Total marks (120)
Attempt questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the value of $4\pi\sqrt{\frac{a}{g}}$ where $a = 4.2$ and $g = 9.8$. Give your answer correct to 2 significant figures. **2**

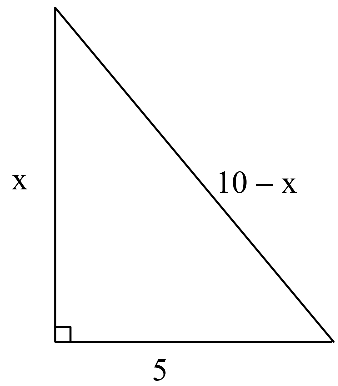
(b) Solve the pair of simultaneous equations **2**

$$x + y = 2$$
$$2x - y = 7$$

(c) Factorise $2x^2 - 3x - 14$ **1**

(d) Write down the exact value of 135^0 in radians **1**

(e) Find the value of x in the right angled triangle. **2**



(f) Rationalise the denominator $\frac{2}{\sqrt{5}-\sqrt{3}}$. Answer in its simplest form. **2**

(g) Find the values of x for which $|x+2| \geq 5$. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Differentiate the following functions:

(i) $y = x\sqrt{x}$ **1**

(ii) $y = x^3 \log_e x$ **2**

(iii) $y = \cos^2 x$ **2**

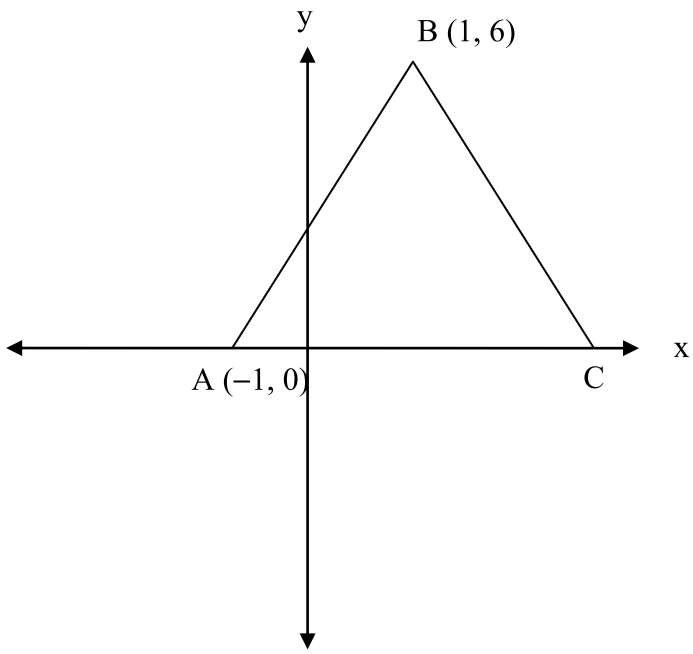
(iv) $y = \dfrac{x^3}{x^2 - 1}$ **2**

(b) Find $\int \sec^2 8x \, dx$ **1**

(c) Evaluate $\int_0^2 e^{3x} + 4 \, dx$ **2**

(d) Find the equation of the tangent to the curve $y = (1+2x)^3$ at the point $(0,1)$. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet



NOT TO SCALE

The diagram shows the points $A(-1, 0)$ and $B(1, 6)$. $\angle BAC = \angle BCA$

Copy the diagram onto your examination paper.

(i)	Find the length of AB in its simplest surd form	**1**
(ii)	Find the midpoint of AB.	**1**
(iii)	Find the gradient of AB.	**1**
(iv)	Find the size of $\angle BAC$, correct to the nearest minute.	**1**
(v)	Find the equation of the perpendicular bisector of AB.	**2**
(vi)	Explain why the gradient of BC is −3.	**1**
(vii)	Show that the equation of BC is $y = -3x + 9$.	**1**
(viii)	Find the coordinates of C.	**1**
(ix)	Find the perpendicular distance from A to BC.	**2**
(x)	Write down the co-ordinates of D such that BCAD is a parallelogram.	**1**

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The gradient function of a curve is given by $\frac{dy}{dx} = 3x^2 - 6x - 9$. The curve passes through the point $(1, -2)$.

(i) Find the equation of the curve. **2**

(ii) Find the co-ordinates of the stationary points and determine their nature. **4**

(iii) Find the co-ordinates of the point of inflexion. **1**

(iv) Sketch the curve showing the stationary points, point of inflexion and the y-intercept. **2**

(b) The equation of a parabola is given by $(x-2)^2 = 2y + 6$

(i) Find the co-ordinates of the vertex. **1**

(ii) Find the co-ordinates of the focus. **1**

(iii) Find the equation of the directrix. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Sketch the graph of $y = \frac{1}{x}$ and $x^2 = 8y$ on the same axes. **2**

(ii) Show the co-ordinates of the point of intersection of the two graphs are $(2, \frac{1}{2})$. **1**

(iii) Find the area of the region bounded by the x-axis and the curves from $x = 0$ to $x = 2e$. **3**

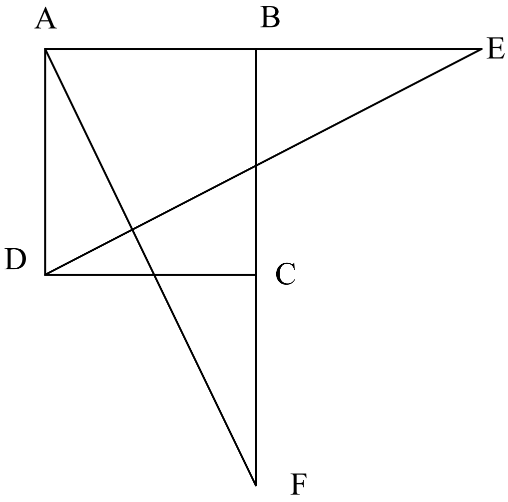
(b) (i) Draw the graphs of $y = 2\sin x$ and $y = \tan x$ for $0 \le x \le 2\pi$ **2**

(ii) Use the graph to find to determine the number of solutions to $2\sin x = \tan x$. **1**

(iii) Find all the values of x where $0 \le x \le 2\pi$ are solutions to the equation $2\sin x = \tan x$. **3**

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) In the diagram ABCD is a square. AB is produced to E so that AB = BE and BC is produced to F so that BC = CF



(i) Copy the diagram on to your examination booklet.

(ii) Prove that Δ AED $\equiv$ ΔBFA **3**

(iii) Hence prove that $\angle AED = \angle BFA$ **1**

(b) A five card hand is dealt from a regulation pack of cards. Find the probability of

(i) a flush (all five cards have the same suit). **2**

(ii) four aces and any other cards. **2**

(c) The mass, *m,* in grams of a radio active substance at the end of *t* years is given by

$m = 10e^{-0.009t}$. Find

(i) the mass, to the nearest gram, after 10 years. **1**

(ii) the rate at which the mass is decreasing after 10 years **1**

(iii) the number of years, to the nearest year, for the substance to reach its half-life. **2**

Question 7 (12 marks) Use a SEPARATE writing booklet

(a) The lengths of the rung of a ladder increase uniformly from 40*cm* in the top rung to 75 *cm* in the bottom rung. If 13.8 *m* of wood are used to make the rungs, many rungs are there? **2**

(b) An author writes a manuscript, so that on the first day he writes 54 pages, on the second day 36 pages and on each succeeding day he writes $\frac{2}{3}$ of the number of pages of the preceding day.

(i) How many pages does he write on the 5^{th} day? **1**

(ii) What is the maximum number of pages he will write? **1**

(c) Solve $\log_3 x + \log_3 (x+8) = 2$ **3**

(d) Given the quadratic equation $x^2 - (2+k)x + 3k = 0$,find the value of k if the product of the roots is four times the sum. **2**

(e) Consider the function given by $f(x) = \dfrac{1}{4+x^2}$ for $0 \le x \le 2$ and that you will be using Simpson's rule with 5 function values **3**

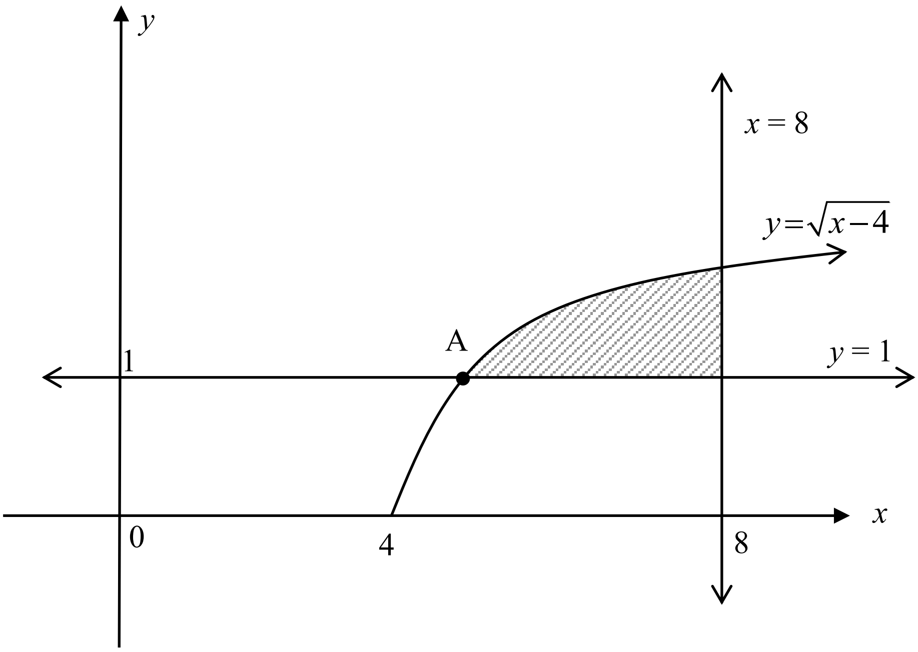
(i) Copy and complete the table of values on your examination paper,

x					
$f(x)$					

(ii) Find an approximation of $\int_0^2 f(x)\,dx$, correct to 3 decimal places.

Question 8 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



The region between $y=\sqrt{x-4}$, $y=1$ and $x=8$ is rotated about the x-axis.

(i) Show that the co-ordinates of A are (5,1). **1**

(ii) Find the volume of the solid of revolution. **3**

(b) The position x *cm* of a particle P moving along the x-axis after t seconds is given by $x=5-9t+6t^2-t^3$

(i) Find expressions for the particle's velocity and acceleration. **2**

(ii) In which direction does the particle initially move? **1**

(iii) When does the particle change direction? **2**

(iv) Where is the particle at these times? **1**

(vi) Find the total distance travelled between t = 0 and t = 3 **2**

Question 9 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Brian borrows \$50000 at 18% p.a. compounded monthly and repays the loan in equal monthly instalments over 5 years.

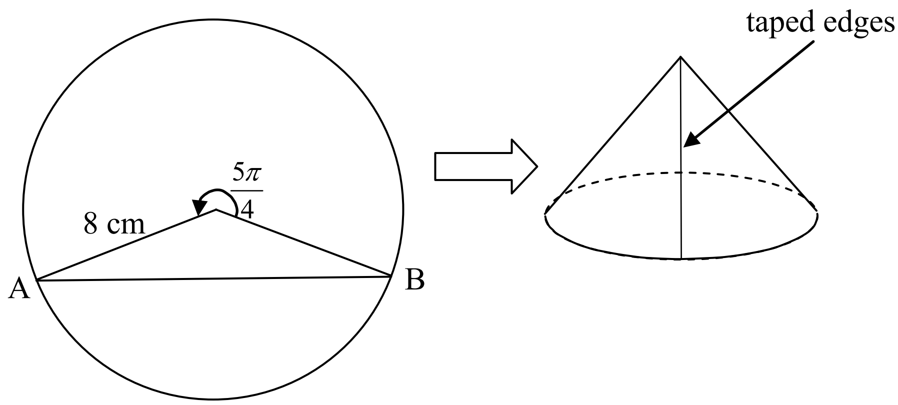
(i) Show that the amount owing after 2 months is **2**

$$A_2 = 50000(1.015)^2 - M(1+1.015)$$

(ii) Find the amount of each monthly repayment. **2**

(iii) How much interest does Brian pay? **2**

(b)



(i) Find the length of the minor arc AB. **1**

(ii) Find the area of the minor sector AB. **1**

(iii) Find the area of the minor segment AB. **2**

(iv) If the major sector was taped to form a cone, what would be the radius of the base of the cone? **1**

(v) What would be the surface area of the cone formed by joining A and B? **1**

Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) At 1:00 p.m. a ship A leaves port P and sails in the direction 030^0 T at 12 *km/h*.

Also at 1:00 p.m. ship B is 100 *km* due east of P and sailing at 8 *km/h* towards P.

Suppose t is the number of hours after 1p.m.

(i) Show that the distance D(t) between the 2 ships is given by **4**

$$D(t) = \sqrt{304t^2 - 2800t + 10000}$$

(ii) Find the minimum value of $\left[D(t)\right]^2$ for all $t \geq 0$ **3**

(iii) At what time, to the nearest minute, are the ships closest? **2**

(b) Differentiate $y = \log_e \left[\dfrac{3x-1}{x+2}\right]$. **2**

(c) Evaluate $y = \displaystyle\int \frac{e^{5x}+4}{e^x}\,dx$. **1**

END OF PAPER

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \,, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

Year 12 2unit Mathematics Trial 2008

Question 1

a) $4\pi\sqrt{\frac{a}{g}} = 4\pi\sqrt{\frac{4.2}{9.8}}$

$\doteq 8.22662065$

$= 8.2$ to 2 sig fig

b) $x+y = 2$ —— ①

$2x-y = 7$ —— ②

①+② $3x = 9$

$x = 3$

sub① $x+y=2$

$3+y=2$

$y=-1$

$\therefore x=3,\ y=-1$

c) $2x^2-3x-14 = (2x-7)(x+2)$

d) $180° = \pi^c$

$1° = \frac{\pi}{180}$

$135° = 135\times\frac{\pi}{180}$

$= \frac{3\pi}{4}^c$

e) $x^2+5^2=(10-x)^2$

$x^2+25=100-20x+x^2$

$20x=75$

$x = 3\frac{3}{4}$

f) $\frac{2}{\sqrt{5}-\sqrt{3}} = \frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \frac{2(\sqrt{5}+\sqrt{3})}{2}$

$= \sqrt{5}+\sqrt{3}$

g) $|x+2| \geq 5$

$x+2 \leq -5$, $x+2\geq 5$

$x \leq -7$, $x \geq 3$

Question 2

a) (i) $y = x\sqrt{x}$

$= x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3}{2}\sqrt{x}$

(ii) $y=x^3\log_e x$

$= u\ v$

$u=x^3$	$v=\log_e x$
$u'=3x^2$	$v'=\frac{1}{x}$

$\frac{dy}{dx} = vu'+uv'$

$= 3x^2\log_e x + x^3\times\frac{1}{x}$

$= 3x^2\log_e x + x^2$

(iii) $y = \cos^2 x$

$= (\cos x)^2$

$\frac{dy}{dx} = -2\cos x\sin x$

iv) $y = \frac{x^3}{x^2-1}$

$= \frac{u}{v}$

$u=x^3$	$v=x^2-1$
$u'=3x^2$	$v'=2x$

$\frac{dy}{dx} = \frac{vu'-uv'}{v^2}$

$= \frac{3x^2(x^2-1)-x^3\times 2x}{(x^2-1)^2}$

$= \frac{3x^4-3x^2-2x^4}{(x^2-1)^2}$

$= \frac{x^2(x^2-3)}{(x^2-1)^2}$

b) $\int \sec^2 8x \, dx = \frac{1}{8}\tan 8x + c$

c) $\int_0^2 e^{3x} + 4 \, dx = \left[\frac{1}{3}e^{3x} + 4x\right]_0^2$

$= \left[\frac{1}{3}e^6 + 8\right] - \left[\frac{1}{3}e^0\right]$

$= \frac{1}{3}e^6 + 7\frac{2}{3}$

d) $y = (1+2x)^3$

$\frac{dy}{dx} = 3(1+2x)^2 \times 2$

$= 6(1+2x)^2$ at $x = 0$

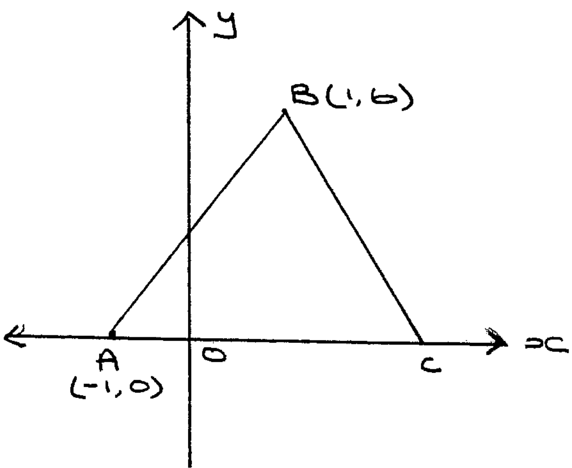
$= 6$

$y - y_1 = m(x - x_1)$ $\quad m = 6, (0,1)$

$y - 1 = 6(x - 0)$

$y = 6x + 1$

Question 3



(i) $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1-1)^2 + (0-6)^2}$

$= \sqrt{4+36}$

$= \sqrt{40}$

$= 2\sqrt{10}$

(ii) Midpoint AB $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-1+1}{2}, \frac{0+6}{2}\right)$

$= (0,3)$

(iii) $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-0}{1--1}$

$= 3$

$\therefore$ gradient AB $= 3$

iv) $m = \tan\theta$

$B\hat{A}C = \tan^{-1} 3$

$\doteqdot 71° 33' 54.18''$

$B\hat{A}C = 71° 34'$ (nearest min)

(v) Midpoint AB = (0,3)

Gradient AB = 3

Gradient of required line $= -\frac{1}{3}$

as $m_1 m_2 = -1$ for perpendicular lines

$\therefore y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{3}(x - 0)$

$3y - 9 = -x$

$x + 3y - 9 = 0$

vi) $B\hat{A}C = B\hat{C}A$

Tangent ratio is negative for angles in the second quadrant

vii) $(1,6)$ $\quad m = -3$

$y - y_1 = m(x - x_1)$

$y - 6 = -3(x - 1)$

$y - 6 = -3x + 3$

$y = -3x + 9$

viii) $y = -3x + 9$ at C, $y = 0$

$0 = -3x + 9$

$3x = 9$

$x = 3$

$\therefore$ C is $(3, 0)$

ix) $3x + y - 9 = 0$ $A(-1, 0)$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|3(-1) + 0 - 9|}{\sqrt{3^2 + 1^2}}$$

$$= \frac{|-12|}{\sqrt{10}}$$

$$= \frac{12}{\sqrt{10}} \text{ or } \frac{6\sqrt{10}}{5}$$

x) D is $(-3, 6$

Question 4

a) (i) $\frac{dy}{dx} = 3x^2 - 6x - 9$

$y = \int (3x^2 - 6x - 9)\,dx$

$y = x^3 - 3x^2 - 9x + c$ at $(1, -2)$

$-2 = (1)^3 - 3(1)^2 - 9(1) + c$

$-2 = 1 - 3 - 9 + c$

$c = 9$

$\therefore y = x^3 - 3x^2 - 9x + 9$

(ii) For stationary points

$\frac{dy}{dx} = 0$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x - 3)(x + 1) = 0$

$x = 3, -1$

when $x = 3$

$y = x^3 - 3x^2 - 9x + 9$

$= (3)^3 - 3(3)^2 - 9(3) + 9$

$= -18$

$\frac{d^2y}{dx^2} = 6x - 6$ at $x = 3$

$= 6(3) - 6$

$= 12$

As $\frac{d^2y}{dx^2} > 0$ the curve is concave up ↺

$\therefore (3, -18)$ is a local minimum

when $x = -1$

$y = x^3 - 3x^2 - 9x + 9$

$= (-1)^3 - 3(-1)^2 - 9(-1) + 9$

$= 14$

$\frac{d^2y}{dx^2} = 6x - 6$ at $x = -1$

$= 6(-1) - 6$

$= -12$

As $\frac{d^2y}{dx^2} < 0$ the curve is concave down ↷

$\therefore (-1, 14)$ is a local maximum

(iii) For points of inflexion $\frac{d^2y}{dx^2} = 0$ and there is a change in concavity

$\frac{d^2y}{dx^2} = 6x - 6 = 0$

$x = 1$

when $x = 1$, $y = x^3 - 3x^2 - 9x + 9$

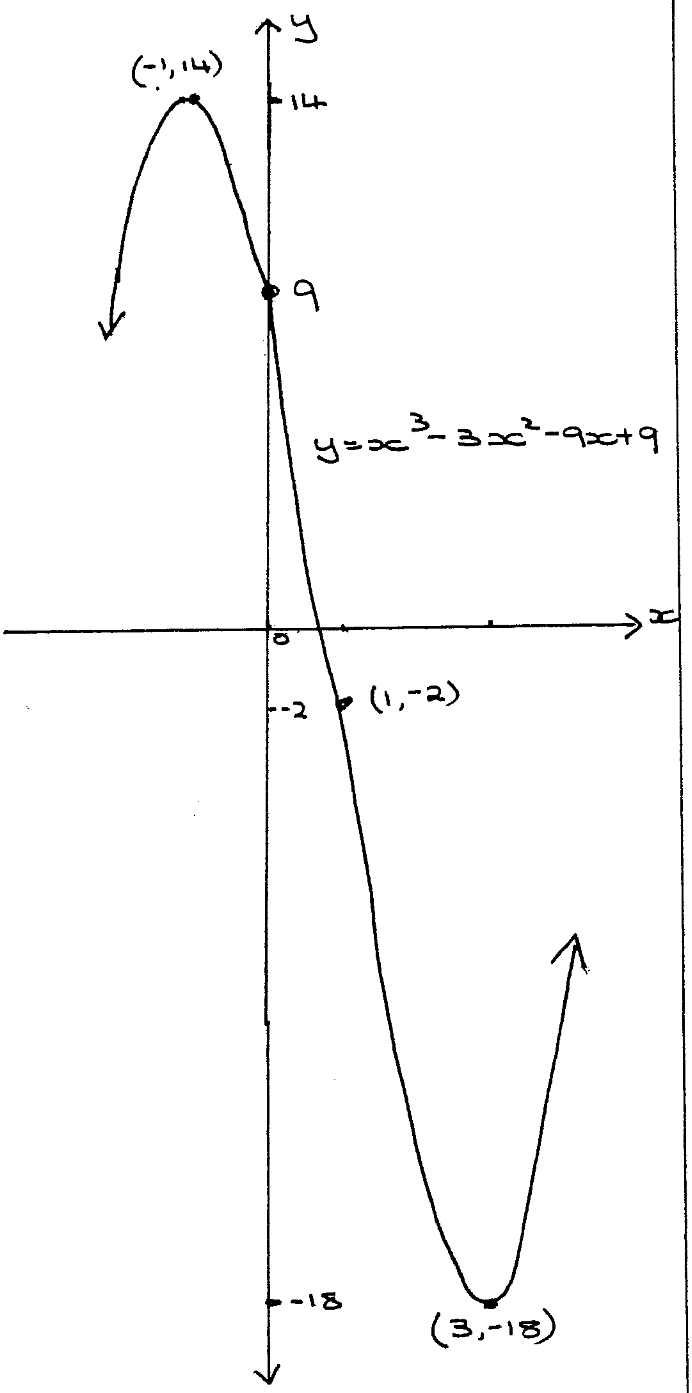
$= (1)^3 - 3(1)^2 - 9(1) + 9$

$= -2$

x	1^-	1	1^+
$\frac{d^2y}{dx^2}$	$-$		$+$

$\therefore$ change in concavity

$\therefore (1, -2)$ is a point of inflexion



b) $(x-2)^2 = 2(y+3)$

$4a = 2$

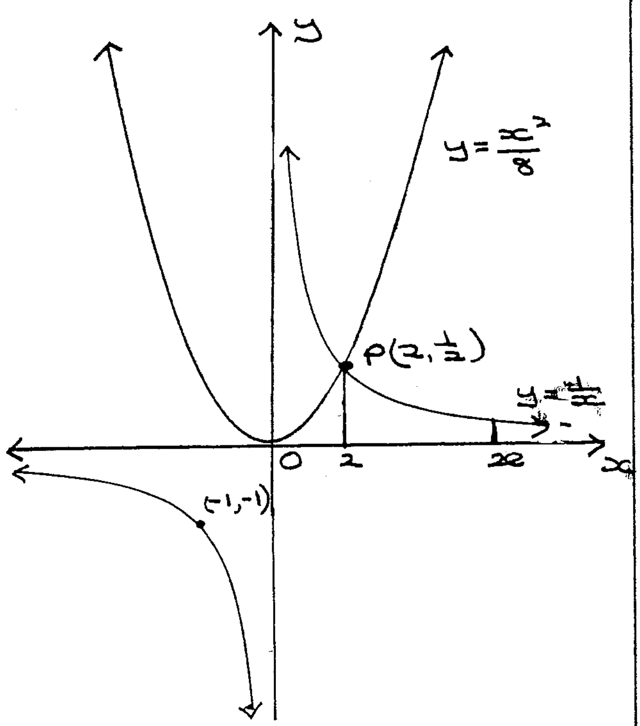
$a = \frac{1}{2}$

(i) vertex $(2, -3)$

(ii) focus $(2, -2\frac{1}{2})$

(iii) directrix $y = -3\frac{1}{2}$

Question 5



ii) $y = \frac{1}{x}$ —— ①

$y = \frac{x^2}{8}$ —— ②

sub ① in ②

$\frac{1}{x} = \frac{x^2}{8}$

$x^3 = 8$

$x = 2$

sub ① $y = \frac{1}{x}$

$= \frac{1}{2}$

$\therefore (2, \frac{1}{2})$ is the point of intersection

iii) $A = \int_0^2 \frac{x^2}{8}\,dx + \int_2^{2e} \frac{1}{x}\,dx$

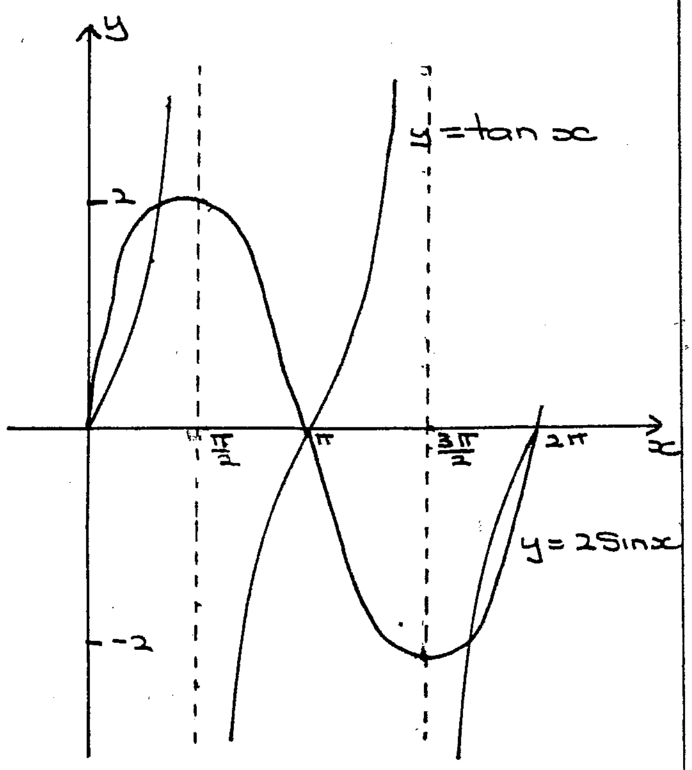
$= \left[\frac{x^3}{24}\right]_0^2 + \left[\ln x\right]_2^{2e}$

$= \left[\frac{2^3}{24} - 0\right] + \left[\ln 2e - \ln 2\right]$

$= \frac{8}{24} + \ln 2 + \ln e - \ln 2$

$= \frac{1}{3} + 1$

Area $= 1\frac{1}{3}$ units^2



(ii) $2\sin x = \tan x$

has 5 solutions

(iii) $2\sin x = \tan x$

$2\sin x - \tan x = 0$

$2\sin x - \frac{\sin x}{\cos x} = 0$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

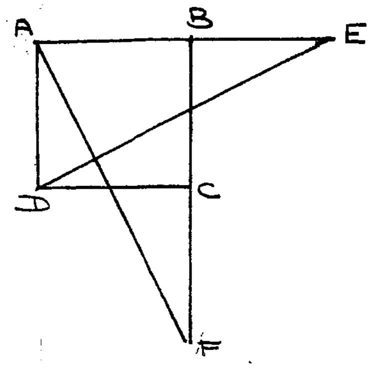
$\sin x = 0$ or $\cos x = \frac{1}{2}$

$x = 0, \pi, 2\pi$ $\quad x = \frac{\pi}{3}, \pi - \frac{\pi}{3}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

Question 6



In $\triangle AED$ and $\triangle BFA$

$AD = AB$ ABCD is a square

$AE = BF$ ABCD is a square, $AB = BE$, $BC = CF$

$D\hat{A}E = A\hat{B}F$ ABCD is a square

$\therefore \triangle AED \equiv \triangle BFA$ SAS

$\therefore A\hat{E}D = B\hat{F}A$

corresponding angles in congruent triangles

b i) $P(\text{flush}) = 1 \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48}$

$= \frac{11880}{5997600}$

$= \frac{33}{16660}$

(ii) $P(4\text{ aces}) = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times 1$

$= \frac{24}{6497400}$

$= \frac{1}{270725}$

c) (i) $m = 10e^{-0.009t}$ $t = 10$

$= 10e^{-0.009\times 10}$

$= 10e^{-0.09}$

$\doteqdot 9.139311853$

Mass $= 9$ grams to nearest gram

(ii) $\frac{dm}{dt} = -0.009 \times 10e^{-0.009t}$

$= -0.09\, e^{-0.09}$

$= -0.082253806$

$\therefore$ Decreasing at the rate of 0.08g per year (to 2dp)

(iii) when $t = 0$ $m_0 = 10$, $m = 5$

$m = 10e^{-0.009t}$

$5 = 10e^{-0.009t}$

$\frac{1}{2} = e^{-0.009t}$

$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-0.009t}\right)$

$-0.009t = \ln\left(\frac{1}{2}\right)$

$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.009}$

$\doteqdot 77.0163534$

The time to reach its half life is 77 years to nearest year

Question 7

a) Let there be n rungs

$a = 40$, $L = 75$ $S_n = 1380$

$S_n = \frac{n}{2}(a + L)$

$1380 = \frac{n}{2}(40 + 75)$

$\frac{n}{2} = \frac{1380}{115}$

$= 12$

$n = 24$

$\therefore$ There are 24 rungs

b) (i) $a = 54$, $r = \frac{2}{3}$, $n = 5$

$T_n = ar^{n-1}$

$T_5 = 54\left(\frac{2}{3}\right)^4$

$= 10\frac{2}{3}$

$\therefore$ He writes $10\frac{2}{3}$ pages

(ii) $S_\infty = \frac{a}{1-r}$ $|r| < 1$

$= \frac{54}{1-\frac{2}{3}}$

$= 162$

$\therefore$ He will write 162 pages

c) $\log_3 x + \log_3 (x+8) = 2$

$\log_3 x(x+8) = 2$

$x(x+8) = 3^2$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x = -9, 1$

But $x > 0$ $\therefore$ $x = 1$ is the only solution

d) $x^2 - (2+k)x + 3k = 0$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = 2 + k$ —— (1)

$\alpha\beta = \frac{c}{a}$

$\alpha\beta = 3k$ —— (2)

$\alpha\beta = 4(\alpha + \beta)$

$3k = 4(2 + k)$

$3k = 8 + 4k$

$k = -8$

e) $f(x) = \frac{1}{4+x^2} \quad 0 \le x \le 2$

x	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
$f(x)$	$\frac{1}{4}$	$\frac{4}{17}$	$\frac{1}{5}$	$\frac{4}{25}$	$\frac{1}{8}$

$$\int_a^b f(x)\,dx \doteq \frac{b-a}{6}\left[f(a)+4f\left(\tfrac{a+b}{2}\right)+f(b)\right]$$

$$\int_0^2 \frac{1}{4+x^2}\,dx \doteq \frac{1-0}{6}\left[\frac{1}{4}+\frac{1}{8}+4\left(\frac{4}{17}+\frac{4}{25}\right)+2\times\frac{1}{5}\right]$$

$$= \frac{1}{6}\left[2.356176471\ldots\right]$$

$$\doteq 0.392696078$$

$$= 0.393 \text{ to } 3dp$$

Question 8

a) i. $y = \sqrt{x-4}$ at $y = 1$

$1 = \sqrt{x-4}$

$1 = x-4$

$x = 5$

$\therefore$ A is $(5,1)$

ii) $V = \pi \int_a^b y^2\,dx$

$= \pi \int_5^8 (\sqrt{x-4})^2\,dx - \pi 1^2 \times 3$

$= \pi \int_5^8 (x-4)\,dx - 3\pi$

$= \pi \left[\frac{x^2}{2} - 4x\right]_5^8 - 3\pi$

$= \pi \left[\frac{8^2}{2} - 4(8)\right] - \left[\frac{5^2}{2} - 4(5)\right] - 3\pi$

$= 4\frac{1}{2}\pi$

$\therefore$ Volume is $4\frac{1}{2}\pi$ units^3

b) $x = 5 - 9t + 6t^2 - t^3$

i) $\dot{x} = -9 + 12t - 3t^2$

$\ddot{x} = 12 - 6t$

ii) Initial $\dot{x} = -9\text{cms}^{-1}$

That is moving with a velocity of 9cms^{-1} is the negative direction ←

iii) Particle changes direction when $\dot{x} = 0$

$-9 + 12t - 3t^2 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 1, 3$

Particle changes direction after 1 second and 3 seconds

(iv) When $t = 1$

$x = 5 - 9t + 6t^2 - t^3$

$= 5 - 9(1) + 6(1)^2 - (1)^3$

$= 5 - 9 + 6 - 1$

$= 1$

when $t = 3$

$x = 5 - 9t + 6t^2 - t^3$

$= 5 - 9(3) + 6(3)^2 - 3^3$

$= 5$

when $t = 1$ the particle is 1cm to the right of O and when $t = 3$ the particle is 5cm to the right of O

v)

t	0	1	2	3
x	5	1	3	5

Total distance $= 4 + 2 + 2$

$= 8\text{cm}$

Question 9

a) Let A_n be the amount owing at the end of the n^{th} month

Let M be the monthly repayment

$n = 5 \times 12 = 60$ $\quad$ $r = \frac{18}{12}\% = 1.5\% = 0.015$

$A_1 = 50000 + 1.5\% \text{ of } 50000 - M$
$= 50000 + 0.015 \times 50000 - m$
$= 50000(1.015) - m$

$A_2 = A_1 + 1.5\% \text{ of } A_1 - m$
$= A_1 + 0.015 \times A_1 - m$
$= A_1(1.015) - m$
$= [50000(1.015) - m](1.015) - m$

$= 50000(1.015)^2 - m(1 + 1.015)$

continuing the pattern

$A_{60} = 50000(1.015)^{60} - m(1 + 1.015 + \dots + 1.015^{59})$

But after 60 months the loan is repaid $\therefore A_{60} = 0$

$50000(1.015)^{60} - m(1 + 1.015 + \dots + 1.015^{59}) = 0$

$$m = \frac{50000(1.015)^{60}}{1 + 1.015 + \dots + 1.015^{59}}$$

$$= \frac{50000(1.015)^{60}}{\frac{1(1.015^{60} - 1)}{1.015 - 1}}$$

$$= \frac{50000(1.015)^{60}(0.015)}{(1.015^{60} - 1)}$$

$\doteqdot 1269.671371$

$= 1269.67$ to 2 dp

$\therefore$ Each monthly instalment is \$1269.67

(iii) $I = 1269.67 \times 60 - 50000$
$= 26180.2$

$\therefore$ Interest paid is \$26180.20

b) (i) $l = r\theta$ $\quad r = 8 \quad \theta = \frac{3\pi}{4}$
$= 8 \times \frac{3\pi}{4}$
$= 6\pi$

length of the arc is 6π cm

(ii) $A = \frac{1}{2}r^2\theta$
$= \frac{1}{2} 8^2 \times \frac{3\pi}{4}$
$= 24\pi$

Area is $24\pi \text{ cm}^2$

(iii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{1}{2} \times 8^2\left(\frac{3\pi}{4} - \sin\frac{3\pi}{4}\right)$
$= 32\left(\frac{3\pi}{4} - \frac{1}{\sqrt{2}}\right)$
$= 24\pi - 16\sqrt{2}$
$= 8(3\pi - 2\sqrt{2})$

$\therefore$ Area is $8(3\pi - 2\sqrt{2}) \text{ cm}^2$
(or 52.77580669 cm^2

iv) Major Arc $= r\theta$
$= 8 \times \frac{5\pi}{4}$
$= 10\pi$

$2\pi r = 10\pi$
$r = 5$

$\therefore$ Radius is 5cm

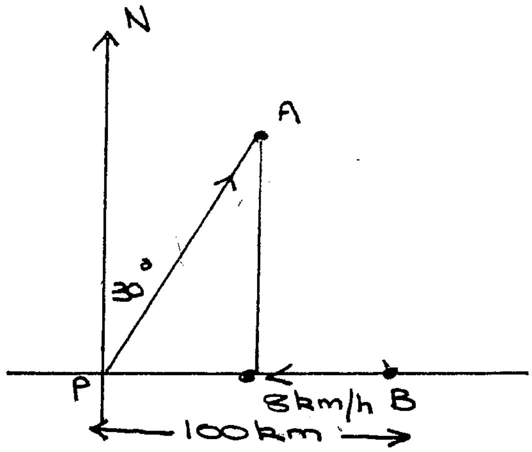
v) $S.A = \pi r l$

$= \pi \times 5 \times 8$

$= 40\pi$

Surface area is $40\pi \text{ cm}^2$

Question 10



At time t, $PA = 12t$

$PB = 100 - 8t$

Using cosine rule

$p^2 = a^2 + b^2 - 2ab\cos P$

$AB^2 = (100-8t)^2 + (12t)^2 - 2(100-8t)(12t)\cos 60$

$= 10000 - 1600t + 64t^2 + 144t^2 - 1200t + 96t^2$

$= 10000 - 2800t + 304t^2$

$\therefore D(t) = \sqrt{304t^2 - 2800t + 10000}$

(ii) $[D(t)]^2 = 304t^2 - 2800t + 10000$

$\frac{d[D(t)]^2}{dt} = 608t - 2800$

$608t - 2800 = 0$

$t = 4\frac{23}{38}$

$[D(t)]^2 = 10000 - 2800t + 304t^2$

$= 10000 - 2800(4\frac{23}{38}) + 304(4\frac{23}{38})^2$

$\doteqdot 3552.631579$

$= 3553$ to nearest km

t	$4\frac{23}{38}^-$	$4\frac{23}{38}$	$4\frac{23}{38}^+$
$\frac{d[D(t)]^2}{dt}$	$-$	0	$+$

$\therefore$ Minimum value of $[D(t)]^2$ is 3553

(iii) 1pm $+ 4\frac{23}{38}$ hr

$\doteqdot$ 1pm + 4 hr 36 min

= 5:36 pm

b) $y = \log_e\left(\frac{3x-1}{x+2}\right)$

$= \log_e(3x-1) - \log_e(x+2)$

$\frac{dy}{dx} = \frac{3}{3x-1} - \frac{1}{x+2}$

$= \frac{3(x+2) - (3x-1)}{(3x-1)(x+2)}$

$= \frac{7}{(3x-1)(x+2)}$

c) $y = \int\left(\frac{e^{5x}+4}{e^x}\right)dx$

$= \int(e^{4x} + 4e^{-x})dx$

$= \frac{1}{4}e^{4x} - 4e^{-x} + C$