Newington College

2004

TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is proved at the back of this paper
- All necessary working should be shown in every question

Total marks: 120

- Attempt Questions 1–10
- All questions are of equal value

Marks

Question 1 (12 marks)

(a) Find the value of **1**

$$\log_e 3.5 - \frac{\pi}{\sqrt{e^3}},$$

correcting your answer to 3 significant figures.

(b) Find the primitive of x^{-2}. **2**

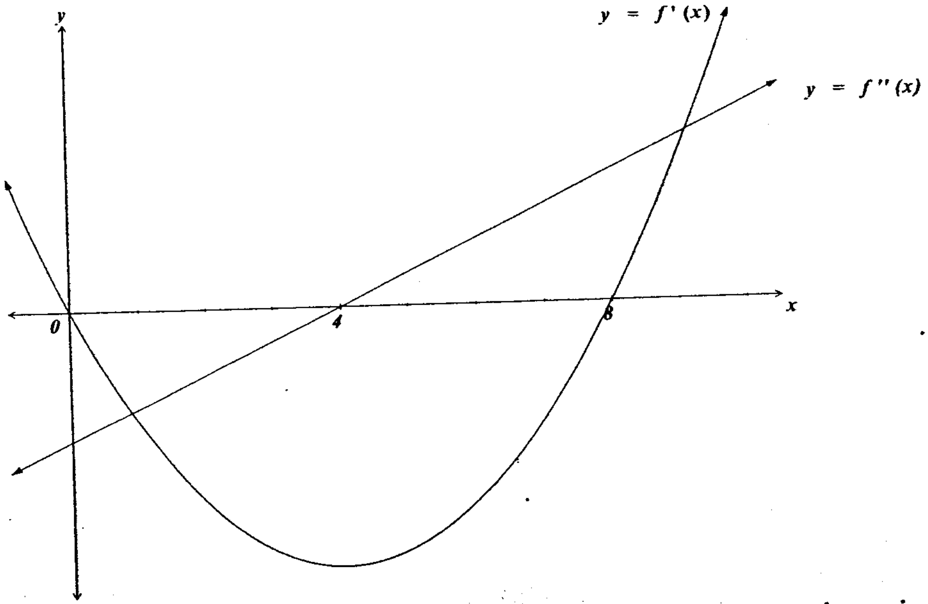
(c) Solve $\frac{3t}{t-5} = \frac{2}{5}$. **2**

(d) Jai sold his car for \$5400, which was 18% less than its original cost. **2**

How much did the car originally cost?

(e) Solve $|2 - 3x| \geq 1$. **3**

(f) **2**



The graphs of the first and second derivatives of the curve $y = f(x)$ are shown in the diagram.

Write down the x coordinates of the stationary points and determine their nature.

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Factorise $1-27a^3$. **1**

(b) Differentiate the following with respect to x:

(i) $y=\sin^3 x$ **1**

(ii) $y=\log_e(3x+1)$ **1**

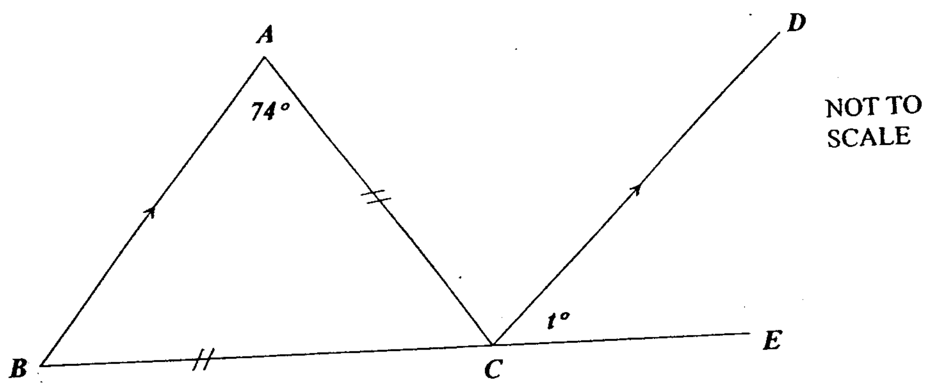
(iii) $y=2e^{-4x}$ **1**

(c) Find the value of p

(i) $\log\left(\frac{1}{x^2}\right)=p\log x$ **1**

(ii) $6^p=3$, correct your answer to 2 decimal places. **2**

(d) Find the value of the pronumeral, giving reasons. **2**

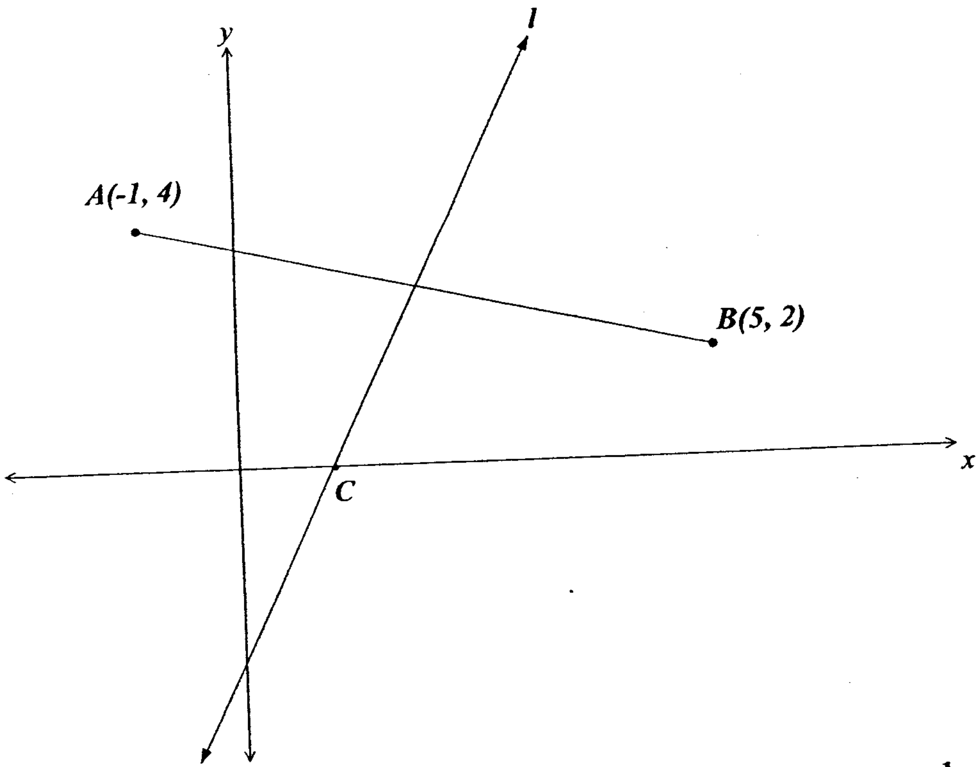


(e) Find all the real numbers x which satisfy the equation **3**

$$x^4=4\left(x^2+8\right)$$

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram below shows the points $A(-1, 4)$ and $B(5, 2)$. The line l has the equation $3x - y - 3 = 0$ and cuts the x-axis at C.



(i) Show that the length of AB is $2\sqrt{10}$ units. **1**

(ii) Find the co-ordinates of M, the midpoint of AB. **1**

(iii) Find the gradient of AB. **1**

(iv) Show that the equation of AB is $x + 3y - 11 = 0$. **1**

(v) Prove that l is the perpendicular bisector of AB. **2**

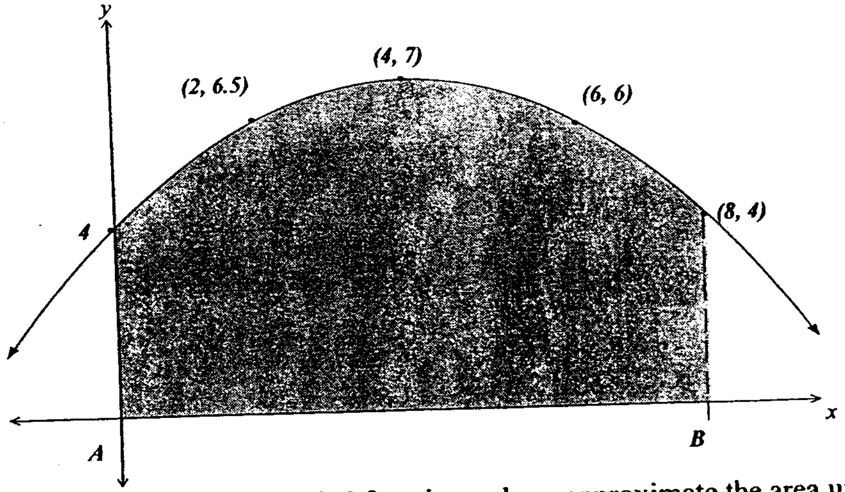
(vi) Find the co-ordinates of C. **1**

(vii) Write down the equation of the circle with AB as the diameter. **1**

Question 3 continues on page 4

Question 3 (continued) **Marks**

(b) **3**



Using Simpson's Rule with 5 function values, approximate the area under the curve $y = f(x)$ from $x = A$ to $x = B$, as shown in the diagram above.

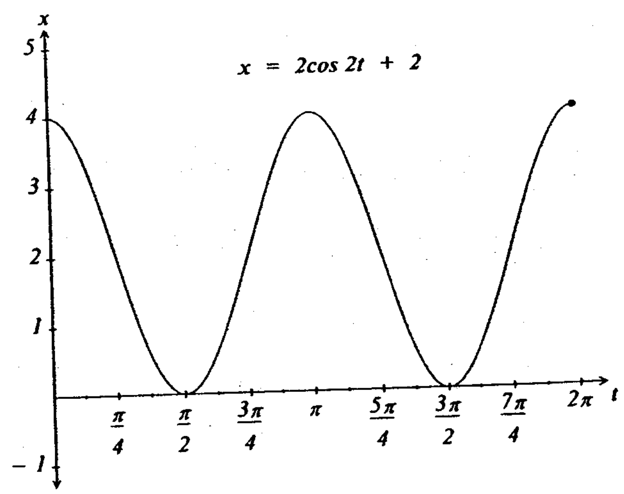
(c) Find $\int \frac{3x^2}{1+x^3}\,dx$. **1**

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) A particle moves according to the displacement function $x = 2\cos 2t + 2$ for $0 \le t \le 2\pi$, for x metres and t seconds, as shown at right: **3**

Find (i) when the particle is at rest.

(ii) when the particle is moving away from the origin.



Question 4 continues on page 5

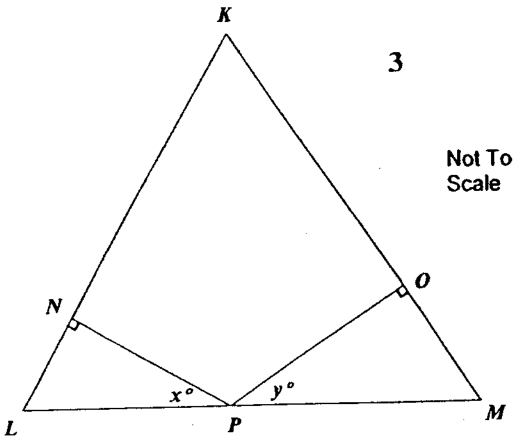
Question 4 (continued) **Marks**

b) A parabola, whose equation is of the form $y = Ax^2$ (where A is a constant) has the line $20x - y + 20 = 0$ as a tangent. **3**

Find the value of A.

(c) Evaluate $\displaystyle\int_1^2 \frac{1}{3}e^{2.5x}\,dx$ **3**

(d) KLM is an isosceles triangle with $KL = KM$. The lines NP and OP are constructed perpendicular to KL and KM respectively, as shown, from the point P, the midpoint of LM. **3**



Prove $x° = y°$.

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = (x-4)(x+2)^2$

(i) Show that $f'(x) = 3x^2 - 12$. **2**

(ii) Find the co-ordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **2**

(iii) Find any point(s) of inflexion. **1**

(iv) Sketch the graph of the curve $y = f(x)$ showing all intercepts, stationary points and points of inflexion. **2**

(v) Find all the values of x for which the graph $y = f(x)$ is concave up. **2**

Question 5 continues on page 6

Marks

Question 5 (continued)

(b) The first two terms of a geometric sequence are $\sqrt{3}-1$ and $\sqrt{3}+1$. **3**

Find the common ratio and the fourth term of this sequence in simplest surd form.

Discuss whether the geometric sequence has a limiting sum.

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) What is the domain and range of $y=\sqrt{5-2x}$? **1**

(b) Find the value of C if the equation $4x^2-12x+C=0$ has equal roots. **2**

(c) If α and β are the roots of the quadratic equation $x^2-5x-4=0$,

find the values of :

(i) $\alpha+\beta$. **1**

(ii) $\alpha\beta$. **1**

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$. **2**

(iv) $\alpha^2+\beta^2$. **2**

(d) For the parabola $x^2=8(y-3)$, find the

(i) co-ordinates of the vertex. **1**

(ii) focal length. **1**

(iii) equation of the directrix. **1**

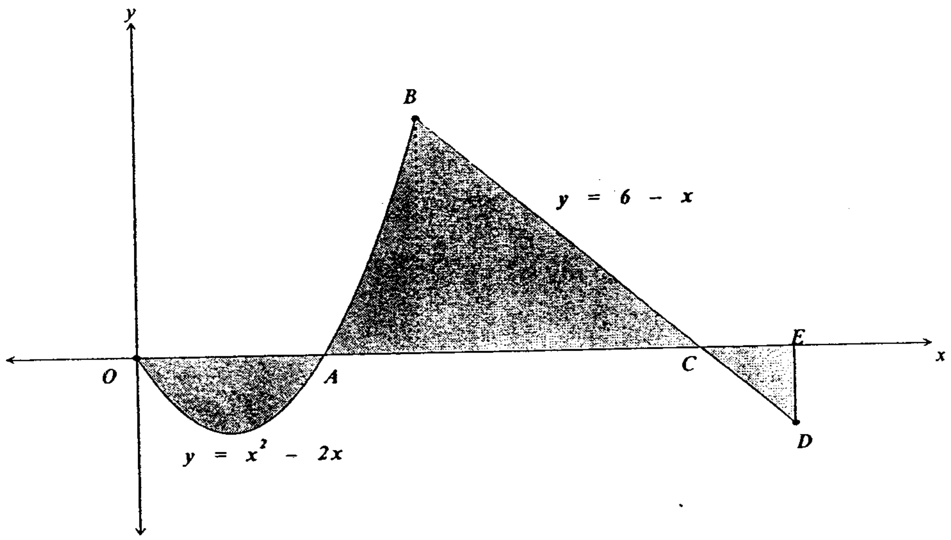
Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The shaded region $OABCDE$ is bounded by the lines $x = 0$ and $x = 7$, the curve $y = x^2 - 2x$, the line $y = 6 - x$ and the x-axis.

(i) Find the co-ordinates of the points A, C and D. **3**

(ii) Show that the co-ordinates of B are (3, 3). **2**

(iii) Calculate the area of the shaded region $OABCDE$. **3**



(b) If $f(x) = x^2$ then $f(x+3) \neq f(x) + f(3)$. **1**

Find a function $g(x)$, such that $g(x+3) = g(x) + g(3)$.

(c) A person saves \$10000 by investing \$100 at the beginning of each month at 6% p.a., compounded monthly.

(i) Show that the expression for the amount saved by the end of the first three months is given by $A_3 = \$100(1.005^3 + 1.005^2 + 1.005)$. **1**

(ii) Hence, or otherwise, find the number of months required to save \$10000. **2**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate $\log_e(\cos^3 x)$ with respect to x. **3**

(b) If $\cos\theta = \dfrac{8}{17}$ and $\sin\theta < 0$, find the exact values for

(i) $\sin\theta$ **2**

(ii) $\cot\theta$ **1**

(c) (i) Find the point of intersection of the two curves $y = x^2$ and $y = \dfrac{8}{x}$. **1**

(ii) Find the gradients of the tangents to both the curves at this point. **2**

(iii) Find the angles of inclination of both tangents to the positive x-axis (Give your answer to the nearest degree). **2**

(iv) Hence find the acute angle between the two tangents. **1**

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) (i) On the same set of axes sketch $y = |2x - 3|$ and $y = -x$. **2**

(ii) Hence, or otherwise, solve $|2x - 3| < -x$. **1**

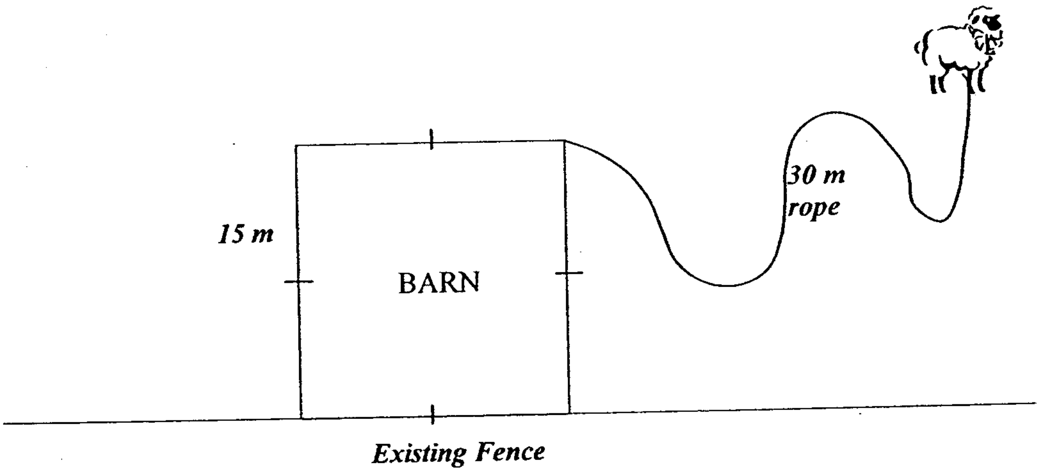
Question 9 continues on page 9

Question 9 (continued) **Marks**

(b) A sheep is tied to the side of a square shaped barn as shown in the diagram. The rope is 30 m long and the side length of the barn is 15m. On one side of the barn is a fence line as shown.

(i) Draw a diagram that shows the entire area over which the sheep can graze. (Not including the Barn) **2**

(ii) If the sheep can graze over the entire area but not inside the barn find the maximum grazing area available for the sheep. **4**



(c) The population of rats in a barn increases exponentially according to the formula $P = P_o e^{kt}$, where P_o is the original population and t is the time in months and k is a constant. **3**

If the initial population of 20 increases to 100 in 3 months, how long will it take to reach a population in excess of 2000 at this rate?

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The probability of tossing a 5 on a biased six-sided die is $\frac{2}{7}$. The die is tossed 3 times. Find the probability of tossing:

(i) one 5. **1**

(ii) no 5's. **1**

(iii) at least one 5. **1**

The biased die is tossed n times.

(iv) Write an expression for the probability of tossing at least one 5. **1**

(v) Find the value of n if the probability of tossing at least one 5 is greater than 99%. **2**

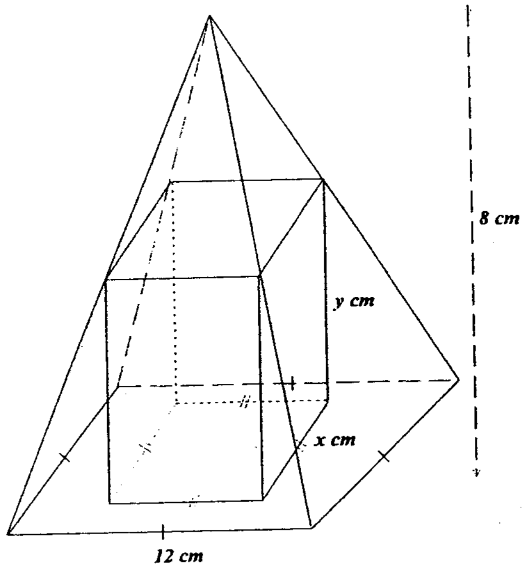
Question 10 continues on page 11

Questio

(b)

Question 10 (continued)

(b) A rectangular prism is constructed within a square-based triangular pyramid as shown in the diagram:



(i) Using similar triangles, or otherwise, show that **2**

$$y = 8 - \frac{2x}{3}.$$

(ii) Let the volume of the rectangular prism be V cm^3, then show that **2**

$$\frac{dV}{dx} = 2x(8 - x).$$

(iii) Hence, find the maximum volume of the rectangular prism. **2**

End of paper

Marks

Question 1 (12 marks)

(a) $\log_e 3.5 - \frac{\pi}{\sqrt{e^3}} = 0.552$ (correct to 3 significant figures) 1

(b)

$$\int x^{-2}\,dx = (-1)x^{-1} + c$$
$$= -\frac{1}{x} + c$$
2

(c)

$$\frac{3t}{t-5} = \frac{2}{5}$$
$$15t = 2(t-5)$$
$$15t = 2t - 10$$
$$13t = -10$$
$$t = -\tfrac{10}{13}$$
2

(d)

82% of original cost $= \$5400$
1% of original cost $= \$5400/82$
100% of original cost $= \$5400 \times 100/82$
$= \$6585.37$ (to nearest cent)
2

(e)

$$|2-3x| \ge 1$$
$2 - 3x \ge 1$ or $-(2-3x) \ge 1$
$3x \le 1$ or $-2 + 3x \ge 1$
$x \le \frac{1}{3}$ or $3x \ge 3$
$x \le \frac{1}{3}$ or $x \ge 1$
3

(f) $f'(x) = 0$ at $x = 0$ and $x = 8$, hence these are stationary points.

At $x = 0$, $f''(x) < 0$, hence this is a maximum turning point, and at $x = 8$,

$f''(x) > 0$, so this is a minimum turning point. 2

Q2…/ page 2

Question 2 (12 marks) Start this question on a new page **Marks**

(a) $1-27a^3=(1-3a)(1+3a+9a^2)$. 1

(b) (i)

$$y=\sin^3 x$$
$$\frac{dy}{dx}=\cos x\times 3\sin^2 x$$
$$\frac{dy}{dx}=3\cos x\sin^2 x$$

1

(ii)

$$y=\log_e(3x+1)$$
$$\frac{dy}{dx}=3\times\frac{1}{3x+1}$$
$$\frac{dy}{dx}=\frac{3}{3x+1}$$

1

(iii)

$$y=2e^{-4x}$$
$$\frac{dy}{dx}=2\times(-4)e^{-4x}$$
$$\frac{dy}{dx}=-8e^{-4x}$$

1

(c)

(i)

$$\log\left(\frac{1}{x^2}\right)=p\log x$$
$$\frac{1}{x^2}=x^p$$
$$x^{-2}=x^p$$
$$p=-2$$

1

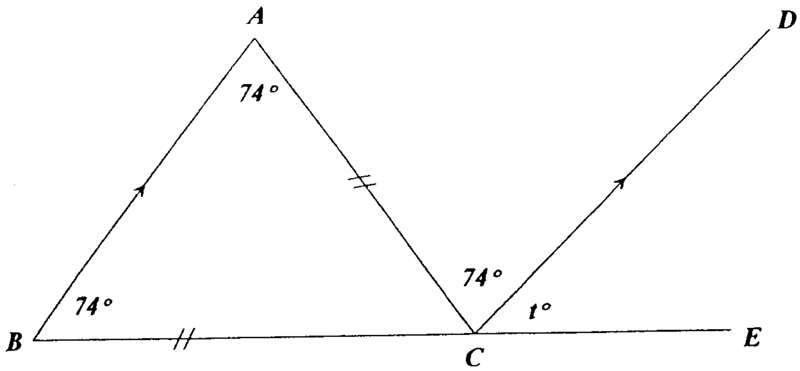
(ii)

$$6^p=3$$
$$p=\frac{\log 3}{\log 6}$$

$p=0.61$ (correct to 2 decimal places)

2

(d)



2

$\angle ABC = \angle BAC = 74°$ (equal angles opp equal sides)

$\angle BAC = \angle ACD = 74°$ (alt angles equal, $BA \parallel CD$)

$\angle DCE = \angle BAC + \angle ABC - \angle ACD$ (exterior angle theorem)

$t = 74$

(e)

$$x^4 = 4\left(x^2 + 8\right)$$
$$x^4 - 4x^2 - 32 = 0$$
$$\left(x^2 - 8\right)\left(x^2 + 4\right) = 0$$
$$\left(x - \sqrt{8}\right)\left(x + \sqrt{8}\right)\left(x^2 + 4\right) = 0$$
$$x = \pm\sqrt{8}$$

3

Question 3 (12 marks)

(a)

(i)

$$d_{AB} = \sqrt{(5+1)^2 + (2-4)^2}$$
$$d_{AB} = \sqrt{36+4}$$
$$d_{AB} = 2\sqrt{10}$$

1

(ii) $M = \left(\frac{-1+5}{2}, \frac{4+2}{2}\right) = (2,3)$ 1

(iii) $m_{AB} = \frac{4-2}{-1-5} = \frac{2}{-6} = -\frac{1}{3}$ 1

Question 3 **(12 marks)**

(a) (iv)

$$y - y_1 = m_{AB}(x - x_1)$$ 1

$$y - 2 = -\frac{1}{3}(x - 5)$$

$$3y - 6 = -x + 5$$

$$x + 3y - 11 = 0$$

(v)

At $M(2,3)$, for $l : 3x - y - 3 = 0$

$LHS = 3(2) - 3 - 3 = 0$ 2

$RHS = 0$

So, M lies on l.

$m_l = 3$

$$m_{AB} \times m_l = 3 \times \left(-\frac{1}{3}\right) = -1$$

So, l is perpendicular bisector.

(vi) At C, let $y = 0$,

$3x - y - 3 = 0$ 1

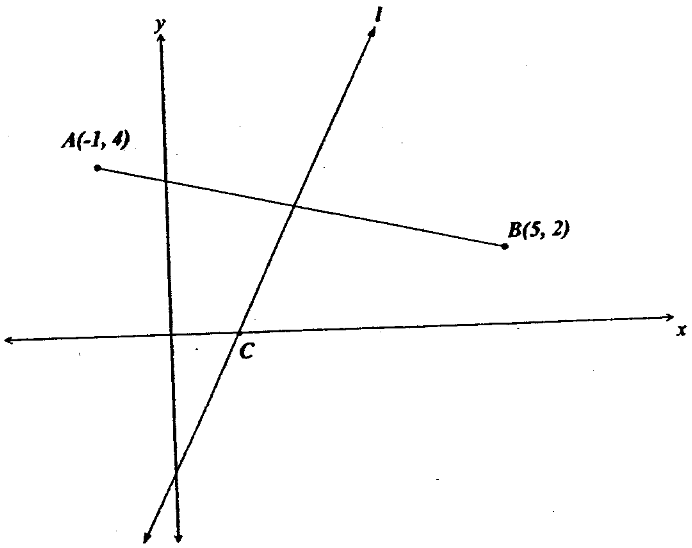
$3x - 3 = 0$

$x = 1$

(vii) If AB is the diameter then M is the centre i.e.(2, 3) and radius is

$\frac{1}{2}AB = \frac{1}{2}\left(2\sqrt{10}\right) = \sqrt{10}$, so equation of circle is 1

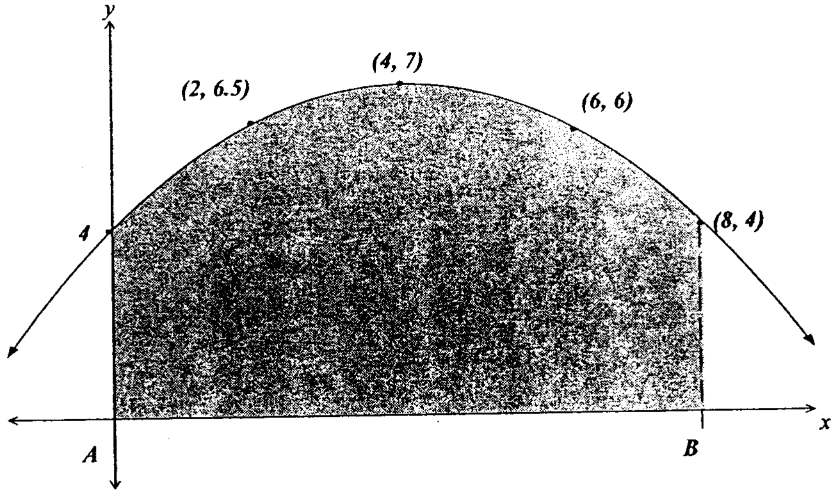
$$(x-2)^2 + (y-3)^2 = 10$$



Q3 cont.../page 4

uestion 3 (cont.)

b)



x	0	2	4	6	8
f(x)	4	6.5	7	6	4

$$Area \cong \frac{h}{3}\{y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\}, \quad \text{where } h = 2$$

$$\cong \frac{2}{3}\{4 + 4\times 6.5 + 2\times 7 + 4\times 6 + 4\}$$ 3

$$\cong \frac{2}{3}\{4 + 26 + 14 + 24 + 4\}$$

$$\cong 48 \text{ units}^2$$

(c) $\int \frac{3x^2}{1+x^3}\,dx = \log_e\left(1+x^3\right) + c.$ 1

Q4... /page6

Question 4 (12 marks) Start this question on a new page

(a)

(i) At rest, if $v = 0$, 3

$x = 2\cos 2t + 2$

$\frac{dx}{dt} = v$

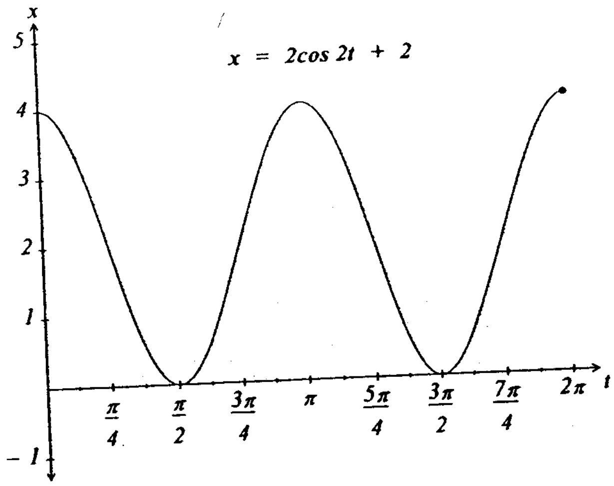
$v = -4\sin 2t$

$-4\sin 2t = 0$

$\sin 2t = 0$

$2t = 0, \pi, 2\pi, 3\pi, 4\pi$

$t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$



(ii) From the graph, if displacement increases then the object is moving away from the origin,

i.e. $\frac{\pi}{2} \le t \le \pi, \frac{3\pi}{2} \le t \le 2\pi$.

(b)

$y = Ax^2$

$y = 20x + 20$

$Ax^2 - 20x - 20 = 0$, has only one solution, i.e. $b^2 - 4ac = 0$ 3

$(-20)^2 + 80A = 0$

$80A = -400$

$A = -5$

(c) 3

$$\int_1^2 \frac{1}{3} e^{2.5x}\, dx = \frac{1}{3}\left[\frac{e^{2.5x}}{2.5}\right]_1^2$$

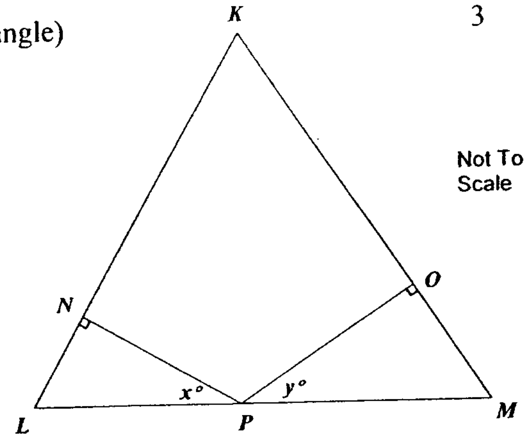
$$= \frac{1}{3}\left[\frac{e^5 - e^{2.5}}{2.5}\right]$$

$= 18.16$ (correct to 2 decimal places)

Q4 cont…/page 7

Question 4 (cont.) **Marks**

(d) 3



$\angle NLP = \angle OMP$ (equal angles in isosceles triangle)

$\angle NLP = 90° - x°$ (complementary angle)

$\angle OMP = 90° - y°$ (complementary angle)

$90° - x° = 90° - y°$

$x° = y°$

Question 5

(a) (i) 2

$$f(x) = (x-4)(x+2)^2$$
$$f(x) = (x-4)(x^2+4x+4)$$
$$f(x) = x^3+4x^2+4x-4x^2-16x-16$$
$$f(x) = x^3-12x-16$$
$$f'(x) = 3x^2-12$$
$$f''(x) = 6x$$

(ii) 2

Let $f'(x) = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

At $x = 2$, $f''(x) = 12 > 0$ so min. turning point.

At $x = -2$, $f''(x) = -12 < 0$ so max. turning point.

(iii) 1

If $f''(x) = 0$ then

$6x = 0$

$x = 0$

$y = (-4)(2)^2$

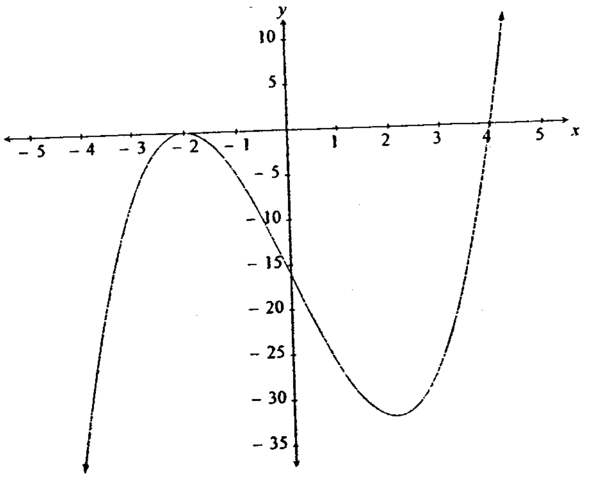
$y = -16$

Point of inflexion at (0,-16)

Q5 cont…/page 5

Question 5 (cont.)

(a) (iv) 2



$y = (x-4)(x+2)^2$

(v) If $f(x)$ is concave up then $f''(x) < 0$, hence $x > 0$. 2

(b) Common ratio $= \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ 3

$$r = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{3+2\sqrt{3}+1}{2}$$

$$= 2+\sqrt{3}$$

$$T_4 = ar^{n-1}$$

$$= \left(\sqrt{3}-1\right)\left(2+\sqrt{3}\right)^3$$

$$= 11\sqrt{3}+19$$

As $|r| = |2+\sqrt{3}| > 1$ then no limiting sum.

Question 6 1

(a) **Domain:** **Range:**

$5-2x \geq 0$

$2x \leq 5$ $y \geq 0$

$x \leq 2\frac{1}{2}$

Q6 cont…/page 6

Question

(b) Fo

Δ

14

16

C

(c) Fo

(i)

(ii

(ii

(i

(d) If

(i

(i

(i

Question 6 (cont.)

(b) For $4x^2 - 12x + C = 0$ to have equal roots, 2

$\Delta = b^2 - 4ac = 0$

$144 - 16C = 0$

$16C = 144$

$C = 9$

(c) For $x^2 - 5x - 4 = 0$,

(i) $\alpha + \beta = -\dfrac{b}{a} = -\dfrac{(-5)}{1} = 5$. 1

(ii) $\alpha\beta = \dfrac{c}{a} = \dfrac{(-4)}{1} = -4$. 1

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha + \beta}{\alpha\beta} = \dfrac{5}{(-4)} = -\dfrac{5}{4}$. 2

(iv) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (5)^2 - 2(-4) = 33$. 2

(d) If $(x-h)^2 = 4a(y-k)$ then for the parabola $x^2 = 8(y-3)$,

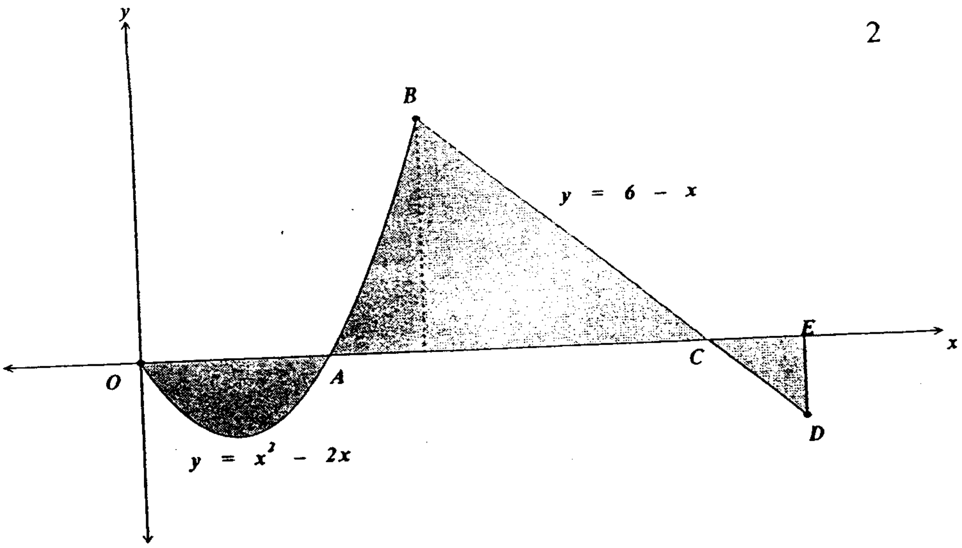
(i) Vertex = $(h, k) = (0, 3)$ 1

(ii) Focal length = $a = 2$. 1

(iii) Equation of the directrix: $y = k - a = 1$ 1

Q7.../page 10

Question 7



2

(a) (i) At A,

$x^2 - 2x = 0$

$x(x-2) = 0$, So, A (2, 0)

$x = 0, 2$

3

At C,

$6 - x = 0$, So C (6, 0)

$x = 6$

At D, $x = 7$

$y = 6 - 7 = -1$, So, D (7, −1)

(ii) At B,

2

$x^2 - 2x = 6 - x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$, So, B (3, 3)

$x = 3, -2$

If $x = 3$, then $y = 3$

Q7 cont…/page11

Question 7 (cont.)

(iii)

$$\text{Area} = \left|\int_0^2 x^2 - 2x \, dx\right| + \int_2^3 x^2 - 2x \, dx + \tfrac{1}{2}\times(6-3)\times 3 + \tfrac{1}{2}\times(7-6)\times 1$$

$$= \left|\left[\frac{x^3}{3} - x^2\right]_0^2\right| + \left[\frac{x^3}{3} - x^2\right]_2^3 + \frac{9}{2} + \frac{1}{2}$$ 3

$$= \left|\left(\frac{8}{3} - 4\right) - 0\right| + \left[\left(\frac{27}{3} - 9\right) - \left(\frac{8}{3} - 4\right)\right] + 5$$

$$= \left|-\frac{4}{3}\right| + \frac{4}{3} + 5$$

$$= 7\tfrac{2}{3} \text{ square units}$$

(b) Typically, $f(x) = kx$, is one possible solution. 1

(c) (i)

$$A_1 = 100(1.005)$$

$$A_2 = (A_1 + 100)(1.005)$$

$$= 100(1.005)^2 + 100(1.005)$$

$$A_3 = (A_2 + 100)(1.005)$$ 1

$$= (\$100(1.005)^2 + \$100(1.005) + \$100)(1.005)$$

$$= \$100(1.005)^3 + \$100(1.005)^2 + \$100(1.005)$$

$$= \$100(1.005^3 + 1.005^2 + 1.005)$$

(ii) If $A_n = \$10000$, then

$$A_n = \$100(1.005^n + 1.005^{n-1} + \ldots + 1.005)$$

$$= \frac{\$100(1.005)(1.005^n - 1)}{0.005}$$

$$\frac{\$100(1.005)(1.005^n - 1)}{0.005} = 10000$$

$$1.005^n = \frac{\$10000 \times 0.005}{\$100(1.005)} + 1$$ 2

$$n = \frac{\log_e\left[\dfrac{\$10000 \times 0.005}{\$100(1.005)} + 1\right]}{\log_e(1.005)}$$

$$n = 80.96$$

$$n = 81 \text{ months}$$

Q8…/page 12

Question 8

(a)

$$\frac{d\left(\log_e\left(\cos^3 x\right)\right)}{dx} = -\sin x \times 3\cos^2 x \times \frac{1}{\cos^3 x}$$ 3

$$= -3\frac{\sin x}{\cos x}$$

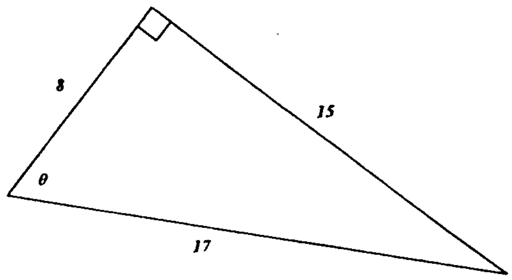
$$= -\tan x$$

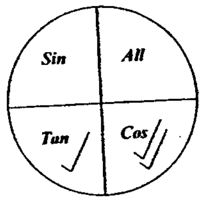
(b) If $\cos\theta = \frac{8}{17}$ and $\sin\theta < 0$, then $\frac{3\pi}{2} \le \theta \le 2\pi$ (see diag)

Using Pythagoras' theorem,

$$x^2 = 17^2 - 8^2$$

$$x = 15$$





2

(i) $\sin\theta = -\frac{15}{17}$

1

(ii) $\cot\theta = -\frac{15}{8}$

(c) (i) For the curves, $y = x^2$ and $y = \frac{8}{x}$,

$$x^2 = \frac{8}{x}$$

$$x^3 = 8$$

So, (2, 4) is the point of intersection. 1

$$x = 2$$

$$y = 4$$

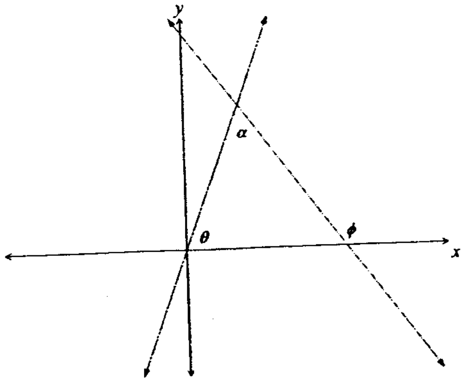
(ii) For $y = \frac{8}{x}$, $\frac{dy}{dx} = -\frac{8}{x^2} = -\frac{8}{4} = -2$ .....[m_1] 2

For $y = x^2$, $\frac{dy}{dx} = 2x = 4$ ....[m_2]

Q8 cont.../page 13

Q8 (cont.)

(c) (iii)



$\tan\theta = 4$ 2

$\theta = 76°$ (to nearest degree)

$\tan\phi = -2$

$\phi = 117°$ (to nearest degree)

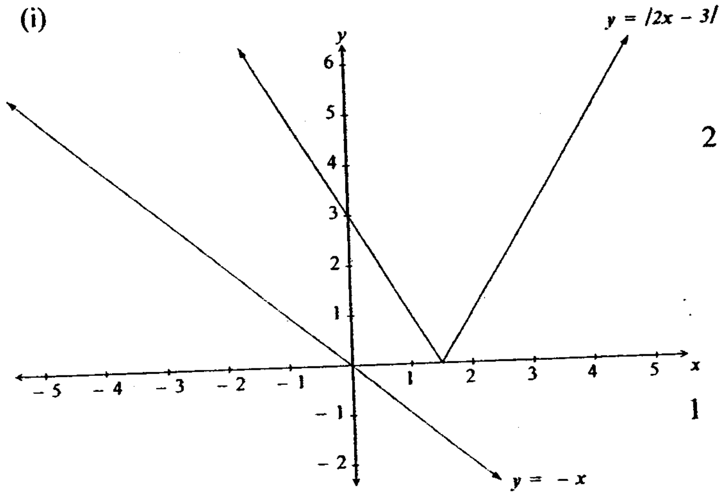
(iv) 1

$\alpha = \phi - \theta$ (see diagram)

$\alpha = 41°$

Question 9

(a) (i) 2



(ii) 1

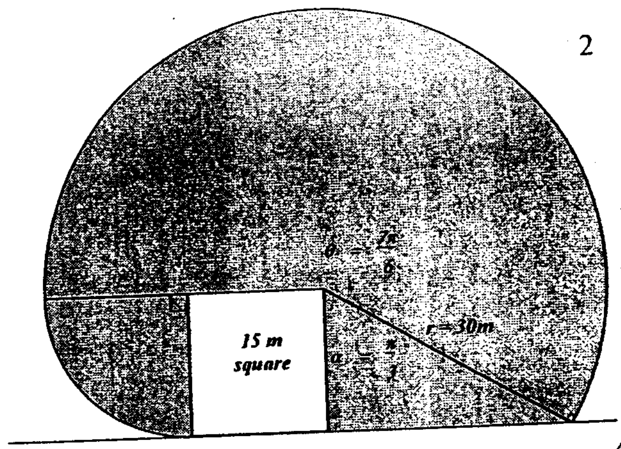
From the graph it can be seen that

$|2x+3| < -x$

has no solutions.

(b)

(i) 2



(ii) 4

Total Area =

$$\frac{\frac{7\pi}{6}\times 30^2}{2}+\frac{15^2\times\pi}{4}+\frac{1}{2}\times 30\times 15\times\sin\frac{\pi}{3}$$

$= 581\frac{1}{4} + 225\sqrt{3}$ sq. m.

Q9 cont…/page 14

Q9 (cont.)

(c) $P = P_o e^{kt}$, if $P_o = 20$, and $P = 100$ when $t = 3$.

$100 = 20e^{3k}$

$e^{3k} = 5$

$k = \dfrac{\ln 5}{3}$ 3

Thus, if $P = 2000$ then

$20e^{kt} = 2000$

$e^{kt} = 100$

$kt = \ln 100$

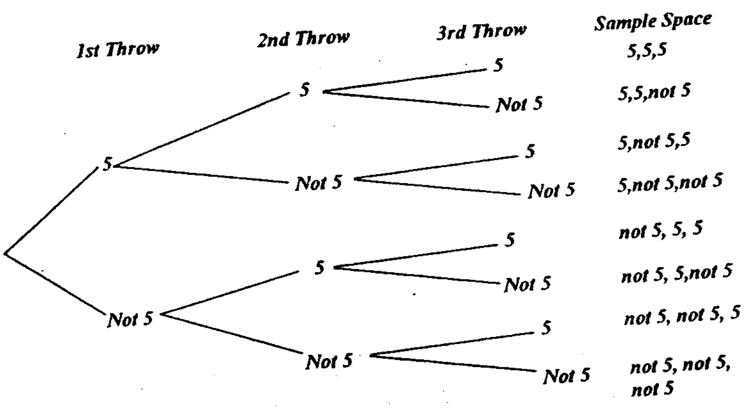
$t = \dfrac{\ln 100}{k}$

$t = \dfrac{3\ln 100}{\ln 5}$

$t \approx 9$ months

Question 10

(a)



(i) P(one 5) $= \frac{2}{7}\times\frac{5}{7}\times\frac{5}{7}+\frac{5}{7}\times\frac{2}{7}\times\frac{5}{7}+\frac{5}{7}\times\frac{5}{7}\times\frac{2}{7}=\frac{150}{343}$ 1

(ii) P(no 5's) $= \frac{5}{7}\times\frac{5}{7}\times\frac{5}{7}=\frac{125}{343}$ 1

(iii) P(at least one 5) $= 1 - P(\text{no 5's}) = 1-\frac{125}{343}=\frac{218}{343}$ 1

(iv) P(at least one 5) $= 1-\left(\frac{5}{7}\right)^n$ 1

Q10 cont…/page 15

Question 9 (cont.)

(v) If P(at least one 5) > 0.99 then

$1-\left(\frac{5}{7}\right)^n > 0.99$

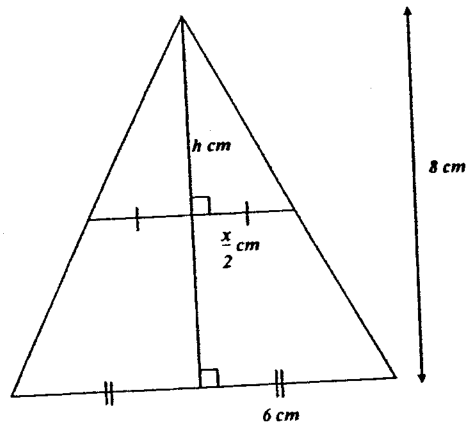
$\left(\frac{5}{7}\right)^n < 0.01$ 2

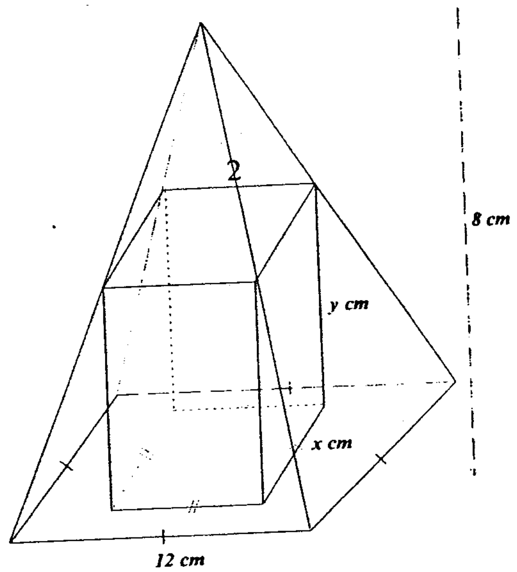
$n = \dfrac{\log 0.01}{\log\left(\frac{5}{7}\right)}$

$n \approx 14$ (nearest whole number)

(b) A rectangular prism is constructed within a square-based triangular pyramid as shown in the diagram:

(i) By taking a vertical section through the prism:





$\dfrac{h}{8} = \dfrac{\frac{x}{2}}{6}$

$h = \dfrac{2x}{3}$ 2

$y = 8 - h$

$y = 8 - \dfrac{2x}{3}$

Q10 cont…/page 16

Q10 (cont.)

(b)

(ii)

$V = x^2 y$

$V = x^2\left(8 - \frac{2x}{3}\right)$ 2

$V = 8x^2 - \frac{2x^3}{3}$

$\frac{dV}{dx} = 16x - 2x^2$

$\frac{dV}{dx} = 2x(8 - x)$

(iii) If

$\frac{dV}{dx} = 0$

$2x(8 - x) = 0$

$x = 0 \text{ or } 8$

$\frac{d^2V}{dx^2} = 16 - 4x$

If $x = 8$ then $\frac{d^2V}{dx^2} < 0$

Maximum volume $= 64(8 - \frac{2 \times 8}{3})$

$= 170\frac{2}{3}\ \text{cm}^3$

END OF EXAMINATION