Newington. 2u Trial. 1989

QUESTION 1 *(10 Marks)*

a) Evaluate $\dfrac{\left(\frac{1}{6}\right)^3 + \left(\frac{4}{9}\right)^2}{\left(\frac{2}{3}\right)^4}$ *(Give your answer correct to two decimal places)*

b) Solve $x^2 + 6x = 91$

c) Solve $|4 - 2x| = 6$

d) If $S = ut + \frac{1}{2}at^2$, make the subject 'a'

e) Find a and b if $(\sqrt{2} + 3\sqrt{3})^2 = a + b\sqrt{6}$

QUESTION 2 *(10 Marks)*

The line $x - y + 4 = 0$ passes through the points $A(-1, 3)$ and $B(3, 7)$. Find:

a) the distance from A to B in exact form

b) the distance of the point $C(0, 0)$ to the line $x - y + 4 = 0$ in exact form

c) the area of the triangle ABC

d) the co-ordinates of the midpoint M between A and B

e) the equation of the line through M perpendicular to $x - y + 4 = 0$

QUESTION 3 *(10 Marks)*

a) Differentiate
i) $(1 - x^2)^5$
ii) $\log_e x^2$
iii) $\cos 4x$

b) For the curve with equation $y = \log_e(x - 1)$, state the largest possible domain

c) Evaluate
i) $\int_0^{\pi} (2\sin x - \sin 2x)\,dx$
ii) $\int_1^4 \frac{1}{2x + 1}\,dx$
(Give your answer correct to 3 decimal places)
iii) $\int_0^1 (e^x - 1)^2\,dx$
(Give your answer correct to 3 decimal places)

/ Q4 ...page 2

QUESTION 4 *(10 Marks)*

a) Which term of the geometric sequence $\frac{1}{9}, \frac{1}{3}, 1, \ldots$ is 2187?

b) i) Write down a single expression for the sum of the first n terms of the series

$1 + x^2 + x^4 + x^6 + \ldots$

ii) What is the sum to infinity when $x = \frac{1}{3}$?

c)Find the first term and the common difference of the arithmetic series in which the 29th term is 40 and the sum of the first 8 terms is 26.

QUESTION 5 *(10 Marks)*

a) Solve for x

i) $\log_5\left(\frac{1}{125}\right) = x$

ii) $\log_{10} x = \log_{10} 8 + \log_{10} 3 - \log_{10} 2$

b) An urn contains six balls, three red, two blue and one yellow. If three balls are chosen from the urn without replacement, what is the probability that:

i) the first ball drawn is red?

ii) all three are red?

iii) not one is red?

c) A biased coin has a probability os $\frac{2}{3}$ of showing a head when tossed. Use a tree diagram or otherwise to find the probability of the coin showing at least two heads in three tosses

/ Q6 ...page 3

QUESTION 6 *(10 Marks)*

a) ABCD is a parallelogram.

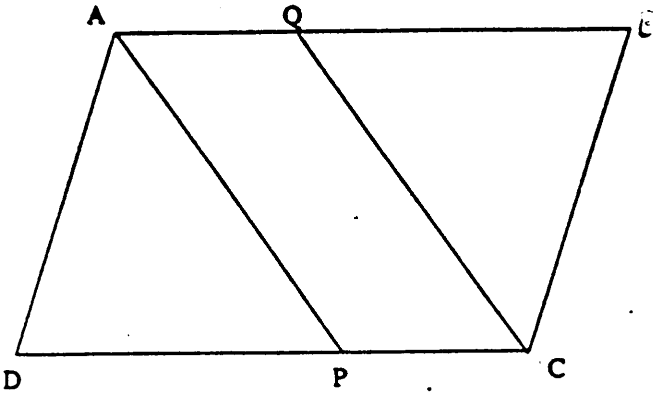
AP bisects $\angle DAB$ and

CQ bisects $\angle BCD$.

Prove that

i) ΔDAP is congruent to ΔBCQ

ii) $AQ = CP$



b) In the figure, OA and OB are radii of length 10 cm, of a circle with centre O. The arc AB of the circle subtends an angle of $\frac{\pi}{3}$ radians at O.

AB is a chord of the circle.

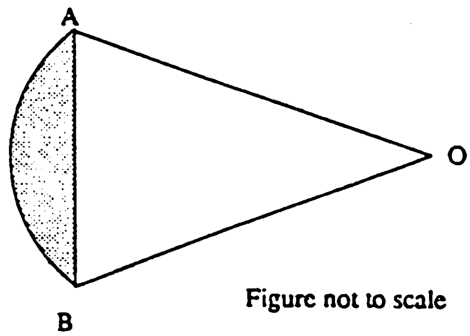


Figure not to scale

i) Calculate the area of sector AOB

ii) Calculate the area of the triangle AOB

iii) Find the area of the segment of the circle shaded in the diagram
(Give all answers correct to 3 decimal places)

QUESTION 7 *(10 Marks)*

a) If α and β are the roots of $x^2 - 6x + 2 = 0$, find:

i) the sum and the product of the roots

ii) the quadratic equation with roots $2\alpha + 1$, $2\beta + 1$

b) Prove that $x^2 - 3x + 5 > 0$ for all values of x

c) For the parabola $x^2 - 4x - 8y - 36 = 0$, find:

i) the co-ordinates of the vertex;

ii) the focal length;

iii) the co-ordinates of the focus;

iv) the equation of the directrix.

/ Q8 ...page 4

QUESTION 8 *(10 Marks)*

a) The Hyat Superannuation Scheme offers compound interest calculated yearly on money invested according to the formula $A = P\left(1 + \frac{r}{100}\right)^t$, where \$A is the amount \$P has grown to after being invested for t years at r % p.a. If Hyat advertises that \$10 000 will yield \$170 000 after 25 years, find the value of r to the nearest whole number

b) The Regent Rollover Fund is operated so that the rate of increase of money invested is governed by the equation $\frac{dP}{dt} = kP$, where k is a constant. If \$10 000 grows to \$60 500 in 15 years in this fund, find the value of k *(to 2 decimal places).*

c) Find the time it takes for \$10 000 to grow to \$170 000 in the Regent Fund and compare the Hyat Scheme with the Regent Fund

QUESTION 9 *(10 Marks)*

a) A particle moves in a horizontal straight line such that its distance x metres from a fixed point O at time t seconds is given by the equation $x = 5 + 4t - t^2$. Find:

i) the initial displacement, velocity and acceleration of the particle

ii) when and where the particle is at rest

iii) when the particle is at the origin

iv) the average velocity during the 3rd second

b) The area between the curve $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, the x-axis and $x = 1$ and $x = 2$ is rotated about the x-axis. Find the volume of the solid of revolution formed, leaving your answer in exact form.

/ Q10 ...page 5

QUESTION 10 *(10 Marks)*

a) i) Sketch the curve $y = 3\sin 2x$ (not on graph paper), for $0 \le x \le 2\pi$

ii) thence solve the equation $\sin 2x = \frac{1}{3}$, for $0 \le x \le 2\pi$, given that the smallest solution in this domain is 0.17

(give all answers correct to two decimal places)

b) i) Find the stationary points for the curve $y = x\,e^x$ and determine their nature.

ii) Sketch the curve $y = x\,e^x$, showing stationary points, intercepts and asymptotes

iii) Hence or otherwise find the values of k for which the equation $x\,e^x = k$ has

A) 2 solutions B) 1 solution

END OF PAPER

(1)

Kensington 1989. 2 Unit Paper.

Q1. (a) 1.02

(b) $x^2+6x=91$

$x^2+6x-91=0$

$(x+13)(x-7)=0$

$x=-13, 7.$

(c) $4-2x=6$ or $-4+2x=6$

$-2x=2$ or $2x=10$

$x=-1$ or $x=5.$

(d) $s=ut+\frac{1}{2}at^2$

$\frac{1}{2}at^2=s-ut$

$at^2=2s-2ut$

$a=\dfrac{2s-2ut}{t^2}$

(e) $a+b\sqrt{6}=(\sqrt{2}+3\sqrt{3})^2$

$=2+27+6\sqrt{6}$

$=29+6\sqrt{6}.$

$\therefore a=29,\ b=6.$

Q2. (a) $\text{length}_{AB}=\sqrt{(7-3)^2+(3--1)^2}$

$=\sqrt{16+16}$

$=4\sqrt{2}$

(b) perp. dist $=\dfrac{|1\times 0+(-1).0+4|}{\sqrt{1^2+1^2}}$

$=\dfrac{4}{\sqrt{2}}$

$=2\sqrt{2}.$

(c) $\therefore$ Area $=\frac{1}{2}\cdot 2\sqrt{2}\cdot 4\sqrt{2}$

$=8$ sq. units.

(d) Coords M $=\left(\frac{3-1}{2},\ \frac{7+3}{2}\right)$

$=(1, 5)$

(e) grad: $x-y+4=0$

$y=x+4$

$m=1.$

perp. grad. $=-1.$

$\therefore$ eqn. through M:

$y-5=-1(x-1)$

$y-5=-x+1$

$y+x-6=0.$

Q3 (a)(i) $\frac{d}{dx}(1-x^2)^5=5(1-x^2)^4\cdot -2x$

$=-10x(1-x^2)^4$

(ii) $\frac{d}{dx}(\log_e x^2)=\dfrac{2x}{x^2}$

$=\dfrac{2}{x}.$

(iii) $\frac{d}{dx}(\cos 4x)=-4\sin 4x.$

(b) $y=\log_e(x-1)$

domain: $x>1.$

(c)(i) $\int_0^{\pi}(2\sin x-\sin 2x)\,dx.$

$=\left[-2\cos x+\frac{1}{2}\cos 2x\right]_0^{\pi}$

$=+2+0.5+2-0.5$

$=4.$

Q3. (ii) $\int_1^4 \frac{1}{2x+1}\,dx$

$= \left[\frac{1}{2}\log(2x+1)\right]_1^4$

$= \left(\frac{1}{2}\log 9 - \frac{1}{2}\log 3\right.$

$= \frac{1}{2}\log 3$

$= 0.549$

(iii) $\int_0^1 (e^x - 1)^2\,dx$

$= \int_0^1 (e^{2x} + 1 - 2e^x)\,dx$

$= \left[\frac{1}{2}e^{2x} + x - 2e^x\right]_0^1$

$= \left[\frac{1}{2}e^2 + \cancel{1} - 2e - \cancel{\frac{1}{2}e^0} \cancel{-0} + \cancel{2e^0}\right]$

$= -1.74\ldots + 2\frac{1}{2}$

$= 0.758$

Q4. (a) $a = \frac{1}{9}$ $r = 3$.

$T_n = ar^{n-1}$

$2187 = 3^{-2} \cdot 3^{n-1}$

$2187 = 3^{n-3}$

$3^7 = 3^{n-3}$

$\therefore n = 10.$

(b) (i) G.P. $a = 1$, $r = x^2$.

$\therefore S_{\text{GP to n terms}} = \frac{1[(x^2)^n - 1]}{x^2 - 1}$

$S_n = \frac{x^{2n} - 1}{x^2 - 1}$

(ii) If $x = \frac{1}{3}$, $x^2 = \frac{1}{9} = r$.

$S_\infty = \frac{a}{1-r}$

$= \frac{1}{1 - \frac{1}{9}}$

$= \frac{9}{8}$ (1.125)

(c) $T_{29} = a + 28d$

$S_8 = 4(2a + 7d)$

$\therefore 40 = a + 28d$ ①

$26 = 8a + 28d$ ②

$\therefore -14 = 7a$

$a = -2$.

from ①: $40 = -2 + 28d$

$42 = 28d$

$d = 1.5$.

Q5. (a) (i) $\log_5(5^{-3}) = x$.

$\therefore -3 = x$.

(ii) $\log_{10} x = \log_{10}\left(\frac{24}{2}\right)$

$\therefore x = 12.$

(b) (i) $P(R)_{\text{once}} = \frac{3}{6}$

$= \frac{1}{2}$.

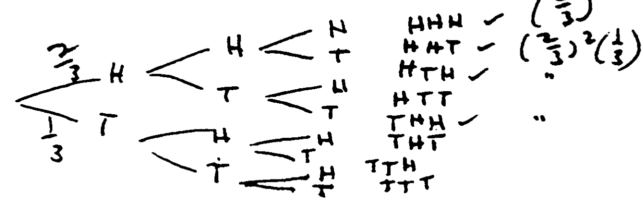
(ii) $P(R)_{\text{three times}} = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4}$

$= \frac{1}{20}$.

(iii) $P(\text{not red}) = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4}$.

$= \frac{1}{20}$.

(c)

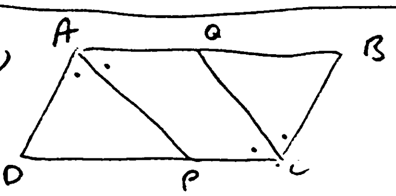


Q5. $\therefore$ total $P = \left(\frac{2}{3}\right)^3 + 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$

$= \frac{8}{27} + \frac{4}{9}$

$= \frac{20}{27} \quad (0.\dot{7}4\dot{0})$

Q6. (a)



(i) $\angle DAB = \angle BCD$ (opp. $\angle$'s parm).

$\therefore \angle DAP = \angle BCQ$.

In Δ's DAP, CBQ:

$\angle DAP = \angle BCQ$ (proved).

$AD = BC$ (opp. sides parm).

$\angle DPA = \angle PAQ = \angle PCQ = \angle QCB) = \angle BQC$, (alt. $\angle$'s, $DP \parallel AB$ and data)

$\therefore \angle DPA = \angle BQC$.

$\therefore \Delta DAP \equiv \Delta BCQ$ (AAS).

(ii) $\therefore DP = QB$ (corresp. sides congruent triangles)

$\therefore AQ = PC$ ($AB = DC$, opp. sides parm).

(b) (i) Area AOB $= \frac{1}{2}r^2\theta$

$= 50\frac{\pi}{3}$

(ii) Area $\Delta AOB = \frac{1}{2}r^2 \sin\theta$

$= 50\frac{\sqrt{3}}{2}$

(iii) $\therefore$ Area segment $= 50\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= 9.059 \text{ cm}^2$

Q7. (a) (i) $\alpha + \beta = \frac{6}{1}$

$= 6$

$\alpha\beta = \frac{2}{1}$

$= 2$.

(ii) $x^2 - \quad x +$

product roots $= 0$.

$x^2 - (2\alpha + 2\beta + 2)x + (2\alpha+1)(2\beta$

$=$

$x^2 - (2(\alpha+\beta)+2)x + (4\alpha\beta + 1 + 2\alpha + 2\beta)$

$=$

$x^2 - (12+2)x + (4\times 2 + 1 + 2(6)) =$

$x^2 - 14x + 21 = 0$.

(b) $x^2 - 3x + 5 > 0$ if

$a > 0$ and $\Delta < 0$.

$a = 1$ $\quad \Delta = b^2 - 4ac$

$b = -3$ $\quad = 9 - 20$

$c = 5$ $\quad = -11$.

$\therefore x^2 - 3x + 5$ is positive definite, >0 for all values of x.

(c) (i) $(x^2 - 4x + 4) = 8y + 36 +$

$(x-2)^2 = 8(y+5)$

$\therefore$ coords vertex $(2, -5)$.

(ii) "a" $= 2$ focal length.

(iii) concave up

$\therefore$ focus is $(2, -3)$.

(iv) directrix: $y = -7$.

Q 8. (a) $A = P\left(1 + \frac{r}{100}\right)^t$

$170\ 000 = 10\ 000\left(1 + \frac{r}{100}\right)^{25}$

$17 = \left(1 + \frac{r}{100}\right)^{25}$

$r = \left(\sqrt[25]{17} - 1\right)100.$

$= 12$ yrs.

(b) $\frac{dP}{dt} = kP.$

$P = P_0 e^{kt}$

$60\ 500 = 10\ 000\, e^{15k}.$

$6.05 = e^{15k}$

$\frac{\log 6.05}{15} = k.$

$k = 0.12.$

(c) $170\ 000 = 10\ 000\, e^{0.12t}$

$17 = e^{0.12t}$

$\frac{\log 17}{0.12} = t$

$t = 23.6$ yrs.

∴ The Hyatt scheme is far superior in generating interest to the Regent Rollover Fund.

Q9. (a) $x = 5 + 4t - t^2$

(i) at $t = 0$, $x = 5$ m

$\dot{x} = 4 - 2t$

at $t = 0$, $\dot{x} = 4\ \text{m s}^{-1}.$

$\ddot{x} = -2$

∴ at $t = 0$, $\ddot{x} = -2\ \text{m s}^{-2}.$

(ii) $\dot{x} = 0$ when $4 - 2t = 0$

$t = 2\ \text{s}.$

at $t = 2\text{s}$, $x = 5 + 8 - 4$

$= 9$ m.

∴ at rest 9m from origin after 2s.

(iii) at origin when $x = 0$

$0 = 5 + 4t - t^2$

$0 = (5 - t)(1 + t)$

∴ $t = 5, (-1\ \text{s})$ ∴ at 5 s.

(iv) $\underset{\text{3rd sec.}}{\text{av. vel.}} = \text{vel.}_{t=3} - \text{vel.}_{t=2}$

$= (4 - 6) - (4 - 4)$

$= -2\ \text{m s}^{-1}.$

(b) $y = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$\text{Vol} = \pi \int_1^2 y^2 \, . dx.$

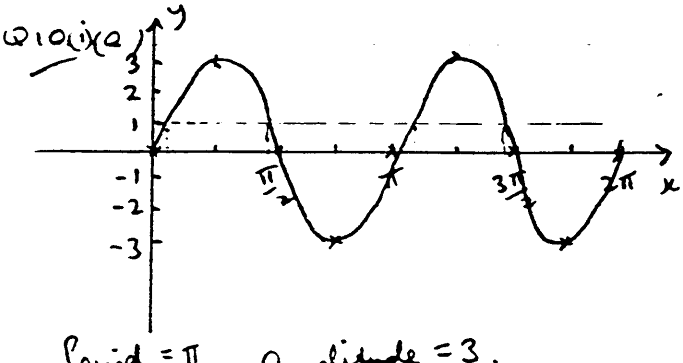
$= \pi \int_1^2 \left[x + \frac{1}{x} + 2\right] . dx.$

$= \pi \left[\frac{x^2}{2} + \log x + 2x\right]_1^2$

$= \pi \left[\frac{4}{2} + \log 2 + 4 - \frac{1}{2} - 0 - 2\right]$

$= \pi \left[3\tfrac{1}{2} + \log 2\right]$ units^3.

Q10(i)(a)



Period $= \pi$, Amplitude $= 3$.

(ii) if $\sin 2x = \frac{1}{3}$

$3 \sin 2x = 1.$

$\therefore$ find simultaneous solution of $y = 3 \sin 2x$ and. $y = 1.$

$\therefore$ solution of $0 \leq x \leq 2\pi$ is $0\cdot17, (\frac{\pi}{2} - 0\cdot17), (\pi + 0\cdot17)$ and $(\frac{3\pi}{2} - 0\cdot17)$.

$= 0\cdot17,\ 1\cdot40,\ 3\cdot31,\ 4\cdot54.$

(b) (i) $y = xe^x$

stat. pts occur when $\frac{dy}{dx} = 0.$

$\frac{dy}{dx} = x.e^x + e^x.$

$= e^x(x+1).$

$\frac{dy}{dx} = 0$ when $e^x = 0$ or $x+1 = 0$

$e^x \neq 0$ $\therefore$ when $x = -1.$

when $x = -1$, $y = \frac{-1}{e}$

$\frac{d^2y}{dx^2} = xe^x + e^x + e^x$

$= e^x(x+2).$

at $x = -1$, $\frac{d^2y}{dx^2} > 0$

$\therefore (-1, \frac{-1}{e})$ is min.

(ii) at $x = 0$, $y = 0$

at $y = 0$, $\therefore x = 0.$

$x \to +\infty$, $x.e^x \to \infty$

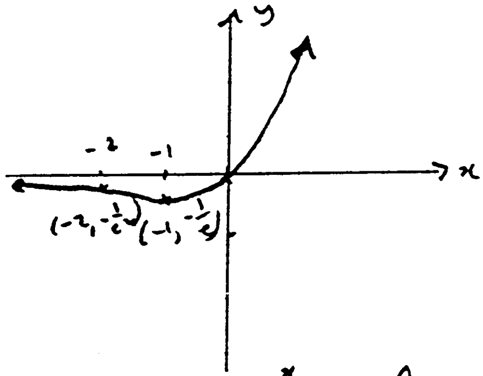
$x \to -\infty$, $x.e^x$ is negative but approaches 0.

$\frac{d^2y}{dx^2} = 0$ when $x+2 = 0$ $(e^x \neq$

$\therefore x = -2$ is pt. of inflexion $(-2, -\frac{1}{e^2})$

$\frac{d^2y}{dx^2} < 0$ for $x < -2$

$\frac{d^2y}{dx^2} > 0$ for $x > -2$.



(iii) $\therefore$ $x.e^x = k$ has 2

(A) solutions for $x < 0$ and $-\frac{1}{e} \leq k < 0.$

(B) $x.e^x = k$ has 1 solut for $x \geq 0$ and $k \geq 0.$