10/18$

$\therefore \theta = \sin^{-1}(5/9)$

$= 33^\circ 44' 56.36'' \dots$

$\therefore \theta = 33^\circ 45'$  (for A)

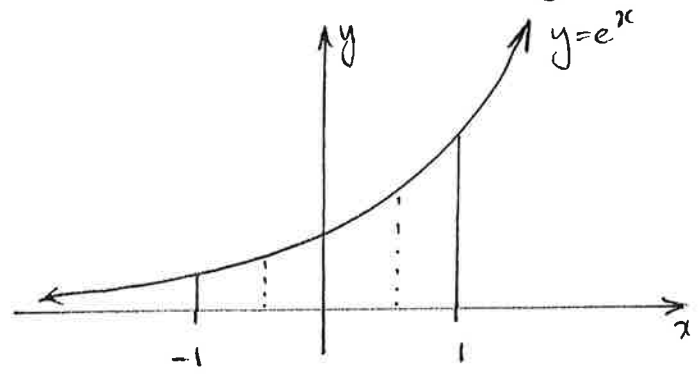
(OR)  $\theta = 180^\circ - 33^\circ 45'$

ie  $\theta = 146^\circ 15'$  (for B)

(i.e. two possible answers, for  $\theta$  acute or obtuse)

(4)

c) i) Sketch (not necessary)



Area :

x	y	x	=
-1	0.3679	1	0.3679
-0.5	0.6065	2	1.2131
0	1	2	2
0.5	1.6487	2	3.2974
1	2.7183	1	2.7183
<u>TOTAL = 9.5967</u>			

3

$$\therefore \text{Area} \doteq \frac{h}{2} \times \text{TOTAL}$$

$$= \frac{0.5}{2} \times 9.5967$$

$$= 2.3991 \dots$$

$$\therefore \text{Area} = 2.399 \text{ units}^2 \text{ (3dp)}$$

ii) Exact Area =  $\int_{-1}^1 e^x dx$

$$= [e^x]_{-1}^1$$

$$= e^1 - e^{-1}$$

$$= 2.350 \text{ units}^2 \text{ (3dp)}$$

$$\therefore \text{Percentage error} = \frac{2.399 - 2.350}{2.350} \times \frac{100}{1}$$

$$= \frac{0.049}{2.35} \times 100$$

$$= 2.08 \dots$$

2

$$\therefore \% \text{ error} = 2.1\% \text{ (1dp)}$$

7 a) i)  $A_1 = \$50000 + 1\% \text{ per month interest}$   
 $+ \$15 - M$

$$= 50000 \times 1.01 - M + 15$$

$$= 50000 \times 1.01 - (M - 15) \text{ (REQ)}$$

ii)  $\therefore A_2 = A_1 \times 1.01 - (M - 15)$

$$= \{50000 \times 1.01 - (M - 15)\} \times 1.01 - (M - 15)$$

$$= 50000 \times 1.01^2 - 1.01(M - 15) - (M - 15)$$

$$= 50000 \times 1.01^2 - (M - 15)(1 + 1.01) \text{ (REQ)}$$

iii) From ii) we "generalise":

$$\therefore A_{96} = 50000 \times 1.01^{96} - (M - 15)(1 + 1.01 + \dots + 1.01^{95})$$

G.P.  $a=1, r=1.01$   
 $n=96$   
 $\therefore S_n = \frac{a(r^n - 1)}{r - 1}$   
 $= \frac{1(1.01^{96} - 1)}{1.01 - 1}$

AND  $A_{96} = 0$  for loan fully repaid

$$\therefore 0 = 50000 \times 1.01^{96} - (M - 15) \frac{(1.01^{96} - 1)}{0.01}$$

$$\therefore M = \left\{ 50000 \times 1.01^{96} \times \frac{0.01}{1.01^{96} - 1} \right\} + 15$$

$$= 827.642 \dots$$

$$\therefore M = \$827.64$$

7 b)  $L = 120(40-t)^2$

i)  $\therefore \text{Rate} = \frac{dL}{dt} = 120 \times 2(40-t) \times -1$   
 $= -240(40-t)$

$\therefore$  at  $t = 6$ :

$\frac{dL}{dt} = -240(40-6)$   
 $= -8160 \text{ l/min}$

ii) For empty tank:  $L = 0$

i.e.  $120(40-t)^2 = 0$   
 $\therefore (40-t)^2 = 0$   
 $\therefore 40-t = 0$   
 $\therefore t = 40 \text{ minutes}$

8 a) i)  $B = 1200e^{kt}$

$\therefore \frac{dB}{dt} = 1200 \times k e^{kt}$   
 $= k(1200e^{kt})$   
i.e.  $\frac{dB}{dt} = kB$  (QED) } from above

ii)  $4200 = 1200e^{6k} \div 1200$   
 $e^{6k} = 7/2$

$\log_e(e^{6k}) = \log_e(7/2)$   
 $\therefore 6k = \log_e(7/2)$   
 $k = \log_e(7/2) \div 6$   
 $= 0.1088 \text{ (4dp)}$

$\therefore$  at  $t = 16$  days

$B = 1200e^{0.2088 \times 16}$   
 $= 33890$  (nearest whole)

b) i)  $y = \frac{\log_e(x^{1/2})}{x}$   
 $= \frac{1/2(\log_e x)}{x}$

3  $= \frac{1}{2} \left( \frac{\log_e x}{x} \right)$   $u = \log_e x, v = x$   
 $u' = \frac{1}{x}, v' = 1$

$\therefore \frac{dy}{dx} = \frac{1}{2} \left( \frac{vu' - uv'}{v^2} \right)$   
 $= \frac{1}{2} \left( \frac{x \times \frac{1}{x} - \log_e x \times 1}{x^2} \right)$   
 $= \frac{1}{2} \left( \frac{1 - \log_e x}{x^2} \right)$  (QED)

ii)  $\therefore \int_1^e \frac{1 - \log_e x}{x^2} dx$   
 $= 2 \int_1^e \frac{1 - \log_e x}{x^2} dx$

$= 2 \left[ \frac{\log_e \sqrt{x}}{x} \right]_1^e$  3  
 $= 2 \left\{ \left( \frac{\log_e e^{1/2}}{e} \right) - \left( \frac{\log_e \sqrt{1}}{1} \right) \right\}$   
 $= 2 \left\{ \frac{1/2}{e} - \frac{0}{1} \right\}$   
 $= 2 \times \frac{1/2}{e}$   
 $= \left( \frac{1}{e} \right)$

9 a)  $2 \log_e x = \log_e (x+6)$

$\therefore \log_e (x^2) = \log_e (x+6)$

$\therefore x^2 = x+6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\therefore x = -2 \text{ or } 3$

BUT by substitution:

at  $x = -2$ :  $2 \log_e (-2) = \log_e (-2+6)$

i.e.  $2 \log_e (-2) = \log_e (4)$

$\log_e x$  undefined

for negative  $x$

$\therefore$  ONLY solution is:  $x = 3$

b) i)  $y - y_1 = m(x - x_1)$

at  $x_1 = a$ ,  $y_1 = e^a$

and  $\frac{dy}{dx} = e^x \therefore m = e^a$

$\therefore y - e^a = e^a(x - a)$  (QED)

ii) For T: let  $y = 0$ :-

i.e.  $0 - e^a = e^a(x - a)$  ( $\div e^a$ )

$-1 = x - a$

$\therefore x = a - 1$

iii) A + T:  $x = a - 1$

A + A:  $x = a$

$\therefore TA = a - (a - 1)$

$= a - a + 1$

$\therefore TA = 1$  unit (QED)

ie constant for all P

(as independent of 'a')

c)  $\int \tan x \, dx$

$= \int \frac{\sin x}{\cos x} \, dx$

$= - \int \frac{-\sin x}{\cos x} \, dx$

$= - [\log_e(\cos x)] + C$

or  $= - \log_e(\cos x) + C$

10 i) A:  $x = 4t + 21 - t^2$

$\therefore \frac{dx}{dt} = v_A = 4 - 2t$

$\therefore$  stationary when  $v_A = 0$

i.e.  $4 - 2t = 0$

$2t = 4$

$t = 2$  seconds

and at  $t = 2$ :  $x = 4 \times 2 + 21 - 2^2$

$x = 25$  m

2

(10) ii)  $x = 4t + 21 - t^2$  } concave down  
 $= -(t^2 - 4t - 21)$   
 $= -(t-7)(t+3)$

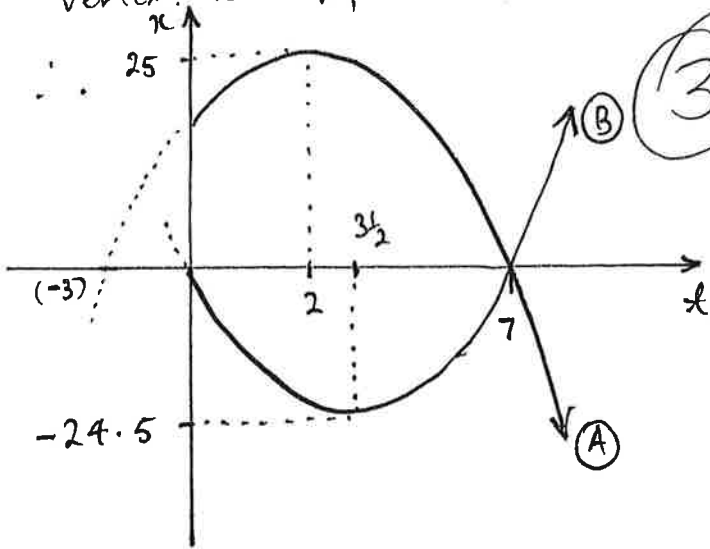
$\therefore t$ -int's:  $t = -3$  or  $7$

Vertex:  $t = 2, x = 25$  (from i)

(B)  $x = 2t(t-7)$

$\therefore t$ -int's:  $t = 0, t = 7$

Vertex:  $t = 3\frac{1}{2}, x = -24.5$



iii) From graph:

Particles meet when  $x$ -values equal  
 i.e. graphs intersect

We see graphs intersect only once  
 at  $t = 7$  (QED)

(OR) Algebra:  $4t + 21 - t^2 = 2t(t-7)$   
 $= 2t^2 - 14t$

$\therefore 3t^2 - 18t - 21 = 0$  ( $\div 3$ )

$t^2 - 6t - 7 = 0$

$(t-7)(t+1) = 0$

$\therefore t = -1$  (not possible)

iv) Distance between particles:  $D$

$D = (A) - (B)$  } for  $t=0$   
 to  $t=7$   
 $= 4t + 21 - t^2 - (2t^2 - 14t)$   
 $= 4t + 21 - t^2 - 2t^2 + 14t$

$\therefore D = 18t + 21 - 3t^2$  (QED) ①

v)  $D_{max}$  at  $\frac{dD}{dt} = 0$

i.e.  $18 - 6t = 0$

$\therefore 6t = 18$

$\therefore t = 3$  seconds)

NOTE  $\frac{d^2D}{dt^2} = -6 < 0$  at  $t = 3$

$\therefore t = 3$  seconds is a maximum

vi) Same velocity at  $v_A = v_B$

and  $x_B = 2t^2 - 14t$

$\therefore v_B = 4t - 14$

$\therefore$  need:  $4 - 2t = 4t - 14$

$18 = 6t$

$\therefore t = 3$  seconds

for same velocities